Electromagnetic Theory
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Lecture No - 06
Introduction to Electric Field

Previously, we have seen Coulombs law which gives us force on one charge because of the presence of another charge. So, if we normally think of the concept of a force you know for example, I take this particular box or item and I want to move this object. So, let me put this object on a particularly flat surface and I want to move this object. Now, how can I move this object?

Well, I can use this pen for example as some agency which if I move, would also move the object. For example, if I take this pen and then apply a force here or any other direction in which I want to move the object then the object will move. So, the work is what I am doing? I am pushing the pen. The pen is in-turn pushing the object. So, such forces in mechanics are called as contact forces and these are the only type of forces that people were familiar in the 18 and 19 century.

However, the Coulombs law that we talked about is actually a force of a different nature here the charge, the source charge which is exerting a force on the test charge or the charge that is placed at the test point that we talked about in the last class there is no contact between the two charges. See, the source charge is not touching or making any contact with the test charge and yet simply because of the fact that these two are charged objects there is a force on the second charge because of the presence of the first charge.

Such laws in which force get imparted from one object to another object even when the two objects are not in contact with each other is called action at a distance. Because this action of influencing, the second object is happening not by any contact between the two objects. The first charge is exerting a force on the second charge but it is not making any contact. So, this action of exerting force is happening without any contact.

And such laws which or the laws which determine such behaviors or which predict this particular force are called action at a distance law. You might of course be familiar with another very famous action at a distance law this is gravitational law. So, in gravitational law you have two objects and they will be attracted towards each other but there is essentially no contact between the two. Now imagine that I have this particular charge over here.

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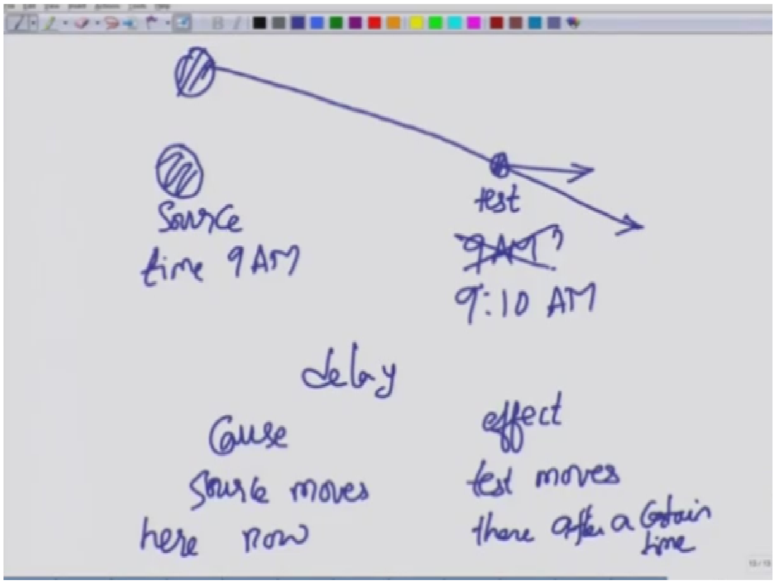

Let me call this as some source charge. The other charge which I am keeping at a certain distance over here let me call this as a test charge. Now Coulombs law, of course tell me that when the source charge is located in this way in the space and a test charge is located at a certain distance in the space there will be a force on the test charge. So, the force on the test charge will be let us say, acting in this particular direction which is obtained from joining the two charges.

So, we of course assume that two charges are point like and the separation is fairly large compared to the size of the object or the size of the charged particles. So it is a force, acting on this one. This is of course the action at a distance phenomenal that is happening. Now imagine that this source charge is moving very slowly. We have to assume that this charge is moving very, very slowly with respect to time and what happens if this charge over a very small velocity or very small speed moves to a different location.

When it moves to a different location, let us say at this location it has moved and of course we will neglect any kind of acceleration that complexities arising due to acceleration that happens when the charge starts moving. So, we will neglect all those other complications this is a qualitative discussion only the actual mechanics of the force that gets transferred is slightly different and we do not want to get into that complexity at this particular point in the course.

So, just imagine that a source has moved at some time. So let us say at time 9 am this particular source charge has moved from one location to another location. Now, what happens or what do you think will happen to the test charge? So, clearly if after a certain time this force has moved to this point, to this new point. The force has moved to this new point.

The force on the test charge would also change its direction because the special position or the position of the source charge is changed. Of course, we will assume that the test charge has not changed. So, the source has changed its position. So the vector that points in the direction of the line that joins the source charge and the test charge will of course be different now. So, now the force would act in a direction that would be different from the earlier direction.

This is something that we would expect. Now the question is and this is true you can calculate how much force that second charge or the test charge will experience because of the new location of the source, you can calculate using Coulombs law there is nothing wrong with that particular calculation. It will give you exact result or at least exact approximately very good result.

The only problem here is that if I ask if at 9 am this source charge started moving, when will this teat charge start to move? Remember when there is force on this test charge, it means that it is getting accelerated. So, when this test charge is moving at what point of time would the test charge start to move? Would it move at 9 am? Answer is no. There is a certain amount of delay. Let us say 9.10 am, of course these numbers please do not take them very seriously.

The numbers are not important. But what is important is that there is a time delay between cause and action. What is the cause? The cause is the source charge movement, source charge moves and the affect is that after a certain time the test charge also moves. The source charge moves and after a certain time the test charge moves. The source moves here now where as the test charge moves there after a certain time.

So, there is some amount of time delay. Now, if you keep this two very far apart then you can see that source charge is moving while the test charge remains stationary after a certain time the test charge start to move. Now to understand these types of phenomenon which was not very commonly understood in the 18 and 19 centuries, physicist invented a concept which initially was a concept that was mathematically convenient.

So, they invented this concept of what we now call as field. Of course at that point also we were calling this as field. So what is a field?

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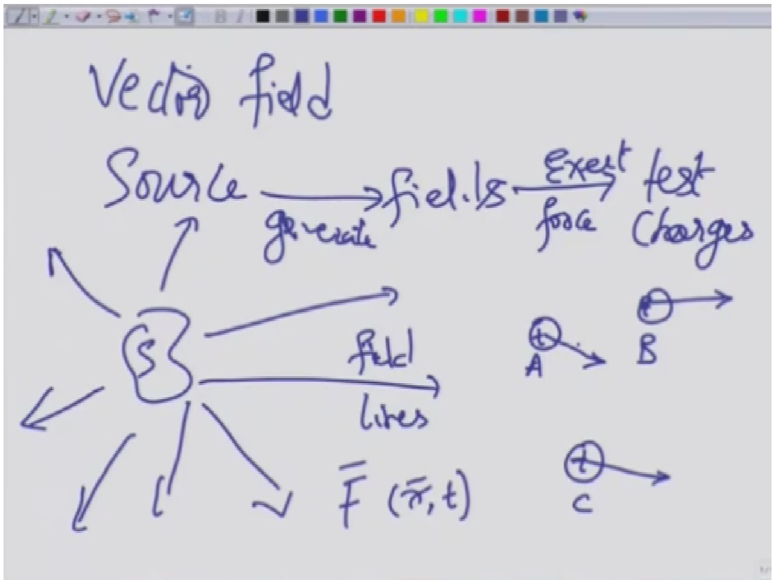

Field essentially embodies this cause and effect relationship. There is something happening now in space here and something else responds because of something that happened over there. So, something happened here like a source charge moved. Now, something else happens in response to this at some other time then? So, field is something that embodies this concept. So, it is kind of intermediary agency, which seem to have care which or which seems to carry the cause and transfers it to the other particular location at a certain time.

In free space, you might approximate this time delay between the cause and effect as l/c where l is the separation between the two charges. So, this intermediary agency although initially introduced only as a mathematically convenience later was seen that it embodies all the real properties such as energy and momentum, making its fields as, as real as particles. Of course, it is possible to actually dispense off with the concept of a field all together.

You can modify Coulombs law and completely eliminate fields and talk about electromagnetic theory in action at a distance more itself, action at a distance phenomenal itself. However, that would be extremely difficult to carryout. Such a program is extremely difficult to carry out and therefore accepted version of approaching electromagnetic theory or understanding electromagnetics is to introduce fields. So, we have source charges which are generating fields.

So, you place some charges somewhere these charges are assumed to generate the fields and these fields in-turn exert force on the test charges. So, you have sources which are generating the field and fields in turn exert force on the test charge. So, this kind of makes electromagnetic theory into a two-layer system. You have source charges generating the fields on their own even when there is no test charge, there is a force field associated with the given source charge.

You can imagine that there is some sort of field around a charge, a field of influence around the source charge and this force field becomes evident or becomes manifest when you place a test charge here. So, this is like earth in the absence of moon for example still has a certain field or you can imagine that intuitively there is a certain field earth which is the gravitational field and when you place an object such as moon or a satellite or a duct particle.

Then there is a force acting on moon, dust particle or a satellite and that force will be because of the field. So you have to imagine that the fields, it’s intuitively easy to imagine that there are fields without any regard to the test charge. The test charge simply manifests what is already there. The field is already there it is real and it is generated by the source charge and that test charge simply brings out the fact that there is already field.

So, what other characteristics do these fields have and what is the field that we are going to talk about? Well, before we talk about the fields here let me just look at the characteristic of the field. I have a source charge of course and which generates fields. There lines you might not be able to understand now but you have to wait for some time to understand them. So, for now imagine that these lines are essentially region of influences or force field kind of a thing.

So, this is your source which I am labeling as S. So, these are the field lines you can imagine that these, field lines as something that is coming out and defining the so called region of influence of the source charge. So, if source charge has generated a certain field. Now, you can place your test charge over here and once you place a test charge there is force acting on that one. Now, you can move the test charger around.

Now you fix the source charge, you can move the test charge from position A to position B you can move it to a different location use of course the same magnitude of the charge that I am using. So, you can move it anywhere you want while you keep the source charge fixed and what would see is that at each point the test charge will experience a certain amount of force.

And we have already seen in the last class that force is an example of a vector quantity which means that I have to attach both magnitude as well as the direction to this particular point where the test charge is located. So, I have test charge located at point A. Let us say the filed direction or the force direction is in this way and the magnitude of the force is given by length of this particular vector here.

At point B, let us say the force is directed along this direction and then again the magnitude of that particular line or of the vector will determine what is the magnitude at this particular point in space you can put a test charge at location C and see what is the direction in which the force is pointing and you will again get the force at that particular position. The important take away from all this is that there are of course as infinite number of points in space.

And you will not be practically taking a point or a test charge and placing it at every point and finding out what is the direction of the force and what the magnitude. You are going to do that one mathematically or numerically, or graphically. But it is important that in principle for every point in location around the source charge I can associate a vector. A vector will have both a magnitude as well as direction which simply mean that I have to attach little arrows indicating the magnitude as well as the direction at each point in space.

Such a distribution of vectors in the region of space is called a vector field. Let me tell that again, a distribution of vectors in space that is attaching one vector at each point in space is called or which defines a vector field. Vector field simply is the result at each position vector r, remember the position vector that we talked about in the last class. At each posting vector r, let me just use a different letter.

At each position you are going to associate a vector which defines a vector field. This vector field could be varying in time or not varying in time? That is, it could be time varying in time or time invariant. When will this be time varying? Well, if the source charge distribution itself is varying then you would expect reasonably that the force at each point would also each point in the space around the source charge will also be changing with time.

So, in that case I have to use a slightly different notation. I will have to say that what is happening to the force at this position, at this time. We will not be meeting the time varying fields for some time now. We are still in the static region of the varying in time electromagnetic which means that our fields, our source chargers and everything that we are talking about will be time invariant.

So, I hope the concept of a vector field is clear now. A vector field is simply the distribution of the vector in the region. You are going to associate a vector at every point it is like someone is making a measurement and attaching an arrow, measurement and an arrow but different points in space. So, this is an example of a vector field.

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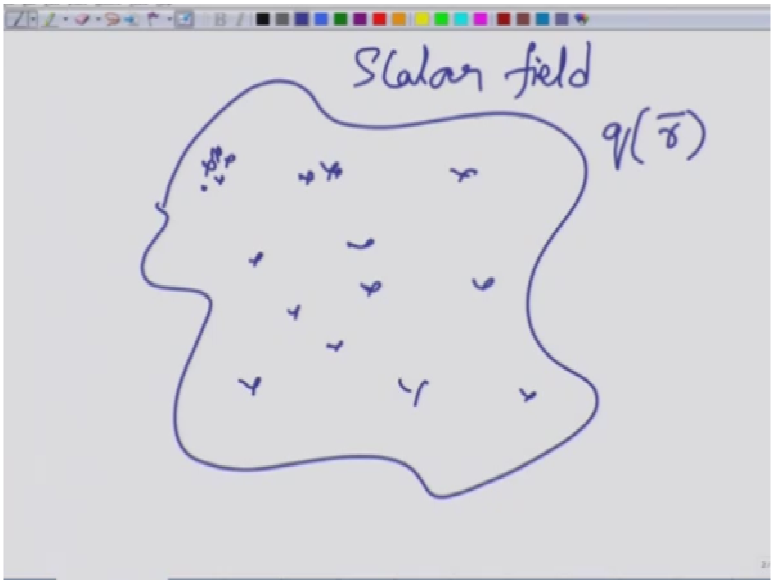

A field of course can also be scalar in the sense that I take a certain region of space for example this could be a curved or a very haphazard curve representing capacitor plate and if I charge this capacitor there will be charges distributed along this particular region in space. This is in the region of a plane. There are charges that are distributed these charges need not be all equal so maybe the charges are concentrated here more.

There are some charges concentrated here but the important point to note is that at each position I can put down a number which gives me the charge. So, at each position in space I can specify what is the charge at that particular or charge that is distributed at that particular point. Such a distribution of numbers in space is called a scalar field. A scalar field is one in which we have a distribution of numbers.

How do I obtain this field or what is this field that I am talking about in electromagnetics? So, let us go back to Coulombs law for now and let us say.

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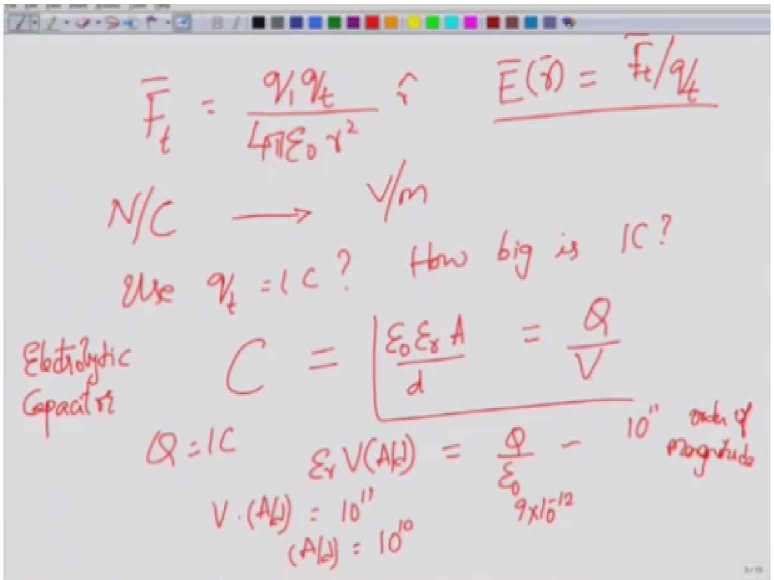

$\bar{F}_t = \frac{q_1 q_t}{4\pi\varepsilon_0 r^2}\hat{r}$ $E(\bar{r}) = \bar{F}_t/q_t$

N/C ⟶ V/m

Use $q_t = 1C$? How big is 1C?

Electrolytic Capacitor

$C = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{Q}{V}$

$Q = 1C$ $\varepsilon_r V(A/d) = \frac{Q}{\varepsilon_0}$ – 10^{11} order of magnitude

9×10^{-12}

$V\cdot(A/d) = 10^{11}$

$(A/d) = 10^{10}$

The force acting on the second charge is given by q1 q2/ 4 pi epsilon 0 r square in a particular direction. We will not worry about what the direction is at this point of time. So, there is a force acting on this charge. Now, instead of talking about this one charge as 1 or 2 let me talk about this as the test charge. So, the force on the test charge is q1 q t/ 4 pi epsilon 0 r square acting in the direction r.

We will talk about what the direction r? or the unit vector r means at a later time. So, this is r, r denotes the unit vector, in the direction in which the force F t is acting. Electric field E, at this particular point r where the test charge is located, please remember that E is located at the test charge point or what we have already introduced as field point. Correct. So we have introduced a field point and the source point.

This electric field is being evaluated at the field point, now you see why we called r as the field point, right. So, this E of r is the electric field at the field point r or the position vector r is now given by the force experienced by the test charge. So, Ft is at r so the force experienced by the test charge divided by the test charge or the force per unit test charge is the field. So, this is the definition of electric field.

And contrary to what you might think that the electric field should be measured as Newton per Coulomb because force is measured in Newton in SI unit and test charges or charges are measured in Coulombs in the SI unit. So, one would expect that electric fields are measured in terms of Newton per Coulomb but you would be surprised a little to realize a little a realize that or to see that electric fields are measured in terms of volts per meter.

In some text books, electric field is also called as electric field intensity. Although, I would not really use the word intensity here simply because intensity has a different meaning in optics so where it represents power whereas here this is representing a field and there is a relationship between field and power which we will talk about it later. So to me, this is electric field I am going to call this as electric field.

But please keep in mind if you are going to refer to text book you might see that some of the text books might refer to this as electric field intensity both are quantities which are vector fields are measured in terms of volt per meter. This is the practical unit for measurement of electromagnetic quantities. So, we are going to use practical units such as volt and amperes. So in contract to Newton per Coulomb.

Electric fields are actually measured in terms of volt per meter in the SI unit. So, this is electric field you might think that determining the electric field experimentally would be very easy. Suppose, there is source charge located some point in space all I have to do is the test charge and move it around. Now, there is a catch the test charge is not innocent, it also has an electric field of its own.

It generates an electric field of its own regardless of the fact that there is a source charge. So, there are two electric fields now. There is one source charge and then electric field because of the source charge and the electric field because of the test charge. If I take a very large magnitude test charge then I might, the field because of the test charge might simply overwhelmed the field because of the source charge thereby giving me incorrect answers.

So, I cannot take qt to be a very large quantity. Ideally, what I should to do is I should take qt to be a very small quantity. Now, can I take qt as equal to zero? Well, the answer is no, why? Because if you remember if you turn the definition of electric field around force is given by filed into charge magnitude. So, if the electric field has a certain magnitude at this particular point in space multiplied by that to the charge magnitude will be the magnitude of the force.

Now, if I take the test charge magnitude to be equal to zero all I am saying is that the force itself is zero. So, two things are ruled out qt tending to zero is ruled out or qt is equal to zero is ruled out. Anyway, it would have made it problem mathematically very difficult because there is nothing like qt is equal to zero. Remember in the last class we mentioned that the smallest possible free charge is often electron.

And electron charge is 1.602 into 10 to the power minus 19 Coulombs. This is small but not zero. So, I don't have the luxury of using qt is equal to zero rather I should use the different value of qt. And again qt, cannot be very large if it is too large then the original field gets disturbed so much that I will not be able to determine the field of the source charge. Now, can I use qt is equal to one Coulomb which seems very nice in mathematically.

So, can I say use qt is equal to one Coulomb? Well, the answer turns out to be no. Because the next question that I am going to ask is, how big do you think is one Coulomb? How big is one Coulomb? Well, you will be surprised to see how big one Coulomb is, to get some idea again this is a qualitative idea do not go by the exact numbers over here. To get an idea of how big really one Coulomb is.

I will use a formula that I am I mean you guys are all hopefully familiar with which is the formula for a capacitor. The capacitance of a parallel plate capacitor is given by epsilon zero, epsilon r A / d where d is the separation between the two parallel plates and A is the area of the plate. We have already seen epsilon zero is permittivity of free space which is of the order of 9 into 10 to the power minus 12 farad per meter.

And epsilon r is the dielectric that fills the parallel plates. Now, if I go back to the definition of capacitor in terms of charge stored on one plate to the potential difference or the voltage that I have applied on the other plate or between the two plates is nothing but q/v. So, q is the charge stored and v is the potential. Now, if I say q is equal to one Coulomb. What would that mean? Or what should I do?

What should the voltage that I have to apply in order to ensure that there is one Coulomb of charge between the two plates of a parallel plate capacitor? So, let us see how much we have to put down as the voltage? So, with q as one Coulomb I can use this equation to solve for certain parameter I am not really interested in the numbers. I am just interested in the magnitude of or orders that you are going to get, order of magnitude results.

So, I have epsilon r V A/d, I am going to put that one in brackets because A/d is the area to distance which I would be or distance of separation between the two plates which I want that will be equal to Q/ epsilon zero, of course q is one Coulomb, I have assumed that and epsilon zero is of the order of 9 into 10 to the power minus 12. So, this goes if you take this the numerator this becomes to the order of 10 to the power 11. Why?

Because there is 1/9 which can be approximated as 1/10 so that is becomes .1 into 10 to the power 12 so that is nothing but 1 into 10 the power 11. So, this is order of magnitude. These are not exact calculations that I am doing. So, go back to this one suppose you want to fill the capacitor with nothing but free space or air then you see that the voltage into A/d quantity must be on the order of 10 to the power 11.

If you want to apply a voltage of 5 volts or 10 volts may be at most. I am making a capacitor for electronic circuit and let us say my voltage could be around 10 volts that is the maximum I can try then A to d ration the area to distance or the separation ratio must be in the order of 10 to the power 10. Whereas the hydrogen atom extent is approximately three angstrom I mean the numbers could differ a little bit but the order is 10 to the power minus 10.

If you use that separation, then the area of the plate that you are looking for will be one meter so you have to take two plates whose area is one meter square and their separation is that of a hydrogen molecule or not an atom hydrogen molecule. So, you need to have such small width and such large area in order to make a one Coulomb of charge. Which means that the charge one Coulomb is an extremely, extremely large quantity.

Then the question is well you might ask I have seen some companies selling me one farad or two farad capacitors and they do not look like 1 meter/ 1 meter, they look very small. How are they manufacturing these capacitors? How are they manufacturing these very high value capacitors? The answer is they used these capacitors are what are called as electrolytic capacitors

And I would suggest that you take a look at electrolytic capacitors which give you a large capacitance values, in internet and would be surprised to find how they actually achieve this large value of charged storage. So, the point of all this is that I can define the electric field e of r but I cannot use qt of one Coulomb qt is actually very large value. Qt has to be chosen sufficiently small but it cannot be zero.

But sufficiently small such that the field because of the original source charge does not get disturbed. I hope that this is very clear and if someone else ask you why you are not using one Coulomb you can respond to by saying that this is the reason why I cannot use one Coulomb of charge. Now, I can find the electric field of one-point charge. I can find the electric field if there are more than one-point charge. How do I find the electric field?

Well I know how to find the force on the test charge if there is more than one source charge. How do I do that? I calculate the force because of the individual charges, due to the individual charges and then add together in the form of vector addition this is superposition principle. So, if the test charge is sitting here and there are many source charges at different locations possibly then I have to calculate the force on this test charge individually.

And then divided the result by test charge. So, in other words I can find the electric field at any given point r because of many, many source charges in order to do that one I simply have to find the electric field from different charges at different locations on the test charge location and I will have to simply add them in vector terms. So, this is a vector addition that I am going to perform and then I am going to get the electric field at this particular location r.

So, in mathematical terms the electric field

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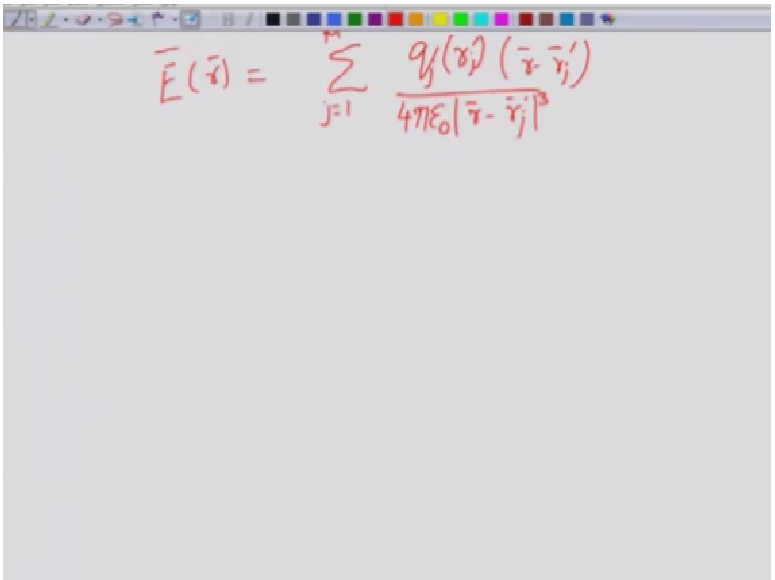

$$\bar{E}(\bar{r}) = \sum_{j=1}^{M} \frac{q_j(\bar{r}_j')\,(\bar{r}-\bar{r}_j')}{4\pi\varepsilon_0 |\bar{r}-\bar{r}_j'|^3}$$

At the point r or at the posting r is obtained by summing up the electric fields because of many, many source charges. So, if the source charges are all located at some point r j prime and this is the amount of the charge that is located at that particular location r j prime as we have seen in the last class I have to sum overall possible charges. So, let us say there are about m charges up there.

So this fellow is 4 pi epsilon zero r minus r j prime which is the location of the source point to the magnitude 3 and in which direction the force will be? The force will be in the direction that joints sorry the electric field will be in the direction that joints the two charges. Now, there is no charge here but you can simply say that the direction is along r minus r j prime. So, this is r minus r j prime. So, this is the electric field at point r because of many, many source charges.

We have already seen that these source charges when they are large number of source charges one can consider them to be continuous charge distribution. Just like one would consider water to be a continuous fluid even though we know that at molecule level they are not really continuous that way. So, we can consider these charges to be continuous fluids and continuous charge distributions.

And we have already seen line charge distribution, surface charge distribution, and volume charge distribution. So, in those cases what is really happening is that the summation is simply becoming an integration of appropriate dimension. We are looking at a line charge density then I need to use one integral over the line integral. If I am looking at the electric field because of the surface charge density, then it would be integral over the surface.

And finally in the general terms it would be an integral over the volume. We are going to calculate the electric field for couple of charge distributions now these are continuous charge distributions. I would request you to pay particular attention to what we are doing in the next few minutes because these are very important and this is how you are going to start learning how to calculate the electric fields.

So, some of those concepts you might have seen earlier if you have seen them just trust me we are going to take it up a notch higher after we introduce some different charges distributions.