Electromagnetic Theory
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Lecture - 44
Faradays Law and Its Application - I

So far in our study if you have been following all the modules in a linear fashion, you will see that we have been studying things which are not dependent on time. We started with charge configuration, that was essentially static right, and then we talked about the electro static field that would arise out of this. Electro static would mean that the electric field is independent of time.

There is no time variation anywhere right and when we let the charges move to form the currents. Currents can only be formed when you have a charge movement and charge movement would mean that there is some time variation involved in the charge configuration. However, as long as the current was steady right, we did not have to worry too much right, because the magneto static fields that were generated, they were all again independent of time.

So, this is the reason why we would call both electro static fields and magneto static fields as static fields okay. These are static in the sense that there is nothing interesting is happening in terms of time. The moment you start varying the current going through a circuit, then very interesting things will begin to happen and to understand that, we need to introduce time dependency to our expressions.

And we do that by discussing first what is perhaps the most important experimental discovery in the nineteenth century called Faradays law, upon which the entire diffuse of electromagnetic is built and once we discuss that we are in a position to understand what happens when time varying fields are present okay. So, we will begin by discussing Faradays law which gives us the link between electric and magnetic fields.

And then we will discuss some applications of Faradays law and tell you what exactly this Faradays law is useful for and finally we will talk about a different form of link between magnetic field and electric field that cannot actually be related from just Faradays law but it can be understood in a broader sense of Faradays law okay. So we will begin by considering what happens when you have a current that is carried by a particular loop okay.

So, we have a current let us assume that we actually have a loop okay.

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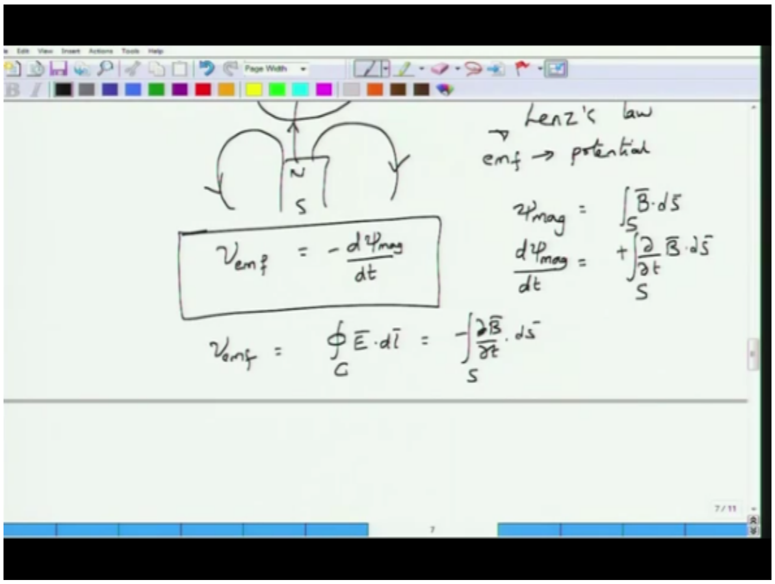

So, we draw a loop here, and this loop is carrying a current i okay, or let us assume that initially the current loop is not carrying any current Ii that would make Faradays law very interesting okay. Now, if the loop is not carrying any current I and we will assume that this gap that we have is very small so we can consider this entire loop as a proper loop, closed loop okay. If this loop is carrying no current, then what happens? There is no magnetic field that is generated.

But is it possible that if I have this loop and then I have a magnet okay nearby. So, I have a magnet which is giving out magnetic fields everywhere right so, this are the fields of the magnet and if this magnet is there and there is a loop that is sitting here will there be any current in the loop? See, we know Ampere's law states that, when there is a current carried in a conductor there will be a magnetic field that is generated.

We have seen the applications of this Ampere's law all along in the last few modules. Now, we turn around and this question if I have a loop and if a loop is subjected to a magnetic field, external magnetic field, will there be a current in the loop? It turns out that 2 people worked on this problem actually many people worked, but 2 of them were very famous and both almost simultaneously got the correct answer.

The correct answer being, yes there will be a current in the loop provided the magnetic field is changing with time okay. If we take this magnet and introduce the loop okay, and both of them are constant with respect to each other that is, they are not varying with their positions then there will be no current induced okay. When will current be induced? You keep the magnet where it is and then take the loop in and out or you keep the loop where it is.

And then take the magnet and push it back and forth. When you are doing this then the magnetic field that is associated with, or the magnetic field that is linking with the loop changes and this change in the loop or change in the flux linking with the loop will induce a current okay. So, this is what essentially 2 people Faraday and Henry found out, Henry was U.S and Faraday was English, but it so happened that Faraday was the first one to publish widely his findings.

And therefore all these laws and the subsequent development is given credit to Faraday okay. Henry is honored by giving the units for inductance, while Faraday has been a very influential scientist on Maths well, who finally unified all of the electro magnetics okay. So, to reiterate what we were discussing, yes it is possible for the loop to get induced by a current okay, whenever there is an external magnetic field, linking to the loop changes.

This is very important, the magnetic field linking or the magnetic flux linking must change okay. So, when that happens there will be current induced in the loop and what should be the direction of the current induced in the loop. Would it be along clockwise direction or would it be along the anticlockwise direction? Well, the answer to that is conveniently comes from what is called as Lenz's law.

And this Lenz law states that, the induced current would generate a magnetic field which would oppose the original magnetic field. Say if it opposes in the sense that, if it subtracts from the original magnetic field then the amount of this one that would be induced would actually be lessen. Now, when current is flowing through a conducting wire right, it would actually give rise to a certain electro motive force or in our words, EMF or potential okay.

In fact, Faradays law does not really talk about current getting induced but rather what he says or what the law states is that there is a EMF produced in the coil, whenever there is a time rate of change of magnetic flux okay. If the magnetic flux is changing with time, then there is an EMF induced, EMF is called electro motive force and this was sort of the force that people considered, which was responsible for the movement of current okay.

So, it is not really the current that we are actually inducing although which is what happens, but it is more importantly the EMF that we are focused on. So, there is an EMF induced whenever the magnetic flux linking to that circuit is changing. Now, why is there a minus sign here? The minus sign is precisely Lenz's law. Minus sign is put so as to make us remember that, the currents must be induced in such a way that the EMF actually reduces right.

The net flux change will be reduced. So, if you choose a current direction, in 1 sense, the EMF could actually or the current would generate its own magnetic field which would add to the original magnetic field which means that the EMF would increase right. However, you can actually do this in a continuous loop kind of a thing right. So, you have a magnetic field, induced current, induced current gives you the induced EMF.

And the induced current will generate a magnetic field which will add to the original magnetic field, increase the EMF, add more magnetic field and this process can go on which obviously will not happen in nature. What actually happens is, the current induced in the coil would actually be in a direction that would reduce the magnetic flux linking to the circuit okay. So, in mathematical terms, this is all that is there for Faradays law okay.

There is an EMF induced in a coil or a circuit which is related to minus time rate of magnetic flux linking the coil. That is what Faradays law states. To put it into slightly better form for us to understand, we express magnetic flux in terms of the magnetic field itself or magnetic flux density B right. What is the magnetic flux linking to a particular circuit? This would be equal to integral of B dot d s, this is something that we saw when we discussed inductance as well.

So, there is a magnetic flux linking to a circuit and that would be given by B dot d s right. So, you can substitute for that. So there is d by d t of magnetic field and that would be getting converted into del by del t of B dot d s okay. I have chosen to take this d by d t inside into this B assuming that, whatever the surface I am integrating, would remain constant okay. So, this is the case where the surface or the coil is actually stationary while the magnetic flux that is linking to it actually changes.

How could it be possible? Well in the coil and the magnet experiment, you place the coil, do not move the coil, but you take the magnet and move it up and down right. So when you move the magnetic field, when the magnetic field recedes, there will be decrease in the magnetic field causing a current and when the magnet approaches the coil, there will be increase in the magnetic field again causing a current right.

So, that depends on how what velocity the magnet is actually moving. As I said, there are 2 aspects to this one right. I keep the magnetic field as it is right. I won't change the magnetic field, but I will let the coil move around right. So, let the coil move into the magnetic field and out of the magnetic field. There will be again an EMF induced okay. Will this EMF be the same as the EMF induced in the other case. Is this also called Faradays law?

Interesting questions to think about okay. We will talk about that second situations slightly later okay. So, for us we can substitute for now what is the magnetic flux okay and then say what is the EMF. Of course, EMF also can be expressed in terms of the electric field correct. EMF is integral over the closed loop or whatever the loop that you are considering E dot d l, that is the EMF over the closed loop.

And this must be equal to minus del B by del t dot d s integral over the surface S which is bounded by this closed curve C okay. So, this is your Faradays law in vector notation. You have a line integral okay of the electric field and that line integral of the electric field which forms the

EMF must be equal to the time rate of magnetic flux that is changing with the circuit okay. So before we discuss other aspects of this, let us look at some examples of Faradays law okay.

Let us understand what directions to choose and what we need to do with this one okay. So, as a simple example first let us consider the case of a loop okay.

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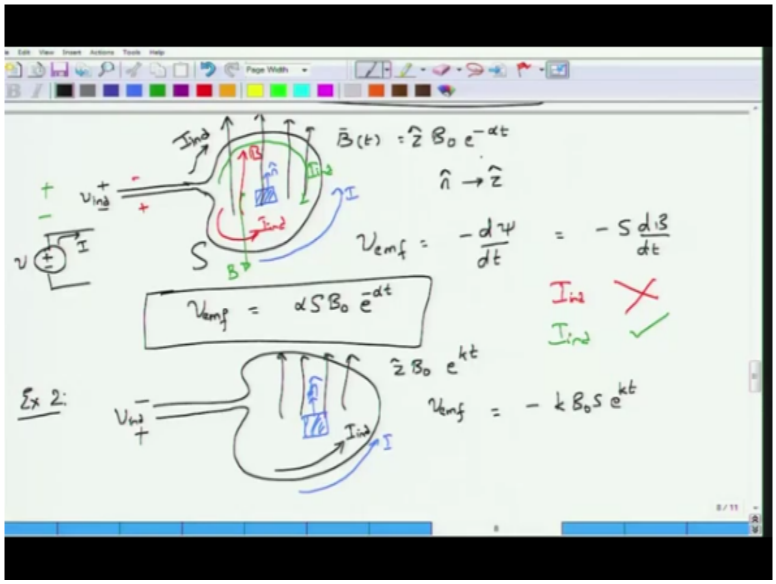

Sorry my loop is not very nicely written so let me just write down the loop like this. I have this loop with the gap being very very small okay and there is a magnetic field which is uniformly perpendicular to the loop okay as the value of B which is given by some B 0 e to the power minus alpha t. Thus the magnetic field associated with this loop is actually in the z direction perpendicular to the loop.

So, if this is my loop, then the magnetic field is in this direction and it is actually decreasing with time okay. So this is the magnetic field. Now, what is the EMF that is induced? If I take this as my specified direction for the EMF okay, what would be the EMF that is induced okay. Now, before we actually look at this one, let us look at a voltage source, the conventional form of writing a voltage source is that, a current leaves voltage source, the positive terminal of the voltage source correct.

So, the current actually leaves the positive terminal of the voltage source and if we have chosen the induced voltage in this way, then there must be a current that is leaving here. Remember this current is not the current that is present in the coil. That is, in the sense that is not initially present, it is the one that is because of the magnetic field that is changing. So, I will write this one as some I ind, to indicate that this is the induced current okay.

And we want to find whether the induced current will be clockwise in the same direction or it would be anticlockwise. If it is anticlockwise, then the terminals for induced voltage would also have to change correct. So, this is what we want to find out now. Before you can do that one, here is a point, here is a typical idea to remember. Let us assume a certain polarity for the surface right, so this an open surface, the surface normally can actually point upwards or downwards.

If they allow the surface element, normal to point in the direction okay, such that, if I curl around my fingers using the right hand rule, the thumb should point along the B region, I mean B fields right. So, if this the convention that we have been following right, so if I have the current in the loop, then the thumb points in the B direction and the loop surface area normal also now points in the direction of B field right. So, my field actually points in the direction of z.

Therefore, the surface normal also points in the direction of z and this would have been the magnetic field that is generated provided there was a current I in the loop correct. There is current I in the loop that current would have generated a magnetic field along z direction okay. If my induced current is in the same direction as that of the current I marked in blue here, it would generate more magnetic field right.

It would keep on generating more magnetic field and the EMF would actually keep on increasing. So, before you solve Faradays law problem, I would suggest you make these simple pointers okay. You establish the surface normal, in such a way that the normal should point in the direction of B field and then establish the direction of the current I, that would actually give you the magnetic field in the applied direction okay.

In any case, you have to only note 2 things. Either your magnetic field is in this way or the magnetic field is in the other way right, in the opposite direction okay. Now, apply Faradays law, EMF induced must be minus d psi by d t correct. And if you assume that the magnetic field B is uniform through the surface and the surface actually has a cross sectional area of S, then this can be written as minus S d B by d t okay. And EMF of course is a scalar quantity.

So, there is no vector out there. So, you can just take the magnitude of the B and then integrate this or whether the part of the B and then integrate this one removing the vector part to this. So, if I do d B by d t what do I get? I get minus alpha into B 0 e power minus alpha t. There is already a minus alpha so, that will cancel with each other. I get alpha S B 0 e to the power minus alpha t. This is the EMF that I am actually getting.

Now, with this EMF what should be the direction of the current? If I take the direction of the current to be along I, terminals for the induced voltage, that would have been reversed correct. If I take the induced current along the same direction and this induced current would leave the positive terminal, then the induced voltage would have been in this polarity. The bottom one would actually have the plus sign and the upper part of the conductor would have the minus sign.

And the induced current would have been in this direction. However, I know that induced current cannot be in this direction why? Because if the induced current is in the same direction as I, then the total magnetic field will actually add. And that is what we do not want okay. So, the induced current is not in the direction of I, rather the induced current is actually in the direction opposite to I okay. So, that the magnetic field produced would actually be going negative.

And therefore the total magnetic field would be reduced. So the induced voltage will be positive with the upper terminal as plus and the lower terminal as minus okay. So, this is what you would obtain from EMF. Although we should realize that this are not really critical because in most cases the magnetic field changes happen in such a way that the EMF would be positive and negative.

It is essentially AC fields that would be applied and therefore the polarity of the induced voltage keeps changing okay from plus to minus, minus to plus, plus to minus and minus to plus. So there is really no point in worrying too much about which direction the induced voltage is there, unless there is a very specific reason for this. When we discuss diamagnets, we talked about, in the module of diamagnets.

We talked about the induced magnetic moment that would be opposing the B field. There we have to careful in writing down what is the direction of the magnetic moment okay. So, here we don't really have to be that careful. However, whatever we discussed for diamagnets, the justification for magnetic moment actually comes from Faradays law and Lenz's law okay. With this, let us consider second example okay, this example is the mirror example of the first example.

The reason I am putting this down is to just indicate which direction the current would be induced okay. So, again I have a loop okay, this loops are increasingly not becoming circular but imagine that this is actually a circular loop and then the magnetic field is still again in uniform along the z direction and it is given by z hat B 0 but this time, I am going to change the file and say e to the power k t so, it is exponentially increasing magnetic field okay.

So, what would be the EMF induced? Again the magnetic field would be generated provided the current I will be in the anticlockwise direction correct. The surface area normal will also be along the z direction for this assumption right and find out what the induced voltage is. So, if this current would have been present in the loop the current would have actually generated the magnetic field along z direction. The normal to the surface is also along the z direction.

But, EMF when u calculate, turns out to be minus k B 0 s e to the power k t right. So, the EMF is turning out to be negative. What does it mean? It means that now, if you look at the current direction and if you assume the current direction to be clockwise, then it would be generating the magnetic field in the negative direction, but my EMF would also be negative right. So because of that, the induced current must be in the same direction as this one right.

So, if the induced current is in the same direction, then it would generate a positive magnetic field. So, that the overall EMF would actually be reduced. So, what would happen to the science of the induced voltage? That would be minus plus v induced voltage okay. So, in the previous case d B by d t was negative and in this case, d B by d t is positive. So, the sign conventions or the current directions for these two would actually change okay. So that is what we have.

As the third example, consider an alternating field applied okay.

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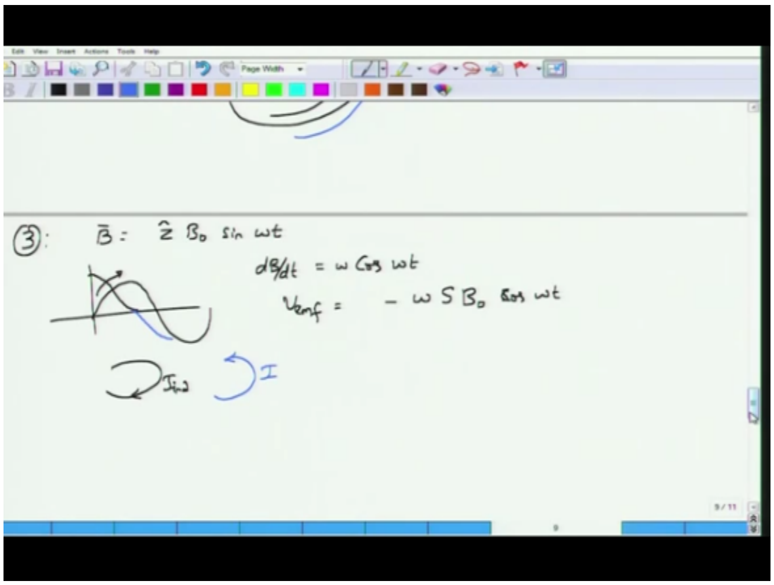

In this, we are going to apply a magnetic field of z hat B 0 sin omega t okay. So, during the first half, at least the magnetic field is actually increasing, which means that d B by d t will be, in this particular case d B by d t will actually be cos omega t and cos omega t is decreasing but it is still positive right. So, during this half, cos omega t is decreasing, but this is actually positive and what would be the EMF induced?

EMF induced will be minus omega so, d B by d t is actually omega cos omega t. So, it would be minus omega cross sectional area is B 0 cos omega t okay. So, again the induced current directions must be in such a way that initially it would be along I, in counter clockwise direction so that, the EMF is reduced. Now, in the second half of the region where d B by d t actually goes down, the current direction should actually be in the counter clockwise direction okay.

As you can see the polarity of the EMF would actually be alternating from plus minus to minus plus okay.

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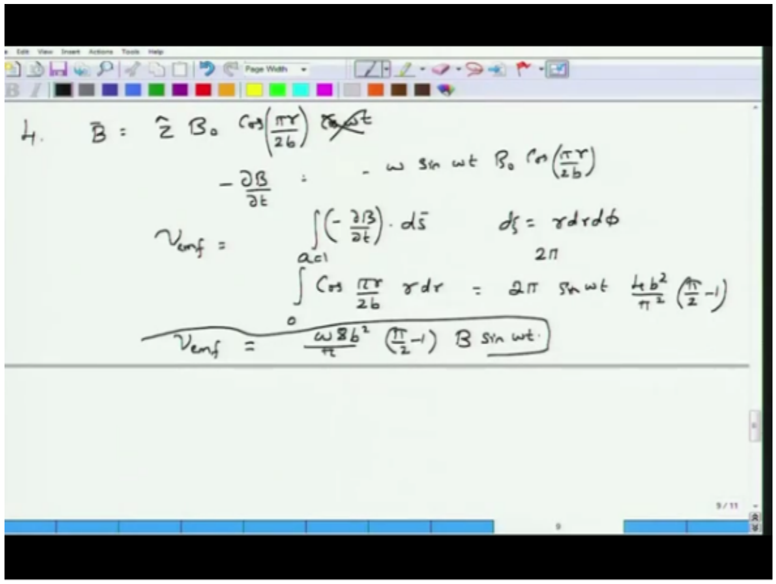

As another example okay, consider a inhomogeneous magnetic field okay, let us assume that B field is given by some z directed field okay. Except that this would be changing with distance okay. So, cos pi r by 2 b cos omega t okay. However, as far as the right hand side of Faradays law is concerned, I am only interested in minus d B by d t right. And this minus d B by d t, assuming uniform cross section for the loop will be given by minus omega sin omega t right.

And there is also the other part of B 0 cos of pi r by 2 b in here correct. Now, what is the EMF produced? EMF produced is integral of this minus d B by d t over the surface right. What is the surface? The surface integral will be the d r d phi surface integral right. D s will be equal to r d r d phi, integration over d phi gives you 2 pi, integration over r should give you a different value. So, it would be actually be cos of pi r by 2 b r d r integral from 0 to a.

If a is the radius of the loop or b is the radius of the loop let us take. So, this would actually be equal to from 2 pi sin omega t and there is some 4 b square by pi square pi by 2 minus 1 or if you take a is equal to 1 in this particular case. So, if you do all this calculation, you will see that the EMF is actually given by some omega 8 b square divided by pi, pi by 2 minus 1 and in terms of B, this would be B sin omega t, because the differential of cos is sin okay.

So, this is the EMF that is induced. What is interesting in this is that except that this was an exercise for finding the flux of a non homogeneous magnetic field. What is interesting is that if we turn off the time variation of the magnetic field and we replace this cos omega t by a constant value, there will be still no magnetic flux link that is changing with time and hence there will not be any current induced correct.

There will not be any current induced, the current will only be induced only when the magnetic field is changing and that magnetic field must be changing with time, it can be for homogeneous field or inhomogeneous field, does not matter okay. As long as time variation is there, only then there will be current induced okay. Again the current direction will be different for the first half, the current direction will be in 1 count a clockwise or count a clock wise.

And for the other one, it would be clockwise direction okay. So, this change would be happening depending on what your sign for d B by d t is okay. So, if you are given the values of B and omega, you can actually find out what will be the total EMF that is induced, but the point here is that, homogeneous field or inhomogeneous field, both will give you induced currents as long as they are changing with time.