Electromagnetic Theory
Prof. Pradeep Kumar K
Department of Electrical Engineering
Indian Institute of Technology – Kanpur

Lecture 53
Waves in Imperfect Dielectrics & Good Conductors

In this module we will discuss wave propagation inside that of a imperfect dielectric and wave propagation inside a good conductor. Wave propagation in good conductor will need us to discuss about skin effect, something that is very important at high frequency wave propagation.

Waves in imperfect dielectrics is something that we would actually see when you drive a capacitor with high frequency signal but then the insulator that is filling the material between the capacitor plates is not perfect. So if the material filling the capacitor plates is not perfect then there is a possibility that there will be some leakage current from one plate of the capacitor to the other plate of the capacitor leading to loses in the form of dissipation of heat inside that of that imperfect dielectric.

So we will like to see what happens to t his loss or how we are going to calculate this loss which is actually material constant dependent as well as what is meant by skin depth or what is meant by skin effect when we consider wave propagation inside a good conductor. So we begin as usual that we began for the case of a lossless or a perfect dielectric wave propagation.

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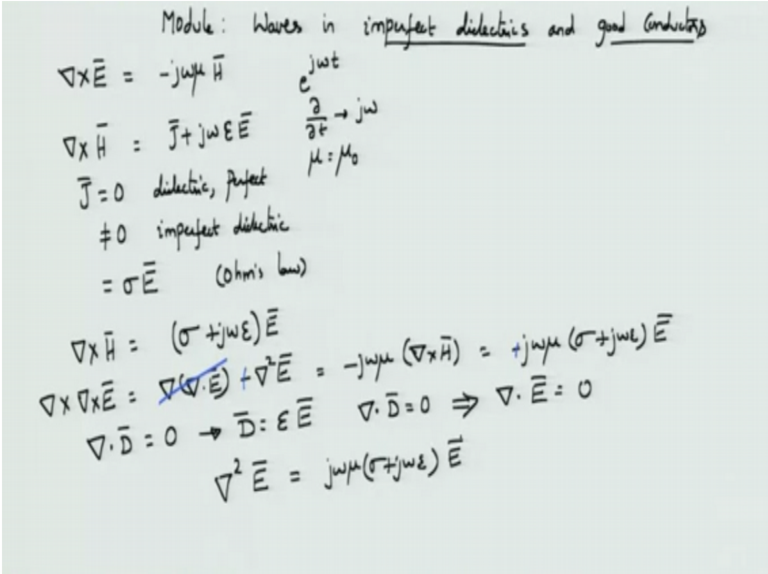

So we begin with Faraday's law and Ampere Maxwell Law. So Faraday's law is straight forward. Curl of electric field is given by minus j omega mu H. Clearly the way I have been writing this equation implies that I am assuming that the time dependence of all the field quantities in this wave propagation with all be in the form of e to the power j omega t. So because of this I can actually suppress the time notation and release that del by del t can be replaced by j omega.

So with this assumed time dependence then Faraday's law becomes curl of electric field being equal to minus j omega mu multiplied by H, okay? So although I am writing mu here in most cases we will be interested only with mu is equal to mu that is we will be interested in the cases where the material medium is explicitly not magnetic, okay? So we will not look at wave propagation inside a magnetic material that is something that is beyond the scope of this particular course.

So although we right knew what you should, realize that mu is equal to mu zero. So with this, this becomes Faraday's law and then what happens to Ampere Maxwell law? Ampere Maxwell law is curl of H is equal to J plus J omega epsilon E. Now in the case of a lossless wave propagation that is propagation inside a lossless material on a perfect dielectric we assume j is equal to zero.

So j is equal to zero for a perfect dielectric because there was no possibility of having any conduction charges free so that they would constitute to a conduction current density J. However, inside that of an imperfect dielectric we cannot assume J is equal to zero because if you assume that then there is nothing different from the lossless case and we will not be able to capture the fact that an imperfect dielectric can actually conduct some amount of electricity.

No doubt that electricity will be small but it can conduct some amount of electricity in the form of a leakage current. So in order to take that imperfect dielectric into account we need J be nonzero. So J is nonzero for an imperfect dielectric. This situation of J not equal to zero will also describe equivalently the good conducting medium such as a metal. So in a metal you will find lot of conduction electrons.

Therefore, J vector the conduction current density will be very high or will be at least quite appreciable comparable to the displacement current in a good conductor. On the other hand, if the medium is imperfect dielectric mainly then conduction current will be there but it will be very small in magnitude, okay? In any other medium the relative magnitudes of conduction current and displacement current determines whether that medium can be considered as an imperfect dielectric or as a good conductor.

So in any case we will assume that J is nonzero in order to capture both the imperfect dielectric as far as good conductor effects. So we have J nonzero, but we also invoke the fact that J is linearly proportional to electric field in the form of Ohm’s law for the field expressions that we described. Earlier J was equal to sigma times E except in this case that sigma cannot be infinity. Sigma is a finite value, okay?

So if you now substitute for J in the form of sigma E, Ampere Maxwell law becomes curl of H is equal to sigma and electric field E is common in both terms. So I can actually put that outside. So I have sigma plus j omega epsilon multiplied by E. Now as before we take curl of Faraday’s law to obtain del of del dot E minus del square E equals minus j omega mu curl of H but I already know what is curl of H, curl of H is nothing.

But minus j omega mu multiplied by sigma plus j omega epsilon multiplied by electric field E. Now we have Gauss’s law which states that del dot d is equal to zero. We will continue to assume that D and E are related by a simple epsilon, okay? So that is how we are actually able to rewrite del by del t of d in the form of j omega epsilon E. So we assume that d is equal to epsilon E and moreover we still assume that del dot D is equal to zero.

So there might be cases of conduction current in an imperfect dielectric but we will assume that at no point inside the imperfect dielectric surface there are some isolated free charges. So because of that del dot d equal to zero still implies del dot E equal to zero and as such this term del of del dot E can be cancelled, can be made equal to zero. There is a minus sign to the right hand side, minus sign here on the left hand side.

They can be cancelled each other, so that what we get is an equation for describing how the electric field E will change inside an imperfect dielectric being given as j omega mu multiplied by sigma plus j omega epsilon times electric field E.

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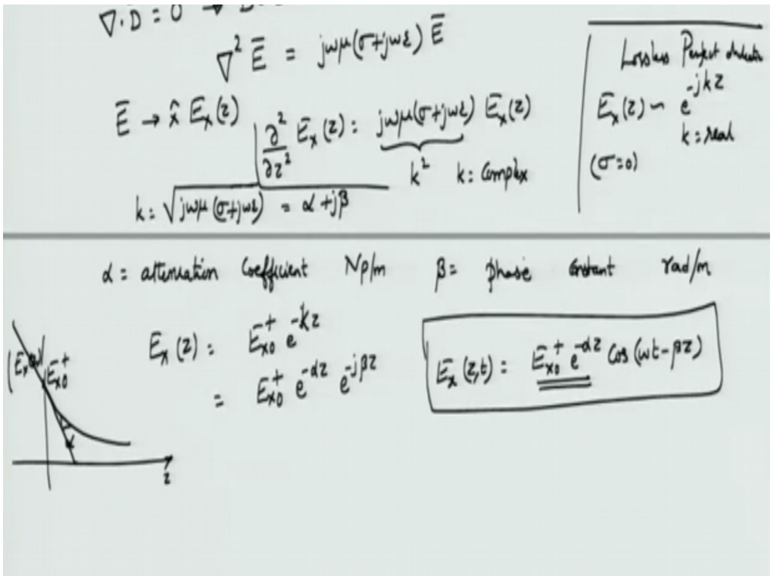

As before we will assume that electric field has only one particular component. So let us assume that electric field E is ex component electric field and it is a function only of z variable, right? So because the way it is propagating along z direction. So in our assumption this del square vector laplacian can be replaced by del square by del z square times only Ex of z which is equal to j omega mu multiplied by sigma plus j omega epsilon times x.

Remember in the case where sigma was equal to zero, this equation was actually minus omega square mu epsilon and we recognize that mu epsilon is nothing but 1 by mu square and therefore we wrote this as minus omega square by v square multiplied by E. And we call this omega square by v square as k square. So therefore we actually had a very simple solution for Ex which was in the form of E power minus j kz and k was a real quantity.

This was for the lossless case or a perfect dielectric case. So this was the case of a lossless perfect dielectric wherein the way it was actually going as a nice cosine wave whose amplitude was not really changing with z in the sense that the amplitude of the electric field would actually remain constant. It would not change with, as you go further. There is no change in the amplitude of the electric field.

It was only propagating as a sinusoidal wave along z direction as you can see in this particular case. This was for the lossless case when sigma was equal to zero. However, now sigma is certainly not zero and this quantity can still be written as sum k square except now that k must become complex, correct? Because k square is equal to j omega mu multiplied y sigma plus g omega epsilon.

However, k will then be equal to square root of j omega mu sigma plus j omega epsilon, okay? And because of this, this is obviously going to be a complex number. This complex number is usually split and written as alpha plus j beta where we call alpha as attenuation constant or attenuation co-efficient. And this is usually measured in nepers per meter. We call this beta as phase constant and this is usually measured in radians per meter.

So what is happening out there? We have an equation which is looking very similar to, at the left hand side is exactly equal to that of a lossless wave propagation but the right hand side I have k square which is actually a complex number, right? So k square is a complex number and k will be equal to alpha plus j beta. The solution of this equation would be that you have two wave, sum Ex plus e to the power minus kz because k is complex I cannot just write this as E power minus j kz, I have to write this as minus kz.

Plus, there would be some backward propagating wave. This backward propagating wave could be generated by a source that is kept or it could be coming off from some reflected wave, okay? So if you do not want to go any concentrate on the backward propagating wave at this point, do not have to consider this one. We will simply assume that the wave is propagating along plus z direction and it is simply going out in that direction.

Now if you look at this expression that we have written here. You will see that this is e power minus kz, but substituting for k in the form of alpha and j beta, you will see that this would be, so let us put a zero substitute to indicate that this is amplitude measured at a particular plane. So you have Ex0 plus E to the power minus alpha z, e power minus j beta z. And if you want to find out what is Ex as a function of both z as well as time, you can then convert this phasor into a real time variable form.

So you will get Ex0 plus E power minus alpha z. I am assuming that this amplitude is still real times cos omega t minus beta z, right? So this equation for the electric field component of this plane wave propagating inside an imperfect dielectric is very similar to that of a perfect dielectric propagation except that the amplitude is actually decaying as a function of z. So you will see that this exponential minus alpha z is there.

This would be something like a current decaying in that of a RC circuit. There we will also have a similar alpha there or rather in terms of tau, the time constant there. So here we have an alpha. So when z becomes much larger than as z increases, the corresponding value of e power minus alpha z actually starts decreasing and therefore the amplitude starts decaying.

So if you actually plot the amplitude alone or the magnitude of the electric field alone, you will find that this would be the magnitude of the electric field you are plotting and you would find that at z equal to zero, it would have the value of Ex0 plus and beyond that it would have a value that is coming through this. But as z increases, as you move away from the source, you will see that the amplitude actually starts decaying with a slope of alpha.

So the initial slope of this line would be alpha and this alpha tells you the rate of decay of the amplitude of the electric field. So this rate of decay of amplitude is measured in neper per meter. Sometimes this alpha or most of the time this alpha attenuation co-efficient is actually given in terms of dB per meter or dB per kilometer and there is a nice conversion between neper per meter to dB per kilometer, something that I do not have enough time to discuss that.

But you can actually take a look at the textbook to see how to convert from natural units of attenuation co-efficient in the form of neper per meter into the more practical units of dB per meter and you can use this one and you can see how the amplitude is actually changing. The point is the amplitude changes or amplitude decays exponentially as you move away from the source.

Okay, so the lesson that we need to learn is that, in an imperfect dielectric, j is not zero, okay? In fact, j will be equal to sigma e and this would not be equal to zero. The second lesson to learn here is that the waves actually attenuate, right, as they begin to move along z. If they are very far away from the source, the wave would still be oscillating with a propagation constant of beta or with a phase constant of beta.

So this beta which tells you how the phase is changing as the wave propagates that would still be there. Except that the amplitude actually starts decaying exponentially inside that of a imperfect dielectric.

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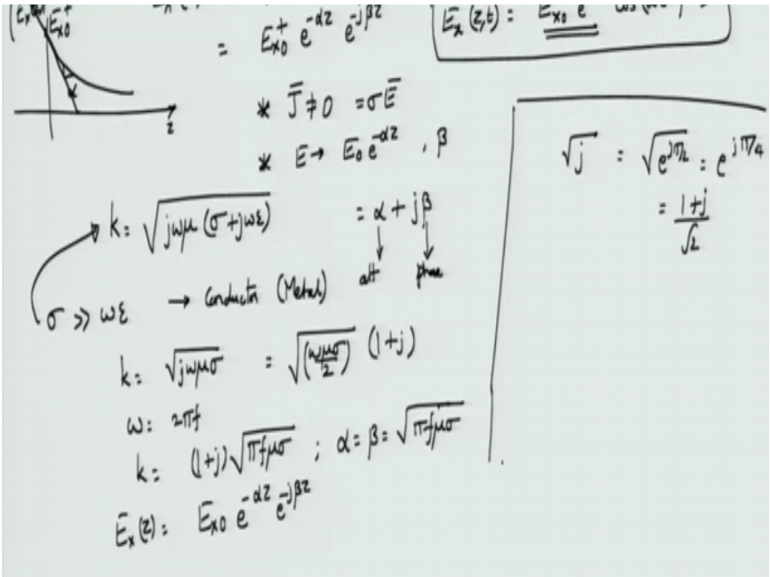

So far what we are looked at is propagation of wave inside an imperfect dielectric and we have seen that this k which is the propagation co-efficient or the propagation constant is actually complex consisting of real part and imaginary part. The real part being the attenuation and this imaginary part giving you the phase change or the phase shift of the wave as it propagates through the material medium.

There are couple of cases that we can consider at this point, which will tell us the relative magnitude of sigma and omega epsilon. First consider the case where sigma is much much larger than omega epsilon. This would be the case of a good conductor that of a metal for example. You take copper or gold, so these metals will have a large conductor current compared to a small displacement current.

Therefore, one can make this approximation of sigma being much larger than omega epsilon and when you put that back into this expression for k, you will see that k is equal to square root of j omega mu sigma. This can be rewritten as by taking the square root of j, can be rewritten as omega mu sigma by 2. This square root actually covers everything and then there is 1 plus j. If you are not convinced with this one you can actually a simple couple of seconds to convince yourself.

What you want is root of j, right? So root of j is nothing but square root of e power j pi by 2, but this is nothing but e power j pi by 4 which is 1 plus j by root 2 because e power j pi by 4 is cos pi by 4 plus j sin pi by 4 and cos and sin pi by 4 are equal to 1 by root 2. So this root 2 goes into this omega mu sigma. But I also know that omega is equal to 2 pi f. Therefore, I can write this k as 1 plus j multiplied by pi f mu sigma under root.

So clearly in this case what we have is alpha being equal to beta both given by pi f mu sigma, okay? So what will happen to the electric field that is propagating. Well, the electric field will be Ex of z given with some initial amplitude Ex0, I am assuming still only forward propagating wave and then you have e power minus alpha z, e power minus j beta z, so both of these are equal to square root of pi epsilon sigma.

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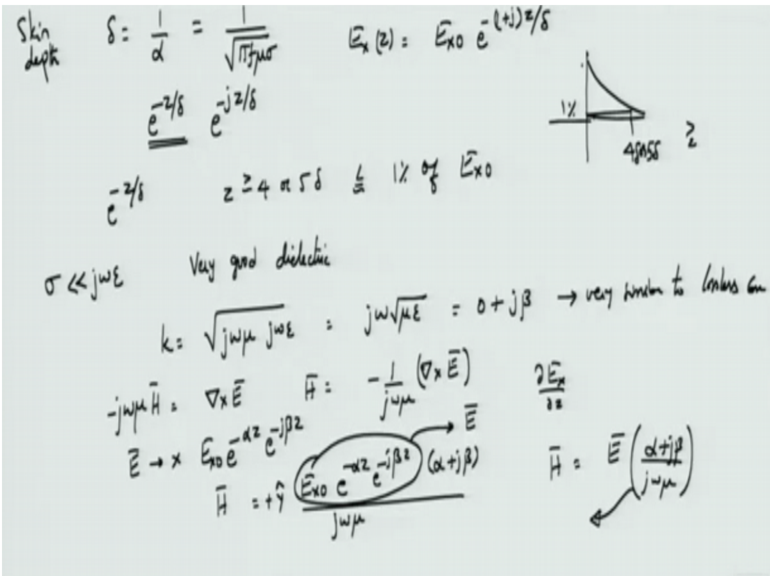

But rather than talking in terms of alpha for a case where you are considering a good conductor, you will actually introduce a different quantity called skin depth. Skin depth is given by 1 by alpha and since alpha is given by square root pi of mu sigma this skin depth delta, let me write this down this as and say this is skin depth delta is given by 1 by square root of pi f mu sigma.

So because of this the electric field propagation inside a good conductor is equal to whatever the surface value that you have or whatever the initial amplitude Ex0 that you have times e to the power minus 1 plus j z by delta, correct? Its amplitude is going as e power minus z by delta and its phase is changing as e power minus j z by delta, okay?

So now we will not discuss about the phase because phase only changes, a phase is not very important at this point, but what we are interested is to see what would be the behavior of this e power minus z by delta. So you have e power minus z by delta and if z is around say 4 or 4 delta and beyond this one, so if this z becomes greater than 4 or 5 delta that is as the way it propagates, if you are, propagate for more than 4 or 5 delta, then the amplitude would have decayed to less than or equal to 99 percent of Ex0.

So this is how the amplitude would have actually decayed. So this decay if you look at this, this is about 4 delta or 5 delta. So 4 delta or 5 delta along z, the amplitude would have actually decayed by 99 percent. So which means that the value of this point would only be about 1 percent of Ex0, okay? So this would have decayed by 99 percent and therefore this would have been around at 1%, okay?

This would happen for 4 delta or 5 delta. That is why this rule of thumb which says that 5 delta units is sufficient for the wave to have been decayed. So the wave would just decay and become very small compared to what its value at surface is. All right, this is skin depth. We will come back to skin depth and some of its ramifications very shortly. Let us for a minute consider the other case around.

We will come back to skin depth in the next module. So for now let us simply consider the other extreme situation where sigma is much much smaller than j omega epsilon. This would be the case where it is a very very good dielectric. So this is a very good dielectric that I am considering. But there is some amount of loss. So there is some small insignificant conduction current which we can neglect in the form of j omega epsilon, I mean in terms of the displacement current. So what will happen to k?

k will be square root of j omega mu j omega epsilon which will give you j omega square root mu epsilon, right? This would be equal to no attenuation and full phase constant, okay? This would be very similar to, this is exactly similar to that of the lossless case. So in the lossless case or in the very good dielectric, if we can neglect the conduction current then the propagation constant will be imaginary, right?

k will be equal to j beta and the wave will only suffer phase shift as it propagates through the material and there is no change in the amplitude. It is also interesting to look at what would be the impedance eta, right? I already know from Faraday's law that minus j omega mu H will be equal to curl of electric field, therefore H will be equal to minus 1 by j omega mu, curl of electric field.

If I assume that electric field E is along x and it is going as e power minus alpha z e power minus j beta z with some initial amplitude of Ex0, right, if this is how the electric field E is going, then H will have only y component and this will be given by H is equal to y hat and then you have Ex0 e power minus alpha z, e power minus j beta z, but this curl of E, the y component is nothing but del Ex by del z.

So I can actually differentiate this Ex by z and I will get alpha plus j beta which would be minus sign here and then divided by j omega mu, correct? So this is what I get. There is already a minus sign therefore that two minus signs will cancel and they are going to get away from this one. This is nothing but the electric field vector itself. This is nothing but the electric field vector E.

Therefore, this H which is given by E multiplied by alpha plus j beta divided by j omega mu, it can be written in this way and therefore this fellow should be in the form of an impedance. So E by H is impedance.

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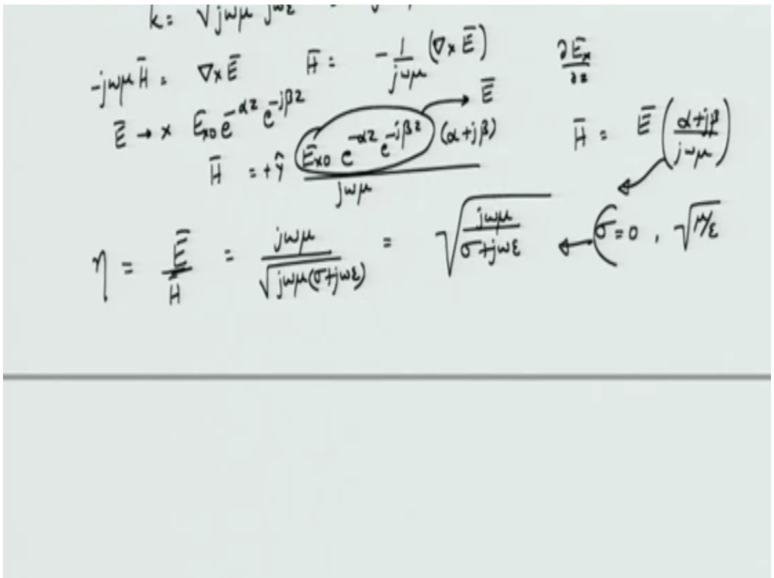

Therefore, the ratio of the electric field to magnetic field in the case of this imperfect dielectric is given by j omega mu divided by alpha plus j beta. Alpha plus j beta is nothing but k and k is nothing but square root of j omega mu multiplied by sigma plus j omega epsilon and this can be further simplified and written as j omega mu in the numerator divided by sigma plus j omega epsilon.

You can convince yourself that this expression must be true because when you take sigma is equal to zero, this ratio of E by H which is basically the ratio of the electric and magnetic field vectors and describes the wave impedance will actually reduce to mu by epsilon and for the case of free space, this will be equal to square root of mu zero by epsilon zero. So this actually equation is correct and that actually checks out with the case that we all ready know how to compute the wave impedance.

But there is actually a significant thing that has happened over here. For the lossless case, eta will always be real indicating that electric field and magnetic field are in phase with each other. They actual carry some amount of energy. Whereas in this case, eta is complex, indicating that E and H are not always in phase with each other, in fact there is a, the phase between the two is not zero indicating that there is some loss in the power.

So the entire power is actually not being able to be carried because eta is not real indicating that E and H will have some amount of phase difference between that and only the power of the order of cause of the phase difference will be carried by the wave. So this loss is inevitable because of the imperfect dielectric or a good conductor that we have assumed.