Electromagnetic Theory
Prof. Pradeep Kumar K
Department of Electrical Engineering
Indian Institute of Technology – Kanpur

Lecture - 62
Application of Smith chart –I

In this module, we will discuss applications of Smith chart for transmission line problems. As we saw in the last modules Smith chart is essentially as a graphical tool that represents reflection coefficient on a real and imaginary plane that is the r gun diagram. And from there, because there is a one-to-one relationship between reflection coefficient and impedance on the transmission line it is possible to represent impedances as well on this particular chart.

As we have seen the mini derivation of the Smith chart, the Smith chart essentially consists of as circles which are constant resistances and semi circles which are constant reactance circles. You the divide the Smith chart into two parts the upper hemisphere and the lower hemisphere or rather the upper plane or the lower plane. In the upper hemisphere, the reactance' are positive therefore or they might or they would correspond to inductive reactance for a given frequency.

In the lower half of the plane or the lower hemisphere, the reactance are negative indicating that for given frequency those would contribute to or those would come from capacitive reactance. Professor to Student conversation starts "So let us look at Smith chart and before we can look at Smith chart and its applications here is what you should do if you want to get full benefit of this module.

Tick a Smith chart-- you can actually download Smith chart from internet, get a printout, get some of those smith chart and be prepared with you." Professor to Student conversation ends. Then as we perform calculations on the Smith chart I will not be actually using the actual numbers. I will actually leave that to you for you to enter those numbers – I will show you the procedures step-by-step.

And I would request you also to carry out the same steps in your Smith chart and in the next module or in the attached pdf documents I will give you the answers to these problems. If you are followed all the steps that we discuss, first understand the steps of using Smith chart and then try out each step for the given values of the numbers that I will be using okay. And then you should obtain answers which I would upload on the course website.

So it should essentially match and if you get match within sat 5 to 10% percentage you are alright because Smith chart is essentially a graphical tool it can only be used mostly to get qualitative analysis. To get qualitative analysis or to get qualitative numbers you should actually plug in the formulas okay. But Smith chart of course gives you a nice intuitive, understanding of how transmission lines would behave under different load condition.

And some of the operations that you would be doing i.e., the values of Smith chart is basically is intuitive and understanding of transmission line problems. Having said this let us know begin to look at some of the applications of Smith chart.

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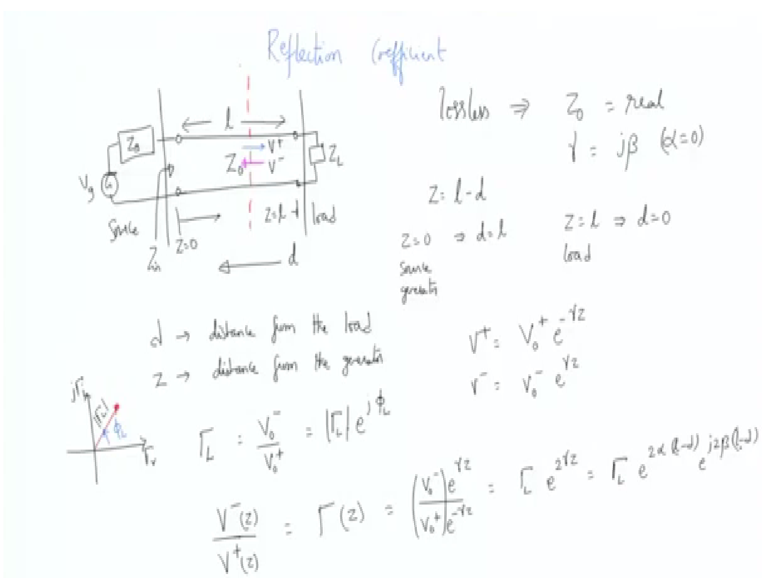

Before we do that one let us review something about Reflection Coefficient, okay. We have looked at reflection coefficient but I would like to just point out a few things about reflection coefficient which actually form the basis of Smith chart as well, okay. So you have a transmission line of some length l let us say, okay this indicate-- this is a two conductor transmission line and this nodes which I left it open circle nodes would indicate the end of the transmission line.

If this is not looking like a uniform transmission line please excuse my drawing, these two lines are straight and supposed to be parallel with each other. To this transmission line you connect a load the load could be complex right. So if the load is frequency dependent as it would be happened if you existing transmission line to sinusoidal signal and for that frequency the load might be inductive capacitive and a conductive or resistive.

So you will have ZL typically to be a complex number and let us assume that Z0 the characteristic impedance of the transmission line is given by Z0 and for all the cases that we are going to consider we will be considering lossless transmission line, okay. So we will assume that transmission line is lossless. This implies that Z not is real okay. And the propagation constant gamma will be purely imaginary that is attenuation coefficient is equal to 0, okay.

Now we will substitute this condition later in a few minutes. To the sending end you might find generator impedance and a generator voltage. These terms of generator impedance, generator voltage and the fact this is the generator comes from an older terminology of transmission lines were the source was usually called as a generator, okay. So on the Smith chart as well you will see scales which talk about wave length towards generator, okay.

Now this is the load portion. The load as I said could be complex. The transmission line as a length L. One of the coordinates that to use would be to measure Z axis from the source end, so at the beginning of the transmission you call this as Z=0, therefore the end of the transmission line will be at Z=L.

Now frequently in the transmission line problems you actually are going to change the length of a transmission line so you want to you know, when you are doing impedance matching or when you are just connecting the load but you do not know-- you want to optimize the length of the transmission line so this L is usually not fixed, okay. Load is usually fixed, so you have an antenna at this portion.

Now you have connected a transmission line, do couple of transmission lines in between so as to match this load to match the input impedance to that of the generator something that we will be talking about later. So the transmission lines could be multiple and the lengths are not usually fixed. So to specify such situations it is common to actually consider the distances from the load by defining a variable D.

How it is relation to Z? Z is L-D, okay. So what happens to Z=0? This implies D=L that is your L units away from the load. So, D axis goes in the opposite direction as that of L, okay. So Z=0 corresponds to a source or the generator, okay which would be L units of distance from the load, okay. So if Z=0 is the source or generator and this is L units from the load.

Z=L corresponds to the load plane because Z=L in our original coordinate system it corresponds to the load plane but Z=L implies the D=0 which means that you are actually at a load itself, okay. So, D measures distance from the load, okay. So keep this in mind, whereas Z represents distance from the generator, okay. In cases where you know, the generator is given and you want to select the load at a particular distance than you actually use the variable Z but in most cases you would be using variable D which would measure the distance from the load.

Now at any point plane on the transmission line there would be incident as well as reflected waves, right. So there would be incident waves which is V+ and there will be reflected waves which is V- phasor and what is V+ phasor, V+ phasor is some amplitude V0+, e to the power -gamma Z, so I am for the time being using gamma but later I will substitute gamma equal to j beta, okay.

And what about V-? V- is V0- which is reflected wave amplitude and it is propagating along -Z direction therefore I am writing this as e to the power gamma Z. Now reflection coefficient at the load is defined as the ratio of the reflected voltage amplitude to incident voltage amplitude, okay. So this is the reflection coefficient at the load this itself would be complex okay because this is actually connected to ZL and Z not and ZL could be complex.

So this reflection coefficient could be complex number which can be written in its magnitude and phase as magnitude of gamma L time phase phi l. Now if you go back to way in which gamma r and gamma i are represented so gamma r and j gamma i are the gamma plane representation of the reflection coefficient you can represent this gamma l e power j phi L by drawing the straight line, okay of the magnitude gamma L and a an angle of phi L as measured from the horizontal axis.

This is how you would actually measure or you would actually represent okay so you would represent the reflection coefficient which is a complex number on gamma r and j gamma i. In fact, this is essentially Smith chart representation as well. Because you remember Smith chart is nothing but circles and semi circles defined on this gamma r j gamma u plane or the complex gamma plane. So this is the load reflection coefficient.

But what is the reflection coefficient at any plane? You imagine taking a particular plane here as shown in the red axis there is an incident where V+ then there is a reflected way V- the ratio of this V- to V+ at any plane Z will give you the reflection coefficient at that particular plane. So substituting for V- and V+ from this expression what you see is this V0- V power gamma Z divided by V0+ e power minus gamma Z.

But I already know that ratio of V0- to V0+ is nothing but gamma L, so this is gamma L e to the power 2 gamma Z substituting for gamma, I can write this as e to the power 2 alpha and substituting for Z, I can write this as l-d e to the power j2 beta and minus d. Now we also utilize the fact that gamma l itself could be complex, therefore I can write this as magnitude of gamma l okay e to the power j phi l then you have e to the power -2 alpha d you have e to the power –j2 beta l and then you have e to the power 2 alpha L e to the power j2 beta L, correct.

Now, there are certain things which are, you know dependent on L, so I can actually group them together so these quantities I can group them together, okay and call this as a gamma with a subscript l please note that this is not the capital l, okay capital l stands for the load and small l stands for this constant phase factor, this is a constant phase factor, right because this is alpha if you fix, beta if you fix and l is the fixed transmission line therefore this would have essentially be some constant phase factor.

To take that into account I can put them down as gamma l, okay. I still have this e to the power -2 alpha D e to the power –j2 beta D, and this is a phase angle which is changing as you go from load because D increases as you go away from load, okay. So this would be the phase angle which will changing but what is the angle there; it is actually minus. So which means that you should be going clockwise, okay as measured from wherever the starting point for gamma l that you would be there which is this gamma l, okay.

So please note that this is actually gamma l, so with this gamma l which is already making an angle of phi l plus two beta l right and has an amplitude of gamma l into 2 alpha l, okay. So this angle is already making and then you move this one away. Now, suppose you assume that line is lossless, okay. Suppose we assume that the line is lossless then what happens to gamma of D?

Because this Z dependence has been replaced by D dependence right we replace Z =1 l - D right. So if you actually write like this, what you will see is that gamma of d is equal to gamma l alpha is 0 therefore this term will be equal to 1, okay. So you have e to the power – j2 beta d. But what is gamma l? gamma l is nothing but gamma l magnitude times e to the power j phi l itself because if you are looking for this one then alpha is equal to 0, right. So there is actually e power j2 beta l, okay.

But this e power j2 beta is essentially a constant phase that does not really matter to us, right. So what actually matters is as you go away for the transmission line what would be the way in which you are magnitude of gamma d changes, right. So you can actually obtain that by taking the magnitude of this one and then write in the phase angle for this, right. So if you take the magnitude you are going to get gamma l magnitude which would essentially be gamma l and when you write the phase of this you would obtain e to the power j phi l – 2 beta d, okay.

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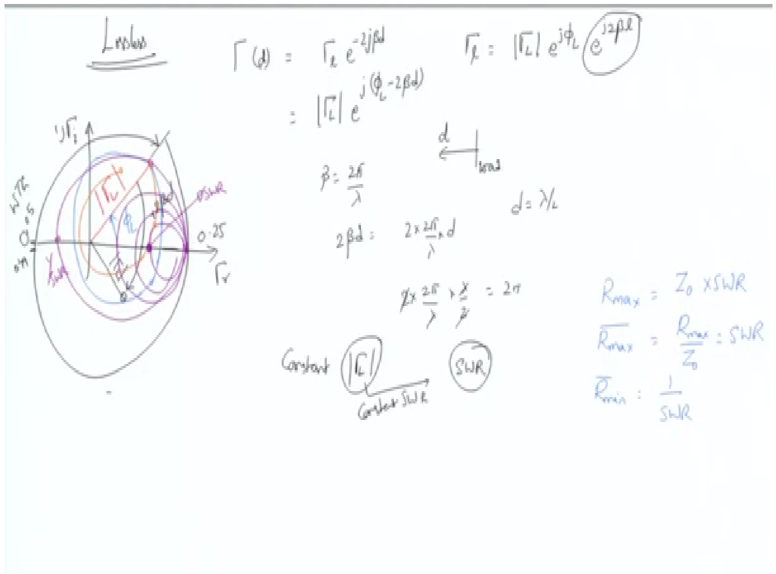

So going back to representation of gamma l on the plane this would correspond to gamma r and this is the imaginary j gamma i, so you draw a line okay of length magnitude of gamma L which would be the load reflection coefficient and the magnitude of gamma L would essentially the same, okay. This initial angle will be phi L okay so that you are actually representing this gamma L magnitude e to the power j phi L.

Now at any point d on the transmission line d units away from the load your new value of gamma will be something that has to be clockwise moved on the constant gamma L circle right. So you assume that there is actually a constant gamma and circle up there, okay. This may not look like circle but please excuse. But the point here is that on this constant gamma L circle because the magnitude is not changing only the phase is changing.

And if you move at a distance d away your total phase angle must be reduced by an angle of 2 beta Z, i.e., therefore your new location would the same magnitude gamma L okay so this is a same magnitude. But a total phase change of -2 beta d, okay. Or rather if I consider minus as clockwise so this would be a total phase of 2 beta d. Immediately you can see certain ramification of this one.

For example, since I know that beta is related to lambda as 2 pi by lambda. So the total phase change as you move a unit d away from node on the transmission line right, the total phase change would be 2 into beta into d which is 2 into 2 pi by lambda into d, okay. Suppose you move d equal's lambda by 2 away from the load, okay. So with d equal to lambda by 2, what would happen to phase change 2 into 2 pi by lambda into lambda by 2. So lambda cancelled, 2 also cancelled and you get 2 pi which means that after moving around, okay. So after you are moved around a lambda by 2 distance from the load you are actually back to the same position.

Now this actually make sense because we know that successive maxima or successive minima are repeating on a transmission line with a periodicity of lambda by two. Therefore, this lambda by 2 bringing back to the same point actually make sense and this is calibrated on the Smith chart, okay. So on the Smith chart if you look this value on the outer scale which is called WTG scale, so on this WTG scale this is 0.

And then as you go around this would be 0.25 and clearly as you come back to here, so somewhere it would be 0.49 and then go all the way up to 0.5 – so 0 and 0.5 were essentially coinciding on each other, okay. So this is essentially what I wanted to tell you as you move on the transmission line your reflection coefficient magnitude does not change. In fact, the movement can be visualized by drawing a constant gamma circle, okay.

By drawing a constant gamma circle, and all the movements are actually made on this constant gamma circle. Now constant gamma actually implies a constant SWR, right. So because there is a relationship between the two, you know that this would essentially correspond to constant SWR therefore constant gamma circle is also constant SWR circle for that particular transmission line, okay.

Again it is important to note that we are dealing with only lossless transmission lines. If not it will not be a circle it would actually be a spiral, okay. For example, your alpha is non-zero then from this point you would actually have dropdown to this point, okay so it would essentially be a spiral and at the end point the distance I mean the length of gamma L would have been reduced by a factor of exponential 2 alpha d, okay.

So that is essentially what this particular spiral thing is, but anyway we are not going to consider the lossless case so this is not anyway important for us, okay. So we are not going to look at this particular lossless case, alright. So bringing back constant gamma l circle is constant SWR circle. Now here are couple of other things that you can immediately note down. What is the maximum resistance on the transmission line?

The maximum resistance or the maximum real part of the resistance would be the transmission line characteristic impedance Z0 which is again assume to be real times SWR, okay. This was the exercise which I gave you in the last module I hope that you would have solved this one and realized this, okay. Because on the Smith chart all impedances are normalized with respect to Z not, what you would actually find is not R max rather R max normalized.

And denoting this normalized by writing a bar over this, this would actually be equal to R max divided by Z not normalizing with respect to Z not and you would see that this is nothing but SWR, right. So the maximum R max would actually be equal to SWR, similarly the minimum impedance or the minimum resistance is given by 1 by SWR normalized, okay. And where would you find SWR, SWR is actually on this particular axis, right.

Because you can actually look at what is the maximum R and maximum R would happen over here right on the Smith chart the maximum impedance, okay occurs on this side. So if you assume that this is the center of the Smith chart then this would be where the constant circles are located on one of these circles there will be an intersection which would correspond to the SWR point, okay.

The minimum points would happen on the circles in which R is less than one and let us say this would happen 1 by SWR corresponding to the normalized minimum impedance. So we have understood much about the reflection coefficient and some basic manipulations on the Smith chart. Now we are going to actually put all these things together to solve certain problems--certain typical problems.

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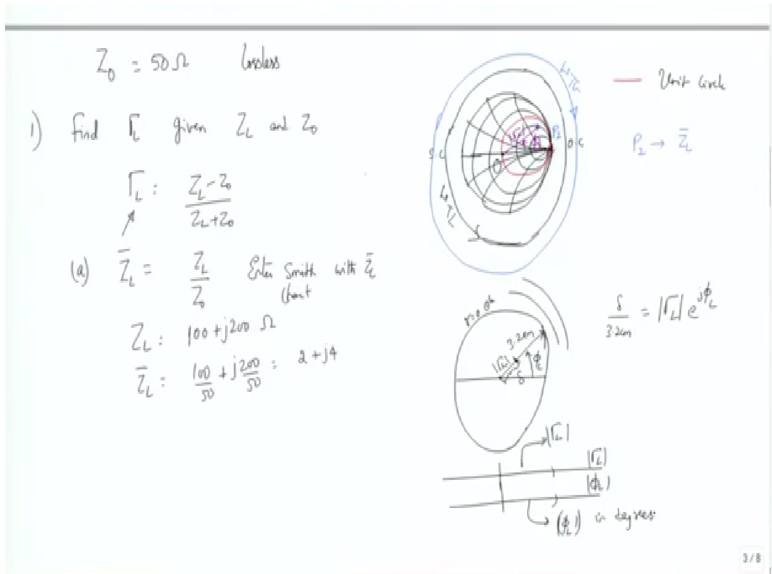

For these problems you may assume that Z0 is 50 ohms, okay. Line is lossless. And I will give you the other numbers as we go along, okay. But first let us look at what kind of problems can we solve, okay. So let us look at the first problem which actually is not really interesting in the sense that we do not normally use Smith chart in order to solve this problem. The problem is find gamma L the reflection coefficient of the load given ZL and Z0.

Are these information enough to solve for this problem? yes. Actually you can use the formula, the formula straight up gives you the answer gamma L is equal to ZL – Z0 by ZL + Z0. But if you want to use Smith chart, okay. So I am going to draw some representative chart over here okay. So this smith charts may not look very nice but they are just used for illustration purposes.

So they are not accurately drawn you should get hold of a Smith chart which is accurately drawn and then follow all these steps that I am talking to you about. Let me also color up one circle which I am going to call this as unit circle. This unit circle means R is equal this red circle which I have drawn is essentially a unit circle, okay. And this point is the center of the Smith chart, okay.

Let us denote that by O, you remember that all these are constant r-circles and this semi circles are constant gamma circle. You should also remember that this axis is the open circuit-- sorry this point is an open circuit and this point with the short circuit impedance. And if you look at Smith chart carefully we will actually see some very interesting things, okay. So on the outer scale you actually find two scales or two circles, okay.

One circle is called wavelength towards load and it would be going the wavelength towards load would be going this way, okay so let us write down like this so this is your wavelength towards load circle, okay. And there is another outer circle which is very important for our case which is the wavelength toward generator. Now why is that-- so this is the wavelength toward generator and it would actually go in this particular direction, okay.

So it would actually go in clockwise direction, okay. Now what is the significances of this wavelength toward generator and toward load? What it simply means is that, since gamma changes in phase as e power – 2j beta d or j2 beta d does not matter this d can be expressed as fractions if lambda because beta is 2 pi by lambda therefore if you express d as fractions of lambda.

Let us say this is 0.2 lambda then that lambda would cancelled and you would get some reasonable number, i.e., all these scales are calibrated with respect to lambda, okay. They are not calibrated with respect to Z not, so you should explicitly calibrate or normalize all of your impedances using Z not, okay alright. So the problem that we want to solve seems to be that I want to find out what is the reflection coefficient given ZL and Z not using Smith chart, okay.

The first step in this is to enter or normalized ZL, okay let us put a bar over this one so you know that this is normalized value or you can use the small case z since that is difficult to distinguish in this screen – I am going to use a bar to indicate that this is normalize quantity. So normalized impedance is nothing but ZL divided by Z not and there is no units for the normalized because Z0 is in ohms, ZL is ohms. Therefore, ZL bar is dimension less quantity, okay. This is just a number, okay.

So you enter Smith chart with ZL bar, okay. Let us consider a simple example let us assume that I am looking at ZL of 100 + j200 ohms, okay. Please note, this is XL and it is in ohms. So this is ZL which is 100 + j200 ohms, i.e., the normalized ZL will be 100 by 50 because that is what the value of Z not that we are assuming plus j200 by 50 which is nothing but 2 + j4. So on the Smith chart you locate r=2 circle.

So let us say this is r=2 circle also locate +j4j. Now +j4 four is inductive reactance therefore it should occur on the upper half of the plate, okay. So let us assume that this is your j4 so your initial point that you would be there on the Smith chart would be this one, okay. So maybe you should use a different color here so let me use this blue color to denote this point, okay. So this point let me also called as P2 and P2 denotes normalized ZL bar; on the Smith chart P2 denotes normalized ZL bar.

“Professor to Student conversations starts” Please note that I am just doing this one on a qualitative basis you should have your charts in hand and you should carry out this steps as we are carrying out, okay. But do not relay on this diagram too much I am just going to show you the steps and you should actually enter the values. I will give you the answers in next, you know in the attached pdf document in the course website, okay. “Professor to student conversations ends”

So you actually end up with first P2 on ZL and from the center of the Smith chart we should now draw line which passes to P2, okay. The angle which this this line make will be the phase angle phi L. Remember that 2 beta L is not of any consequence to us therefore I have removed that 2 beta L from consideration. So, whatever the angle that we measure of this line which would also give you the magnitude of gamma L, okay.

So how does it give you the magnitude of gamma L? We know that outer circle on the Smith chart must be-- sorry this is not even looking like a circle, so let us say this is the outer circle on the Smith chart, this has the maximum radius. So on the Smith chart let us say this is about 3.2 cm point okay so decided to print this Smith chart on a paper and then on the paper it came out to be 3.2 centimeter.

Now this magnitude of gamma L would be the fraction of this one, right. So let us say this is magnitude of gamma L, okay. Measure this using a scale and then-- so let us say you have measured this one and you found that the value here is delta and then normalized this one by the radius of the outermost circle.

Please note down, the smith chart there are two outermost circle, one is WTG scale and other one is WTN scale, we are not talking about those circle, we are taking about the Smith chart R=1 circle, okay. So outer, there are two more perimeter circle do not worry about those okay these are WTL and WTG scales. So once you get delta normalized with respect to 3.2 centimeters—I mean I am assuming that this radius is 3.2 centimeter.

But on your Smith chart you have to measure what would be the radius, okay so this is on the R=0 circle, okay. So this would be delta by say 3.2 centimeter and that would give you the magnitude of gamma L. The angle that this would make will give you the phase so your reflection coefficient is gamma L e power j phi L. Do not use this procedure, where are also – calibrated down which tells you the magnitude of gamma L as well as the phase angle phi L.

So all you could do is take the compass okay measure this distance and then cut this two axis with that particular compass distance, okay so when you do that one you can directly get out what is the value of magnitude gamma L and the phase angle phi L that would be in degrees, okay. So this is an application of Smith chart not widely used because the formula is more convenient in this case.