Electromagnetic Theory
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Lecture – 39
Magnetic Material – I

So in this module, we will discuss magnetic materials. We will first begin by looking at the traditional view of magnetism that is classical electromagnetic view of magnetism. And then we will use those concepts to discuss various types of magnetic materials in a very brief manner. Because to discuss magnetic materials it itself is a huge area and we would not like to do that one in this course.

But we will give you the brief idea of what different magnetic materials are and what are their corresponding properties. So to begin with let us first look at what is the traditional view of magnetism. Before we can do that let us actually recall what we have studied in the last few classes in terms of what magnetic fields are produced right. So we have actually discussed the use of Ampere's law or Biot-Savart Law in order to obtain magnetic field of current carrying conductors in various configuration.

One of the simplest configuration that we consider was that of a long straight conductor which was carrying a current I, right. And then the magnetic fields around this long straight conductor was all in the direction of phi and magnetic field we know was given by I by two pi r the magnetic field H was given by I by 2 pi r.

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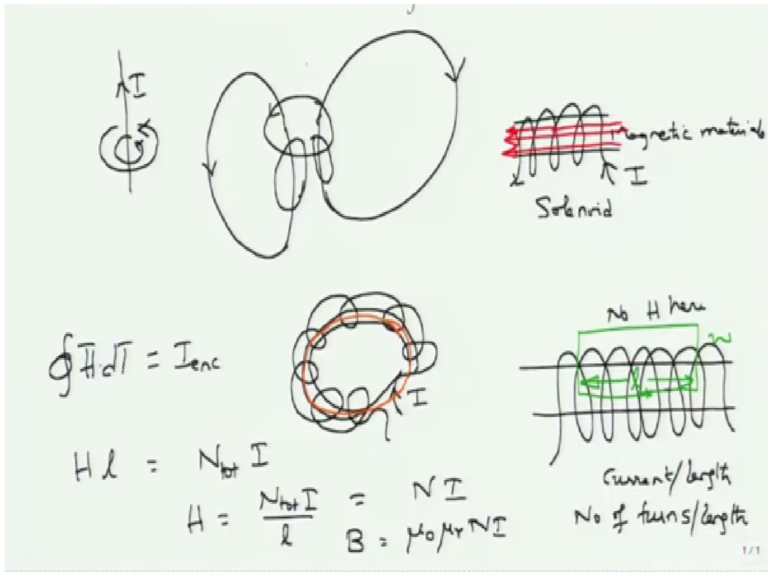

We also consider a different geometry of that of a current that is carried around a loop after a certain radius a. And here the magnetic field lines especially the magnetic fields which are around at very far distance right. They would all form this loops. And we have shown that this field which are at very far away distance was essentially equivalent to that of a or in fact the nature of the fields was exactly equal to that of an electric dipole right an equivalent of an electric dipole of course the-- near region.

We were not in the calculation; we look that this magnetic field is in the far field region. There are a few additional geometries that are used to produce especially very high magnetic field okay. So these are what are called as solenoids and toroids, they actually consist of a magnetic material which we are going to discuss sometime later essentially a ferromagnetic material, they consist of current I and then the current I is actually wound on a wire and there is a magnetic material here, okay.

So this is the magnetic material around which the wires are which carry the current I are tightly wound and with a very, very narrow spacing between the wires. And for the ideal case of an infinitely long solenoid this is as a solenoid we have not discussed in the field of a solenoid which we will do very shortly. The fields of a solenoid are entirely inside that of the magnetic material and they are all forming a line along the access of the solenoid.

Solenoid is an essentially that fill in the, so they all form lines okay and these are horizontal lines and there are no field outside this when the field is sorry when the solenoid is infinitely large. At different form of magnetic field generation that is used in some of the magnetic circuit is that of a toroid. A toroid, essentially consist of a coil that has been wound in the form of a magnetic material that is bent, okay.

So it is a bent coil around which you are actually winding the current carrying wire. So when you do that the magnetic field turns out to be completely within this coil. There are no magnetic field outside the coil, okay. So this toroid is especially useful for confining magnetic fields inside a given coil configuration. So to give you what is a magnetic field of a solenoid consider this following Ampere law, so assume that there is actually an infinite solenoid here okay, o there is an infinite solenoid which carries a current I.

But since this is an infinite solenoid we can actually characterize this by talking about what is the current per width or current per length or equivalently the number of turns right each turn carries a current I so you can actually call this current per length instead of that you can also talk about number of turns per length. Okay. So this multiplied by I would give you the amount of current per length. So if you have say, 10 turns per one meter.

And current is around one ampere okay these are just random numbers which I am picking from my head these are not the actual values so just to give you what the idea here is. So here if the number of turns is 10 per meter and then there are about then there is a current of 10 ampere in each turn then the current per meter will be 10*10 100 meters per-- sorry hundred amperes per meter.

So you can actually think of this solenoid with its wires which are wound tightly around the solenoid on the outer surface of the solenoid has some sort of a sheet current flowing around the solenoid, okay. So for such a solenoid if you want to find out the fields remember again the solenoid length is very, very large which I am approximating as a infinitely long solenoid. So an infinitely long solenoid if you are at any point right there would be an equal contribution of the current from one half.

And there will be an equal contribution of a current from one half and there will be an equal contribution of your current from the other half sorry the magnetic field from the other the half. And if you think of this solenoid as consisting of current carrying wires which are staked one upon the other you can see that magnetic field on the axis is coming from this would actually be along this way and there would also be an another turn at the top which will produce a magnetic field along this okay.

So if you look at since the current direction would be opposite here the total magnetic field would be along the axis of the solenoid. So this is how you can actually calculate the fields of a solenoid which we are not going to do here what we will do is we will assume an Amperian loop here okay of some length l does not matter. We will assume an Amperian loop of length l and a certain width w.

So in this loop what you have to observe is if this is an infinitely long solenoid then the magnetic field is of only this path there will not be any magnetic field outside so no H here tells us that there is no magnetic field outside that of the solenoid and all the magnetic field is there inside the solenoid. This is an approximation which is very good when your length of the solenoid is very, very large especially compare to diameter or its this one.

So with this as the Amperian loop what would be my total current? So if the length of the path is l then the total current will be NI where N is a number of turns per meter multiplied by l will be the total number turn and then you can talk-- write this as NIL. Or if you were to consider N as a total number of turns inside this meter then it would be N*I. So there is a different ways of writing the same expression.

What you have to understand is that each turn if contributing a current of I Amperes and therefore if there are total of N turns in the each turn if contributing a current of I Amperian loop that you have considered the total current will be N total into I, okay. And what would be the left hand side of the Ampere's law if you have forgotten Ampere's law this is Ampere's law right integral of H. dl is equal to the total current enclosed.

So we have actually obtained right hand side total current enclosed. For the left hand side you can observe that there is no magnetic field along the path for w so because the field is all perpendicular and along the axis. So there will only be one field which would be H and the length of that one is H into l. So this would be equal to the total current enclosed and therefore the magnetic field H which would be along the axis of the solenoid will be given by total turns in that loop l I by l okay or you can talk in term of the total current or turns per length.

And then say N into I. If you want to obtain the B field assuming that the magnetic material inside is field with some mu 0 or mu r then the magnetic field will be equal to mu 0 mu r N into I. So if you take this solenoid and instead of making it an infinitely long solenoid you just take the solenoid and bend it around forming a certain radius A, then you can see that the magnetic field would essentially more or less remind enclosed or more or less remain confined into the magnetic material itself.

So there are no magnetic fields outside here. So with this brief introduction to solenoid.

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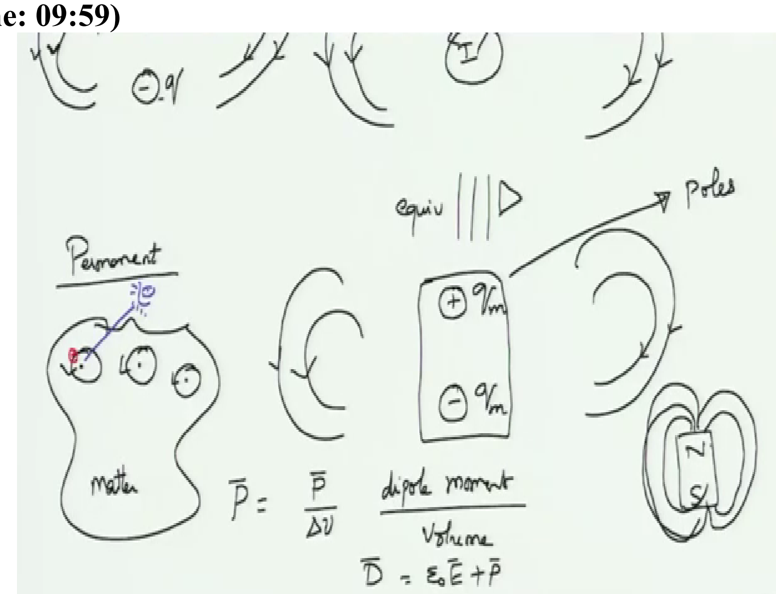

Let us go ahead and look at the magnetic materials, we have already discussed dipoles if you remember what a dipole was the electric dipole was that of two charges which are separate by a certain distance and the field far away from the dipole who are evaluated and it was given by certain expression and interestingly the same situation actually occurred for that of wire that was carrying a current I.

Again, the far fields were all essentially given by the same expression—the expression were all the same as that of the dipole okay. So which actually means that I can actually think of it current carrying loop as an equivalent dipole except that now I have you know I am calling this as an equivalent dipole except that I do not have electric charges right, what I have is fictitious magnetic charges.

So I have fictitious magnetic charges of plus qm and minus qm okay. And these charges would produce a magnetic field which was exactly the same as that of the current carrying loop the far field are essentially the same. So these are actually two ways in which people think of magnetic materials or magnetic dipole especially and diamagnetic materials are made up of magnetic dipole. One is to think of them as tiny current loops okay, carrying a certain current I and generating the magnetic field okay.

So if you take a material you will find lot of such loops inside. Or you can think of them as equivalent magnetic charges, no doubt these magnetic charges are fictitious because they cannot be isolated and found in nature. But you can think of this magnetic charges, fictitious magnetic charges as the same configuration as that of an electric dipole which would produce exactly the same field identically the field as that of the current loop.

In fact, the colloquial term for this magnetic charges use to be poles and this is how the colloquial law of attraction was formulated early light poles, repel and unlike pole attract, so pole would actually refer to the poles of a magnetic material. So if you think of a matter right any magnetic material just as we did for the dielectric material you can think of the magnetic material as consisting of tiny, tiny current loops.

Where are these current loops coming from? Well, matter is made up of atoms right and atoms have an inner nuclear which is positively charged and an outer electron which is negatively charged but it would be moving around classical view of an atom that of atom moving around in a given orbit right. So it would be moving around in an orbit generating a tiny loop of current.

Because if you imagine yourself at this particular point where you are seeing yourself at this point right, you will see that the charge is moving with a certain velocity or a charge is moving with a certain frequency and crossing this particular point corresponding to a current, okay. And there will be of course billions and billions of such atoms inside any acrostically large magnetic material.

When you take a piece of magnetic material there will be a large number of billions and billions of such atoms okay. And each atom is essentially a tiny current loop. so the magnetic fields that are produced by each of this current loop will add up in the case of permanent magnets okay permanent magnets are those which poses this tiny current loop producing a magnetic field that would actually add up coherently.

So if you take a magnet which is what you know a fridge magnet or refrigerator magnet then it will actually attract and if you try to take some iron filings and then place this magnet slightly above it then they would actually form a certain pattern this how we would actually visualize the magnetic lines of force or magnetic field by taking the magnet okay, a bar magnet or a round magnet.

And then placing them with iron filing and you will that the iron filing would actually form a certain loop around this bar magnet right. So this is because there is some magnetic field generated by this permanent magnet. There are no currents here that is there are no externally currents here. What we have instead are those tiny, tiny little loops inside this magnetic material contributing to a overall magnetic field.

Of course to contribute to an overall magnetic field you would of course expect that these tiny, tiny current loops must be aligned in some way, right. So this must be aligned in some way okay only when they are aligned then there will be an net magnetic material if they are not aligned they are completely randomly oriented the magnetic moment of each of this current loop is completely oriented randomly, then there will not be any net magnetic field.

Permanent magnets are those which actually have some magnetic field and the reason is simply because they have this tiny, tiny current loops is said which are all aligned with each other. But please remember if you take a piece of magnetic material you will actually find a large number billions and billions of such magnetic dipoles so each current carrying loop is actually a dipole okay.

Of course not many such permanent magnets are found and even those permanent magnets that we find are dependent on temperature that is if you start increasing the temperature by heating up the magnet after a certain critical temperature okay the magnetism actually breaks down the magnetic material does not remind magnetic anymore, okay. So this is a problem that would happen even for a permanent magnet.

So in that sense there is no permanent magnet-- independent of temperature but for room temperature or for the temperature of operation that we are interested the magnetic movements are permanent in the sense that there would be contributing to an external magnetic field, okay. However not many materials are lucky enough to have their movements aligned with each other so therefore they do not actually poses magnetic fields.

So these two ways of thinking about magnetic materials is to think one of them as current loops having a certain magnetic moment or you think of them in terms of a fictitious bar magnet with plus magnetic charges fictitious charges at one end and negative magnetic charge minus qm again fictitious at the other end. The far fields that are produced by these two configurations are exactly the same.

And for our purposes one can model this magnetic field either by postulating a current loop which is what one would find in the magnetic material made up of matter, okay. Or you can equivalently think of them as charges. Now one might ask, why should I think of them as charges? The reason is mathematical. Of course the reason is also historical but if you forget history the reason is mathematical.

Because when you think of magnetic charges then whatever that we studied in terms of Laplace's equation or Poisson's equation they can be applied directly to evaluate the magnetic fields of these configurations, right. Well with one caveat the fact that the magnetic field the del cross H is not 0 means that this potential concept cannot be applied everywhere in the space. They should be applied in only those cases where the magnetic field is 0, okay.

So with these ideas to why one should consider the two types of views of magnetism we will then look at a little more detail into the modeling of magnetic material, okay. So to Model Magnetic Material, we introduce just as we did for Dielectric Material, remember what we had this dielectric material we had these dielectric moments and then we found or then we said that we introduce a quantity called polarization P which was suppose to be dipole moment—electric dipole moment per unit volume, right.

So we introduced this P and then we introduce a new vector called D and we related D to E and P in this expression. This is what we did for a dielectric material, now we are going to do this for magnetic material, so which means that we are going to look at magnetic dipole moment per unit volume called that has a certain vector M just as we have polarization, we will have magnetization vector and that magnetization vector will relate B H, okay. So let us do that.

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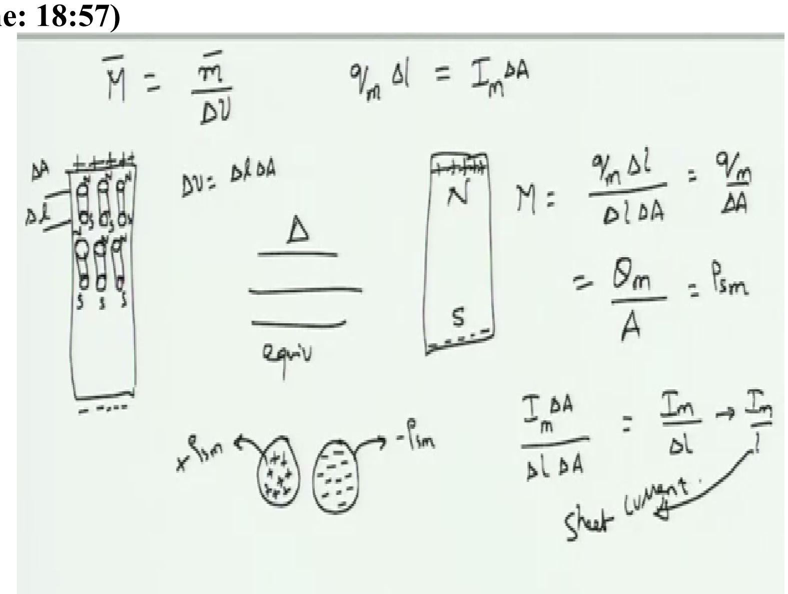

So we define magnetization M as magnetic dipole moment per unit volume. And for the case of permanent magnet such as a bar magnet you can imagine this you know remember this equivalent dipole think that we were talking about so you can imagine them having a positive and a negative fictitious charge forming a dipole moment, right and each of them can be thought of as North South Pole this is the traditional view of magnetization having two poles north and south.

North corresponds to fictitious positive charges and South corresponds to fictitious negative magnetic charges. So for this one if you assume that each of them as a length l or delta l and each of them has a area delta A then the volume occupied by each dipole will be delta l into delta A, or one cannot actually talk of a group of dipole, okay. So you are not really looking at individual one but you are looking at a group of dipoles which are average.

So this delta l and delta A are pertaining to those small regions small enough to be considered as microscopic but large enough to consider several magnetic dipoles together, okay. So with that the volume will be delta l delta A this is tiny enough for us to define this magnetization M, okay. So we can say that delta V is equal to delta l into delta A and the magnetic dipole moment the equivalent magnetic dipole movement is qm into delta l, right.

So this is exactly the same expression that you would have written for the electric matter so you have magnetic materials qm delta l being the dipole moment, this of course is actually equal to current I into delta A. This is the current that is carried by the loop multiplied by delta A that would be the magnitude of the dipole moment. So when you actually look at this lot of tiny, tiny magnets inside a magnetic material.

You will see that the north and south poles essentially cancel out each other and what you are left with is some sort of a big dipole with all fictitious charges at the top and all negative charges at the bottom, okay. So this top charges are the north pole and the bottom charges at the bottom of this one is the south pole. So what is a magnetic moment? Magnetic moment is qm into delta l. So what would be the magnetization vector M?

The magnitude of the vector M will be qm delta L into delta L delta A giving me qm by delta A, right. And if you consider the total such magnetic charges this is was for a single dipole or a small enough region of a dipole. If you consider the total area right and total number of charges total charge as qm which is a sum of all this tiny, tiny dipole charges and over the area A which is a sum of all delta A's what you essentially get is a magnetic surface charge density.

Because this is charge per unit area. So you get a magnetic surface charge density and if you look at the top and the bottom surfaces of the bar magnitudes you would see them populated with magnetic charges with a magnetic charge density plus rho sm and the south pole is all populated with negative magnetic charges or with a magnetic charge density of minus rho sm. A similar equation can be obtained in terms of the current as well.

Because the magnetic moment is I into delta A just so that this is a magnetic moment let us put a m subscript for the current carried by the magnetic dipole, okay so this would be Im into delta A divided by delta L into delta A, right. So if you look at this one you will see that this is given by Im by delta L and when you consider the entire length of the material this would actually become Im by L.

And what is this Im by L? What are the units of Im by L? The units of Im by L is current per meter. And this is preciously the sheet current that we talked about. Okay.

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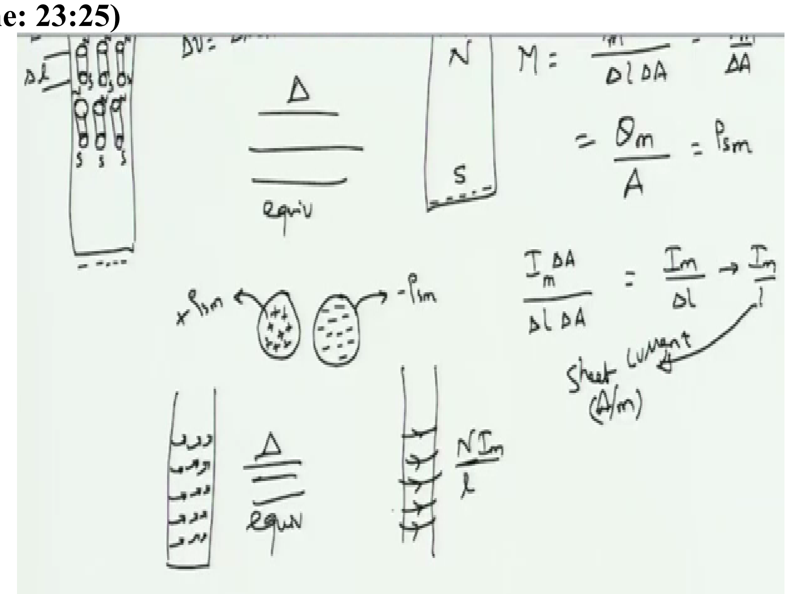

So the sheet current of measured in ampere per meter is another equivalent way of looking at the magnetic material. So where does this come from? This actually comes from imagining tiny, tiny loops, right. All these loops in the interior of the magnetic materials cancel with each other leaving you the dipoles at the surface which means an equivalent of a long solenoid with currents.

So with currents such that the current per or if there are number of turns of N then the number of turns per meter into L into I will be the current enclosed per meter right so it would be NIM by L will be the current that is enclosed up there. Okay. We will now look at the relationship between B mu H and N.

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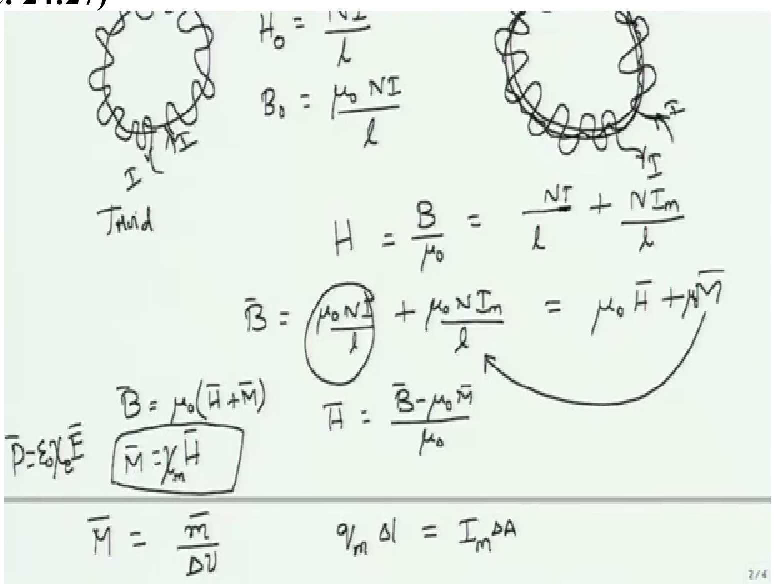

We have previously introduced you to two fields B and H and we did not really say anything about this relationship between the fields B and H which we said that well in free space it so happens that B is equal to mu not into H, okay. So H is measured in ampere per meter if you remember and B was measured in Weber per meter square which was equivalent of Gauss which also had an equivalent of Tesla.

So this was the traditional think in that we have introduced you to but we did not really stress upon what was the relationship between B and H except telling you that this is equal to mu 0 into H. What we can actually do is we can define experimental situation or we can design an experiment which will actually bring out the relationship between B and H for free space and for magnetic matter.

So to do that consider this solenoid and then you wind up solenoid with wire okay. And the material that the solenoid is made up of is air. If you assume that the material made up of-- sorry the toroid actually. So if the material that is making up this toroid is actually air or free space then the magnetic field we saw from the application of Ampere's law or we could apply Ampere's law to give you that magnetic field as NI by L.

Because there is only externally applied current I and H0 is NI by L, and because in the free space H is related to H by a factor of mu 0 B 0 will be equal to mu 0 NI by L. Now take the toroid but except fill that toroid instead of with free space fill it with a magnetic material, that is take a magnetic material bended inside in the form of a rod and then wind it up with current carrying wires of the same turns and then apply the same current I.

So you apply the same current I. What will happen the field produced must be of two different reasons. There is a field that is produced because of the external current which is mu NI by L, okay no surprise out there. But this tiny loops inside the magnetic material itself which will align because this is a permanent magnetic-- will align and then it will contribute an extra field of its own, okay. And that extra field of its own will be some NIM by L.

Note that I am not put any mu here, okay we do not want to put a mu here we just want to say that Im is the magnetic current or the current produced because of the tiny current looks inside the magnetic material. And if there are end turns and L is the length and the total H field produce will be NIM by L. So this should actually be equal to the field B. So one second let us actually write down this in a straightly different manner.

So initially let us put this mu 0 out here. So we say B is equal to mu 0 into NI by L just to focus on the fields produced by the current, so this is actually H field so instead writing this B by mu 0 you could have written as H field. And in the h field there is no mu 0 right H is suppose to be independent of the magnetic material.

So H is given by NI by L plus NIM by L right. This H field will be equal to B by mu 0, okay. We assume that this H and B are still characterized by this mu 0, of course we will find that this is not exactly the same as H0 and B0. So H is B by mu 0 and this particular expression which will give you B as mu 0 NI by L no surprise here but you also get mu 0 NIM by L. Now, we know that B and H must be related in a certain way and this mu 0 NI by L.

We have already seen that it is equal to the H field so we get B is equal to mu not H plus something else and this something else is the magnetization M. So what we have here is-- and since there is a mu 0 sitting inside, so we need to multiply this one by mu 0 as well. So what we actually have is the relation which says B is equal to mu 0 into H plus M equivalently H can be written as B minus mu 0 M divided by mu 0, okay.

So if there is no magnetization that is the free space whatever H is there that would be equal to B by mu 0, okay. However, in the presence of magnetic materials the magnetic field B would actually change from simple value of B to B minus mu 0 into m and divided by that mu 0 will be equal to H. In most cases just like the case of dielectric material the relationship between M and H is that of linear.

Well, magnetic material is actually poses even more exotic properties but we will not go into those properties at this moment, okay. So M is linear related to H and this relationship is defined by this parameter chi M-- this is chi M is the magnetic susceptibility just like you had-- sorry P is equal to epsilon 0 there was an epsilon 0 at that because of whatever the reason is, so epsilon 0 times chi e into electric field, okay.

We in fact evaluated the relationship between P and E, I am sure that it was linear for small electric fields, right we looked at electric polarizability in terms of the atomic polarizability, ionic polarizability and oriental polarizability. So similarly, we will look at the relation between m and h for two classes of material called diamagnets and paramagnets. We will keep the discussion brief but give you the basic ideas of those materials. So you have M is equal to chi M into H, again M is a magnetic susceptibility