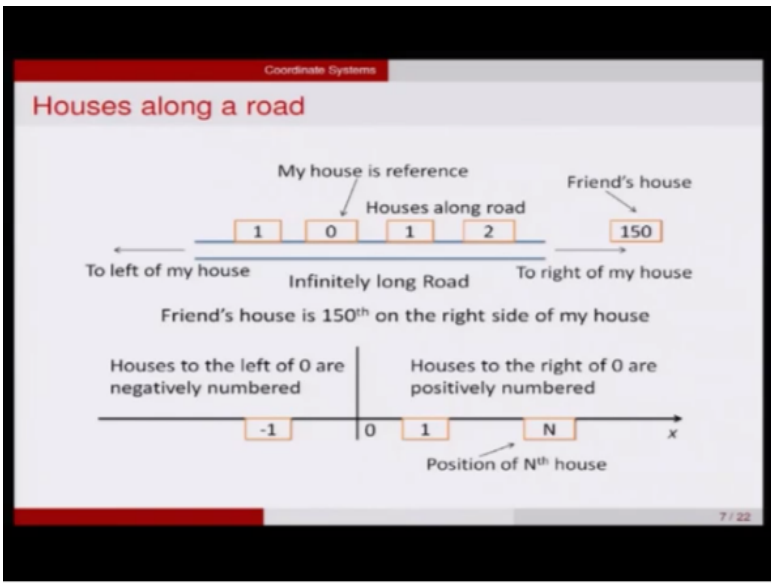
Electromagnetic Theory
Prof. Pradeep Kumar K
Department of Electrical Engineering
Indian Institute of Technology - Kanpur

Lecture - 04
Rectangular Co-ordinate System

Now, to introduce coordinates let me start with a simplest case of one-dimensional coordinate systems, okay. Let us assume that I have an infinitely long parallel road okay, which is running along this way and there are houses situated along the road, okay.

(Refer Slide Time: 00:21)



You can imagine a highway or something where the road is long and there are houses situated along one edge of the road, okay. Now, any road could be taken as a reference because this is an infinitely long road, there are an infinite number of houses and you can take any house as a reference, but it might be very easy for me to take my house as a reference, okay. So, I will take my house as a reference and I call my house as house number 0, okay.

Of course I could have called this house as 150 or 200 it does not matter, but 0 seems to be giving you a nice intuitive feeling that this is the origin, this is the reference. So, the reference could be set to 0 and I take my house number to 0, okay. Now, what I start doing is there are different houses along the road, I start labeling all these houses right. So, I label the houses 1, 2, 3, 4, 5 for example my friend's house happens to be 150th house from my house, okay.

So, this friend's house is 150th house from the reference house which is my house. So, I number them as 150. Then, I have 151, 152 and so on and it will continue like that. There are houses to the left of my house as well. So, I will label them also 1, 2 ,3 ,4, 5. Now, if I say house number 5 you would be confused. Why would you be confused because I am not specifying whether this house number 5 is to the right of my house or to the left of my house right.

So, once I fix the reference I also need to specify the direction in which I have to find this house. So, for example we have a mutual friend and I want to direct that mutual friend to one of my other friend's house right to my friend's house which is at 150 to the right of my house. I have to specify go along right of my house and stop at 150th house, okay.

So, I have to specify both direction as well as the distance or the house number which I have to go. So, this is a one-dimensional system. Now, we can mathematically represent them that is we can convert this housing analogy to a coordinate system by drawing a line and we label this line as x axis and we call this point say my house as a reference house with a number 0.

I associate my house with 0 and then I start numbering all the other houses the house to distinguish between the house to the right and houses to the left, I can say the houses to the right are positively numbered and houses to the left are negatively numbered. So, the house number one which was to the left of my house has now become minus one now I do not have to specify you know go to the right or go to the left.

I just have to specify minus 150th house or plus 150th house may be there is a house in between 150 and 151 this house could be 150.5. So, I could say “hey go to the location of 150.5”. May be there is a landmark at 203.5 or 203.8 right. So, I could then say go to the landmark 203.8 or go to the landmark at minus 153.6 right. So, I can specify that and x is the axis that I would be calling, okay.

So, what this house analogy has told you is that I can associate on this road any point right I can specify that point by giving it a number. So, I have converted or I have found the method in which I can locate a point in space by a number. I can specify that point in space by a number. For example, the position of the Nth house can be specified by giving a number N. Now, let us say, I have a friend okay who does not agree to consider my house as a reference I mean everyone would like to consider their house as the reference.

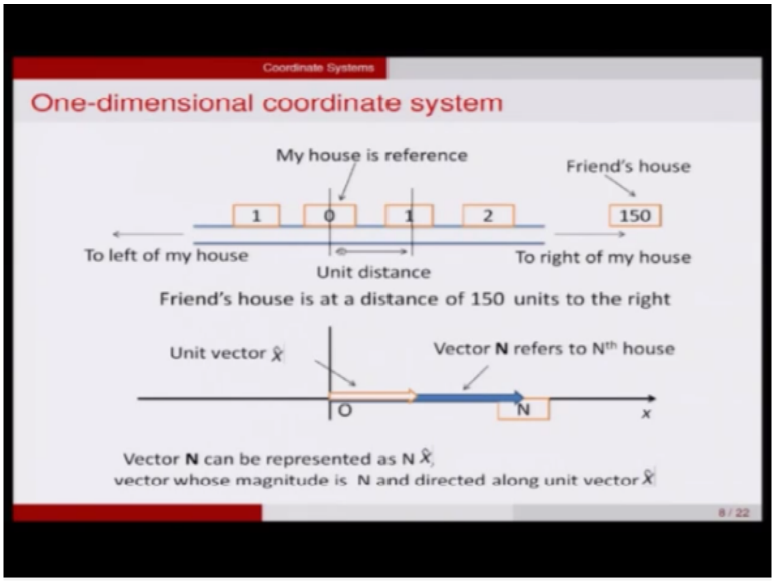
So, I have a friend who wants to consider his house which happens to be house number 1 as the reference. So, what he or she does is that they keep this house 1 as the reference house, okay. So, in their coordinate system or in their method of measuring the locations or specifying the locations of houses one becomes 0, N becomes N minus one.

Does that mean that the Nth house has actually physically changed? No, the house is situated where it was, it's number that we are specifying has become different because in my mind 0 is the reference house and for his coordinate system, 1 is the reference or 1 is the 0 for that one. So, because the origins are different the number that we come up with are different but physically this is the same house.

The house has not changed just because I have decided or my friend decided that the origin needs to be different, okay. So, I hope you get the difference between specifying a coordinate system and specifying a number, okay. This is closely related to the fact that vectors, we will soon see how to connect vectors here. So, this is closely related to the fact that vectors are independent whereas the number that we associate with a vector is dependent on the coordinate system that I choose.

So, now how do we associate vectors to this coordinate system, we have already established a coordinate system, okay and Let us say the coordinate system that I have established should allow me to measure distances.

(Refer Slide Time: 05:58)



So, I do not want to say the position of 150th house I want to say "hey my friend's house which is at 150 is actually at 150 kilometers" or it is at 150 meters or it is at 150 centimeters, okay. So, there is a landmark at 203.8 kilometers so I want to specify distances out there but you will see what I did over here, I had a distance of kilometer, meter, centimeter. How was this distance or how was this unit of measure formulated? Let us look at this. I have all these houses, it is again see that I am using my coordinate systems.

So, I can use my coordinate system and say I measure the distance from my house to my next house along the right which is say 1 and I call this as the unit distance. So, all other distances along x axis are now measured in units of this distance, okay. So this is 1 unit, so 150th house will be 150 units away from 0, again there is no specific reason why we have to specify this as 1 unit distance.

I could specify the distance between midway from my house to my friend's house or to my neighbor's house as 1 unit, then the 150th house which is my friend's house will be at a distance of 150 into two which is 300 units if I half the unit. So, it is clearly my control as to what I should call as a unit and we pick whatever the unit that would correspond to us, and we just stick to that unit and this kilometer, meter, centimeter are all the choices that we make.

So, if I take 1 kilometer as the unit, then all the other distances can be measured and will be measured as some multiple of kilometer. That multiple need not be integral, it need not always be 1 kilometer, 2 kilometer, 3 kilometer. Our landmark could be at .23 kilometers, okay. So, I hope that distinction is very well understood by you.

Now, this is a coordinate system along to the right, there is x axis, I have also obtained the units on which I am going to measure now, how do I define a vector? Consider the Nth house okay. I can specify the Nth house by giving its distance from the origin, origin being my house which I am taking as the origin or the reference I can give the position of this Nth house as N units away from 0. I can equivalently draw a vector whose length is N and whose origin is at 0 and which terminates at N.

Now, you can imagine that I am first removing the coordinate system, I just have a vector which is lying horizontally of this length 0 to N, okayay. I mean of length this line and now if I want to attach a coordinate system, I can simply imagine a coordinate system over here in which the origin coincides with the tail of the blue vector, okay. Which is what the vector N is, okay. So, this vector N can be represented as N times x hat.

What is this funny looking x hat over here? X hat tells me a unit vector along the direction of x. What is a unit vector? It is a vector which has unit magnitude and points in the direction of the coordinate system. Here, I have a one-dimensional coordinate system along x, so this is pointing along x direction has a magnitude of one, so this unit vector have indicated by this orange colored vector, okay.

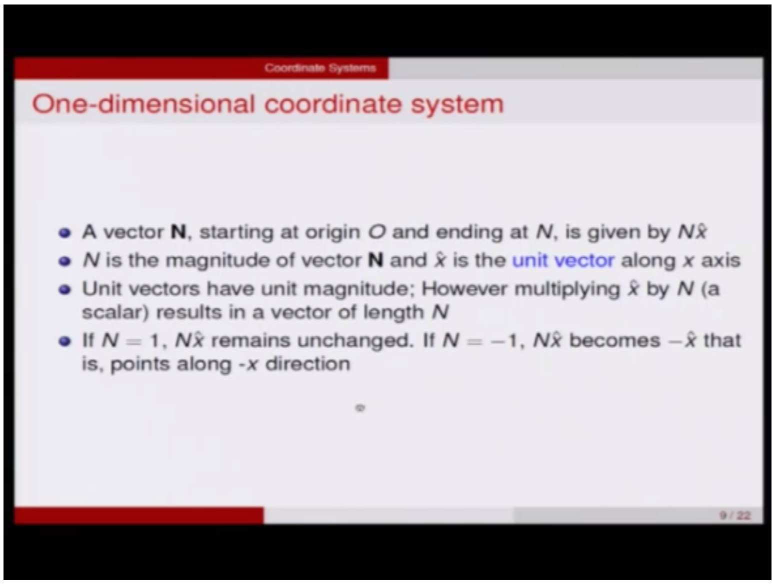
So, orange colored line arrow is a unit vector and if I take a unit vector and multiply it by a number N then I get the vector N, which has the magnitude of N and it will not be pointing along the x direction, okay. So, now, I have shown you two things one there is a position which is the location of the space along the x axis that can be specified by giving it a number.

So, if you imagine that there is no vector over here, if you imagine that there is no vector here, but just a number N that is sitting here on the x axis then the location of this point where N is sitting is Nm okay. You can have another position which would be M which would be along the x axis. Now, I have a vector with no specific reference to the coordinate system. However, I can introduce a coordinate system and say that this vector can be represented as N x hat where N represents the magnitude of the vector and x hat represents the direction of the unit vector.

So, in this case the unit vectors can only be directed along x. Now, suppose I want to represent a vector which has same magnitude N, but it will be pointing in the negative x direction, how would I represent that? All you have to do is you have to represent this as minus N x hat. You could of course take a unit vector which will be pointing in the negative x direction you can call that as some minus x hat.

Then, I can multiply that by the required magnitude say the value of N and then obtain minus N x hat. What we preferred to do is that we keep the unit vectors fixed. If I want to refer to a negative direction I simply multiply that one by a sign number, if the sign number is positive, it will be pointing in the same direction as the unit vector, if the sign number is negative right, the multiplier is negative, then it will be pointing in the opposite direction, okay.

(Refer Slide Time: 11:52)

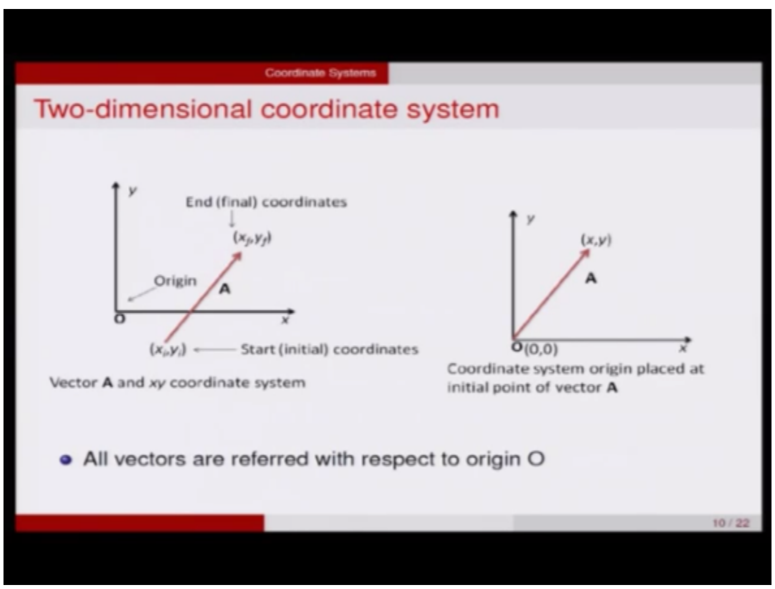


So, this was the one-dimensional coordinate system a vector N. So, let me recap this, a vector N which is starting at origin O and ending at a point N is given by N x hat. Please note the notation that we are going to use. This will be the notation that we will be using throughout our study, okay. Here, the number N which can be positive or negative, okay. If we associate and sign along with this one if I do not have then N is a positive number.

So, N is a magnitude of the vector N and x is the unit vector along x axis. If I want to represent a vector, which is acting along minus x direction or which is along the negative x direction then I have to assign sign here okay. I have to say minus N x hat which will then point to a vector whose magnitude is N that is whose length is N, but it will be acting in the minus x direction, negative x direction, okay.

If I take a unit vector and then multiply it by any number I am essentially stretching the unit vector, okay. If the multiplier is less than 1 then I am compressing the vector. If the multiplier is equal to minus 1 then I am taking the vector and changing the direction from plus to minus, okay. Now, let us go to two-dimensional coordinate system. Whatever fun we had to have with one-dimensions is all over now let us go to two-dimensional scenarios, okay. Now, here again the notion of coordinate system as well as the vector will be different, okay.

(Refer Slide Time: 13:17)



So, you can have a vector which is independent of the coordinate system and then you choose a coordinate system such that the numbers that you get will be unambiguously defining a vector and it will be simpler to work with. So, you have an unambiguous definition of a vector in the chosen coordinate system as well as an easier method of handling the numbers, okay. Just to bring out the distinction look at this figure, you have a vector which I am calling as vector A which is given by red colored arrow over here.

This red colored arrow as in the previous vectors will have 2 points, there is a starting or the initial points. Because this is a two-dimensional coordinate system you might expect that every point in the position will be specified, positioning space will be specified by 2 numbers. So, this origin point of the vector A will be specified by 2 numbers x i and y i. Similarly, the end point of the vector will be specified by 2 numbers which are x f and y f, okay.

What are these x and y? These are the 2 lines or the directions which are needed to completely specify any point in the coordinate system. In the two-dimensional system, I have to specify the location of any point by 2 numbers. One along x and one along y. Imagine that, I have this plane and then there are houses scattered all over.

Now, if I want to reach a house which is at some particular point in this space over here, I can say “hey go along horizontally 10 units of houses” or "10th house and from that house go vertically to 20th house to reach the house which is numbered 10, 20" okay. This is the meaning of giving 2 coordinates and you can notice that the directions horizontal and vertical are perpendicular to each other.

We will talk a little bit about perpendicular, the requirement of perpendicularity later, okay. At any point we can be considered as origin so we consider this point o as the origin here. So, anything to the increasing a side of x will be positive, this would be positive if you go below this line, you will be going minus y if you go to the left, you will be moving along minus x direction, okay. So, any point in this two-dimensional space or a plane can be specified by 2 numbers, okay.

So, here is a vector A with its initial and final points. Here is a coordinate system that my friend has come up with. Now, every time I specify the vector A I have to specify both the initial as well as the final points. Now, this might become little tricky okay because every vector needs to be specified by 4 numbers now, one for the starting point and one for the end point. On the other hand, it would be wonderful if I take the same origin okay for all the vectors.

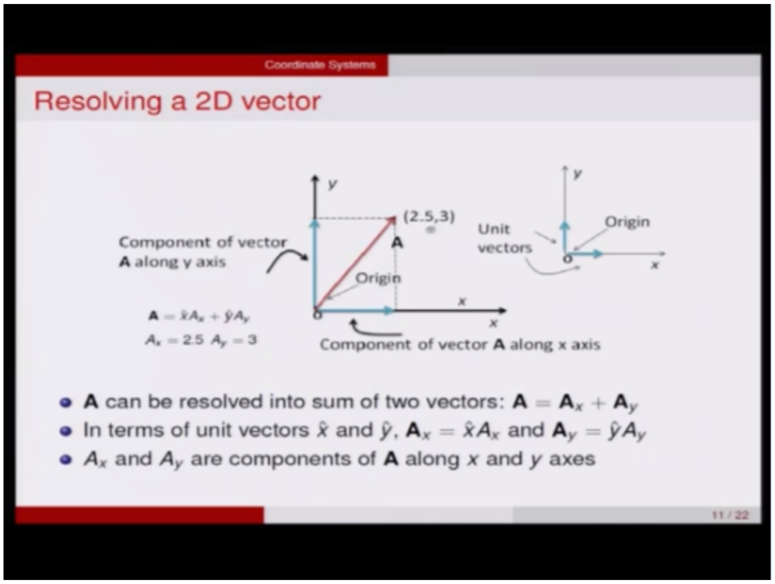
What am I getting by doing so? I am eliminating this need of specifying this x i and y i all the time. I do not have to specify x i and y i all the time. If I choose my coordinate system as coinciding with the tail or the origin of this vector, but the coordinate system does not seem to be coinciding. What is the solution? I cannot move the vector, of course I can move the vector but normally what we think of is we do not move the vector, we move the coordinate system.

There are 2 equivalent ways of doing the same thing. Either you move the vector parallelly until you reach the origin of the coordinate system, okay. We can do that one or we simply move the coordinate system until you reach the origin and moving the coordinate system not only means parallelly moving them you can also twist and rotate the coordinate systems, okay. Some of these things will become important later.

At this point, let us not clutter too much about that so what I am going to do is that I am going to redraw my lines y and x such that the origin of the coordinate system O right coincides with the origin of the vector A. Thus a vector A will now be specified only by 2 points x and y. Why because this 0 and 0 the origin is implicitly understood. If I had another vector, I could draw a line from the origin to the other point and that other vector could also be specified by only 2 numbers.

This is the advantage of coinciding or making your origin of the coordinate system coincide with the tail of all the vectors. How about the vector difference? Well you have to wait for a few slides to see how to define the sum and addition of vectors when you have 2 different vectors here, okay. But for now please remember that you can draw a vector from the origin to any other point in the two-dimensional space. All vectors are referred with respect to origin, okay.

(Refer Slide Time: 18:16)



So, you have a vector A which is referenced with the origin O here and Let us say this vector A is terminating at a point 2.5 and 3. What is this 2.5 and 3 mean? If you forget the vector A, this simply gives you the location of the point 2.5 and 3 in the two-dimensional space that is described by this vector x and y correct. So, I have a vector x and y and all points are specified by 2 coordinates.

So, in this particular case the vector A goes from the origin and terminates at 2.5 and 3. Now, to go from O to A, I can of course go along the red line or I can go along the horizontal direction until I reach this point. You can see that where I am reaching this one and then continue in the vertical direction.

Now if you remember how we added two vectors, you had one vector tail origin to that you added another vector that is you brought in another vector by parallelly translating it such that the tail of the second vector coincided with the head of the first vector. Now, I have a tail of this vector which is now getting added to the vector which is horizontal. So, I have a horizontal vector then a vertical vector.

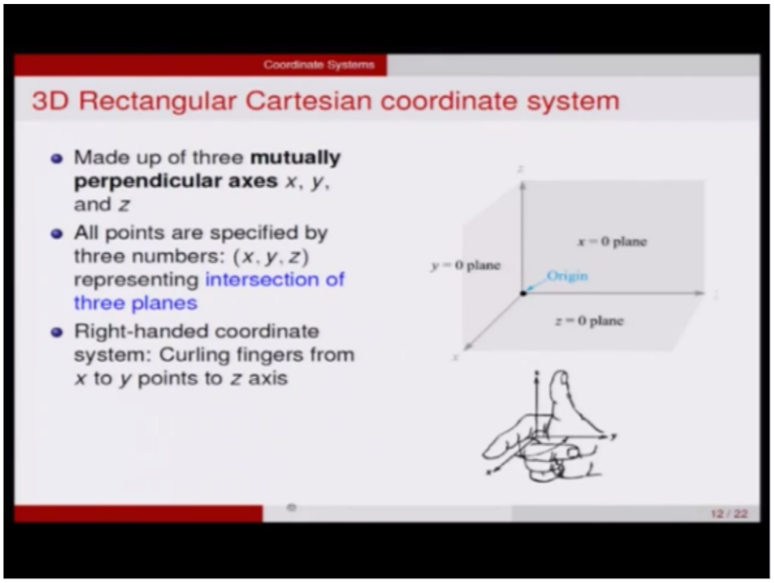
The sum of these 2 vectors is obviously the vector O A which you can see very clearly over here. What are the 2 vectors that I am adding? The horizontal vector which is given by this blue line and then a vertical vector which is given by this blue line. What is the magnitude of the horizontal vector? It is exactly this 2.5 and in which direction does it point? It points along the x direction. So, I can represent this vector itself as 2.5 x hat right. This vector is 2.5 x hat and this is precisely the vector that is giving you this horizontal vector.

So, this horizontal vector is characterized by x hat A x where x hat is the direction of the vector, which is along the x axis and A x is the magnitude of that vector that is of the horizontal vector which is 2.5, so this is another way of saying that you move 2.5 units along x and then move 3 units along y and if you move along y you are actually creating a vector here which again is nothing but a parallelly transmitted vector y hat A y and what is A y here? A y is 3.

So, this original vector O A has been resolved or decomposed into 2 vectors, the sum of 2 vectors both this vectors are, so one of the vectors is along x, the other vector is along y. These two themselves are perpendicular to each other. You can see that from the graph the vector A x x hat is perpendicular to the vector A y y hat, okay. Similarly, as we defined a unit vector for the x axis I can define a unit vector for the y axis and I have actually used that definition of unit vector of the y axis when I am writing this vector y hat A y, okay.

So, the vector O A has been resolved into 2 components that is sum of two vectors, one vector along x and the other vector along y. So, I have two vectors one along x and one along y. The sum of these two is giving me the vector O A okay. Keep this in mind okay.

(Refer Slide Time: 21:52)



Now, we move from two-dimensional coordinate system to three-dimensional coordinate system. Now, this is why electromagnetic is sometimes thought to be very abstract mathematical and at of subject because you have to work with three-dimensions. However, your work will be simplified if you understand the coordinate systems and if you choose an appropriate coordinate system for your problem, okay.

We will see the tragic consequences of choosing a bad coordinate system to solve a particular electromagnetic problem sometime later. For now, let us try to understand the three-dimensional coordinate system. If you have followed the discussion of one-dimension and two-dimensional coordinate system extending this to three-dimensions should not be a problem except that it will be little mentally taxing in visualizing the three-dimensional vector.

If you go back to two-dimensional case, you had two lines may be we can take this as the two-dimensional coordinate system example so you had 2 lines which were perpendicular to each other and these 2 lines are mutually perpendicular to each other and you define a unit vector along one and you define another unit vector along another axis. So you have a unit vector x hat, you have a unit vector y hat, okay.

So, any point was actually given by intersection of 2 lines. For example, you look at this point 2.5, 3 this point at which the vector A is terminating is actually the intersection of 2 lines. This line, this horizontal line that is shown here is the horizontal line which corresponds to the constant value of x, okay. So, this line is x is equal to 2.5. Whereas this horizontal line that I have shown here dash is the line y is equal to 3, okay.

This line is x is equal to 2.5 you can actually stretch this along this direction and no matter where you are on this point the value of x will always be the same. The value of x along this dashed vertical line will always be same and it will be equal to 2.5. The value of y along this horizontal line will be the same and it will be equal to 3. Only thing is x will be changing in the horizontal line whereas x will be constant here and y will be changing as you move up and down, okay.

So, any point in two-dimensional coordinate system was represented as intersection of 2 points. Now, on a three-dimensional coordinate system you are looking at intersection of mutually perpendicular axes which form 3 planes, okay. What is this plane? See, now look at this x is equal to 0 plane, I have 3 mutually perpendicular axes that means that axis x will be perpendicular to axis y, axis y will be perpendicular to axis z, okay. Of course, axis z is perpendicular to axis x. Look at x is equal to 0 plane okay.

All points on this plane have the x value which is equal to 0 and if you look at this horizontal plane, this is z is equal to 0 plane because all points on this plane have a constant value of z which is 0 in this case. Similarly, you have another plane which is y is equal to 0, you can go up which means z is changing you can go along the direction of x which means x is changing but you are not moving away from the plane which means y is constant.

So you have y is equal to 0 or y is equal to constant. In this case, the constant is 0 plane. You have x is equal to 0 plane, you have z is equal to 0 plane and the intersection point of all the 3 planes defines the origin. Now, if I want to specify a general x , y, z point how do I specify? Well you have to put up 3 mutually perpendicular planes there you have to put up. X is equal to constant plane, y is equal to constant plane and z is equal to constant plane.

So, you have to put up 3 mutually perpendicular axes composed of 3 different planes so as to specify any point. So, clearly any point in a three-dimensional space will have to be specified by

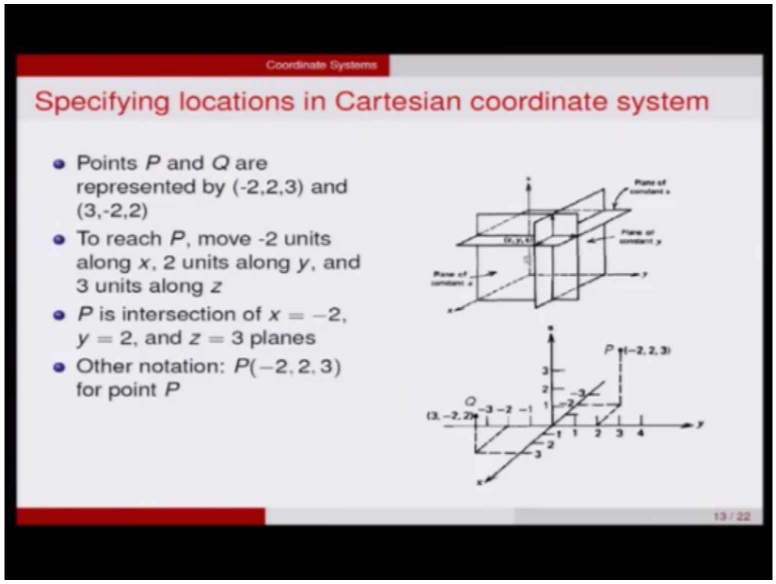
3 points, okay. Note the direction of the vector arrows over here, I have a vector x which is increasing in this direction, I have a vector y which is increasing in this direction and I have a vector z which is increasing in this direction, okay.

The specific directions were chosen to satisfy what is called as right-handed coordinate system such that the resultant coordinate system is right-handed coordinate system. What is right-handed coordinate system? Right-handed coordinate system is shown here. You imagine yourself as using your thumb finger to point along the z direction, okay.

So, if you imagine yourself as using the thumbed point along the z direction you will see that the index finger will be pointing along the x and the curved fingers will be pointing along y direction okay. So, you have thumb pointing along z direction and the index finger pointing along the x direction and the curling is pointing along the y direction. The curled finger would be pointing along the y direction.

So, in another words if you go from x to y you will be going up along the z direction. You can imagine a different coordinate system in which if you go from x to y you will be moving along the minus z direction, okay. Minus z with respect to this particular z I am talking about that would be a coordinate system that would be looking like this, okay. It would be an inverted coordinate system and sometimes called as the left-handed coordinate system.

(Refer Slide Time: 27:30)



Let us look at how to specify the points on a three-dimensional coordinate system. To specify a point which is x, y and z okay I need 3 planes as I have just told you I need 3 planes. So, if you look at this plane, this plane should be erected at x is equal to some constant okay whatever the value that is given to me and this plane you can see here is to be erected at y is equal to constant. So, whatever the value of y that I need to specify I have to put up a plane here.

The intersection of the planes x is equal to constant and y is equal to constant will give me 2 coordinates x and y. To get the third coordinate, I have to now put up a plane which is z is equal to constant, okay. So, 2 planes will give me 2 coordinates the third plane will determine the final point or the final coordinate of the location in space. Again, you have three mutually perpendicular directions which is one vector pointing along x, one pointing along y and one pointing along z direction.

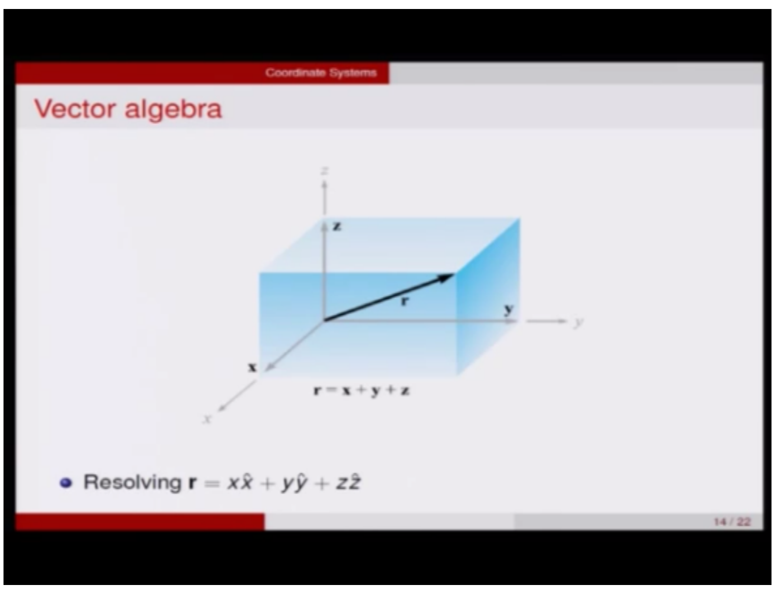
For example, consider point p and q, Point p is -2, 2 and 3 which means that I have to put up a plane at x is equal to -2, x is equal to -2 occurs beyond this. For example, this is the x is equal to -2 and then I have to move 2 point 2 on y axis. So I can actually move along the x line to the y axis to get me this point, okay. This is equivalent of putting up a plane at y is equal to 2 and plane at x is equal to -2, the intersection of those two will be at -2 and 2.

Now, if you move up because this is 3, if you move up to the point 3 or the plane where z is equal to 3, you will end up with point P. So, to get to point P you move -2 along x, 2 along y and 3 units along z. Similarly, to get to y which is 3, -2 and 2 you have to first move along x is equal to 3 that is to move to the plane x is equal to 3. So, I moved to this plane x is equal to 3.

On this plane, y and z values could be anything but y is equal to 2 is given. So I need to now move to y is equal to minus 2 planes. So, I need to setup y is equal to minus 2 plane. Now, when I bring these 2 planes together I am at a point which is 3 and 2. Now I have to move to point 2 along z so which means I have to erect a plane at z is equal to 2. So, I have moved to point 2. So, this Q is defined by 3 numbers 3, - 2 and 2, okay.

A different notation is to just give you the vector I mean just give you the point P and give the coordinates of the point. So, P and in brackets you give the coordinates say in this case it is -2, 2 and 3 and for Q it would be 3, -2 and 2, okay.

(Refer Slide Time: 30:31)



So, this is how to specify locations on a three-dimensional coordinate system. I hope that you have understood this particular point and I suggest that you try to graph more number of points to become familiar with this particular coordinate system.