Electromagnetic Theory
Prof. Pradeep Kumar K
Department of Electrical Engineering
Indian Institute of Technology – Kanpur

Lecture - 52
Wave Reflections (Normal incidence)

In this module we will discuss wave reflection from a perfect conducting medium when the wave is incident normally on to the surface of the perfect conductor. So far we have assumed that there is only one kind of wave propagating which was the positive straight going wave. We said that mathematically there can be situation or a there can be an expression in which the electric field would also propagate or the wave would also propagate along minus x direction, right?

But we did not elaborate how one can produce a minus z propagating wave unless there is a source of a wave which would propagate the waves along minus z direction. One way of getting this negative propagating wave is to actually make the wave reflect of a surface.

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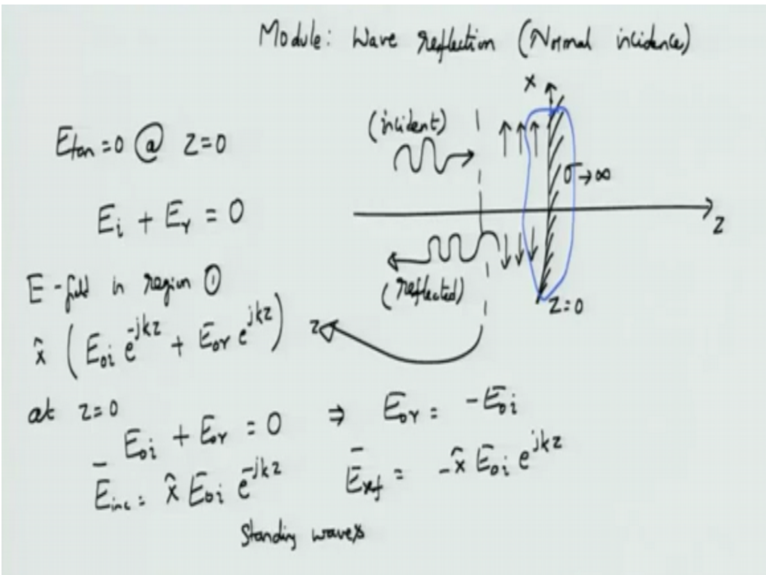

So accordingly consider this surface that I have. Assume that this entire surface of this entire region is a perfect conductor. We will not worry about what happens inside this region and how the effect of imperfect conductivity would come into effect here. We will assume that medium is perfectly conducting.

Therefore, its sigma is going off to infinity. And this is of course the direction of the wave propagation along the z direction and have kept this particular conducting surface or rather thick conducting surface at z is equal to zero. We can also mark off one of the axis as x axis, right? So the conducting surface if you imagine like this.

Then this would be the direction of the wave propagation, right, according to the direction the way it is propagating and this my finger would point along the x axis and this thumb would point along the y axis forming up a plane which is kept at z is equal to zero and to the right of this plane is all a conducting medium and to the left of this plane we can assume it to be a dielectric, a perfect dielectric or a free space.

So a wave would actually propagate starting from very far away case. This is a plane wave which is coming in here and it would hit the conductor and what would one expect to happen? Can you just have a situation where the wave would propagate and there is nothing that is happening to the left hand side? A moment's reflection should tell you that this cannot be the situation because for this situation to occur there have to be electric fields inside a conductor.

We have shown in previous model once you know like when we discussed electrostatics although we discussed the case for electrostatics it is true that you cannot have electric fields whether it is static or dynamic inside that of a conductor. So what really happens is that there will not be any wave inside the conductor but rather the entire wave must fall back or must be reflected of the conducting surface. So the wave would come in.

The electric field vectors are all directed along the x axis. The electric field vectors would come in here and then they would get generated with electric field vectors along this particular direction. That is there would be reflected fields that are set up or reflected wave that is set up. Accordingly, since we have two different kinds of waves we can call this wave as incident wave, the wave which is incident from a far away source and reaching the conductor and this wave as reflected wave.

At this boundary, very close to boundary I have two electric fields sitting here and the sum of these two electric fields because they are tangential to this surface, remember these are tangential to the surface and we know that from the boundary condition the tangential electric field must be equal to zero you will have since the tangential electric field must be equal to zero at the boundary z is equal to zero.

We will have e incident amplitude plus E reflected must be equal to zero. This is the amplitude of the incident and the amplitude of the reflected field. Together they must add up to zero because there is no tangential component of the electric field. Of course at any other region of space that you consider this would actually be the total field in the region one. The total E field in region one will be sum of incident field with an amplitude.

Let us say Eoi e to the power minus j kz plus amplitude of the reflected fields and going as E power j kz, right? Why should this be per j kz because this is the way which is propagating along minus z direction, whereas this incident wave is propagating along plus z direction. So this would be the total electric field at any z in the region. So for example at this point of z this could be the electric field.

At the boundary at z is equal to zero, the sum of the electric field vectors must be equal to zero because this entire electric field is along x axis and the field component which is along x axis on to the surface of the conductor means that this would be tangential and therefore at the boundary at z is equal to zero, that is a substitute for z is equal to zero in this expression. What you will find is that Eoi plus Eor the amplitudes must be equal to zero from which it is very clear that Eor must be equal to minus Eoi.

So clearly the amplitude of the reflected wave is minus amplitude of the incident wave, if the incident wave can be written as the x polarized but z propagating wave and write this as E power minus j kz, the reflected wave can be written as minus x hat Eoi, remember Eoi is equal to Eor or rather minus Eoi is equal to Eor, E to the power j kz. So you can see that the reflective field would be directed along minus x direction, having the same amplitude Eoi.

So you have an equal amplitude wave but another wave which is 180 degree opposite to that one. So clearly in the region before the conductor that is in the region in front of the conductor you will actually have an interference of the wave created, right? So you have one wave which is propagating in this way and there is another way which is propagating in the backward direction.

These two waves are at 180 degree face difference with each other. Therefore, they will interfere and then form what is called as a standing wave pattern. So they form what is called as standing waves or standing wave pattern. So this is what would happen in front of the conductor.

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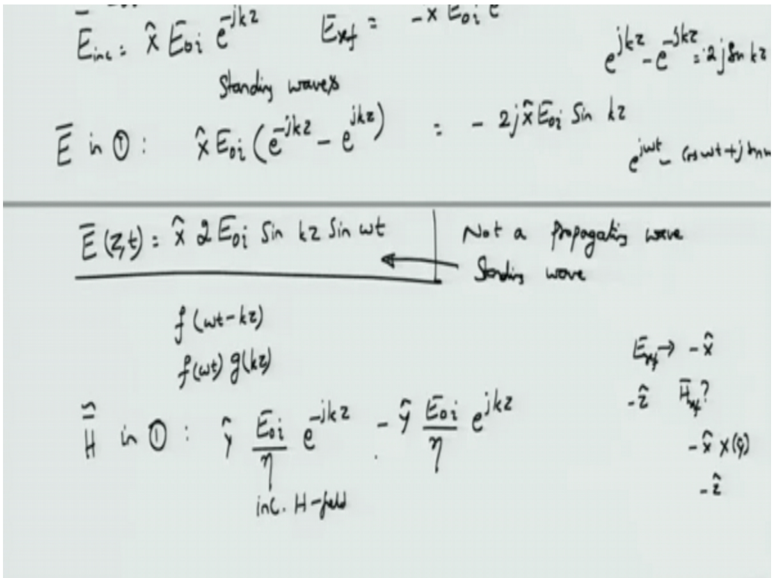

So you can actually see that this standing waves are formed by writing down the expression for the electric field, right? Electric field in region one is given by x hat Eoi is a common factor, I will take this out. E power minus j kz minus E power j kz and utilizing the relationship for sin and exponential of j kz quantities, you can actually see that, E to the power j kz minus E power minus j kz is actually 2 j sin kz and there is a minus sign here.

So therefore this become minus 2 j x hat Eoi, sin kz. You must say where is the standing wave happening. Well, this is just the phasor that we have written down. So this is the phasor component of the electric field, but if you write down the full electric field E, that would be obtained by multiplying this by E to the power J omega t and then taking the real part. So when you multiply this expression E power minus 2 j x hat Eoi sin kz with E power j omega t which is actually cos omega t, plus j sin omega t and then take the real part.

You will see that cos omega t 2 j term will be imaginary. Therefore, that will not be present and j and minus j will be plus and the electric field vector that you are going to obtain will be 2 along x axis of course, this would be 2 Eoi sin kz sin omega t. This is clearly not a propagating wave, right? This is clearly not a propagating wave. This is in fact standing wave, right?

A propagating wave is characterized by some function of omega t minus kz where as the standing wave is characterized by some function of omega t and some function of z. In this particular case this is a standing wave. What is the amplitude of the standing wave? The standing wave amplitude is 2 times Eoi. Does it make sense? Yes, because this is simply addition of two out of phase waves.

So when you look at that one, the maximum amplitude can go up to 2 Eoi and there is no surprise out there and this is the expression for the electric field. What about the expression for the magnetic field? To obtain the expression for the magnetic field let us actually write down the corresponding components of the magnetic field. We know that electric field is along x direction, the magnetic field must be along y direction.

Your magnetic field component, let us first write down the phasor form. That would be easier for us to write before this particular case. So the magnetic field in region one will have two components. One component is the incident component which would be associated by writing this amplitude as Eoi by eta, the direction being y, correct? And then you have E power minus j kz. So this is the incident magnetic field, right?

Now there is also reflected magnetic field. Reflected magnetic field will be directed along minus y hat direction. It will have an amplitude of Eoi by eta, correct? Only thing here is that now if the electric field is along minus x direction for the reflected field, so this reflected field is along minus x direction, propagation should happen along minus z direction, right? So what should be the component of H reflected?

Well, you have minus x hat cross something must be along minus z hat. If H is along y then this would be along minus x cross y which would be along which would be along minus z direction and therefore you can write this in this particular fashion. So you have reflected field as Eoi by eta, E power j kz, but Eoi is actually minus Eor.

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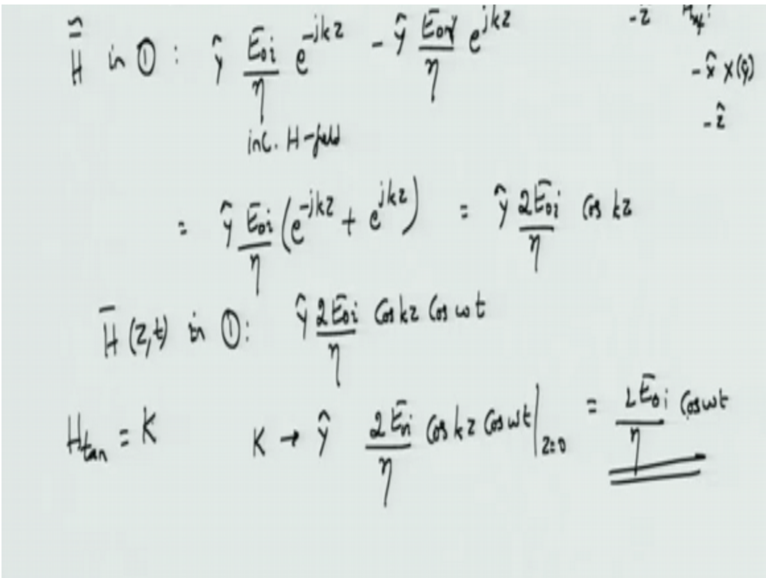

Therefore, you have to write this thing as y hat Eoi, sorry I should have written this as Eor to begin with, you have y hat Eoi by eta, E to the power minus j kz, plus E power j kz because Eor is minus Eoi, right? So this would be the expression for the magnetic field phasor. Again this E power minus j kz plus E power j kz can be written in terms of cos kz, so this becomes two times.

So two times Eoi by eta and cos kz, again this is an example of a standing wave because you can actually write down the complete magnetic field H as a function of both z and t in region one and that will be equal to y hat 2 Eoi divided by eta cos kz cos omega t. So this is the expression for magnetic field and what would be the situation for magnetic field at z is equal to zero, will it be zero or not.

Well, it turns out that it need not be zero because the condition for magnetic field is that the tangential continuity of H must be equal to the corresponding current density or the sheet current density that could be sustained on the conductor surface, right? So you have a conducting surface and there is a magnetic field, the tangential magnetic field. The discontinuity in the magnetic fields will be equal to the sheet current density k.

And this is possible because we have a conductor, there will be current in that conductor that is induced and the fact that H tangential 2 is zero does not mean that H tangential 1 is also zero. In fact, tangential 1 will actually be equal to current density or the sheet current density k. So you actually have the sheet current k along y direction having an actual value equal to or the numerical value equal to 2 Eoi by eta cos kz cos omega t.

Of course this must be evaluated at z equal to zero, so this would be 2 Eoi divided by eta cos omega t. So there is actually a sinusoidal current sheet right on the conducting surface which would be nonzero. In fact, this current sheet would generate a magnetic field of its own and then it would induce what is called as a radiation pressure on the conducting surface. We do not have to go into those details at this point.

But just remember that boundary conditions indicate that tangential H continuity must be maintained by the sheet current density k and in fact this is precisely what you are going to get. The magnetic field discontinuity will manifest in the form of sheet current density k, okay? So you can actually have sketch at this particular point how would the fields look like away from the conductor.

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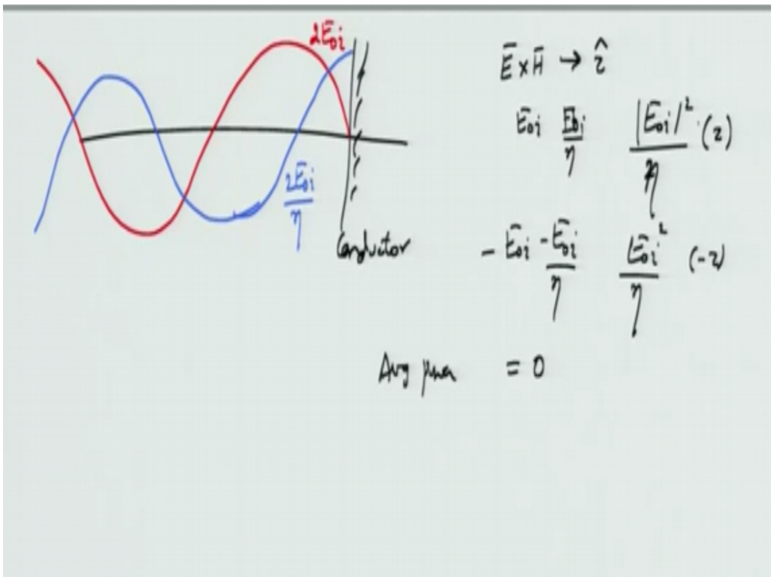

So this is my conductor surface that I have considered and then if you look at the field, for the electric field the field must be going to zero at the origin or at the conducting surface. Therefore, the electric field would go something like that with a maximum amplitude of 2 Eoi whereas the magnetic field would actually be nonzero at this point and then it would go around like this.

So I hope I am drawing them nicely, so it would actually go around like this with a maximum amplitude of 2 Eoi by eta. What can you comment about the power carried by the wave? Well, the incident wave actually carries some power, right, because you have E cross H along the plus direction. E is Eoi, H is Eoi divided by eta. So the power that is carried by or the poynting vector that is incident, associated with the incident wave is given by Eoi mod square divided by, and if you assume Eoi to be real then you can actually replace this magnitude square.

So Eoi square divided by eta, this would be the amplitude of the poynting vector. What about the amplitude of the reflected vector? I have Eoi, I have Eoi minus eta, right, minus Eoi and minus Eoi by eta is coming from the backward propagating and the amplitude of that would also be Eoi square by eta. Except that this would be minus Eoi by eta, except that this is minus z propagating, this is plus z propagating.

The total power carried, the average power that is carried by the sum of the electric field and magnetic fields or the waves in front of the conductor will be equal to zero. So the average power carried will be equal to zero.

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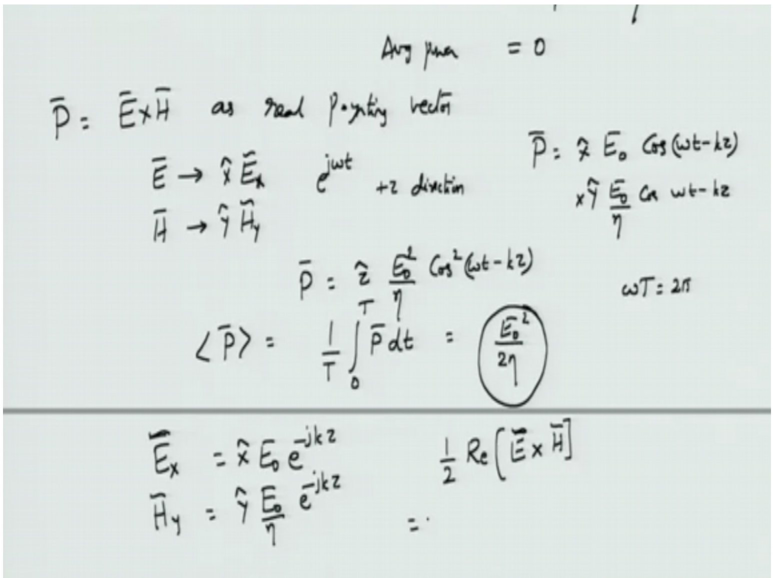

Now, how did I actually come up to this average power? Well, we have only seen one form of the poynting vector; P is equal to E cross H as the real poynting vector where electric fields and magnetic fields are actually real. This is the real poynting vector and in fact this is the only type of vector that would exist. However, when you express electric field and magnetic field in terms of their phasors then the poynting vector expression can be simplified.

With the help of phasors if you write E as the corresponding phasor and H as the corresponding phasor and for now let us consider them to be simple x directed and y directed phasors, x and y directed phasors, corresponding to a wave plane oscillating at a frequency E power j omega t and propagating along z axis let us say, so this is propagating along plus z direction.

Now for such a wave that I have, the poynting vector P will be equal to, with an amplitude of E0 it would be E0 in x direction, or if I go to the full poynting vector, the real poynting vector would be cost omega t minus kz, cross y hat E0 by eta cos omega t minus kz. So the poynting vector P would actually be z directed with an amplitude of E0 square by eta and then I have cos square omega t minus kz.

Now if I ask what is the average power that is carried by this wave to obtain the average power I have to integrate this one, right? So if I integrate this one over one-time period where omega multiplied by t is equal to 2 pi, where omega is 2 pi by t. So if I integrate this poynting vector P with respect to time I will see that integral of cos square omega t minus kz will give you half and this fellow will become E0 square by 2 eta.

So if I am looking at the average power that is carried by this and then I am looking at the average power in terms of the man it is not only the vector, so you will see that this average power carried by a wave which is oscillating with a frequency omega and propagating along the plus direction is given by E0 square by 2 eta. Now how can I obtain the same expression or same relation when electric field and magnetic fields are represented in terms of their phasor quantities.

Well, in terms of the phasor, electric field will be Ex phasor which is E0 this is still x hat up here. So E0 E power minus j kz and the magnetic field phasor will be Hy that would be along y, E0 by eta, this is not changed, E power minus j kz. Now what you form is, you form half of real part of E phasor times H phasor.

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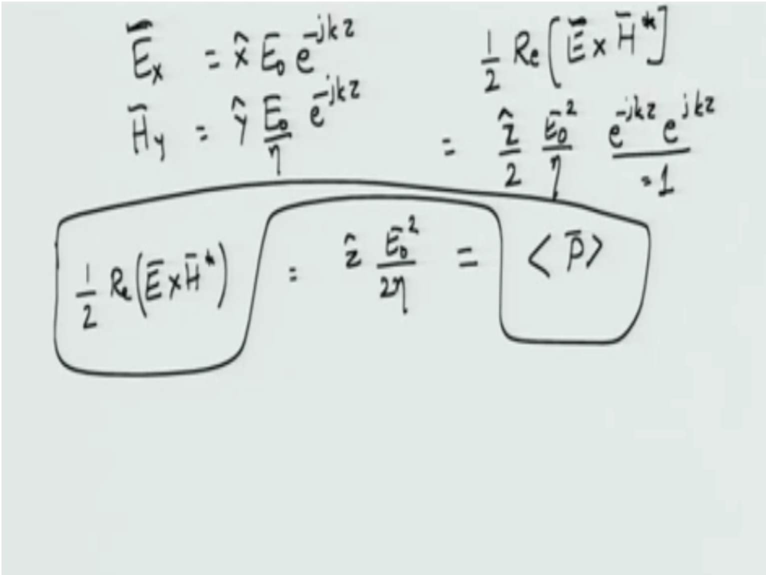

So when you form this quantity what you see here is half, not this E cross H phasor, what you actually form is E cross H complex conjugate. So you take H complex conjugate and then cross it with E and then take the real part. Well, if you substitute for electric field phasor and magnetic field phasor from these two expressions that I have given here you will see that it would still be x cross y that would be along the z axis.

And then you have E field given by E0, H given by E0 by eta therefore E0 square by eta will come out and assuming that E0 is real this entire thing will come out. Eta is also assumed to be real. So everything has come out. And what you are left inside is E power minus j kz multiplied by E to the power j kz's complex conjugate which is E power plus j kz, right? So this fellow is equal to one and what I get is z hat E0 square by 2 eta. What is this quantity?

This is half real part of E cross H complex conjugate and this quantity is precisely equal to the average pointing vector. This is exactly equal to the average poynting vector and you can actually, if you want to obtain the average poynting vector and you have only the phasor relationships you can actually obtain that. So this expression is very important and if you look at this expression now in terms of what we have obtained for the electric and magnetic field of incident and reflected bill, you will then appreciate that.

The average poynting vector will be with an amplitude of half here that I need to add it would be Eoi square by 2 eta carried along by the incident wave, Eoi square by 2 eta carried along by the reflected wave, the sum of these two will actually turnout to be equal to zero. So there is no net average power carried by the total electric field in front of the conductor when there is a reflection, okay?

Well, we wrote down the reflection for wave, however in many cases we will be interested in finding out what is the ratio of the electric field that is reflected to the incident field and what is the ratio of the magnetic field incident to the magnetic field. So this ratio of electric field reflected to electric field incident and ratio of electric field transmitted to the electric field incident define what is called as reflection co-efficient and transmission co-efficient.

To really see that reflection and transmission co-efficient, we need to move away from this normal incidence conducting type of surface to normal incidence, dielectric-dielectric kind of surfaces.

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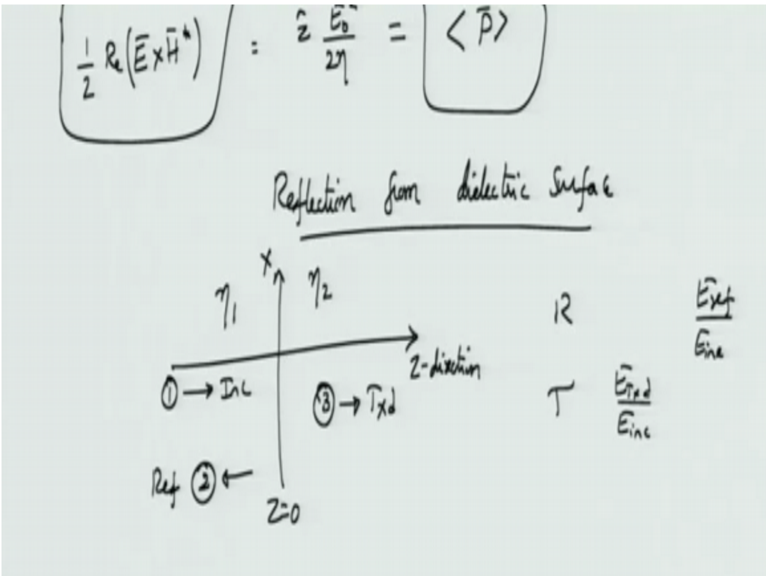

So our next job would be to see what happens when you have reflection from dielectric interface. So if I look at reflection from a dielectric surface what would that be? Assume that as before at z is equal to zero there is one dielectric kept to the entire right hand side, so this dielectric is characterized by eta 2, this dielectric is characterized by eta 1. Now I am not characterizing them by mu and epsilon.

I am actually characterizing them in terms of impedances. Eta 2 and eta 1 and let us now see what happens to the wave when it is incident normally as before let us assume that the way with plane polarized, it is a uniform plane wave which is polarized along x direction and then travelling along the z direction. So this is the z direction and the wave is propagating along this and this meets as initially it would be propagating in a medium of impedance eta 1.

Then it meets a medium of impedance eta 2 and let us see what really happens, okay? First you have an incident wave, because eta 1 and eta 2 are not the same, there will be some reflected wave. However, in this case now, because in a dielectric region you can actually have electric fields, there will be some electric field into the second medium, right? So because of this at this interface you actually have incident, reflected and transmitted waves.

So you have incident, you have reflected and you have transmitted waves into the second region. Accordingly, you have the reflection co-efficient which is the ratio of power reflected to power incident. You have transmission co-efficient or equivalently in terms of the electric field reflection co-efficient also you can write and you have transmission co-efficient in terms of transmitted electric field to incident electric field.

The corresponding power functions are called as reflectance and transmittance. So these are reflection co-efficient and transmission co-efficient. So how do I obtain this reflect co-efficient, transmission co-efficient? Well it all boils down to writing the expressions and applying boundary conditions.

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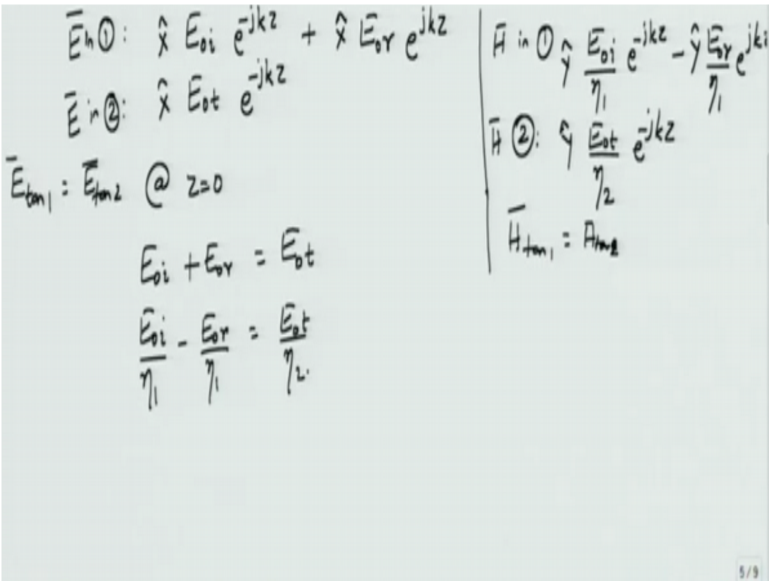

Consider medium one, inside this medium what is the total electric field, well it will be reflected field. So E in region one will be the sum of incident and reflected wave. What is incident wave? Let us assume that the amplitude of the incident wave is Eoi. It is polarized along x direction and then it is propagating along z direction, plus I have now the reflected field. Reflected field will also be polarized along x.

It cannot have a different polarization. This is a fact that you will have to confirm yourself and then the amplitude would be some Eor. At this point I do not know what is the relationship between Eor and Eoi, right? So I am going to write this as Eor. But the wave is propagating along minus z direction, therefore I can write this as e power j kz. What would be the electric field in region two?

In region two I have electric field in the same polarization x but this time it would be transmitted. So I have Eot e power minus j kz because the way it is propagating along z direction. Now what is the boundary condition that I can write? The boundary condition is that the total tangential electric field must be continuous across the boundary. Since x directed electric fields are tangential to the dielectric surface at z equal to zero.

This condition at z equal to zero boundary simply means that Eoi plus Eor must be equal to Eot. I got one equation. Is that sufficient? Unfortunately, no. I need to get one more equation. Where do I get that one? I can actually use magnetic fields, right? What is the magnetic field in region one? Wave is directed along x direction. So I can write this as Eoi by eta, this time it is along y direction, E power minus j kz.

And then for the magnetic field since we know that it has to propagate along minus z direction, let us reverse the direction of magnetic field, so this become minus y hat, Eor by eta, E to the power j kz, so that x directed reflected field cross minus y directed magnetic field will propagate in the minus z direction as I want. What is the magnetic field in region two? Magnetic field in the region two will be y hat Eot by eta 2, right?

So I should actually clarify this one also as in the incident region as eta 1 and eta 1 and for this as eta 2. In medium one it is the ratio of electric field to magnetic field is eta one. Therefore, it becomes Eoi by eta 1 and the same thing for reflected field as well. It is Eor by eta 1, but in medium two it is the second medium impedance eta 2 that must be entered here and then this will still be propagating along z direction.

I have also assumed that there are no additional phase factors anywhere in this relationship. Now what is the condition for magnetic field? Well, the tangential magnetic fields, clearly this is still tangential to the z equal to zero surface must again be continuous now, right? There is no possibility of having any surface current as in the case of conductor, right? There is no possibility of having a surface current between dielectric to dielectric.

Because dielectric, dielectric is assumed to be perfect and free of charges and currents. I have boundary condition for magnetic field as tangential component H1 must be equal to tangential component, H tan 2 and being continuous. So if this has to be continuous at z equal to 0 of course, what would be the expression for this continuity expression, that continuity equation that the boundary condition that would be E01 by eta one minus Eor by eta 1 must be equal to Eot by eta 2.

Now I have all the required equations to go forward and calculate my reflection co-efficient and transmission co-efficient. How do I do that one?

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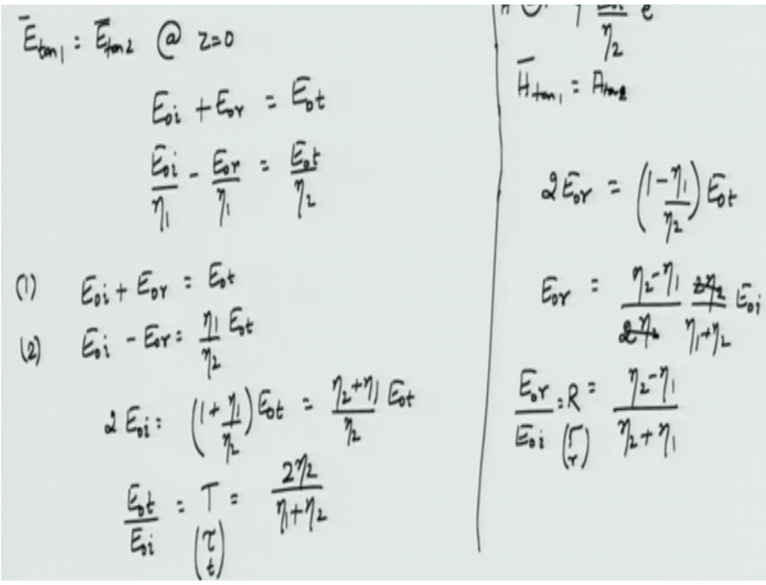

Let us suppose I take this eta 1 to the right hand side and then I am left with two equations, Eoi plus Eor equals Eot, first equation, then Eoi minus Eor is equal to eta 1 by eta 2 Eot. If I add these two equations I get 2 Eoi equals 1 plus eta 1 by eta 2 multiplied by Eot. Simplifying this I get eta 2 plus eta 1 divided by eta 2 and the ratio of the transmitted electric field to the incident electric field which we call as the transmission co-efficient T.

Some people use small tau, small case tau or use small letter t. All these are equivalent, do not really both about which one it is. The point is that this transmission co-efficient is their field transmission ratios. That is, it is the ratio of electric field transmitted to incident electric field and this is given by eta 2 divided by eta 1 plus eta 2 What about the reflection co-efficient.

I can obtain the reflection co-efficient from these two equations by subtracting 2 from 1. If I subtract 2 from 1 what do I get, I get 2 times Eor because Eoi gets subtracted out, must be equal to, one minus eta 1 divided by eta 2 multiplied by Eot. This is nothing but eta 2 minus eta 1 divided by eta 2, right? Eor, 2 I can bring it down. But Eot I can write in terms of Eoi, right? So Eot will be equal to 2 eta 2 divided by eta 1 plus eta 2 times Eoi.

Two eta 2 in the numerator and denominator cancel and I am left with ratio of Eor to Eoi which I will call as the reflection co-efficient R. Some people use gamma as the reflection co-efficient, or some people use small r as a reflection co-efficient, does not really matter what you have used. So Eor by Eoi is given by eta 2 minus eta 1 divided by eta 2 plus eta 1. So this is the ratio of electric field reflected to incident electric field and this is the reflection co-efficient.

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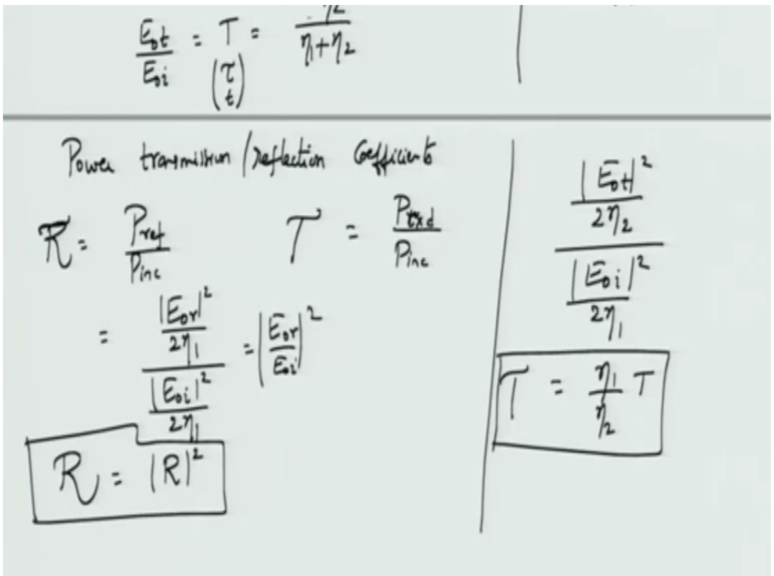

Now before leaving of this topic, let us also calculate the power transmission and power reflection co-efficient. What do I mean by power transmission, power reflection co-efficient? Well, what I mean is that what is the ratio of reflected power to incident power. This we call as the transmittivity, maybe we can write this as T, script T and similarly R as the reflectance which is, sorry, this has to be R which is the power reflected to power incident.

And transmittivity or transmittance as power transmitted to power incidence. What is the power reflected? Power reflected is the poynting power density that I am looking at. If I consider a certain region of space, then the power reflected can be written as directly proportional to poynting vector of the reflected field. What is the poynting vector of the reflected field?

That would be Eor magnitude square divided by 2 eta 1 divided by incident power is Eoi magnitude square divided by 2 eta 1. 2 eta 1 cancels with each other and this is 2 eta 1 because they are in the same medium and I get Eor by Eoi magnitude square, which is nothing but reflectance or reflection co-efficient magnitude square, okay? So the reflectance is actually equal to reflection co-efficient magnitude square.

What about now the transmittance or the transmission power to incident power? Well, what would be the power transmitted? Power transmitted would be Eot magnitude square by 2 eta 2 because this is in the second medium divided by incident power will be Eoi magnitude square by 2 eta 1. So if you form this ratio, the transmittance is given by 2 cancels out, then you have eta 1 on to the top.

So you have eta 1 by eta 2 times Eot magnitude square by Eoi square which is nothing but transmission co-efficient T. So this is the power reflection and power transmission co-efficient.

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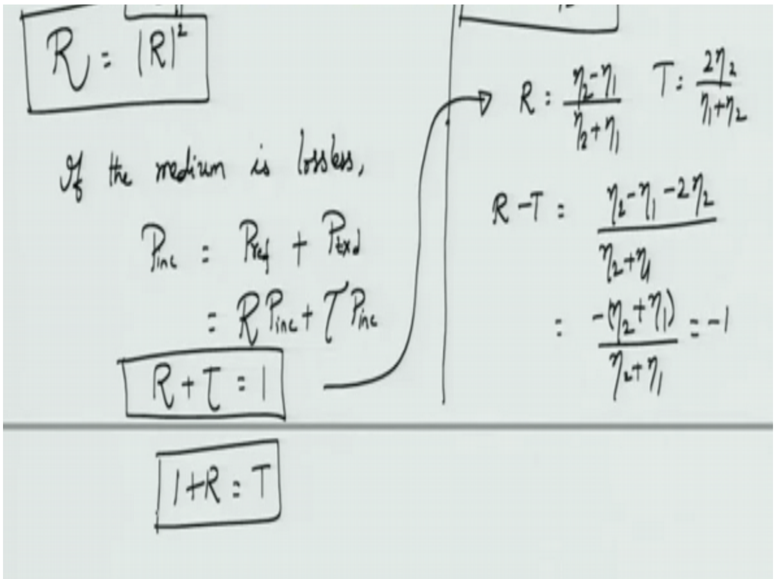

If the medium is lossless, then what should happen? Then the total incident power must be conserved. So you cannot have any power loss which means that some power can be reflected, some power can be transmitted, but the sum of this reflected and transmitted powers must add up to the total input power. So the power incidence must be equal to power reflected by power transmitted.

So power reflected is actually written in terms of reflectance r times P incidence and power transmitted is written in terms of the transmittance T or transmittivity T times power incidence. Therefore, what this implies is that R reflectance plus transmittance must be equal to one. Fortunately, this same kind of relationship very similar kind of relationship actually hold even for the individual reflection co-efficient and transmission co-efficient.

The expression should actually have been 2 eta 2 by eta 1 plus eta 2. So what is reflection co-efficient, R? R minus T, what would that be? That would be eta 2 minus eta 1 minus 2 eta 2 divided by eta 2 plus eta 1, right? And this eta 2 minus 2 eta 2 is nothing but minus eta 2, minus eta 1 or I can put the minus sign common divided by eta 2 plus eta 1, that would be equal to minus 1.

Now interchanging this minus T on to the left hand and the right hand side what I have is, 1 plus R is actually equal to T.

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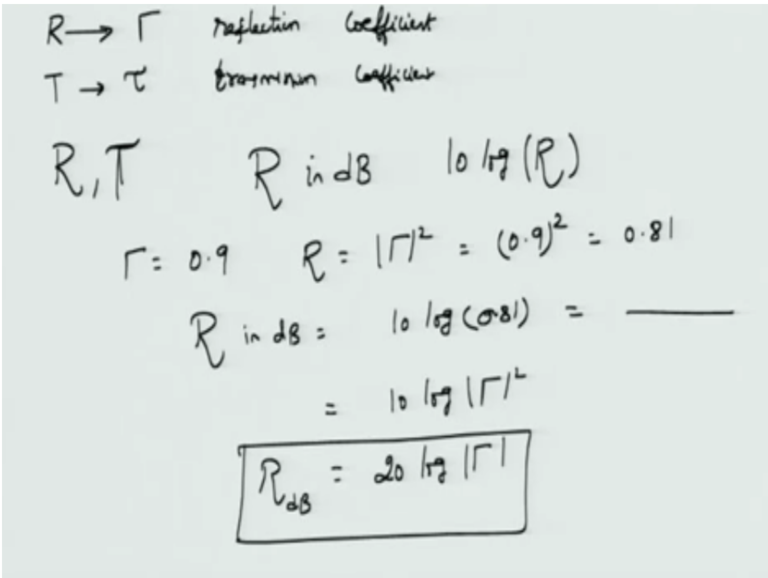

From now on we will actually replace this R by gamma, standing for reflection co-efficient and this T by tau standing for transmission co-efficient. In many microwave circuits you are actually interested in this reflectance and transmittance which actually tell you the ratio of the power that is reflected to incident power and ratio of power transmitted to this one and this reflectance or transmittance.

And mostly reflectance that you are interested you measure this one, not in linear scale but rather in decibel scale. That is you actually take this reflectance and then form log of this reflectance. So when you do that one you are expressing this reflectance in terms of dB and this is what you would obtain. So for example if a certain microwave circuit or I mean, microwave circuit actually is reflecting the electromagnetic wave to the reflection co-efficient gamma of say 0.9, right?

This would mean reflectance R as magnitude gamma square which is 0.9 square which would be 0.81, okay? And then reflectance in dB will be equal to 10 log reflectance, reflectance would be 0.81, so whatever the value that you are going to get you can actually write this one down. That is not really important. What I wanted to say was, this 10 log, when you go back to the reflection are, that would be magnitude gamma square.

So this square because of this square you can actually bring this two on to the left hand side, I mean out of the log thing and write this as 20 log magnitude of gamma. This would be the reflectance measured in terms of dB and this is something that we will be using later when we discuss transmission line and then associated parameters and it is something that was also used in microwave circuits.