Electromagnetic Theory
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Lecture No - 45
Faradays law and its application - II

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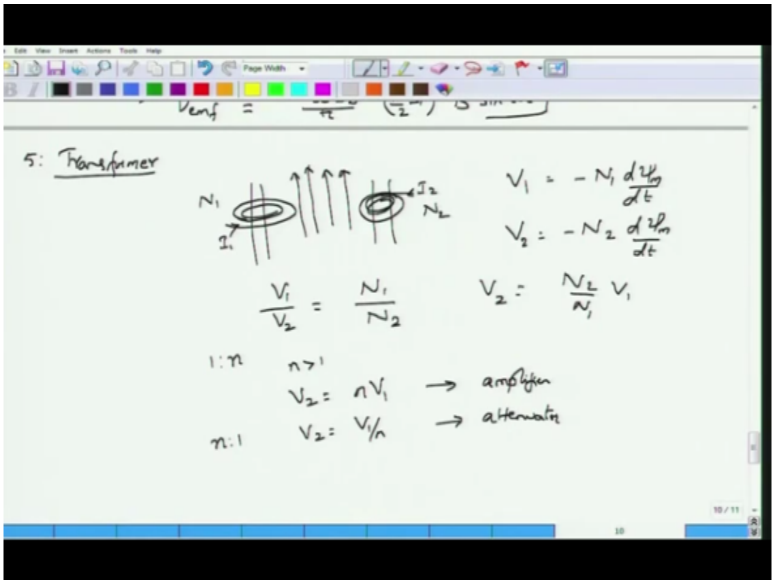

As a final example, let us consider a very important circuit element called transformer. What does the transformer do? It is actually used to change the voltage and current ratio right. So, it has a primary coil, which is carrying a current of I 1 and is bound by turns N 1 and then there is a secondary which is carrying a current of I 2, but there is a winding of N 2 right. And what would happen to the B field?

The B field which actually be completely contained in the core of the transformer provided that the transformer core is having a larger permeability okay. So, if you have a large permeability, then B will be uniform upwards inside the core and the voltage that is produced or the EMF that is produced in the first or in the primary will be the amount of magnetic flux that is changing within that right.

And for the voltage that is produced or the EMF that is generated in the second coil will be minus N 2 d psi m by d t. Where psi m is the magnetic field that is linking circuit primary coil and the secondary coil right. So, with magnetic field assume to be uniform right, so you can imagine that if I am applying the current I 2 then, there will be a current I 1 that would be generated right.

So, you can actually see what is the ratio of V 1 by V 2 and the ratio of V 1 by V 2 will be N 1 by N 2 right, and V 2 will be equal to N 2 by N 1 into V 1. Normally we are concerned with transformers which are given by 1 is to N right. That is to say the (()) (02:00) ratio is taken in such a way that N 2 by N 1 will be equal to small n and n will be larger than 1. So that V 2 will be equal to some N times V 1.

In this case, the transformer is actually acting as a voltage amplifier okay. On the other hand, if the transformer ratio is specified as n to 1, then V 2 will be equal to V 1 by n okay. In this case it would actually be acting as an attenuator. A real transformer has lot of complexity, it has surface currents and it has mutual inductance, we are neglecting all these effects okay. For a loss less transformer however, the total power must be conserved.

V 1 I 1 must be equal to V 2 I 2, this also gives you a result in terms of I 2 by I 1 okay. I 2 by I 1 will be equal to V 1 by V 2 which is equal to N 1 by N 2 okay. And if you define V 1 by I 1 as the resistance of the primary and V 2 by I 2 as the resistance of the secondary, then this primary and secondary resistances are given in the ratio of primary to secondary ratio is actually given by N 1 by N 2 square okay.

And if you take a 1 is to end transformer, then this would be 1 by n whole square okay. So, this was about transformer and again what I want to emphasize is the transformer works on Faradays law okay. The magnetic field associated would be changing. Here see what is common about all these examples is that, for the transformer or for the coil that is kept with non homogeneous or homogeneous magnetic fields, the coil was not actually moving right, the coil was not moving okay.

It was the magnetic field that was associated with that coils or the wires that was actually changing with time leading you to some EMF produced.

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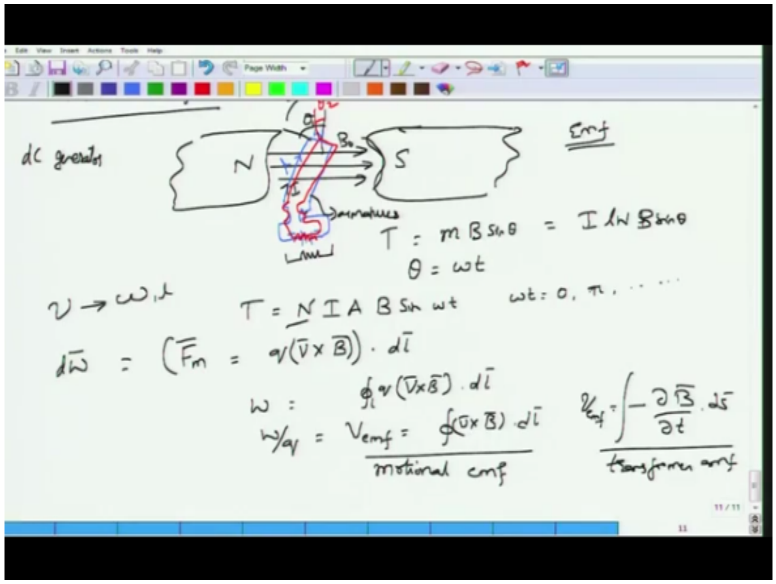

Now, take the other case, in the other case, which is called as motional EMF okay, motional EMF is because of the movement of a wire okay. Consider north and south poles okay of a magnet. So, this is a north pole and this is a south pole of the magnet. So there will be magnetic field in between right, so there will be a magnet field now, what I do is I take a loop okay of some length l and some width w okay.

And then I connect this loop to 2 brushes which are called as commutators and slip rings and I connect these commutators to a battery okay. I connect this to a battery. I have not really written down nicely and if there is a battery or a volt meter more precise if there is a volt meter here, this volt meter would actually register a reading. When will it register a reading? It will register a reading when the loop is actually moved.

You could actually move this loop by hand you know taking mechanical motion, you can actually rotate the loop okay with a certain angular velocity and when you do that, there will be an EMF induced okay. The EMF induced is called motional EMF and it actually comes because there is a magnetic force acting on the electrons that are carrying this current okay. So, if there is a current I, either you can supply the current I by connecting a battery here.

Or you can remove the battery and put a load resister okay. So in 1 case, you are actually generating DC. So, in this case when you put a load resister, actually generating DC voltage okay. The commutators are actually chosen in such a way that when this happens, when there is a half rotation of the coil, the commutators actually switch places in such a way that the overall direction of the voltage would actually be in the same direction.

Otherwise, the voltage would be 180 degrees opposite to the original voltage okay. You would actually get an AC voltage but if you want to get a DC voltage, you flip the commutators at a right time such that the voltage polarity is always unidirectional okay. So, how do we understand this motional EMF? Imagine that I have the loop is at an angle theta okay. So the loop is an angle theta and there is a torque on this loop correct.

If this is a current carrying wire or the loop, there is a magnetic moment and the torque is given by m B sin theta and in this case, the length of the magnetic element is also need to be considered. So the torque is r cross f or m cross B right. So, it would be m B sin theta and what is m? M is the current times l into w right. So this the torque that is applied on the electrons or applied on the charge carriers which will sustain the current okay.

So, this torque can also be related to the work that is required okay and you can find out what is the EMF that is induced. Because the torque is given by I l w sin theta. Hence the loop is actually varying with an angular velocity of omega t, then the torque induced will be I and l w can be written as A okay. So, I have B sin omega t and if instead of 1 loop you have multiple loops, you have n loops this would be the overall torque okay.

So, if you have the torque here now instead of talking about torque let us look at what is the force that is acting on the charges right. So, from the torque what we observe is that, torque is actually proportional sign omega t and this you can see that at some point at omega t is equal to 0 or not really 0 because, you are starting something at 0, but at any integer multiple of pi and so on, the torque will actually be equal to 0.

Now torque is a rotating motion right. So, torque actually causes the coil to rotate. When you supply a current and then make the loop rotate, the loop is actually rotating, so it is trying to convert the mechanical energy of rotation into the EMF that is induced on to the loop. What we have seen is that, if you start the rotation and leave it, then this torque because of the magnetic field actually causes the loop to rotate but at some point the torque is 0 right.

So, when the torque is 0, the battery terminals are actually momentarily shorted, because there is no EMF produced and the battery terminals are momentarily shorted. However, because of the initial build up the moment of the torque where it is actually 0 is very small and it is actually it will go over the rotation energy will actually go over the torque and flip the loop okay.

So, that is what even though torque is supposed to be 0 at omega t is equal to integer multiples right, multiples of pi, this will be only momentarily okay before it actually over comes this rest position okay. So, the torque actually is sort of continuous except for the brief moment around pi where the torque is actually going to 0 but, although that happens that does not really have a too much of an effect on how the voltage is actually induced on the load resister okay.

Now, to understand what is the motional EMF or develop and expression for the motional EMF, let us go back to the magnetic force acting on the charges right. So, with the positive current in this direction which I have taken and I am assuming that the current is all positive charge carriers okay. The magnetic force acting on the charge carriers is q v cross B right. Where v is the velocity with which the charges are moving.

In this case it is the angular rotation right. So, you need to connect this velocity to omega right. Omega being the angular velocity in turns or radiance per second or just per second and v is the linear velocity. So, you have to connect v omega and l, and when you connect that you can actually see how fast or how quickly the charge q is moving. And this force acting on that charge will be given by q v cross B okay.

So, if you now take this force and then say that the loop goes over 2 you know over small distance right. So, initially let the loop is at this angle theta, call this as theta 1 okay and then let the loop be at an angle which is say, I am trying to write down a loop here okay, a loop with a resistance up here. Let the angle here may be theta 2. So, initially I have an angle theta 1 and an angle theta 2 now after a movement of the loop right.

So in doing so, it has actually moved a small distance of d l correct. So what is the work done on this current loop so that it has actually moved a distance of d l and that work will be force into distance right. So, F m being q v cross B when you take this and then integrate over the small distance d l or multiply this one with d l and you have to keep in mind because this are angles I mean these are vectors, you have to use a dot product over here.

So, force into distance will give you the work that is done. This is the differential amount of work done by the magnetic field in order to move the charge in such a way that there is a movement of d l right. So, it is actually pushing the torque is actually being applied over here to rotate the charges right. So, what would be the total work done? Total work done will be q v cross B so, q actually applies to both v and B dot d l right over the complete loop.

Over the complete loop, this would be the work that is being done. Now, I know that work done per charge gives me the potential or the EMF and therefore this EMF is given by v cross B dot d l okay. This would be the closed loop integral and this integral which you have obtained is called the motional EMF. This EMF is different from the changing magnetic field okay getting linked to the corresponding circuit and producing a EMF.

So, this EMF which is sometimes called as the transformer EMF, because here the coils are stationary, it is the magnetic field which is changing with respect to time and this EMF which is called as a motional EMF in which the magnetic field is stationary that is magnetic field is uniform, it is not changing, it is always B 0, where as it is the coil which is actually moving okay. This example was that of a DC generator.

One can actually make an AC generator also out of this. In a typical DC generator, there will be multiple loops. These loop are called as armatures, there will be multiple windings if the armatures okay. Here I have shown only one wire, otherwise you need to multiply everywhere by value of N okay. So, here is the number of armatures. So, there is actually a distinction to be made between two types of EMF.

1 EMF is called as the motional EMF, the other EMF is called as the transformer EMF. Technically these 2 are distinct phenomena okay. 1 phenomena can be explained without using Faradays law at all. The motional EMF does not require Faradays law okay. However, it is possible to view this as a special case of Faradays law, so that we do not have to deal all the time with motional EMF and transformer EMF.

Although, the subject of which one is motional EMF and which one is transformer EMF is very very interesting and there are lot of paradoxes associated with that. Primarily which motive, which agency to apply, which agency to actually consider has the one that is responsible for producing an EMF in circuits okay. There are lots of interesting and paradoxical situations where it is not very clear which one is really operating.

Fortunately, turns out that whether this is motional EMF or transformer EMF the net result is that there will actually be an EMF induced. So, if you just think of the EMF induced and do not distinguish between motional and transformer, then you will be fine. However, in most cases it is very difficult to determine. Because there will be component of motional EMF as well as transformer EMF okay.

And Faradays law has been extended in such a way that both can be considered as special cases of Faradays law okay. In the case where transformer EMF is there, the magnetic flux that is changing is because of the changing magnetic field right, or the loop is stationary right, you can move the partial differentiation inside and say that this minus del B by del t which is changing and being integrated over d s is the one that is responsible for EMF induced okay.

Motional EMF can be sort of incorporated into Faradays law if you keep this minus d by d t outside and then let the surface change over okay. So, what it means is that, the total flux linking would actually be changing right with respect to time. So, in this case of DC generator, the total flux will be l w into B 0. But that would not be constant. That would be changing right. Because the surface itself is changing, it is rotating, there will be factor of sin theta into there.

So, both methods or both interpretations will give you the same EMF, but it is true that motional EMF has actually nothing to do with Faradays law, only that we have considered extended Faradays law so that motional EMF can be absorbed into it rather that dealing all the time with motional EMF okay. We will not be dealing with this motional EMF in this course in fact we will not even be dealing with transformer EMF.

What you have to realize is that, Faradays law simply states that, there will be a EMF induced whenever there is a changing magnetic field okay. And EMF is electric field. What if electric field is also changing, that is an interesting question. We will come to that question later okay.

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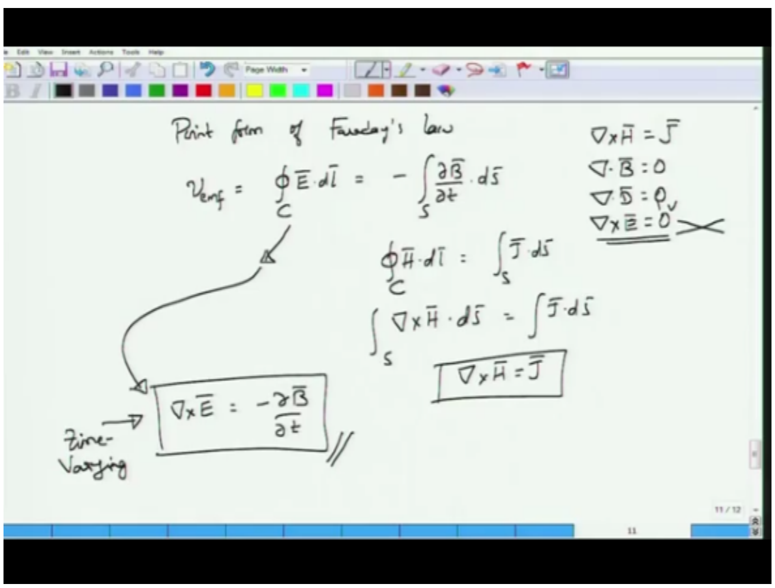

So, now we want to develop point form of or differential form of Faradays law okay. So, I want to develop this one because I want to complete Maxwell's equation so this is the third Maxwell's equation that we are looking at. We have already relation which says del cross H is equal to J right, the current enclosed or the current density J and then we have del dot B equal to 0, del dot D equal to 0, we also have del cross E is equal to 0 right.

We had del cross E is equal to 0 because this was for the electro static case. However, we have seen several cases where del cross E is equal to 0 may not exactly be true and this one of the case where it is not true okay. So, we begin again by writing down what is EMF? EMF around a closed loop is E dot d l right over the closed loop is E dot d l. So, here del dot D is actually rho V which is that volume charge density.

So, coming back to the EMF that is generated, EMF is given by the line integral of the electric field right over the closed curve C, so it is integral of E dot d l. This should be equal to minus integral of del B by del t dot d s correct. To develop the point form, you need to see that if you take integral of E dot d l over the closed curve or on a short closed path and then divide by d s, what you are going to get will be whatever that is there on the right hand side right.

See, I will not develop this one in that detail that we developed it for the magnetic field okay. We will go by the analogy between these two equations. So, in these 2 equations, if you see, the left hand side is a closed line integral of a corresponding vector quantity and the right hand side is an open surface integral okay and if you now write down the point form for this integral of H dot d l equal to integral of J dot d s.

The left hand side can be rewritten as del cross H dot d s right over the same surface, this would be equal to j dot d s and from here, we wrote that del cross H is equal to J, this was by making use of Stokes' theorem right. Similarly, I can write down for the left hand side of this one as curl of E right after integrating over the same surface or applying Stokes' theorem and get the same thing. So, curl of E will be equal to minus del B by del t okay.

So, clearly the equation that we were using earlier del cross E is equal to 0 is no longer true, because there is magnetic field that is present and this magnetic field is the source of the EMF. The changing magnetic field is the source of the EMF and therefore this equation had to be modified from the electrostatic case and this is the point form or the differential form of the equation for Faradays law okay.

And this equation is valid for time varying as well as time invariant cases. When time is not varying or when the fields are not varying with time, then the right hand side will be equal to 0, which means that electric and magnetic fields are separately considered. There is no interaction between the 2. However, when the magnetic field is changing, it will act as a source of EMF or, it will act as a source of the electric field which means that they are now related to each other okay.

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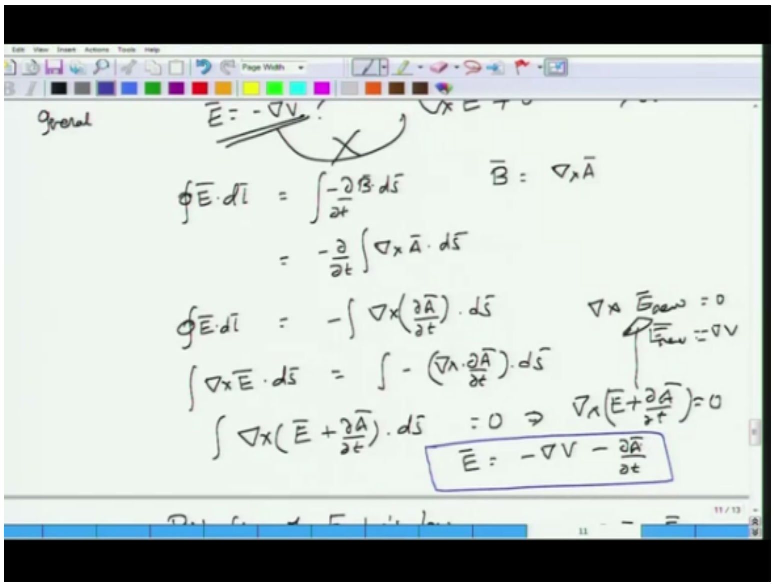

We want to develop one additional form for the electric field which is interesting, because we have seen that electric field is equal to minus gradient of V earlier right. For the electro static case, we were able to write this electric field as minus gradient of V because del cross E was equal to 0 okay. This was for the static electric field case you were able to write this one. But now, del cross E is no longer 0, in fact this is actually equal to minus del B by del t.

So, can I still write E as minus gradient of V? Turns out that I cannot do this right. In the general case, I cannot consider the electric field as a gradient of a time independent potential. The physical reason should be quiet obvious. This potential function V was independent of time, it was only function of spatial coordinates so no matter how or what operation I do on this V, I will never be able to get the time variation.

So, clearly this equation needs to be modified. So how do I modify that equation? Let us go back to the Faradays law right in the integral form so write down this as integral of E dot d l which is the EMF produced given by minus del by del t of B dot d s okay. Now, I know what is B in terms of the vector potential A, so, B is equal to curl of A. So, I can write down this as minus del by del t.

I can pull this minus del by del t outside of the integral and have del cross A dot d s okay. I can also push this del by del t inside and write this as minus del cross del A by del t dot d s, now I have one close line integral and a surface integral. I can actually convert one of them into a fame integral by applying Stokes' theorem.

So, I can convert the left hand side into writing this as del cross E dot d s. Curl of E, which would be equal to minus del cross del A by del t dot d s and push this two into the left hand side, you get del cross E plus del A by del t dot d s over the integral will be equal to 0. This simply implies that curl of E plus del A by del t is equal to 0. Now I seem to have obtained a new type of an electric field right.

This new electric field actually is the old electric field plus the time variation or the rate of change of the magnetic vector potential itself okay. So, for this new field curl of E new is equal to 0. Which means that, E new can be expressed in terms of gradient of a scalar potential okay. Thus, you can actually rewrite electric field as minus gradient of v minus del A by del t and this will be the general expression for the electric field having a spatial dependent form only when the time variations are 0 and time variation included okay.

So, this is the expression for electric field in the most general case okay. So you have a time independent parameter and time dependent parameter. When there are no time variations, when there are no magnetic fields, we will have only the gradient of the potential, when there are magnetic fields, obviously this would not be the same thing okay. So this is about the electric field in general in terms of the potential okay.

So, let us consider one example of point form of Faradays law and then we will stop discussing Faradays law okay.

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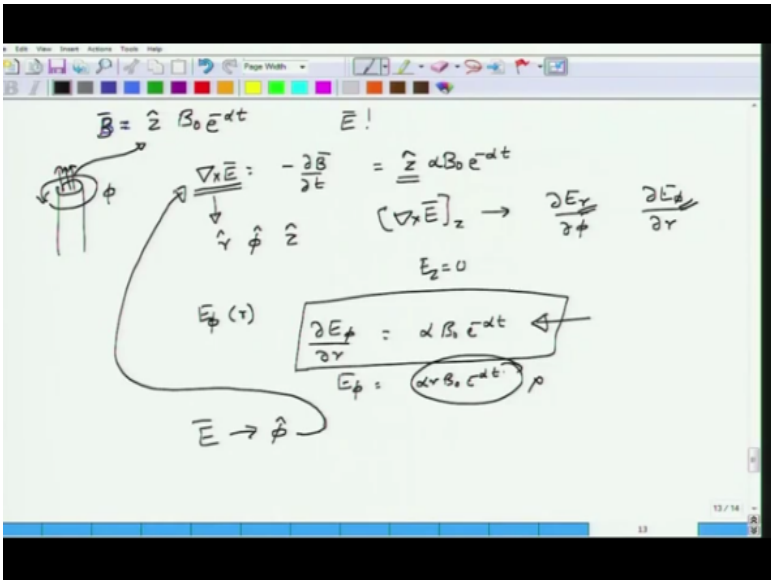

So, the example that I would like to consider is that B field is given to be some uniform Z hat and B 0 e to the power minus alpha t, we have already obtained what is the induced EMF for this case right. We have seen that the induced EMF is positive and it has a current in a particular direction for the loop that we have obtained. What we want to find is determine the electric field E.

How do I determine the electric field E? I know from point form that curl of E is equal to minus del B by del t right. So, if I differentiate B with respect to time, I get alpha, there is a minus sign so therefore that minus and minus sign will go away. So, I get z hat alpha B 0 e power minus alpha t okay. So, this is the curl of electric field okay and you have to now apply the curl operation in the appropriate geometry.

Remember the geometry that we were considering was the cylindrical geometry right. It was actually the loop that we were considering or you could think of this as a coaxial cable with the magnetic field coming out in this way okay and the magnetic field will be given by this expression. So, it does not really matter you have to think this one in terms of the circular loop and if you write down what is the expression for curl of E.

You will have three components r phi and z, but clearly because the right hand side has only the z component, you will have to have only the z component for curl of E okay. The z component of curl of E will involve E r and E phi components okay appropriately differential with respect to phi or differential with respect to r right, there would be a 1 by r or minus 1 by r that would be there.

But point here is that, you have radial electric field component and phi component here clearly E z will be equal to 0. There will not be any E z possible in this situation, because the right hand side is, I mean the curl of E has only this component right. Now, amongst the radial and phi component, by symmetry of this problem, because I know that the loop is symmetric around phi right.

So the loop is symmetric around phi, azimuthal symmetry but if I change the loop radius, then the value of the EMF induced will change, because the cross sectional area will change right. So, clearly I should have E phi component and this should be only be a function of r okay. And for that you can substitute for the values, so you will probably get something like del E phi by del r is equal to alpha B 0 e power minus alpha t.

And calculate E phi by integrating this expression okay over whatever the value that you want to, just integrate this expression you are going to get alpha r B 0 e power minus alpha t okay. This I am assuming that my expression for del E phi by del r is correct. Otherwise you have to actually put up the proper value for del E phi by del r proper expression for the curl of the electric field and then find the electric field okay.

Just to sum up, the electric field will be along the phi component, do not exactly worry about what the actual value of the electric field is okay. This can be done if you know what is the expression for curl of electric field okay. So, I have electric field along the phi component okay and this was, and I can actually evaluate what is the value of this electric field by using this point form of Faraday’s law.