Electromagnetic Theory
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Lecture - 65
Transients on Transmission line – I

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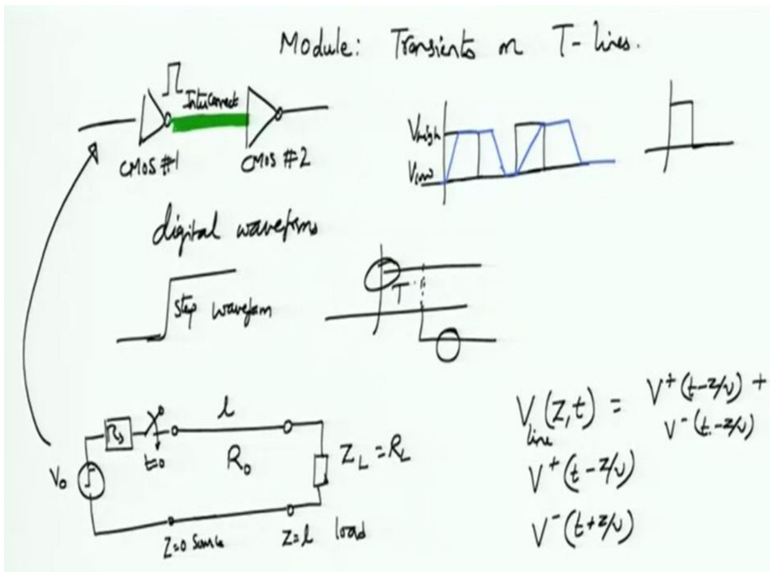

In this module we will look at transients on transmission line. From the earlier modules on transmission lines we considered sinusoidal excitation that is we took sinusoidal voltage source of a particular frequency or a given wave length connected that to a transmission line circuit that could have loads that could have different links different characteristic impedance all these things we saw.

But we never really although we know that the voltages on the transmission lines and other parts of the circuit that actually varying with time but this time variation was all in the form of a sine or a cosine wave right. It was a sinusoidal waveform. There was no explicit requirement for us to mention anything about time because the waveforms were all going to be sinusoidal. The transmission line circuits that we analyzed.

So far were all linear. I mean all the waveforms that we had were all sinusoidal waveforms. Therefore, there was no requirement for us to explicitly mention the time dependent nature of voltages and currents okay. But in this module we are not going to do a steady state analysis. So the analysis we did so far was called as steady state which means that the voltage sources were assumed to be connected for a long time.

So that all the initial effects of connection you know the moment to connect there is some initial charge. You know initially charging of the transmission line, initial current flowing, and initial voltages changing all those initial things have died down. You know all these things have been done and now the circuits have reached a steady state. That is what the analysis so far we have done.

However, in many cases we are actually interested in those transient that is we are interested what happens when you take a transmission line. And then connect it to in between say two driver and a load and if the driver voltage waveform is changing with respect to time then how would the transmission line respond in time domain to this changing voltages. This is something you would see quiet often when you are looking at high speed digital designs okay.

Consider a typical high speed IC which will have multiple you know things would be there but this is the simplest thing which I would like to discuss. You have a CMOS inverter okay formed from some MOS and other things. We have another CMOS inverter okay. One CMOS inverter is driving another CMOS inverter okay. It may be intentional or unintentional but one is driving the other.

It need not be a CMOS inverter itself it could be a capacitor, it could be a resistor, it could be something else that could be driving. But the point I want to highlight is that the signals that are going through this particular circuit or not the sinusoidal ones. They are in fact what are called as digital signals or digital waveforms. How would these digital waveforms look? Well these digital waveforms if you see are what we call as the switching waveforms sometimes in the older literature.

They would actually going from V high to V low right so in a case of TTL circuit these high and lows have a certain values and in case of CMOS there is different values for V high and V low.

Regardless of all that there would be the switching right. So for example this can be considered to obtain a clock then this output should actually continuously been switching compared to the input okay. So there is actually one more inverter is required but that is another point.

I have written something over here as interconnect on a high speed IC the leads that connect one component to another components acts as a transmission line. Even though you do not want it to act like a transmission line it does act like a transmission line. Because the distances involved are small but the frequencies involved or the wave length involved are also small. So wave length is small frequency is high and even a small piece of wire or a contact this one would act like a transmission line okay.

So it is just a copper thing which would act like a transmission line at almost all high speed digital circuits. Of course all in the analog circuits as well. So on these circuits or on this interconnects you have the source side signals which are switching from high to low and they are to be transported across to the second CMOS inverter which is acting like the load in order for this one to be allowed to switch okay.

Of course in practice you do not get change from low to high in zero time what you see is much milder versions. So you will actually see something like this right. So you would see with a certain rise time and a certain fall time. These are the type of signals that you are going to see okay. These are much more practical but nevertheless for this simple analysis. We will assume that the signals on the interconnect obtained at the output of one component, which we call as a source and to be transported to the load or switching like this okay.

We will initially not concern the pulse excitation you will assume initially that the transmission line or this source guy was not having any voltage initially at some t is equal to 0 then orbitally it has actually changed it is value okay. There is a step waveform that is launched on the transmission line now it is understandable right. So you understand how the transmission line would respond to a step.

Since pulse can be considered as one positive step and another negative step applied after the pulse period right so you have a negative going step. So if I know how the transmission line behaves for this step voltage I know how it behaves for this one as well so I can actually use super position too simply understand how the transmission line would respond to a pulse okay. So this is the step waveform that we have launched at the input of the transmission line.

Now from this somewhat realistic kind of a behavior we are going to idealized kind of a behavior. We will consider that there is a transmission line and then there is a load okay. This load is that CMOS load that we were characterizing. We will assume that this load is real okay for this analysis we will assume that the load is real so Z L is actually some R L okay. The transmission line is also lossless having real characteristic impedance R 0 has a certain length l.

Okay and we will also put down Z is equal to l as the load point okay and Z is equal to 0 as the source point okay. So here let us assume that you have R S which is the source resistant internal resistance of the source and then the source itself is a step waveform having amplitude of V 0. Of course there is actually a switch which we will imagine has to connect after R S. So there is R S over here and then there is a switch okay.

The switch is initially not connected thereafter at time t is equal to 0 the waveform is switched and then the circuit is complete okay. This is exactly what we have discussed early but now I have replaced this CMOS driver by a voltage source and its internal impedance R S. And I have replaced this load CMOS gate by this load R L. The interconnect is represented by a lossless transmission line having a characteristic impedance of R 0 okay.

So this is our prototype circuit for analysis of the transients on the transmission line okay. Now before t is equal to 0, there is nothing happening. So the voltage across the load is zero but at t is equal to 0, when we connect the source to the transmission line what would happen. We know that voltages on the transmission line are functions of both Z and t right. In fact for a forward propagating wave the voltage would be something like V + (t – Z) divided by V.

And for the backward propagating or for the reflected voltage it is V - t + Z divided by V. Therefore, the total voltage on the transmission line let us call this as V line will be sum of forward and backward voltages okay. What is this V here? V is the phase velocity.

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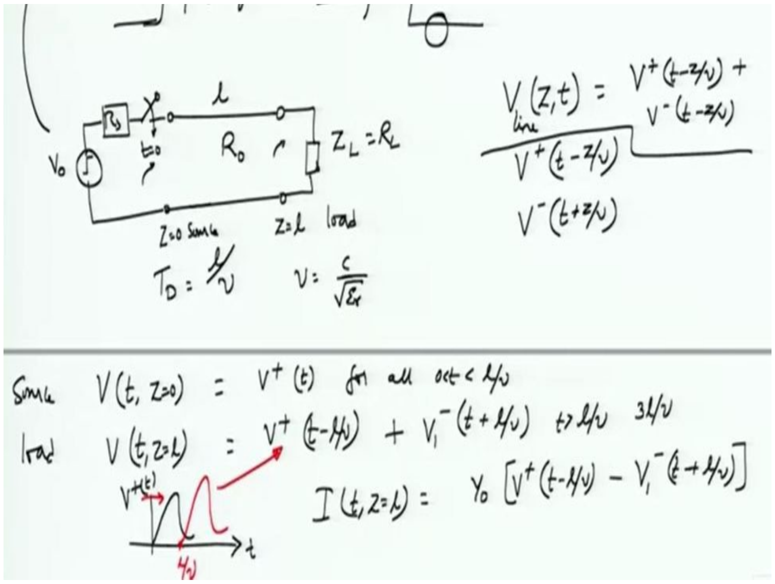

Let us not get in to the reason why this is called a phase velocity and whether this is the same as the one we defined earlier. We will differ all those discussions sometime later. What we want to show is that this V is related to l okay because the delay of the transmission line is given by l by v okay. V can itself be written as C by square root Epsilon r. So if the transmission line is filled with insulating material that is not air.

You have to calculate what is the velocity of light on the transmission line? And that would be C by square epsilon r. So the basic idea here depends here you know imagine that there is a length with something happens over the source end. It will take T D time for this effect to be seen at the load side. So something happens l distance way then it will take some time l by v in order to be observed at the load point okay.

So this is the basic relationship between delay on the transmission line, length of the transmission line and velocity of wave propagation or pulse propagation okay. So I hope this part is alright okay. We can similarly find out what is the since the line voltage have to be continuous right so that line voltage that is coming in here. This voltage must exactly be equal to the voltage at the source. This must be equal to the voltage at the load side right.

So at the source side what is the voltage on the transmission line? Source occurs at Z is equal to 0 okay. So the source voltage is basically given by V + (t) for all t such that this is less than l by v. Actually it would be up to 2 l by v but let us not go there again. So at the source side until for the time delay the source voltage will be equal to only V + (t). This would only be the forward voltage. There is nothing changing out there.

At the load side what would be the voltage? The load voltage would actually be at Z is equal to l and the load voltage is given V + (t – l by v) which is precisely this voltage V + (t) that has been delayed by l by v so if you have to plot this as a function of time then this would V + (t). So let us assume that this is V + (t). Then what would be V + (t – lv). V + (t- lv) will be starting at a time l by v okay. And then it would essentially be the same thing.

Although I have shown this to be two different amplitudes, this amplitude is exactly the same thing. So this is V + (t-lv) which is simply a delayed version of your signal V + (t) okay. This is your V + (t). But this is not the only way. The moment you have some voltage arriving and if R L is not equal to R 0 there will be reflects developed at the load correct. So these reflections which will develop will create a negative travelling wave right.

So the wave which this is going towards the source. What is that voltage? That is some V - but it is supposed to be flowing in the backward direction. Therefore, it is V – (t + lv) but this has to start only after t is greater than l by v. That is only after you have received some voltage can the reflected voltage start to appear okay so that load voltage is V + (t – l by v) + v – (t + lv). What would be the current total at the load?

Current will be I (t) Z is equal to l given by Y 0 which is the admittance of the transmission line 1 by R 0 at times V + (t – l by v) - V- (t + l by v) correct. So we will also right down this one in a slightly different way since this is the first time we are seeing some reflection so we can actually write this as V 1-, nothing has changed in terms of anything. So all that we have done here is to remind ourselves that this V 1- actually is the voltage that is developed in response to this V + that arrived at a time l by v.

And for the time from l by v all the way up to 3(l by v) these things will be quiet at the load side which means to say that there are no more reflections that are being generated okay. So this is the current. Current is given by Y 0 in to this thing which is the forward going voltage plus negative travelling reflected voltage divided by Z 0.

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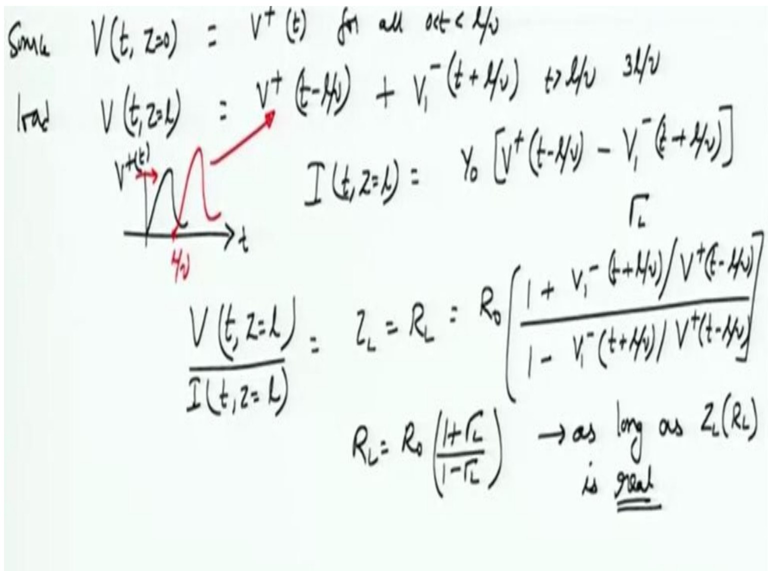

That is 1 by R 0 or is equal to Y 0 okay. If you now look at the line voltage at Z is equal to l and look at the current on the line the ratio of this two must give you the load impedance Z L which is equal to R L. And you can actually do this substitution and what you will see is the R L is equal to R 0 multiplied by (1 + V 1- (t + l by v) by V + (t - l by v) by 1- V 1- (t + l by v) by V+ (t-l by v)). If you look at this equation you will see something that is very similar.

This is the ratio of reflected voltage V 1- appearing at the load side divided by the input voltage or the voltage reaching the load okay which is V + (t – l by v). So the ratio of reflected to incident must be the ratio gamma L and for passive loads that we are considering this will be independent of time okay. So as long as this Z L is purely resistive not just passive it is purely resistive then gamma L will be a real quantity.

And you can actually write down this expression as Z L or is equal to R L you know is equal to R 0 multiplied by 1 + gamma L by 1 - gamma L. And gamma L will also not be complex it will be real as long as Z L or equivalently R L is real okay. It could be greater than Z 0 or less than Z 0 greater than R 0 or less than R 0 but it has to be real.

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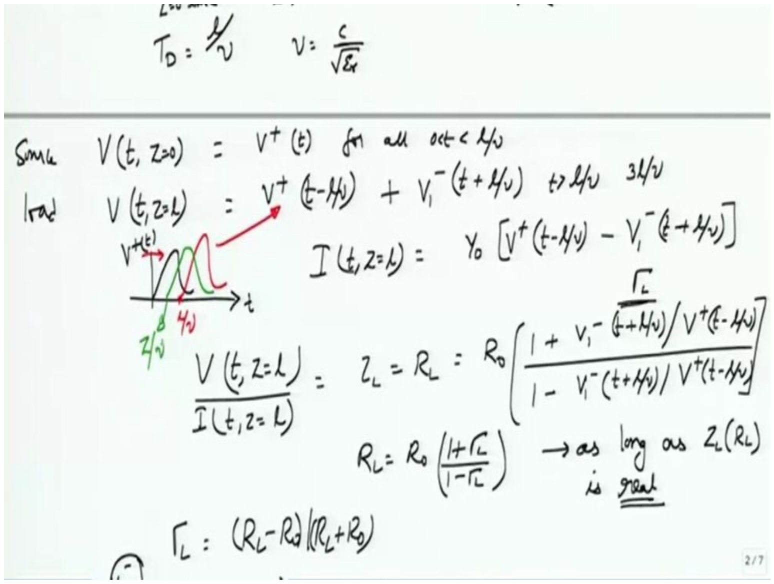

So which means that only for resistive loads this kind of a reflection coefficient can be developed okay. One can actually perform an analysis and obtain the same value for gamma L as we did for the steady state analysis but provided you have loads which are purely real okay. If you have a capacity or an inductive load or a combination of those loads this equation will not be true and you have to perform that in a slightly different way.

Alright so the reason I went through this set of equations is to convince you that gamma L does not lose it is importance when you are analyzing digital waveforms on the transmission line as long as load as well as the transmission line as well as the source impedances are all real okay. So you can similarly define something at the generator side or the source side and call that as gamma G. Of course what is gamma L? Gamma L is R L - R 0 by R L + R 0 right.

And similarly you can define gamma g as R g - R 0. You imagine that the source impedance now acts like a load for the reflected voltage. So here the reflected incident voltage is V 1- the reflected voltage would be say V 2+ okay. And this ratio would give you gamma g and that would be equal to R g - R 0 by R g + R 0 okay. So this is something that we wanted to write here.

Now we have all the tools that are required for us to understand how the wave propagates. So you understand this equation right. So whatever the source voltage that we have launched after a certain time delay that voltage appears at the load. What happens if you are looking at the intermediate point? So you place an oscilloscope between the source and the load on the transmission line.

Well in that case the voltage would start at a slightly earlier time right. So what time should pass it would be Z by v because this V + (t - Z by v) will be the voltage V + (t) which is delayed by Z by v okay.

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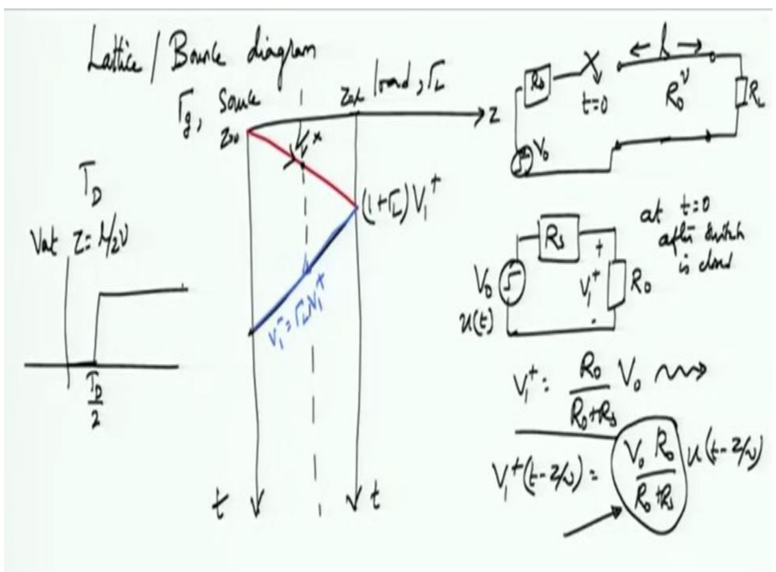

The way to analyze these transients or this digital step waveform on the transmission line is to write down what is called as lattice or bounce diagram okay. This is a nice graphical approach of course more complicated analysis has to be carried out numerically but for this course we will assume that things are simple and we will understand how to use this lattice or bounce diagram. If nothing else this will at least tell us how to actually approach the problem.

And obtain some intuitive understanding of the problem itself okay. So what is this lattice or bounce diagram? The idea is simple you have a source side right. So there is something that is happening along time. So this vertical axis down is the time axis okay. Sometimes you find in literature, time is normalized with respect to the delay time of the transmission line. We can do that or you do not have to do that one.

There is no problem out there so let us call either this time as absolute time or one can imagine that that there is time delay normalization you can do that as well. So there is a similar time diagram at the load side as well. So you have load end and you have a source or the generator end. Load end is characterized by gamma L and generator end or the source end is characterized by gamma g okay.

And this length would correspond to the length on the transmission line. So with Z is equal to 0 over here and Z is equal to L corresponds to the load. This is called space-time diagram or lattice diagram because on the horizontal axis you have the distance between the source and the load on the transmission line and on the vertical you have the time coordinates okay. So you have space as well as time coordinates therefore this is called as space-time or lattice diagrams okay.

What happens to that circuit okay at t is equal to 0 when you switch the source, okay. When you switch the source what happens to the circuit? So there is step source waveform okay and then you have a transmission line connected to the load as well. So this is R L, this is R 0. This transmission line has a length l. The propagation velocity is v okay. So we have everything that is characterizing the circuit.

But what we have not understood is that if an initial step voltage of the amplitude V 0 is applied at t is equal to 0 what would actually be the voltage on the transmission line? Now it is very simple to recognize that the moment the switch is closed R L is not visible to the source write. The source cannot see the R L. What it sees instead is that it will only see the equivalent impedance of transmission line.

And that is nothing but the characteristic impedance of the transmission line R 0 right. So this is what this particular step voltage sees just at t is equal to 0 after the switch has been closed okay. What is the initial voltage? What would be the initial voltage across this one? This initial voltage let us call this as V 1+ okay. The one thing that you would actually propagate along the transmission line.

And this V 1+ from voltage divider analysis is simply R 0 by R 0 + R s into the amplitude of the step voltage waveform V 0. So this is the voltage that is actually launched on the transmission line and this amplitude actually begins to travel okay. Since these step waveforms cannot be represented mathematically by unit step waveform u (t). The voltage V 1+ as a function of time and this one is actually this amplitude V 0 multiplied by R 0 by R 0 + R s u (t – Z by v) right.

So this is the voltage that is actually been transported from the transmission lines from source to the load end. And what we do on this lattice diagram is that we look at this amplitude and then we draw a line which starts at Z is equal to 0 and reaches Z is equal to l. And we also mark the amplitude here as say V 1+. This is the amplitude that we have V 1+ is R 0 by R 0 + R s multiplied by V 0 okay.

And we put an arrow indicating that this moment of this voltage is from source to load. So the source actually carries a unit step or a step voltage of amplitude V 1+ on the transmission line. Well at this point there is a reflection coefficient therefore this set up a reflected voltage right.
So this sets up a reflected voltage with an amplitude of gamma L V 1+. This would be equal to the amplitude of the initial reflected voltage V 1-.

And this would be propagating from load side to the source side. What would be total voltage at this point? The total voltage will have to be 1 + gamma L multiplied by V 1+. Why is that so? The voltage at any point on the transmission line has to be sum of incident and reflected voltage. Now if you were to put oscilloscope over here and say what is the voltage at this point? We can see that if you have put this one half way through between then if this distance gives you a time delay of T D.

Then until T D by 2, there is no voltage. But at T D you will actually see a voltage of V 1+ appearing on the oscilloscope right. So at the oscilloscope at Z is equal to l by 2 v which is half way between you will see that until T D by 2 you would not see any voltage then the voltage actually appears.