Electromagnetic Theory
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Lecture No -57
Transmission Line

In this module, we will begin discussing a one of the most fertile meeting grounds of field and circuit theory known as transmission lines.

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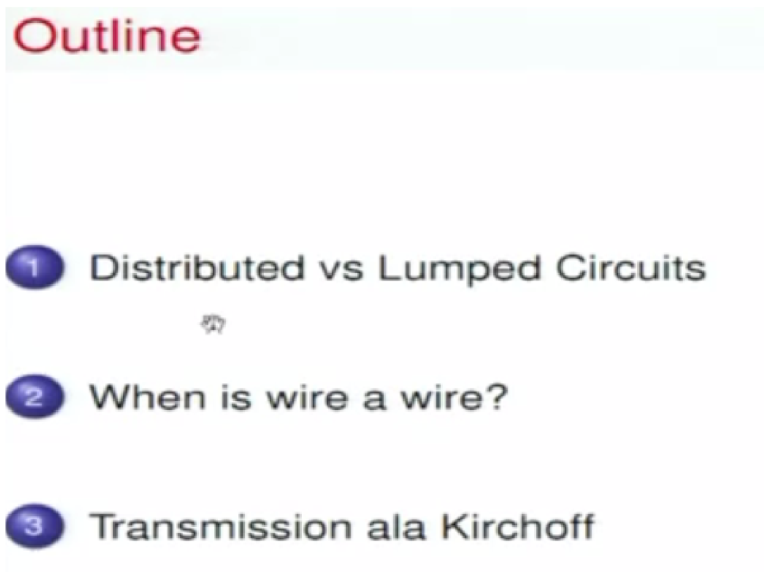

The outline for this module will be that we first discuss distributed versus lumped circuits. Lumped circuits is something that you must have studied earlier in your engineering course whereas distributed circuits is what an example of transmission lines are other distributed circuits are waveguides and antennas. Then we ask a question when is a wire, a wire. So in our earlier studies in electrical engineering we come across wire.

And then we do not bother too much about what a wire is? Is just a piece of metal or a conductor that is used to connect two different circuit elements and that is all that is there to a wire. In circuit diagrams, we do not really about wire, we bend the wire, we twist it, we can put it in any way that we want. But, we will not be talking about those kind of things but we will ask a question as to how long can we ignore the reality of a wire?

A wire is actually made up of a conductor and then this conductor would respond in a certain way to electromagnetic fields we have seen this one earlier. So, we cannot ignore that reality of a conductant, wire being an actual made up of natural conductor. So, we will be talking about the question is to when is wire a wire and when we need to actually model wire itself. And then we will consider some transmission theory along Kirchhoff's law.

And show that Kirchhoff's laws are nothing more than Maxwell's equation under certain conditions.

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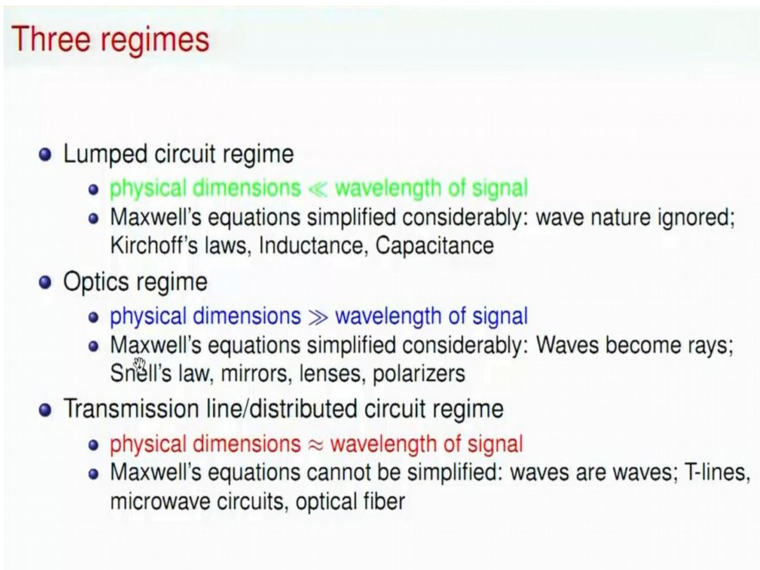

When you discuss any circuit you typically end up looking at 3 different regimes. The most familiar regime for you would be the lumped circuit regime. In lumped circuit regime, you conveniently ignore whatever the physical dimension of the components are. So, you have physical dimensions of a resistor, a capacitor or an inductor or a transistor. You know, diode; you can think of anything and also think of the wires that connect these elements.

So, if you actually have done some experiments in the lab. You would actually put your components on a breadboard and then you take piece of wires and connect them up together. But you rarely give any thought to the dimensions of the wire. Right? So the voltage at any point on a circuit will be dependent on that particular point on the circuit. However, within a given wire itself the voltage will not change from one point to another point.

Of course on a transistor circuit, the voltage at the base is different, emitter is different and the connector is different. But at the base .1 cm away from the base or 1 cm away from the base, if there are no circuit elements connected in that region that voltage will actually be the same. Right? So that is what we mean. We do not really mean that voltages when they are different at different points, we simply do not mean that in that sense.

So, what we mean is that even on a wire, in a lumped circuit regime, the voltage at all points is the same. Now, if you have looked at the modules on electrostatics you would recognize this statement to be nothing more than the fact that in electrostatic condition, a piece of conductor will form a equipotential surface. That is potential will be the same at all points on the surface of a conductor. However, that is no longer true in other regimes as we will see.

To recapitulate in lumped circuit regime you really concerned with dimensions of the circuit because the dimensions of the circuit will be very very very very small compared to wavelength of a signal. For example, if you are running a transistor circuit at 100 kilohertz. Right? So you can quickly calculate what would be the wavelength of this one. So assuming speed of light the wavelength will be 3 into 10 to the power 8 divided 100 into 10 to the power 3.

So, 3 into 10 to the power 8 divided by 1 into 10 to the power 5, will still give you wavelength to be much more than 300 meters or in fact it would be around 3000 meters, 3 kilometers. So, compared to the 3 kilometer wavelength the size of a transistor is so small that you can completely neglect its dimensions. This is what the lumped circuit regime is. In lumped circuit regime Maxwell’s equation become very simple.

Wave nature is completely ignored, voltages are voltages, currents are currents. And the corresponding fields are simply fields. They are not waves, they are simply static fields or quasistatic fields and it is the region where we define inductance, capacitance and apply Kirchhoff's laws in order to a solve circuit problem. The other extreme regime that you might be familiar with is, what is called as optics regime.

So, when you actually discussed optics in your high school and 10th and 1st and 2nd year intermediate classes you would rarely be concerned about the wavelength. You would imagine light to be in the form of rays coming in and hitting a particular surface getting reflected or getting refracted. Because in this regime, in the optics regime the physical dimensions are much larger than the wavelength of a signal.

If you are played anytime, you know with these convex lenses and then try to light up a piece of paper or something by focusing light rays from Sun onto the piece of paper, you would have noticed that light rays are coming in with a wavelength of about 600 nanometers. Around that range or .6 micrometer, whereas your lens that you would be using will have radius of about 1 or 2 inches.

So with this large dimension in 1 or 2 inches is so large compared to this micrometer wavelength of the light that is coming in, that you can actually simplify Maxwell's equation nicely. You can think of waves as being rays. We know that light is actually in the form of wave but these waves become rays. Electromagnetic waves can be replaced by rays and it is where you have these mirrors, lenses, polarizers all these optical elements being work.

You know being used and then this is the region where Snell's law actually holds. In between these two regimes, in the lumped circuit versus optics region in between is where what we call as a distributed circuit or transmission line regime. In the transmission line regime what happens is that the physical dimensions are neither too small compared to wavelength or too large compared to wavelength. They are in fact very close to being the size of wavelength of the signal.

So if the frequency is 1 gigahertz then the wavelength is approximately 0.3 meters. So 00.3 meters if you consider piece of wire to be, this point 3 meters is nothing but 30 centimeters. So, 30 or 40 centimeter wire or even you a 15 centimeter wire will have its dimensions very close to wavelength of the signal. Now what does it mean to us? It means that Maxwell’s equations cannot be simplified.

You have to treat this piece of wire as transmission line which carries waves. Waves cannot become rays and waves nature cannot be ignored waves (()) (07:39) this is where we actually have transmission lines, optical fiber, microwave circuits all this into this one. It might be little surprising for you to find optical fibers in transmission line regime. But this is true, because for a single mode optical fiber the diameter of that one or the radius of that single mode fiber is around 5 microns. Whereas the light wavelength is around 1.5 microns.

So, this difference is not very large. So, one cannot ignore this transmission line regime. So, Maxwell’s equations cannot be ignored and in fact they are used to predict the modes inside an optical fiber. And that is precisely the reason why we need to include optical fiber in the transmission line regime. Although it's not usually thought of as a transmission line.

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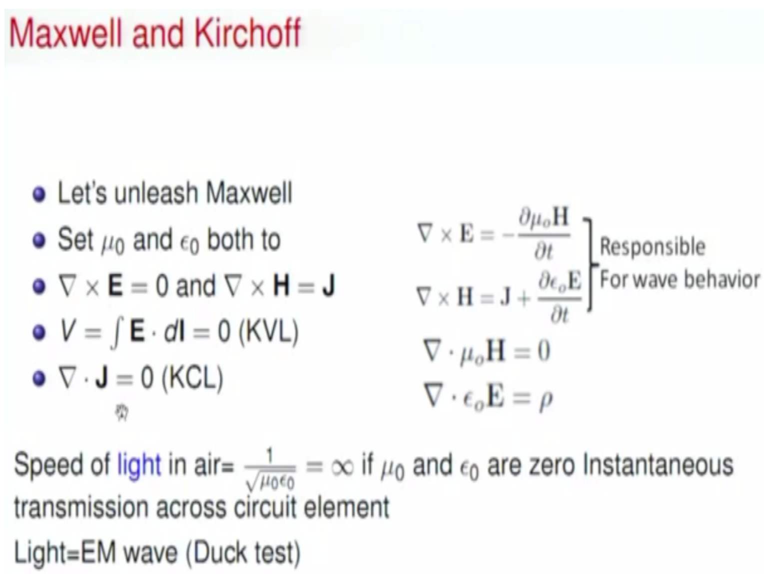

The next thing which I want to discuss very quickly, because you already know all these relations, is to actually show you the relationship between Maxwell and Kirchhoff. Right. So one normally associates Maxwell’s equation with electromagnetic fields and Kirchhoff's laws with circuit variables of voltage and current. But we already know that circuit variables are in fact defined in terms of the field variables.

So if you, for example consider Maxwell’s equation of Curl of electric field and Curl of H and then set the constants Mu 0 and Epsilon 0 to 0. So if you set both of them to 0, what would happen is this right hand side would go away; this right hand side of Del Epsilon E / Del t will also go away. And if you remove these two, what you have is electric and magnetic field being decoupled with each other.

These two elements or these two terms Del Mu 0 H, which is actually Del B / Del t and Del (()) (09:21) / Del t were the terms, which were responsible for wave behavior and when you set Mu 0 and Epsilon 0 to 0, then these two terms will go away and then your equations will get very simplified. So, you now have Curl of E equal to 0, which means the electric field is almost static or behaves like static and Curl of H is equal to J.

Only the conduction current which again means that H will be time invariant magnetic field. So, this is very close to static fields and this equation of Curl of E equal to 0 implies that I can actually write down V, you know, which is the potential difference as the line integral of electric field. Curl of E is equal to 0 allows me to define a potential and for that potential if you take a potential around the closed path that will be equal to 0.

Now, potential around a closed path equal to 0 is nothing but Kirchhoff's Voltage law. So, this is KVL in this fashion. KVL is intimately connected to electrostatic fields. Similarly, if you go to Del dot J the current continuity equation and we have already seen that the right hand side must be equal to 0, this simply indicates that there is no point in any circuit where current can be or charges can be accumulated. So, charges have to be continuously moving.

This is another statement of the fact that the current leaving at a particular node, all currents leaving at a particular node must be equal to 0. The algebraic sum of currents leaving a node equal to 0. So, this is what KCL means. And what we have shown here is that when you set Mu 0 and Epsilon 0 to 0, then Maxwell equations become equal to Kirchhoff's laws. There is another artifact of setting Mu 0 and Epsilon 0 to 0.

What it means is that the speed of light in air, which is 1 / square root of Mu 0 into Epsilon 0 that becomes infinite. Which means that the moment you connect a voltage source, you know in the form of a generator to a circuit, then the voltage immediately spreads to the load side or to the point where you are connecting. All the circuit quantities will actually be immediately energized. There is no delay in any of that and this is precisely what we mean by saying ignore the wave nature.

And light is also an Electromagnetic wave because it kind of behaves very similar to the electromagnetic field, which is something that we have seen earlier as well.

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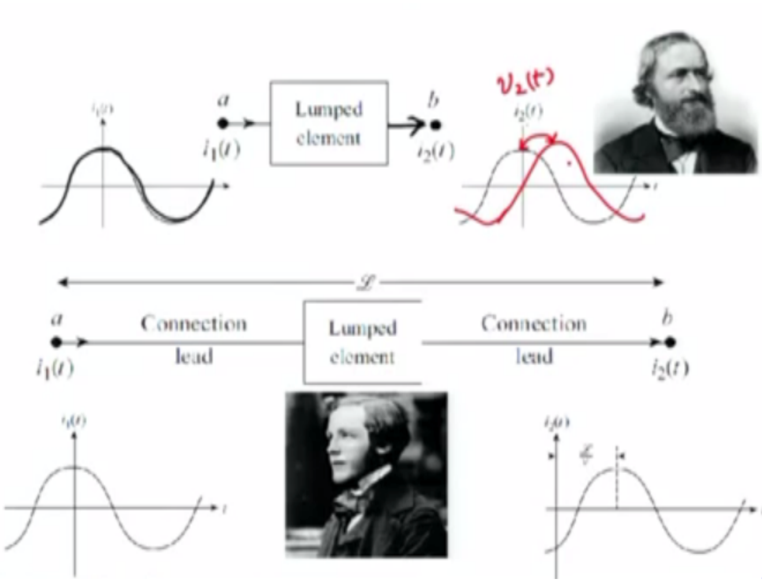

To bring out the difference between a lumped element and a circuit element, let us look at this picture. Here you have a lumped element. So a lumped element means that, it could be a resistor, it could be a capacitor, it could be something else. So in this case you might assume that this is a resistor and these wires from node ‘a’ to the connection to the lumped element is the connecting lead here and this is the second connecting lead.

And these connecting leads that you have considered will actually have no effect because you see the current i 1 of t at this point, immediately as you put the current i 1 of t. So you can see that the leads which we have drawn for the lumped circuit actually has no effect, because as you can see the moment you turn on the current i 1 of t. So, you can see that the current i 1 of t that is turned on will immediately be available at the output of this lumped element.

Of course you can actually have a situation where the current would be out of phase. It might actually come out like this. However, this out of phase thing is not really important because that is only telling you the phase. But if for example this was actually not current but rather voltage then you could actually have some amount of phase change but that does not mean that this is delaying actually coming of the introduced by the lumped element.

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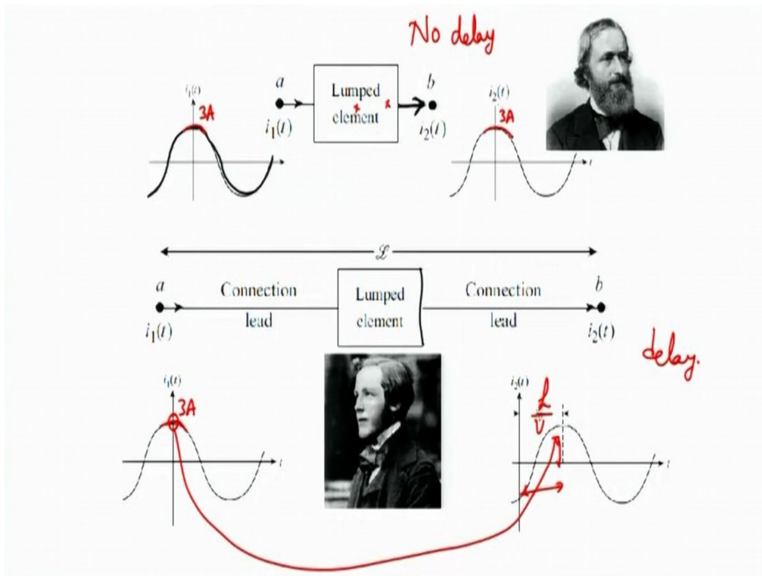

What is really happening is that if you look carefully. So you see that as the current i 1 of t which must be equal to the current i 2 of t. So, as you can see that the moment the current actually starts to become, you know, as it goes from some value over here, you can see that i 2 of t also goes through the same values. So if for example, this is some 3 ampere this would also be the value of the current at 3 ampere.

And at this point, we have nowhere mentioned that whether this current i 2 of t is taken at this point on the lumped circuit element or on this point on the lumped circuit element. So, what we mean is that the moment you connect the current source i 1 of t to this lumped element, the current i 2 of t will follow the same current i 1 of t without any delay. There is actually no delay in current i 2 of t following the current i 1 of t. Now, that is quite important out here.

This is what is the characteristic of a lumped element is. In contrast, you consider this case where we actually have a lumped element, the same lumped element that we used earlier. But now, let us actually put down a connecting lead. The connection lead we will put down and then we will make this wire to be much longer than what we have considered over here. In this case what happens is that if you actually get hold of a nice oscilloscope.

And you can actually connect this lead, this point ‘a’ to oscilloscope as well as the point ‘b’ to the oscilloscope and trigger them with the same source, you will see that as the current changes over here, so say the same 3 ampere current. This current change will not immediately appear here. There will actually be some amount of delay in this change of the current appearing at this point, so or rather at this point.

You can see that there is some amount of delay in current i 2 to appear when you have current i 1 connected out there. This delay is given by approximately l / v where l is the length of the connection leads. We assume that the lumped element itself is of zero length. Whereas the connection leads together will have a length l then the delay that you would see the current at the output of this current.

I mean output of this lumped element which is connected through this long connecting leads is given by l / V. So there is this amount of delay which you cannot explain away by resorting to lumped circuit theory. Because lumped circuit elements do not consider this kind of delay in picture. Alright, so this was the difference between lumped circuit element and a connection lead.

So, what is critical over here is that whenever you have a lumped connection lead over here, which is longer than what is usually, then there will always be some amount of delay in the output. So, if you consider this i 2 of t as the output and i 1 of t as the input, there is some amount of delay between i2 and i1. So, this delay between output and input just because you have connected a wire is the characteristic, which makes a wire into a wire.

So, you cannot ignore a delay nature, which is actually because of the wave nature that we will soon see of the wire itself. So, the wire is the one which is contributing to this amount of delay.

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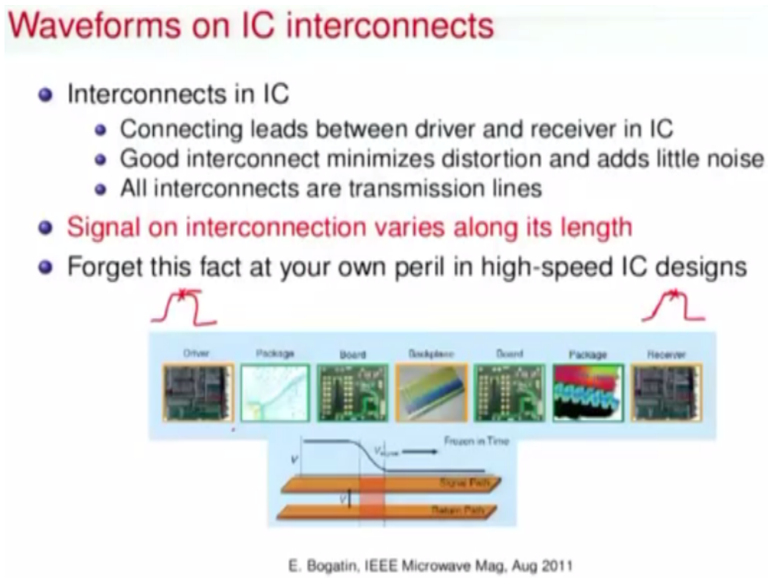

Now, why are we interested in characterizing this delay or why are we interested in characterizing these waveforms. Because, or an integrated circuit for example, you have what are called as interconnects. An interconnect is something that simply connects a driver and a receiver. For example, this would be a driver of a given package. There is some package board, backplane board, passage and then finally there would be some amount of receiver, I mean there is some receiver out there.

And then there is a waveform, which is switching over at the driver side and that has to be a corresponding switch at the receiver. It could be either plus positive going pulse or negative going pulse depending on what the driver and receiver are actually doing. What is the logic in between? But the point is that, there if this entire thing work to be connected with interconnects of zero length, then there would have been no delay in these two.

However, because these are not connected as zero connection length, there will be some amount of delay between when the driver switches and when the receiver, you know switches back or switches changes its value. And that is something that you would have seen. You know as you increase the frequency you would start seeing this kind of behavior where the receiver is not changing at the same time as the driver.

And this change implies that if you do not consider this explicitly then you might have designed a logic gate in which the driver is running at a certain speed and the receiver is not changing as in synchronized position with the driver and your entire logic circuit might really fail. You should not forget the fact that waveforms are (()) (18:28) interconnect will actually go as a wave rather than as a lumped circuit variable.

So, we just saw that waveforms of an IC interconnect is actually in the form of a wave, it actually propagates in the form of a wave and therefore you need to consider all of this interconnects as transmission lines.

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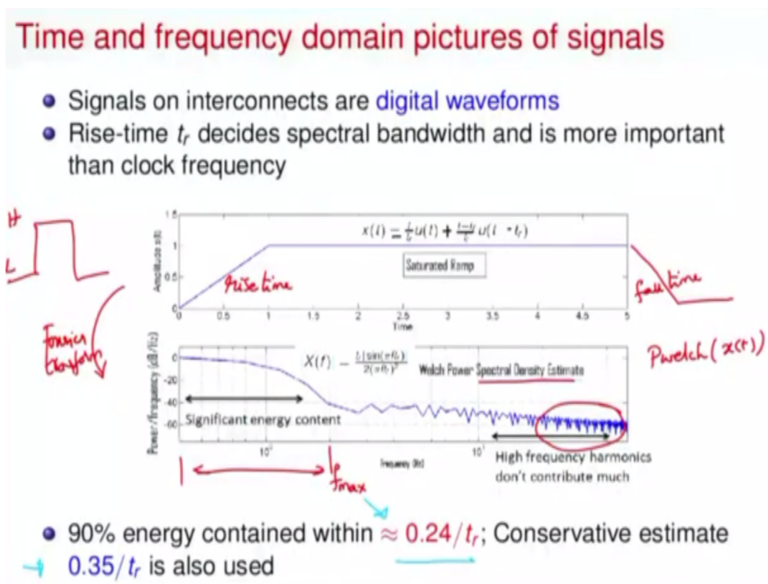

You cannot ignore this transmission line behavior but maybe there is another way of actually looking at how these signals propagate on a transmission line and then see whether we need to actually invoke transmission line or not. To see that one let us look at how the time and frequency domain pictures of signals look like. We have set that signals on an interconnect are digital waveforms.

So they are waveforms which were going from logic 0 to logic 1. Although this is what you would imagine on a typical interconnect, you know, or a typical logic circuit, you have a logic low and logic high. You would imagine that this change from logic low to logic high happens with 0 time delay. But in practice that cannot be true. In practice what happens is that there is actually a small amount of time over which the logic level changes from 0 to high.

This time over which this change happen is called as the rise time of the signal. So this is called the rise time of the signal. Similarly, when there is a change from logic high to logic low there will be a fall time in the signal. So, typically this rise time and fall times are not even equal with rise time being slightly higher, rise time being shorter and fall time being typically longer. But, for our purposes we do not have to worry about this fall time or anything.

So we will consider a simplified case, where we are considering what is called as a Saturated Ramp? What do you mean by a Saturated Ramp? The voltage level changes from lower value say 0 volts or it could be a current, it doesn't really matter. So, this voltage or current level changes from 0 all the way up to maximum value and then get saturated out there. So, this is the time domain picture of a switching waveform.

We will assume that, the on period of the pulse is much much longer then the rise-time or the fall time. Therefore, you can see that as far as the driver is concerned, this switching waveform is almost looking like a saturated ramp. So you can see here that for a Saturated Ramp, where we have some amount of Rise-time, the expression for this time domain signal can be written as x of t equals some t / t r U (t) + t - t r / t r into U (t - t r) .

So you can actually think of this as two-unit step functions with appropriate amplitudes that has been used here so as to scale up correctly. So here you can see that this consists of one-unit step which starts at t equal to 0. And then you have another unit step, which starts at t equal to t r together giving you this Saturated Ramp waveform. This Saturated Ramp waveform, which we have drawn here in this upper panel is actually a function of time.

We might actually get slightly interesting behavior or interesting observations when we go from time to frequency domain. Now, how do you go from time to frequency domain? You go from time to frequency domain by taking the Fourier transform. So, if you take the Fourier transform you will be able to go from time to frequency domain and when you do that one rather than looking at the Fourier transform we will be looking at what is called as the power spectral density.

Power spectral Density will tell you, what is the power that is spread over all the different frequency contents. So, if this was obtained from MATLABs Welch Power Spectral Density estimate function, so we actually use the function call Pwelch in order to get this time domain waveform into, use the time domain waveform, to give us the power spectral density estimate. So, if you look at power spectral density or energy spectral density.

And then you see that the power spectral density is significant. So, this axis is actually measured in terms of d B per hertz. This is power per frequency. This is measured in d B per Hertz. Therefore, and if you take some 20 or 40 d B to be significant energy content, then you can see that until this particular frequency, for this given case there is significant energy content. There after the corresponding amplitude, you know are the power spectral density actually goes down and become smaller and smaller.

There are certain high frequency components as you can see here. But these high frequency harmonics do not really contribute much because their energy levels are down by factor of 60 d B compared to the amplitude over here. So you see here that one can describe time domain waveform in terms of the frequency domain waveform. And if you ask what is the typical, you know frequency over which 90% of energy is contained.

And you can see that the typical energies is around 0.24 / t r, where t r is the Rise-time of this waveform. So about 0.24 / t r is the frequency over which 90% of the energy is contained. If you are not happy with 90%, you can go to 99% and then this number will actually slightly change. And conservatively we typically estimate around 0.35 / t r to be the total to be the maximum frequency within which are the bandwidth within which the entire energy of the waveforms are conserved.

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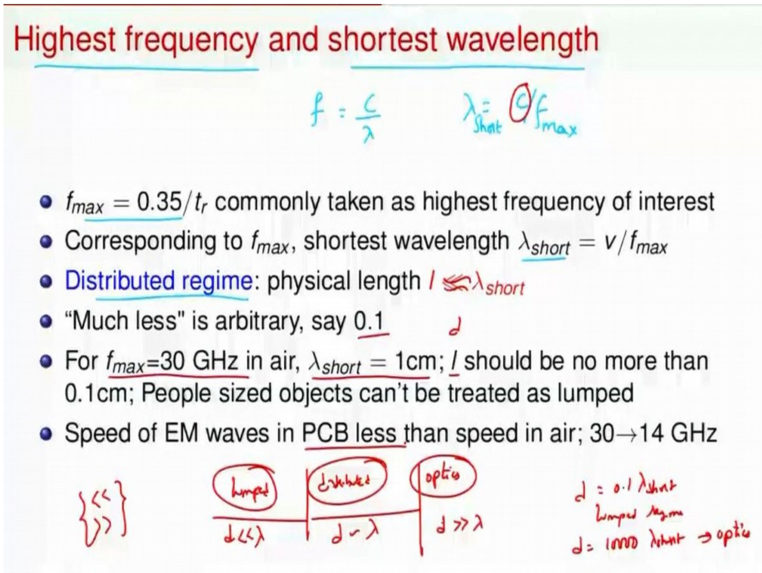

So, what we have seen is that if you go to time and frequency domain pictures, we will end up having a highest frequency and correspondingly there will be a shortest wavelength. What is the relation between frequency and wavelength f is equal to C / Lambda. So, you have Lambda equals C / f, will give you for a corresponding maximum frequency, there will be a shortest wavelength Lambda.

And if you consider this as to be the maximum frequency, then you can correspondingly find out what is the shortest wavelength and this shortest wavelength is given by V / f max. So, I used here C / f max assuming that we are considering with free space propagation. But in case of a PCB, the velocity of waves will be much less than velocity of light. So, velocity of waves will be much less than the value of C. So, you need to use the appropriate value for the velocity.

So, you have the shortest wavelength to be given by V / f max and we say that we are in the distributed regime, whenever we are physical length is actually close to this shortest wavelength. This is actually close to shortest wavelength. So, this is close to shortest wavelength. So, if your physical length is much much smaller than shortest wavelength then you will be in the lumped regime.

If on the other hand, the physical wavelength is much much larger than shortest wavelength, then that it would be in the optics regime. When 'l' is close to short wavelength then you are in the distributed regime. This distributed regime is kind of arbitrary. You know we actually do not have, we cannot really say that this is lumped and at this critical frequency you are in the distributed regime and thereafter at this point you are in the optics regime.

These differences are mostly qualitative. They are giving you the general idea that your physical dimensions must be much smaller than Lambda. Whereas in the distributed circuit your d to be around Lambda and in the Optics regime d must be much larger than Lambda. But these much smaller or much larger are subjective terms and they are quite arbitrary.

So if you for example consider it to be 0.1 that is to say if the physical dimension is about 0.1 times the shortest wavelength, then we will call this in the lumped regime. Otherwise we will call this in the distributed regime. Of course, if d is equal to some 1000 or 10000 times Lambda short, then we might be in the optics regime. So, anywhere between these two values, we will be there in the distributed regime.

So, if you consider this as 0.1, much less to as 0.1 and assume that maximum frequency that you are working with is 30 gigahertz and speed is air, you know like you are looking at a wire which is carrying 30 gigahertz signals, then the shortest wavelength of interest will be 1 centimeter. This means that if we want to treat our circuit as a lumped circuit, then the length of the circuit or you know, dimension d of the circuit should be no more than 0.1 centimeter.

If it is more than 0.1 cm, then you cannot treat them as lumped circuits or to put it in more colloquial way. People sized objects cannot be treated as lumped. Again people sized objects mean at this particular wavelength, people sized objects cannot be treated as lumped regime. Of course, you might want to treat them as optics regime but for that people have to be much much taller and you know the dimension must be much larger.

So, again I would just would like to emphasize that in air the speed of light is C or speed of electromagnetic wave is C, whereas in PCB that would be much less.