Electromagnetic Theory
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Lecture - 41
Magnetic materials – I (contd) and Boundary condition for Magnetic fields

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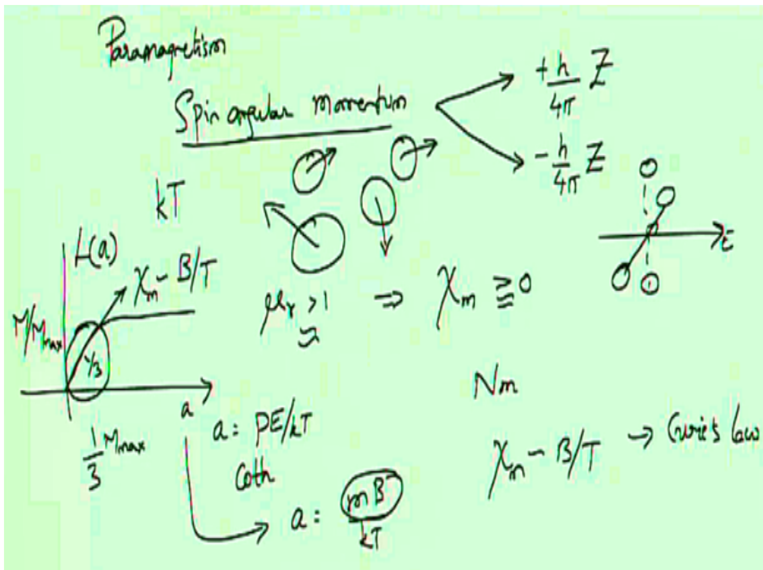

A second type of material is one which actually is called paramagnetic material. Paramagnetism arises because of a different type of spinning of an electron. One spin that we have talked about is how the electron would revolve around the nucleus. However, if you look at quantum mechanically, each electron possesses in addition to this angular or vital angular momentum, at different angular momentum called as spin angular momentum.

This spin angular momentum, there is no classical analogy for this one. Although some ideas of spinning top has been used, but there is essentially no classical way of explaining why there is spin angular momentum. And the surprising fact about spin angular momentum is that, it can only come in two flavored. It can be plus h/4pi or it can be minus h/4pi or its integer multiples. It can only come in these two different flavours or these kind of multiples of h/4pi.

And if you consider the ground state of this spin, then it would be h/4pi and minus h/4pi, with Z is equal to one and Z being the integer. These spin angular momentum imparts a certain permanent magnetic dipole to the paramagnetic materials. However, the paramagnetic materials under room temperature possess so much of thermal energy that this dipole moment would be completely randomized.

So, if thermal energy is basically kT with T being the temperature and this value is so large for paramagnetic materials at room temperature that the effect of spin angular momentum giving you a dipole is totally lost. This dipoles, if there was no thermal energy, you would think that they would all align nicely and paramagnet essentially becomes a magnet, but unfortunately at room temperature.

And when the temperature starts increasing, the energy that they receive is so much that the dipole moments, all are completely, randomly oriented giving you a net magnetic moment or a net magnetic field, which is very, very weak. The only difference is that mu r is greater than one. But even there mu r is very, very close to one, except that mu r is positive, but very close to one. This of course also implies that chi m is slightly greater than zero.

There is actually a very nice interesting way, if you remember what we did for dielectric material, we actually had a function called Langevin function. Langevin function was PE/kT, in terms of PE/kT. For small values of a, this was one over three and for larger values of a, it kind of saturated up there. It was some hyperbolic cot or something that we actually found that out. This is for the increasing values of a.

A similar expression and a similar equation applies here, there is a maximum alignment that is possible for the paramagnetic materials. But this maximum magnetization is never, you know, it is actually very difficult because the larger the value of a they quicker they would actually go to the maximum alignment. And the funny part is the maximum alignment is not the maximum that you can actually extract from.

We will not go into the details, but if N is the density and m is the maximum magnetization, for most paramagnetic materials you are basically sitting in this region. So there is a one by third maximum that you can actually obtain, not any more. And instead of PE/kT, where PE was the dipole moment multiplied by E, in this magnetic case a is given by mB/kT. Now, you could easily see that m is the magnetic dipole moment and B is the magnetic field.

And this multiplication would actually give you the change in the energy when you apply a magnetic field. If you remember that dipole moment, the dipole moment was like this and it would, when you applied an electric field, it would actually have align from its initial position of being perpendicular to the electric field, so similar thing over here. The spin angular momentum can be thought of as initially aligned perpendicular to the applied field.

But under the action of the field they would get slightly aligned with itself, but not much more because of the increasing kT value. So the value of kT is so large that the numerator component is quite small. And the value of a is small, for which you have one by three. If you draw a line here, you see that linear relationship coming out. And for the linear relationship, you will see that chi is essentially B/T.

The magnetic moment is B/T because this is a. a is mB/kT. So, a is proportional to B/T. And the susceptibility is also proportional to B/T. This relationship of susceptibility being B/T is called as Curie's law. So this is all about paramagnetic materials, not much we can talk about it or at least in the contest here. We will briefly look at another magnetic material called as Ferro magnets. As I said, Ferro magnets by themselves are very important in magnetic materials and magnetic circuit design.

But their understanding of Ferro magnetic materials is actually quite a bit intricated.

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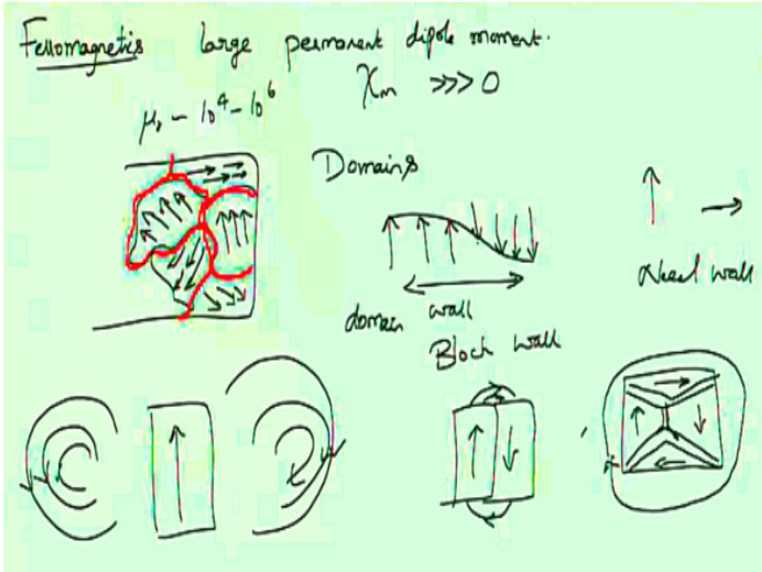

So for our purposes, what are Ferro magnets? Ferro magnets are those, when you actually talk of a magnet, you are actually talking of a Ferro magnet. Ferromagnetics have those which possess a large permanent magnetic dipole. Their magnetization is very, very large in the order of ten to the power four to ten to the power six. So, their mu rs are in the order of ten to the power four to ten to the power six.

Meaning, they possess a large value of chi m and a large permanent dipole moment. Chi m is positive, much, much larger than zero. If you look at the internal structure of a Ferro magnet though, you will see that a very interesting thing happens. There are ways in which you can actually measure this. So, the dipole moments inside a Ferro magnetic material actually spontaneously align themselves.

But they do not align in the same direction. You can actually see groups of similarly aligned dipole moments. And then from one group to another group, they would actually change. Within each group, there all aligned in the proper, in a same direction. So, these different directions and from one group to another group, they would actually change over. In the absence of any external magnetic field, this is how the Ferro magnet look.

It still produces a magnetic field. It still produces a magnetic field because there are groups, which are aligned in this way. However, even a weak external magnetic field can actually align the dipole moments of a Ferro magnet nicely, so that the overall magnetic field is greatly enhanced. It actually concentrates all the flex inside the magnetic material. Well, we said groups, but in magnetic material literature, these are called as domains.

Each domain is characterized by having a magnetic dipole moment aligned in a given direction. But from domain to domain, their directions will differ. Of course, I have shown in a very crude way over here. There is actually a wall between each of these domains wherein the magnetization actually changes smoothly from one value to another value.

The way it would change is that change happens to be like, in one domain the magnetic field is, I know in this direction and in the other domain the magnetic field is in the other direction, downward direction. So, you need to actually go in this way. So, you have a magnetic field that is changing in this way, smoothly changing from positive to negative let us say or from up to down.

This such a wall or such a transition is actually a certain wall width and this is called as the domain wall. So, you could actually have two variations. One is this way. The other one is only ninety-degree change. So, accordingly one is called as a kneel wall and other is called as a block wall. I, kind of tend to forget which one is which. But that is not really important because we are not going to discuss any more of this.

The important point is, you know Ferro magnet, the dipole moments aligns spontaneously into the form of domains, one domain to another domain there is a change in the magnetization. The wall width is quite small. However, the wall can also be made to move by application of external fields. The way the domains are formed can be understood, if you look at the bar magnet. So if you have a uniformly magnetized bar magnet.

If you look at the fields outside, there will be a large amount of fields. However, what Ferro magnet does is to try and minimize this external field. In the absence of any magnetic material, they would try to minimize this external magnetic field. And the way it thinks it can be minimized is, if you take two magnets with opposite dipole moments, what would happen with this, the external fields would be minimized, the field would be weak.

And it would still be there, it would not be that it is gone, but it would essentially be very weak outside. However, if you take a bar magnet, I mean, if you know instead of having only two poles of dipole moments, if you assumed that there are additional types, so in this the dipoles are aligned horizontally. Here they are aligned vertically downwards, here they are aligned left, here they are aligned to the right.

They essentially form a current loop in such a way that the magnetization vector simply rotates inside and there would not be any external magnetic field. These are the domain walls that we have written here. This is how the domains are qualitatively formed. Of course, you need to really look at the way they are analyzed quantum mechanically to understand completely how Ferro magnets work.

In terms of that, compared to electric fields, magnetic fields are really, really quantum mechanical in nature.

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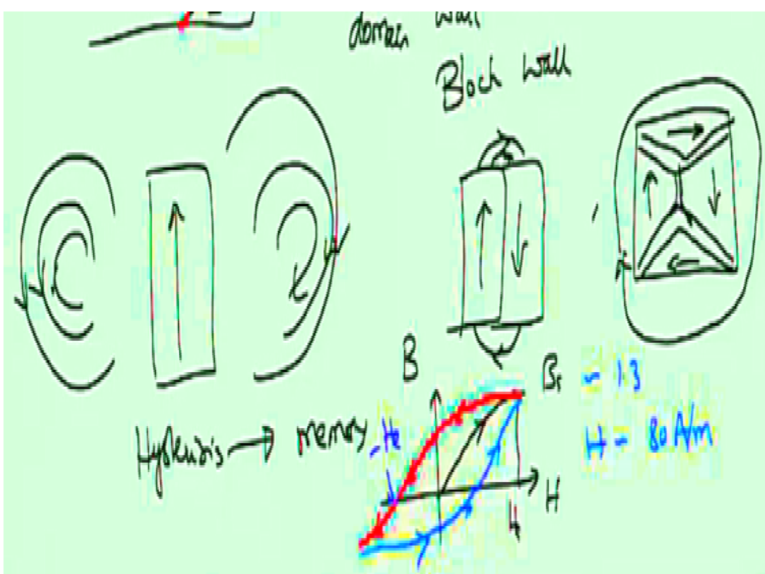

The second important aspect of a Ferro magnet is what we call as Hysteresis. Hysteresis is sometimes colloquially called as memory. We say that magnet has a memory. But what it means is that if you take the B Verses H curve for a Ferro magnet and initially start increasing the magnetic field H, you would go along a particular curve, you would reach a maximum value called the saturation magnetization B s, at saturation H field H s.

There would not be exact saturation. You will not be able to reach the saturation. But for all practical purposes, you are very close to it. Then what happens as you start decreasing the magnetic field H? What happens is that, it would not follow the previous curve, but it would rather go in a slightly different direction, still reduces, but it reduces in this way. Note where the magnetic field is going to zero.

That is if you actually go back, that is if you went from origin, increase the magnetic field until H s came back your magnetic field B will not be zero. To make magnetic field B go to zero, you need to apply an extra magnetic field what is called as coercive field, in order to coerce the magnetization B to zero. Then when you start re increasing the magnetic field, it would actually go into a different direction. This is called as the Hysteresis curve of a magnetic material.

This is for the pure iron, so Bs is around 1.3 weber/meter square and an H value of about, just about eighty amperes/meter is sufficient to drive the magnetic material into saturation. So, these are different magnetic materials. There are additional magnetic materials called as anti-Ferro magnetic materials and Ferry magnetic materials. We will not discuss them. This is not the right course for that. Instead, we will now look at the boundary conditions for the magnetic materials.

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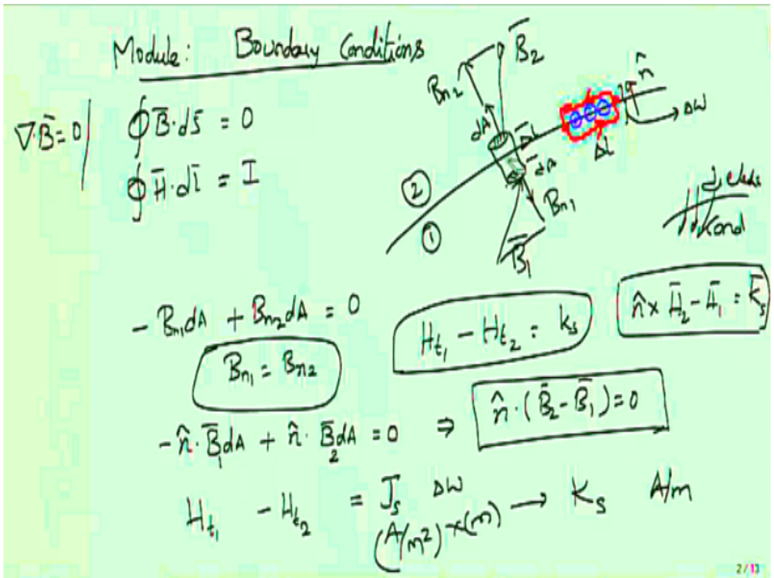

So in this module we will briefly look at boundary conditions for the magnetic fields. Remember boundary conditions for electric fields? For electric fields, we have two laws, the Gauss's law for the vector D and the Curl law for E, are their equal integral forms. We applied those integral forms across a boundary and we found how the electric and electric flux density vectors, electric field and electric flux density vectors would vary across a boundary.

Similarly, we have two laws here. We are going to use them. One law states that B dot ds is equal to zero in the integral form. This is basically del dot B is equal to zero in the point found. And the second law is that, integral of H dot DL is equal to I, whatever the current that is enclosed. So these two laws, how would they describe the behavior of B field or H field at the boundary is what we want to consider.

So consider a medium. We can assume that you have these two medium. And how did these boundary conditions come from? You remember there were two points and two points were separated by a line and the line was bisected by a plane. That is how we described boundary conditions, very true, we are going to do that here also. So, if you apply integral of B dot ds is equal to zero, to apply that you have to imagine a Pepsi can.

So, you have to imagine a Pepsi can. And then there is this area dA. This area dA is pointing in the direction of two. This is pointing in this direction. If you assume the normal goes from one to two or may be normal goes from two to one, then dA in the second medium is one to two. So, if the normal is going from one to two, you can see that dA in the second medium is in the same direction as n, in the region one it is in the opposite direction.

So, we will assume that this length delta L goes to zero later on. So, if you have a magnetic field that could be two different values over here, what is the component of the magnetic field that is important to us in this expression? Only the normal component because I am considering the integral over surface area. So, only the normal component, which means I need to consider B n 1 here, I need to consider B n 2 here.

So, what will happen to this integral of B dot ds? That would be B n 1 dA, which is in the minus direction plus B n 2 dA is equal to zero. It is zero because this volume of the Pepsi can that we are considering does not enclose any magnetic charges. There are no magnetic charges, although in the previous module we used fictitious magnetic charges. So, there is no real magnetic charges. Therefore, this will be equal to zero.

This simply indicates that B n 1 is equal to B n 2. So in the vector form, you can write this as minus n dot BdA plus n hat dot Bd, sorry this is B 1 and this is B 2, this is equal to zero, which implies that n hat dot B 2 minus B 1 is equal to zero. The normal component of B is continuous across a boundary. Now, the second equation is telling us that the integral of h dot dL must be equal to the total current enclosed.

Now, when can we have a current enclosed? Only when there is a thin conductive layer. So, you can have two dielectric medium in between. But if there is one conductor and one dielectric, then there can be a current. This is on the plane of the separation between dielectric to conductor. You can have a sheet current. So, we are going to consider this particular case to derive the boundary condition.

We will assume that there are these currents, which are on the surface, constituting a surface current density or surface current sheet. So with that, if you now apply the ampere’s law, this is essentially ampere’s law, to this loop let us say in this way, what you should see is that, if there are currents coming out of this page, then there will be a magnetic field. So, if the current is coming out, then imagine that these current lines are all coming out to you like this.

Then there will be a magnetic field, which would be circulating like this. So, currents are coming out, so the magnetic field is in the direction in which we have written them down like this. So, apply now the ampere's law. So, inside the first medium, H will be the tangential component of H, will be H tan for example, along some length let us say this is delta L or and this is delta W, the width of this one is delta W. As before we will actually make this delta W go to zero.

So, you have H t 1 delta L minus H t 2 delta L, must be equal to the current density on the surface, which could be represented as J s, multiplied by delta L delta W. Of course, you will divide this both sides by delta L and delta L is non zero, so that goes away. Sorry, this H t 2. And you will left with J s into W, delta W. J s is ampere/meter square, which is the surface current, multiplied by delta W, which is in meters.

So the net result is that you get a sheet current density or current sheet, which is measured in ampere/meter. So, the second ampere's law of integral of H dot dL has actually given us the second boundary condition, which states that H t 1 minus H t 2 is equal to K s. The corresponding expression in terms of n would be n cross H 2 minus H 1, will be equal to the sheet current density K s.

So, these are the corresponding boundary conditions that we were looking for. If one medium is magnetic and the other medium is nonmagnetic, we will see that essentially the same kind of relationships would exist. Because if one is magnetic, B 2 will be zero, so the magnetic field would be just continuous across that one. The normal component of the magnetic field be must go to zero there. It does not mean that the tangential component has gone to zero there.

However, outside if there are no currents, if between the magnetic material and the nonmagnetic material there are no currents, then the tangential H field will be continuous. But that would not be really happen for, there will be a current that is induced and that current will maintain the continuity for H field, because outside the magnetic material, H t 2 will also be equal to zero. So, whatever the H field that could be there, that would result in an induced current sheet.

It is obvious. We talked about magnetic matter having been composed of this tiny, tiny current loops. And on the surface, what are these current loops doing? These current loops are giving you a effective current sheet on the top layer and on the bottom layer and on the surface, depending on what kind of magnetization is there inside, you end up having a surface current or a sheet current because of this magnetic materials.

So this was about the boundary conditions. There are some fairly simple problems that you can solve.