Electromagnetic Theory
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Lecture - 40
Magnetic Materials – I (contd) & Magnetic Moment

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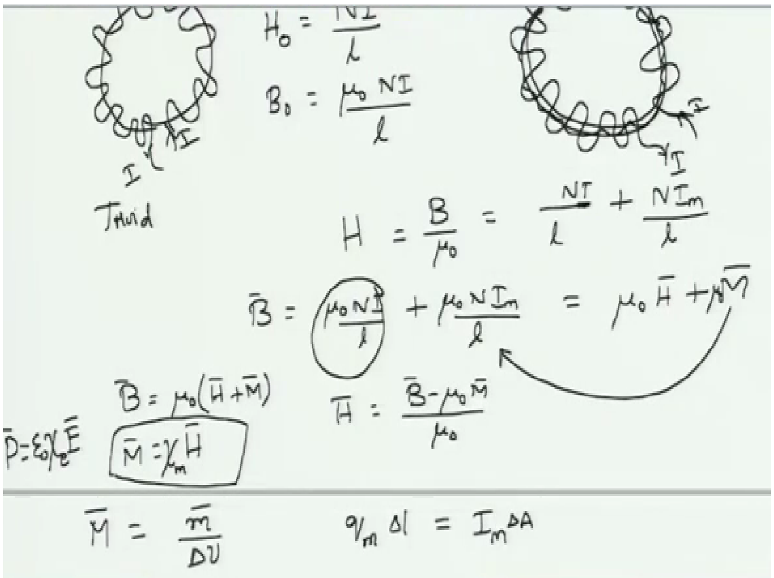

Before we can actually discuss this different Magnetic material we need to very briefly talk about slightly more about the magnetization vector m itself.

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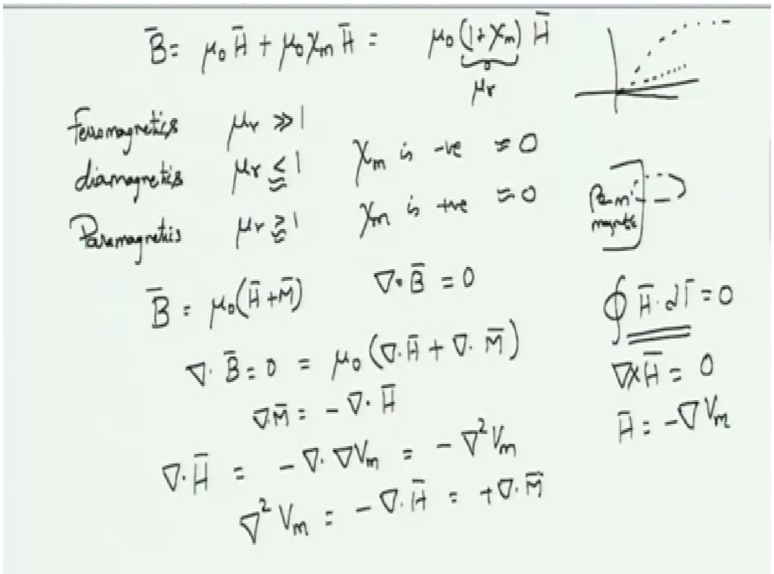

Now before doing that finally let us look at relationship between B and H. B is mu 0 into H + mu 0 into m but if you take m is equal to chi m into H this becomes mu 0 chi m into H and this becomes mu 0 into 1 + chi m and you can call this factor in brackets as mu r the relative permeability.

And this relative permeability has his different value for different type of materials for ferromagnetic materials which are what you normally mean when you call a magnetic material mu r is very, very large compare to one. It is a very large value of mu and this is the reason why they can actually generate magnetic fields. Of course, above a certain temperature ferromagnets lose your magnetism but we do not worry about that now.

It turns out that all materials in nature are intrinsically diamagnets. Diamagnets are characterized by having there mu r value actually less than but very close to 1. When can mu r be less than 1? When chi m is negative okay. But it is very close to 0. So that the overall mu r is very close to one but it is actually slightly less than 1 about point .999 that would be a value for a diamagnet, okay.

And finally you have Paramagnets, Paramagnet materials have a value of mu r which is greater but very slightly greater than one, chi m is positive here but it is very merely equal to 0, okay. So if you look at the graph for chi m versus H chi m for a diamagnetic is been negative; chi m for paramagnet will be positive and that of the ferromagnets it would be would be large, okay. So this is how you differentiate between these three class of materials.

We will consider diamagnet, paramagnet and ferromagnets but before doing so we need to consider a small detail about the magnetization vector m. Now, recall B give by mu 0 H + m okay. I also know from the point form of Gauss's law for magnetic fields that del dot B is equal to 0 which means that applying the divergence on both sides you get, correct? And what this implies is that del dot m is equal to –del dot H, okay.

In a permanent magnet what happens is that there are no any external currents so if you look at the relation Ampere’s law it becomes clear that H dot dl will be equal to 0 because there are not currents to enclose. This equation is actually true even when you consider a magnet, so this is a permanent magnet that I am considering. Even if you are loop goes through some area of the magnetic material inside.

There will not be any current enclosed and therefore this integral of H dot dl will still be equal to 0, okay. So because of this and the corresponding point form relationship that Gauss's law of H is equal to 0 you can actually express H as the scalar magnetic potential--

gradient of a scalar magnetic potential Vm, okay. And you can actually derive for similar equation for H and Vm just as you would have derived it for a electric material and called it has position for Laplace's equation.

So that is del dot H will be equal to –del. gradient of Vm this should actually be equal to –del square Vm. And you also know how to solve this del square Vm is equal to minus del dot H, so you have this Poisson's equation which is minus-- sorry del square Vm is equal to –del dot H and del dot H is actually equal to –del dot M so del square Vm will be equal to del dot M, right.

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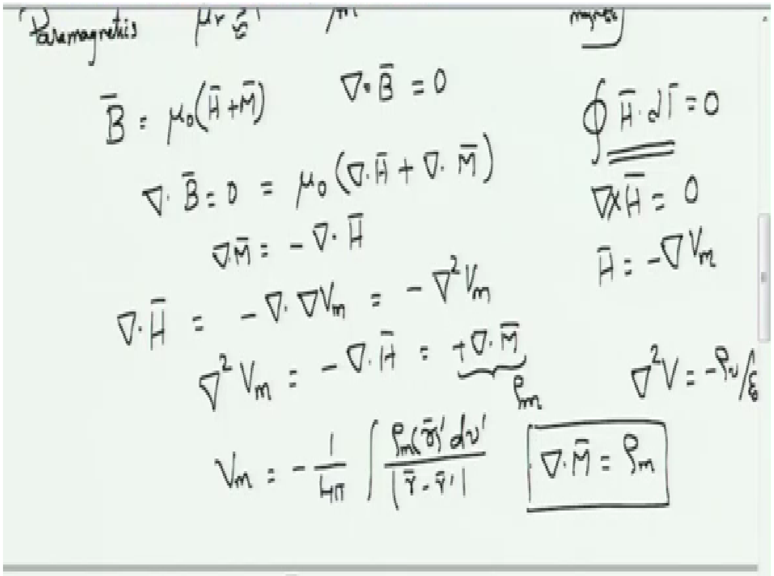

If you think of this as some charge density-- magnetic charge density rho m then the expression for Vm the magnetic scalar potential will be -1/ 4 pi integral of rho m at the source point dv prime divided by r-r prime. Why is there a minus sign here because if you remember the solution for electric potential del square V is equal to – rho v—rho v by epsilon 0 you would have seen that there was no minus sign there.

So because there is no minus sign in this expression del square Vm is equal to del dot m this expression will be having a minus sign. Of course rho m of r prime is actually divergence of magnetization vector m, so clearly what we have is del dot m is equal to rho m, the volume magnetic charge density okay.

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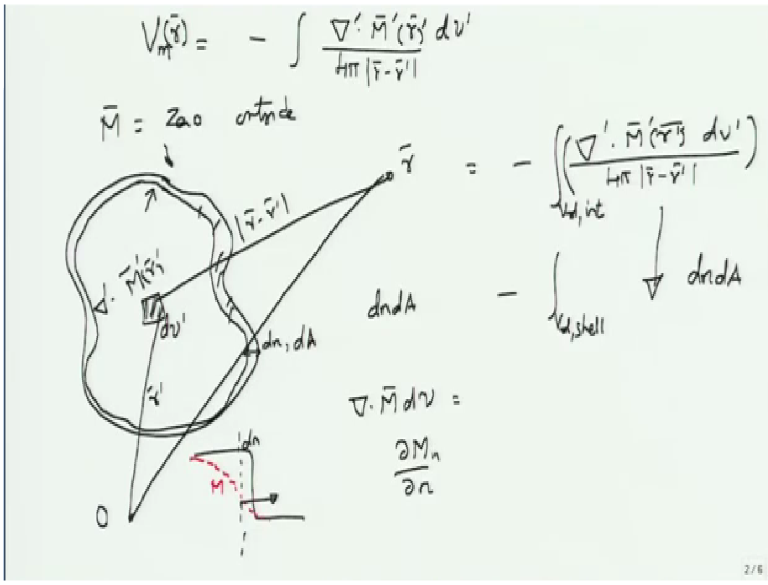

So with Vm written in terms of the magnetic charge density are equivalent in terms of the magnetization vector del.M and this dash actually indicates that this is consider to be the operator with respect to the field-- with respect to the source and not with respect to the field. So you have dv prime divided by 4 pi r-r prime this would be the magnetic potential at the field point r.

For a magnetic material turns out that M is equal to 0 outside of the material, so if you consider magnetic material M is equal to 0. This poses a slight trouble in interpreting this equation. Why? Consider a certain magnetic material and there is in the interior if you consider any volume element dv prime and you are looking at the magnetic field that this point r r.

So you can with respect to all of these measure with respect to the origin r then this is r-r prime, again we are going assume that this r is must larger than r prime. That is you are looking at a very far away distance from the magnetic material. Now if your volume element lies entirely this side the magnetic material than this is no problem okay. So your equation can be split into two ways one equation simply works inside so volume interior where it has del dot M prime, r prime, dv prime divided by 4 pi r-r prime.

There is no problem here, so here looking at the value of M prime or rather del dot M prime at this point at r prime. And then you are summing the contributions because of this interior volume magnetic charge density. However, at the surface what happens is that you are now in a slightly difficult position because magnetic material is 0 outside but magnetization-- magnetic material is 0 outside but magnetization inside it is not 0.

So there is some sort of a jump discontinuity that is happening. To consider the jump discontinuity to describe how to work around with that one you imagine a very small shell that you are placing; this shell has a width of dn which is direction—which is the small change along the normal to the surface at this each point of this magnetic material that is a normal to the surface and around that normal to the surface imagine that there is a very small distance dn, okay.

And there is a area of this shell itself to be dA. So what is the volume of this thin shell? The volume of this thin shell is dn into dA. And this integral we have done – one part of the integral entirely the interior and there is a next part of the integral which is on the shell which is essentially the surface integral that you have. Again, the integral will not change so this would essential be the same integrant except that you are integrating with respect to dndA rather than dv prime.

And remember magnetic-- magnetization is completely 0 outside the magnets and it would be non-zero inside so there is that jump discontinuity. If you are lucky then the jump discontinuity would be of this form, it would very smoothly go to 0 so the magnetization M around the boundary would go very smoothly to 0. In practice, what happens is this is the sort of jump discontinuity that you are going to get.

So if you imagine a small area dn which is what you are looking at then the change in the magnetization over the small shell along this length dn is given by del dot m dv. If you remember what is divergence, divergence is essentially del M by del of whatever the ways so here you are looking at how the normal component of the M changes as you go along the normal to the surface, okay because we are always interested in the normal component.

If you apply the divergence theorem here you can convert the del dot M dv integral into M.ds there you will see it is only the normal component that is of interest to us, so this becomes del dot M dv is actually given by del Mn the normal component of the magnetization divided by del n which gives you the change in the magnetization Mn which is the divergence multiply this one by the volume of the shell which is del dndA.

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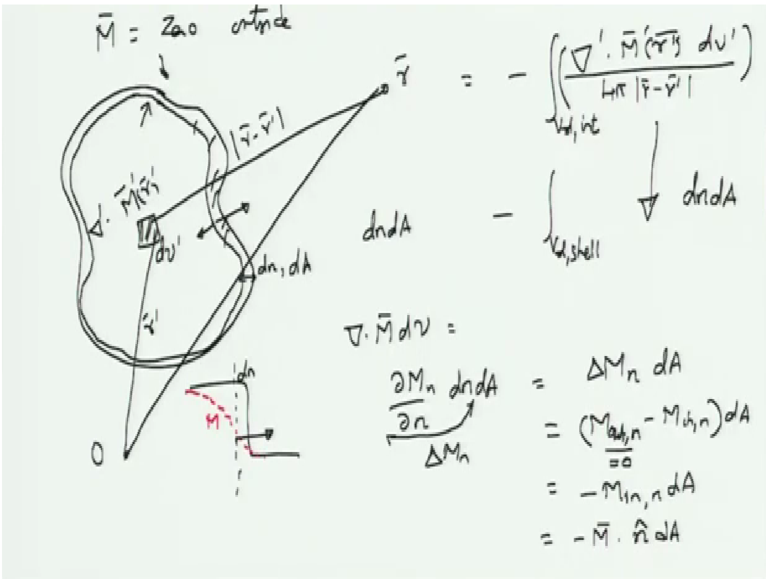

Now this is nothing but the total change in the normal component of the magnetization multiplied by dA delta mn is this quantity, this is delta Mn. And I already know that this change in magnetization have to happen only because of the internal magnetic field, right. So this delta Mn which is the normal component the outside of the shell minus normal component of the inside of the shell multiplied by dA, right this would be the delta Mn into dA.

But I know that this quantity is equal to 0 so what I actually have is magnetization in the interior near the shell the normal component of magnetization in the interior near the shell multiplied by dA. So you can actually rewrite this and say –M dot n dA where the normal is prompting outside but –M dot n actually points inside, okay. And this fellow if you substitute into the second expression will become the surface charge equivalent surface magnetic charge, so it actually become--

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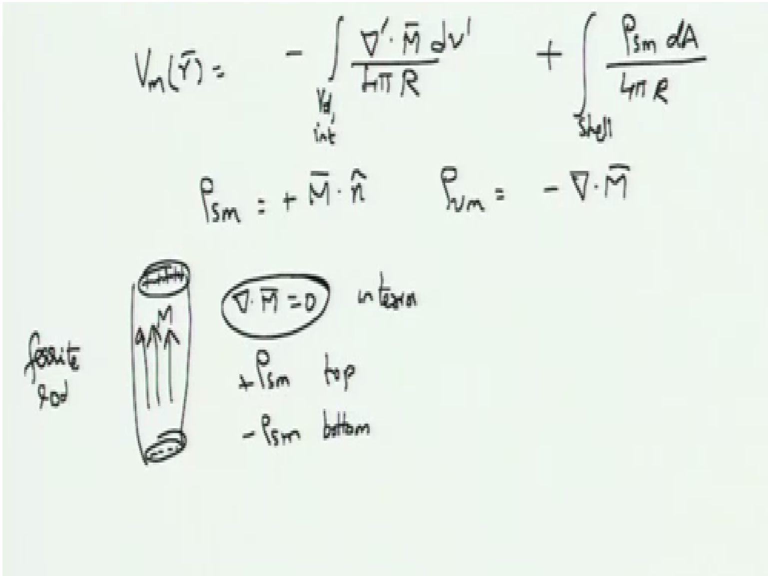

The first integral will not change so I have Vm of r given by minus integral of del prime dot n dv prime; I have removed that m of r prime and everything and then you have 4 pi r okay this is the interior volume integral that you are doing plus there is a corresponding M. n dA, right so you had this integral of the volume that would change to M dot n dA l dot M becomes M dot n dA.

So this becomes the surface charge density of rho sm over the surface integral of the shell 4 pi r this is the shell surface integral. So you have actually obtained equivalent surface charge density which is M dot n + M dot n, minus sign is counted for the divergences and this one. So you also have obtained the volume charge density which is –del dot M. Clearly, if you take a rod magnetic material which is uniform magnetization.

There will not be any divergence of M inside so divergence of M is equal to 0 in the interior of this rod which would be ferrite rod for example, okay. And all the charges that you are considering will be the equivalent charges at the boundary, so they will only be plus rho sm at the top and minus rho sm at the bottom okay. So this would be the equivalent surface charge density that you will have.

We have talked about magnetization, magnetic dipole moment and we have talked about equivalent magnetic charges and equivalent magnetic-- volume charges and surface charges. And exactly similar equivalents-- equivalent relationship can be written for currents as well, okay. So in that case we will see that it would essentially turn out to be an inside volumetric current and, on the surface there will be a current density the surface current density or the sheet current, okay.

"Professor to Student conversation starts" I am not going to do that part of the development for us this is sufficient okay. This part is sufficient, so we are not going to look at that other one, alright. We will now study the three types of magnetic materials that we talked about in the last module. "Professor to Student conversation ends"

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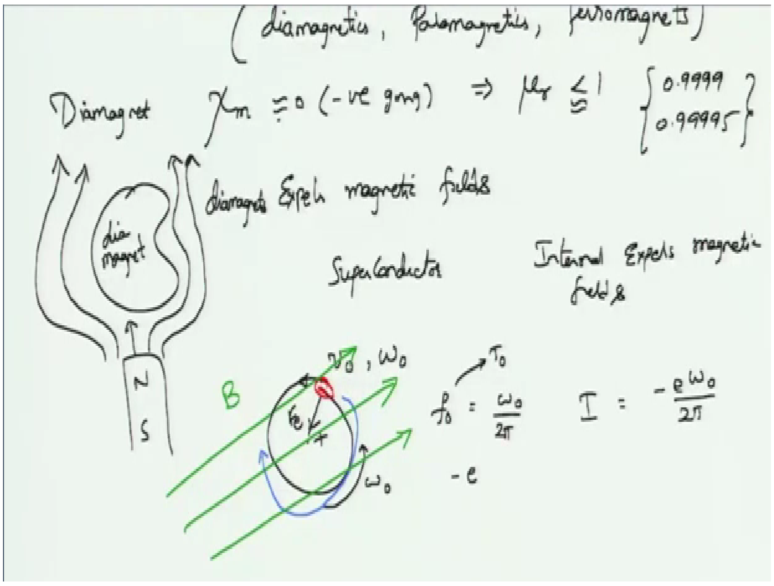

So our module here talks about magnetic material primarily concentrating on diamagnets okay diamagnetic materials, paramagnets we will just give you the brief idea and ferromagnetic also will give you the brief idea because these ferromagnets are a subject of their own, they are pretty huge subject of their own, okay. Consider a diamagnet, we have talked about a diamagnet.

And we have said that a diamagnet is a material which exhibit almost no magnetization, okay they are not magnetic materials there magnetic susceptibility is actually close to 0 but of course it is slightly negative, okay so it is negative going but very close to 0 which implies that the relative permittivity of this materials-- sorry relative permeability of this material is actually less than one but it of course it is very close to 1.

Typically values include .999 or .99995. So there is small amount of value of mu r less than 1 and these magnets or these materials are not at all magnetic material. So if you actually take a diamagnetic material and you have a magnetic material that is permanent magnet producing magnetic field what will happen with the diamagnet is that the fields would actually curl around and go outside of that magnetic material.

So the diamagnet for this reason we actually say that a diamagnet expels magnetic fields, okay. Very extreme case of diamagnet is that of a super conductor, a super conductor is an extreme form of diamagnet in the sense that the internal magnetic field is completely 0, it completely expels all of the magnetic material—a magnetic fields, okay. So what could be the reason for such a behavior? Why is chi n negative?

To really understand the origins of negative susceptibility you need to actually look at quantum mechanics but you can go reasonably far by talking about the Classical Electromagnetic Theory and that applying it to understand diamagnet, okay. So we are going to do that one. A diamagnet essentially has no permanent magnetic materials even when you apply an external magnetic material it does not relegate magnetized.

And the reason is that the magnet material actually consists of tiny, tiny current loops as we said and this current loops are all because of the electron which is circulating the nucleus right so you have an electron which is circulating the nucleus, okay. The magnetic movements are completely randomly oriented they are not arranged in any form of an alignment, there is no inherent arrangement of the dipoles, okay.

So-- which means that the net magnetic affect will not be seen. So if there is no magnetic field because of the diamagnets. Assume that this electrons are moving or revolving around the nucleus with a certain velocity v0 with given velocity v0 you can also characterized them by having a certain frequency of revaluation omega 0, right. So this is corresponding frequency of revaluation.

Now what keeps this electrons in this orbit the classical picture tells us that it is the centripetal source that is acting by-- acted by the nucleus that keeps this electron in the orbit. It is like we taking rope and spinning the bucket, okay. So the centripetal force which is directed inward radial will keep this electron in the orbit and this electron will be moving which a velocity v0 or an angular frequency of omega 0 radiance per second.

What is the corresponding frequency f0 of the revolution in so many cycles per second that would be omega 0 by 2 pi. What is the charge that is going around the charge is that of minus e being the electron charge so what will be the current I? Current I will be -e omega 0 by 2 pi. If this charge is moving with this frequency the time it takes to complete one revolution is 2 pi by omega 0.

So if there is a charge per time that is what the coulombs per second-- the definition of an ampere is, so if you apply that you are going to see that the current is -e omega 0 by 2 pi, okay. This is the electron current. The conventional current will be directed opposite to the moment of the electron conventional current is directed clockwise for example. So for such a direction what will happen to the magnetic field that are generated.

They would have actually have to point—sorry the magnetic moment to this should point along into the page, right. So in this case point into the page, okay. Briefly this is what happens. We will apply an external magnetic field, okay. So let us apply an external magnetic field. We consider two cases, one the external magnetic field is in the same direction as the magnetic moment and two the magnetic moment is opposite to the direction of the magnetic field applied. For these two cases two things happen.

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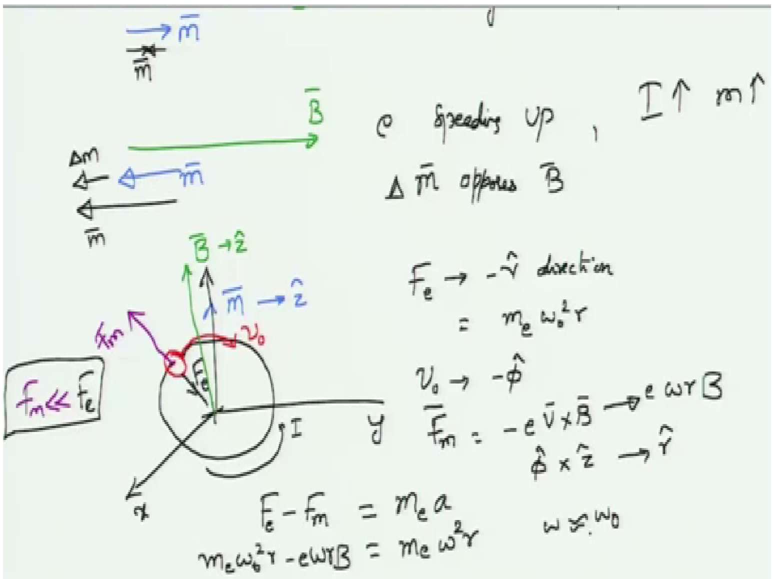

If the magnetization is in the same direction, so if this is the applied magnetic field B and this is the magnetic dipole moment m, okay then the electron basically response by slowing down okay electron response by slowing down. Now what is the effect of slowing down it means that it is angular frequency is reduced. If the angular frequency reduces because current is directly propositional to angular frequency this electrons slowing down simply means that current has reduced.

If the current is reduced magnetization m also gets reduced, right. So e slows down, I reduces, the magnetic moment m also reduces. However, the component of the magnetic moment m that decreases will be in the direction opposite to B. So if this is the magnetic movement initially then after the application to a dielectric material this will become the magnetic material.

So the change in the magnetization which was necessary to bring about this reduction in the magnetic moment is directed opposite to B, okay. What happens if you consider the other case? Now with the other case, the magnetization, the magnetic field will be is in a particular direction but the initial magnetic movement is in the opposite direction. The electron actually responds by speeding up which means that current goes up, m also goes up.

Why should m go up? So if m goes up then it will produce a magnetic field that will oppose the magnetic field that is applied, okay. So that total magnetic field acting, so you have a large magnitude over here another large magnetic over here but after the application the magnitude opposite has increased further which means that the net magnetic moment has or net magnetic field has decreased, okay.

Again, we can show that the magnetization m will actually be in the direction that would be opposite to d so m opposes d or rather the change in the magnetic component opposes B. So what you have is diamagnets are made up of magnetic material—sorry diamagnetic are those magnetic materials which you can think of as having atomic configuration in such a way that there are two electrons which are orbiting in the opposite direction.

But when an external magnetic field is applied one of them slows down the other ones picks up a speed and this change in the magnetic moment actually generates a certain equivalent dipole, okay. And we will see what happens to that will be the result of m and h very shortly. So we have seen that delta m opposes the magnetic field, okay. So using this fact that the change in the magnetic moment.

The component of the magnetic moment is in such a way that net magnetic field across the atom reduces, we will not try to derive a relationship between magnetization and the applied magnetic field H, okay. So to basically expand upon an idea that delta m opposes B consider a slightly more detailed. So I have a loop lying in the x and y plane which I am showing in this particular way. So the magnetic moment will be pointing in the direction of z.

So magnetic field is along z direction for this electron that is revolving in the x y plane, so we assume that orbit is in the x y. So imagine that the electron is actually moving in the clockwise direction, so let us write that one, so electron is moving in the clockwise direction which means that so this is my electron and this electron is moving in the clockwise direction with a velocity of v0.

The radius of this orbit is r and this velocity is v0 there is a corresponding omega 0 also. So as this electron moves it is held by the nucleus which is sitting at the center by a certain centripetal force fe. What is the magnitude of the centripetal force fe? First of all fe is in the direction of radial so it is along +r hat direction and this centripetal force is basically mass of the electron m e into its acceleration omega 0 square into r.

Now here is a slight conceptual point that you have to note down this electron mass is not exactly equivalent to the physical electron mass, this is the effective mass of the electron, okay. When you consider multi electron atoms it turns out that you have to consider the coulomb field that exists between the electrons themselves; electrons are arranged in certain shells and these electrons in different shells will actually have a coulomb field in between.

So there is essentially only a an effective field that you are going to see and that effective field is captured by specifying the effective mass, okay rather than the physical mass of the electron. Anyway that is a minor point of more that we wanted to write down. So this centripetal force is actually directed inwards and its magnitude is m e omega 0 square r and in this is one that is responsible for holding the electron in the orbit.

This is the classical picture; this is the one that is holding in this one. Now, the electron itself is moving in the direction of –phi hat, this is because the phi would actually correspond to clockwise, contour clockwise direction we assumed that electron is going in the clockwise direction so v0 is going along -phi hat. Now if I apply a magnetic field in the z direction, okay. So I will apply a magnetic field in the z direction. Now is this consistent?

The magnetic moment is along z direction. Let us look at this. While the electron is revolving clockwise the conventional current actually flows in the anticlockwise direction, contour clockwise direction and the magnetic moment for this would be along z direction, right. So x to y a magnetic moment should be along z direction, so all are finds at this point are correct.

Now with v0 going along -phi hat direction there is a second force that gets acted on the electron the moment you supply a magnetic field; the moment you apply a magnetic field there will be a magnetic force set up and this magnetic force is given by -e v cross B. V is the velocity of this electron, e is the charge in the electron and this is the magnetic force that is set up on the electron because of the applied magnetic field B.

So what direction fm will be? Well V is along -phi there is a minus sign with respect to e already so this will be phi hat and there is z direction for the magnetic field so you get phi hat cross z which will be in the radial direction; it will be outward radial direction, okay. So there is a corresponding magnetic force here which would be in the direction that would be outward and it would be the one that would be trying to pull the electron away into the radial direction okay.

If this fm was in the same direction as Fe then you try to pull the electron into the nucleus. You need to understand that this Fm magnitude is actually very, very small compare to Fe. So the centripetal force is the one that is actually dominating and this Fm the magnetic force is actually very, very tiny compare to Fe. Although if you look at the way that I have written it seems that Fm is as large as Fe, but in practices or in the atomic case Fm is very, very small compare to Fe, okay.

So that we can continue to assume that the electron is in the same orbit only that its velocity will slow down. Why should its velocity slowdown? Because the angular momentum is a velocity dependent quantity, and this angular moment have to change because the force acting on this has change.

The total force acting on the electrons has change which changes the angular velocity or the angular movement, angular movement change means the angular velocity change because the radius of the orbit has not been changed, okay. So there is essentially a total force now of Fe plus Fm or Fe minus Fm depending whether Fm is in the same direction as Fe or in the opposite direction of Fe, right.

So, for our case Fm is in the opposite direction to Fe and Fe is larger for us we have assumed so the total force acting on this electron is Fe minus Fm this must be equal to m e into a which is by Newton's law mass into acceleration. So what is the acceleration? omega square r, right. So this is m e omega square r. What would be Fe, Fe is m e omega 0 square r. What is Fm? Well, Fm magnitude is e omega r and applied field B, this the magnitude of the magnetic force.

Now you might have seen here that I actually have two different omegas, so you would actually ask why am I using two different omegas. Remember? Before the magnetic field is applied the force acting on the electron is just se and because of that, that is characterized by an angular velocity omega 0 okay and that is what the centripetal force is depending on. The centripetal force is depending on the physical act of this electron moving around the circle or the orbit and that value is me omega 0 square into r.

We are assuming that omega 0 has not appreciably changed it is not like omega 0 is phi mega hats and because of the applied magnetic field omega 0 changes to some 15 Giga huts, okay. It is essentially very close to the same value it only changes slightly. In this particular case it would increase slightly or decrease slightly depending on which way we are, so here it would in fact decrease because Fm is in the opposite direction to Fe.

So the after the application the magnetic field is actually dependent on the actual angular velocity omega, so that is why we have two different omegas into this expression but remember that omega is very, very close to omega 0.

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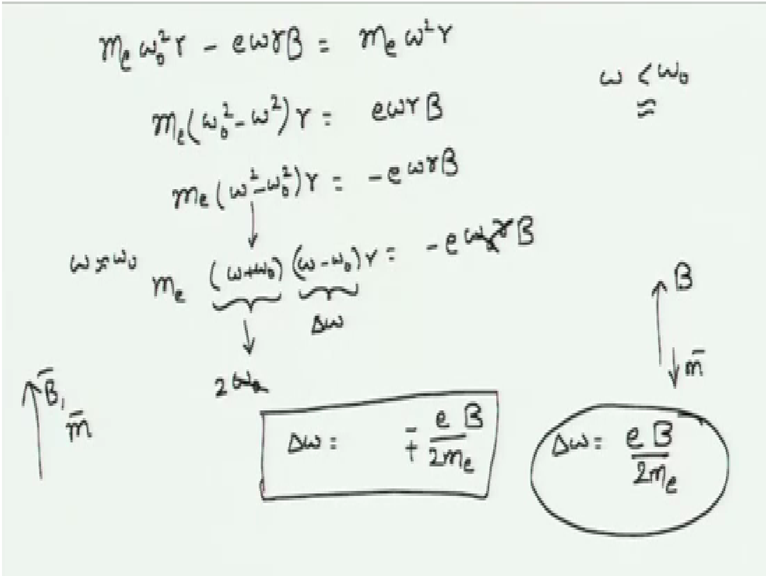

So if you now solve this equation or if you now manipulate this equation further you will see that the centripetal force was m e omega 0 square r minus e omega rB is equal to m e omega square r, okay. So I can bring this omega into this left hand side pull this e omega rB to the right hand side and say m e omega 0 square minus omega square into r is equal to e omega rB and I know that omega is actually less than omega 0 although it is very close to omega 0 it would actually slightly less than omega 0.

So I can rearrange this equation by writing this as omega square minus omega 0 square into r okay this becomes e minus omega rB. Now because omega is very close to omega zero and omega square minus omega 0 square can be written as omega plus omega 0 into omega minus omega 0. Let us call this change in the angular velocity as delta omega and because omega is very close to omega 0 the sum of this would essentially become two omega 0 okay it is like 10 and 10.

The difference between the two will be .01 but 10 + 10.1 is very close to 20 that is 10 * 2, okay. So this must be equal to -e and we are going to assume that omega is very close to omega 0 so this would be omega 0 rB, okay. So omega 0 can be canceled on both sides, r goes on both sides okay so luckily what we are now left is delta omega is -e by 2 m e into b, this is the reduction in the angular velocity when magnetization and the magnetic moment are in the same direction.

By following a similar logic, you can actually see that when magnetization is initially opposing or the magnetization is – the magnetic field is in such a way that opposes a initial magnetization m or the magnetic dipole moment m then this delta omega actually becomes plus, okay. So delta omega can be minus or plus and if you look at the magnitude, of course magnitude of change delta omega will be eB by 2me, keep this in mind because later we will see that this has a certain additional affect on the materials.

So we have written down what is angular velocity but what we want is magnetic moment.

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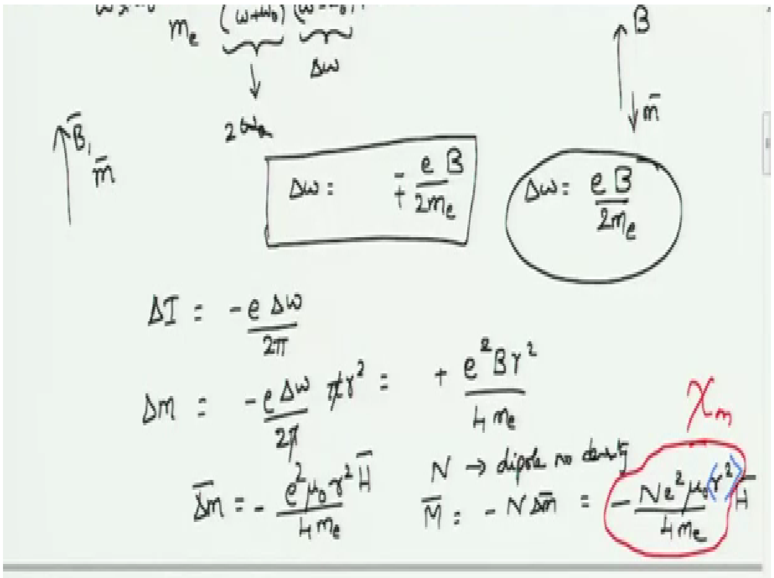

Magnetic moment is now because of the change in the current. If this is the case B is in the same direction as m the current reduces; in the other case the current increases-- in any case the change in the current given by -e delta omega by 2 pi delta omega being the change in the angular velocity divided by 2 pi is the frequency multiplied by -e and the magnetic moment change delta m will be the change in the current into the area of the orbit, the area orbit is pi r square pi goes on both numerator and denominator so you have now –e.

And if you are considering the case where delta omega is negative the magnetite of delta m becomes minus of minus that become plus and you get e square Br square divided by 2me into 2 is 4me, okay this is the net magnetic moment that you see for this case where B and m are in the same direction. In the other case, where B and m are in the opposite direction delta m will have same magnitude but it will still be opposite to the B direction, right we have seen that one.

So in terms of the vector delta m can be written as e square mu 0, this is B and H relation; r square H divided by 4me. And if there are m such magnetic dipole moments per unit volume so m is dipole density or number density dipole number density the number of dipoles per unit volume than the magnetization m can be written as n into delta m which is or rather a minus n delta m oh sorry-- yeah sorry here I have to put the minus here delta m the magnetic moment is actually -e square mu 0 r square by four m e into H.

So m is equal to minus Ne square mu 0 r square divided by 4ne into H and clearly this quantity that we have written here will be the susceptibility chi m and we can now see that susceptibility is independent of temperature, okay? This is the characteristic of a diamagent, susceptibility is independent of the temperature and it is basically negative and proportional to H-- sorry independent of H.

So that mu value will be sum 1- Ne square mu 0 r square by 4me and that mu will be less than mu 0, right. So we have shown that for diamagnetic material this chi m will be negative. Now there is one additional think that you must have realized what was happening, in this entire development we have sort of saying well there can only two directions that we are considering in one where the magnetic moment is aligned with B or magnetic moment is opposite to the B, right?

And in practice of course magnetic material there are this random orientation this cannot be true all the time, so what should happen? What will happen there will be an average angle that you need to consider or equivalently if you consider these magnetic moment orientations over the entire 2 pi or a pi angle than you are basically looking at different values of r; the effective orbit r will have to be changed.

If you want to mechanically, we consider this or even classically also we consider this affect by averaging over different values of r square, okay. So this averaging when you do it for diamagnetic material turns out that chi n is so small very close to 0, so this averaging or the randomized averaging over the random oriented dipole shows that the average will be 0 and because of this reason the value of mu r is lately less than 1, okay.

So this is all about diamagnetic material except one small additional factor that we wanted to talk about. Well we have the orbit spinning the electron circulating in an orbit, right? Then we applied a magnetic field there will also be a torque on the electron. What would be the effect of the torque? So if you imagine a loop and then you imagine applying a uniform magnetic field the effect is to actually tilt the loop. So the there will be a tilt in the loop itself the electron is now revolving the loop itself will tilt and the magnetic moment vector m will be actually for performing a precession, okay.

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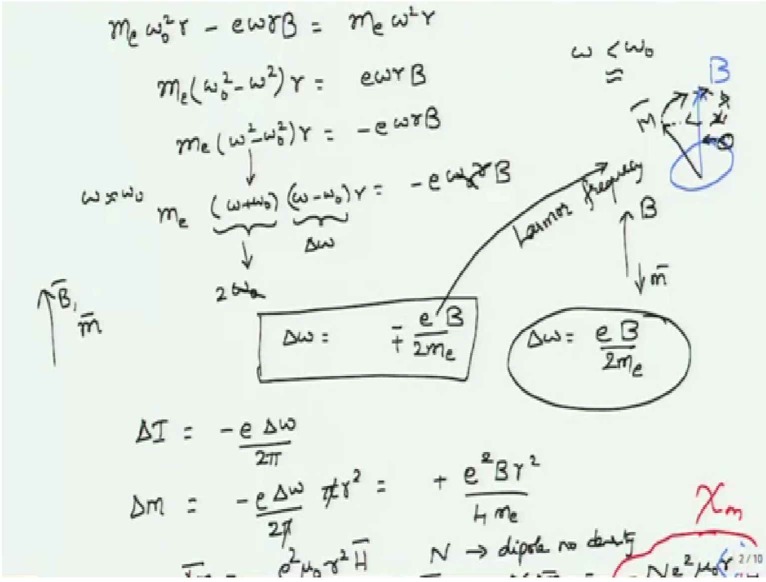

So if this is the external magnetic field that you have applied there will be a tilt in the orbit of the electron okay such that the magnetization vector m which is basically the sum of or the net effect of all the dipoles inside that magnetic material will itself start to change, okay. So there will-- it itself will start to precess around, okay. So if the electron is moving around like this than the magnetization would move around in this way.

What is the frequency of this precession is given by eB by 2me and this precession is--precession frequency is called as Larmor Precession Frequency or the Larmor Precession and this called as a Larmor Frequency. So this was all about diamagnetic materials. They are not of an interest of magnetic application because they do not possess any magnetic movement and in fact they would actually expel all the magnetic field.

Of course they actually form a nice magnetic shield if you want but as for generating magnetic field they are completely useless.