Electromagnetic Theory
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Lecture 36
Magnetic Vector Potential

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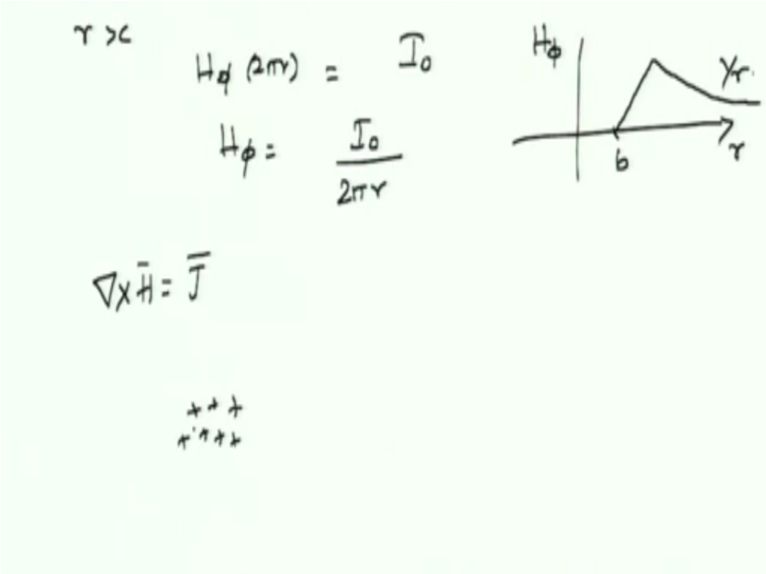

We have discussed one-point form of the law which was curl of H is equal to J. Now, before, so there is another law which we need to get. What is that law? Well, I am not going to derive that law here. I will give you are qualitative idea. Consider a certain charge distribution.

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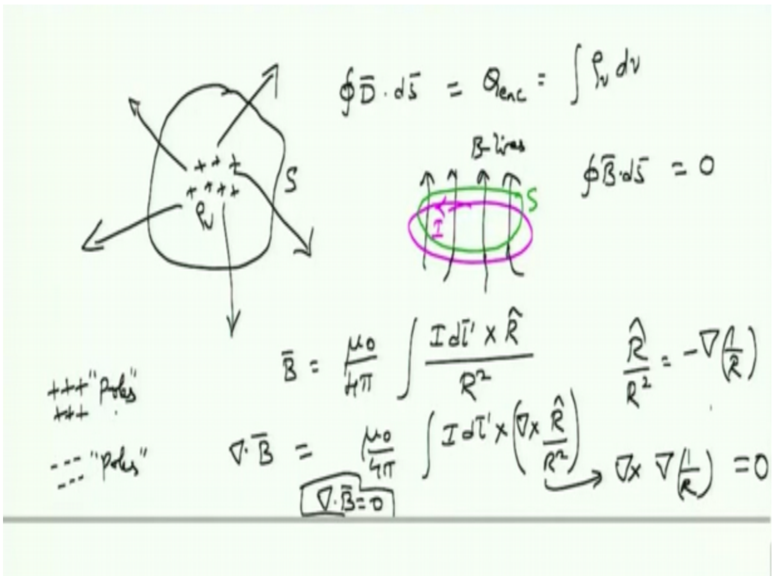

Consider a certain charge distribution in such a way that if I imagine putting up a surface around this charge distribution assume a certain surface S and a corresponding volume V and the charge distribution has a volume charge density of rho v. Now what did our Gauss's law say. Gauss law said that, the electric flux density vector D integrated over the surface element ds, integrated over the closed surface s, was actually equal to the total charge enclosed, right, which was integral of rho v over dv.

That is the volume integral of the charge density. Now imagine a situation where I have the magnetic fields that are coming out, okay? And this magnetic field is obviously coming out by having some sort of a loop of current I, correct? So I have a loop of current I and we have just shown the magnetic fields to be constant and they would all be coming out like this.

Now if I imagine covering up these B lines using some surface s, these are the B field lines that are coming out and over the surface S you will see that if I try to find out what is B dot ds over this surface, this will actually be equal to zero. There is as many field lines as entering as those field lines are leaving the surface. So you cannot find any surface in such a way that if you enclose it you will only have outgoing magnetic field lines.

Magnetic field lines cannot just exist as isolated field lines coming out of a point. Just as an electric field lines would be, so if you enclose this, you would actually find the total charge enclosed and you can see that the field lines emanate from the charges, there is no such thing for magnetic fields. Mathematically you can show this, you can actually go back to this expression for B using Biot-Savart Law, write down this.

Mu zero by 4 pi, integration of I dl prime cross R hat divided by R square and then you can show, to get the point form, you can find del dot B which is the divergence of B and remember that del dot actually goes inside the integral because this gradient is being taken or the del operator is operating on the field point. So you will actually get this mu zero by 4 pi integral of I dl prime and del cross R hat by R square, okay?

There is a cross here and this quantity R hat by R square, we have already seen to be gradient of 1 by R. We have already seen to be minus gradient of 1 by R. So this becomes curl of gradient of 1 by R, there is a minus sign, that is not important for us and we know from vector calculus that curl of gradient would be equal to zero. We have shown it for time independent case that is stationary case but this equation of delta B this operation is actually true for time varying case also and even in that this turns out to be equal to zero.

So the integrant is actually zero which means the integral is also zero and what you get is del dot B is equal to zero. The physical idea behind this is that, if I consider the B field lines coming out of any region of space and you try to surround it by a surface, I will never be able to isolate these lines. The lines will always close or at least there would be as many lines that are entering, as many lines are leaving.

So indicating that there is no possibility of at least there are no possibility of existing a positive poles, just like positive charges or a negative charges, there is no possibility of positive or negative poles of a magnet.

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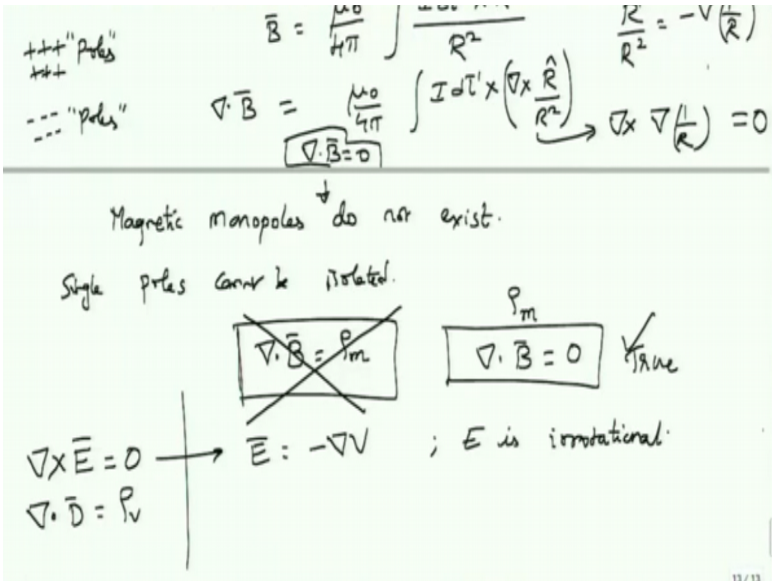

So this statement is sometimes indicated as magnetic monopoles do not exist. So in typical interviews when we ask for what is del dot B is equal to zero without really thinking about it more students would answer magnetic monopoles do not exist. They seem to think that somehow delta B is equal to zero simply indicates magnetic monopoles do not exist. So the only physical relation that people give us is that, magnetic monopoles do not exist.

So when we ask what are magnetic monopoles they are unable to answer that. The idea is that mono is a single, okay? A pole is one of the oldest terminologies for charge type of situation. You had a magnetic, you have a magnet of north pole and south pole and similarly you know this statement of like poles repel and unlike poles attract was actually derived from this earlier terminology. Pole being that agent which would attract or repel, okay?

So what we mean when we say magnetic monopoles do not exist is that, single poles cannot be isolated. So far we have not found any case where we have a magnetic charge density rho m. If there was any magnetic charge density in the universe, then the magnetic form of the law would actually read del dot B is equal to rho m. Mathematically this might be alright to show that this del dot B is equal to zero, but if it is physically not okay, that is physically rho m does exist then you have to change that law into del dot B equal to rho m.

Otherwise in so far what we have seen, del dot B is actually equal to zero for all the time. So del dot B is equal to zero, but even if there are one magnetic monopoles, then this law has to be modified. You have to distinguish between this law as a physical rule and this law as a mathematical tool. As a mathematical tool sometime it is advantageous to include this magnetic charge.

In most case, in some of the literature you would find people using this magnetic charge because it simplifies the calculation and then setting this quantity rho m equal to zero after the calculation, okay? You can do that but we will not be doing any of that for us, okay? So for us del dot B equal to zero will always be true and many many experiments have been conducted to actually try and isolate magnetic charges so that try to find magnetic monopoles and so far there has been no positive result in finding such a magnetic monopole.

So for all practical purposes magnetic field lines B can be thought of as being closed up on themselves in such a way that the line integral of B over the closed surface would always be equal to zero or the divergence of the magnetic field at all points is equal to zero, okay? So this is the physical law that we wanted to write down. Although I have shown this del dot B equal to zero in a quite hurried manner, I would suggest you carry out this calculation for yourself.

It is just couple of vector identities that you need to involve or use and you would find the del dot B equal to zero. So let us summarize four relationships that we have obtained. One relationship was del cross E is equal to zero. This is from the electrostatic case and del dot D is equal to rho v. Because del cross E is equal to zero we were able to write down E as minus gradient of V and we said that E is actually irrotational, okay?

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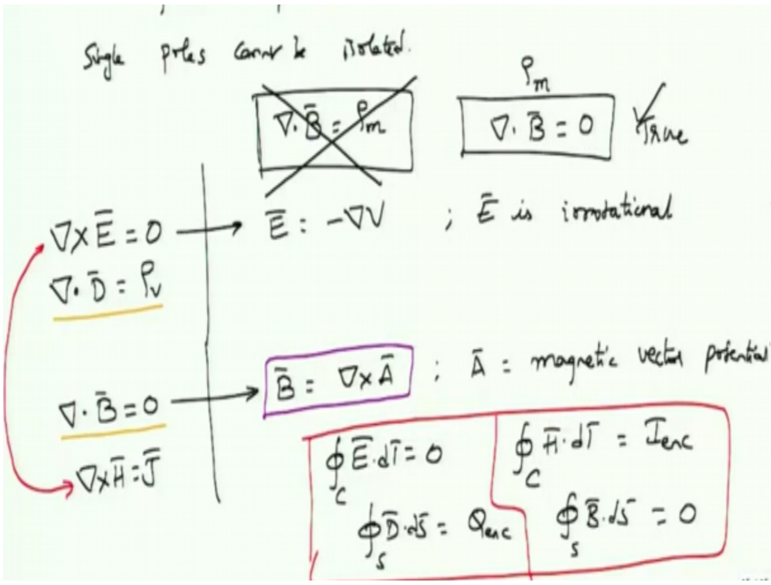

When we discussed curl, we said that E is irrotational. For the magnetic fields we have del dot v equal to zero which actually allows us to write B as curl of some other quantity A and this quantity A is called magnetic or sometimes magnetic is dropped but we will keep that one, magnetic vector potential. And del cross H is equal to J, this is an incomplete equation. We will come back to its completeness and this is ampere's law.

So the curl of H will be equal to the current enclosed, okay? Whereas curl of electric field was actually equal to zero. We will later see that this equation and this equation needs to be modified in the time varying case. Whereas these two equations, these two equations are perfectly valid for both time varying as well as time invariant cases. So what are the corresponding integral forms of this equation?

Well, the corresponding integral forms are, for del cross E is equal to zero, integral of E dot dl equal to zero and integral of H dot dl over the closed curve or the closed loop is equal to the total current enclosed. The dot product law which are called as electric and magnetic Gauss's law basically saying that integral of D dot ds over the closed surface is equal to total charge enclosed or integral of B dot ds, the magnetic flux density vector integrated over surface is actually equal to zero.

So you would be sometimes, I mean you would be using integral forms sometimes and you would be using the point form sometimes. In most numerical methods you would actually be using the point form to develop the numerical methods and if you want to understand boundary conditions you would be using the integral form, okay? Alright, so here we actually very sneakily suggested something, right?

Because del dot B is equal to zero, B can be written as del cross A. We did not mention too much about what that was and that is the idea of, we are going to do that one now. What we mean here is that when the divergence of B is zero, you can express B as curl of A because del dot curl of A will always be equal to zero.

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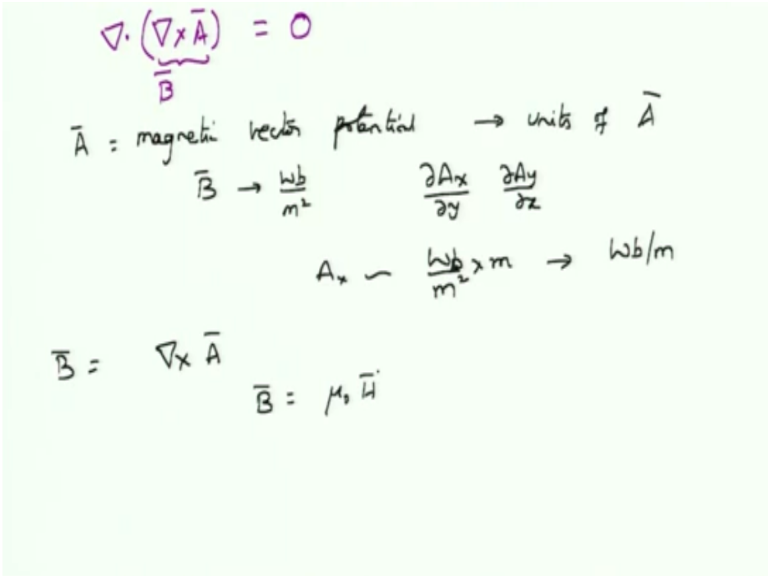

This is a general relationship which exists for any vector field A, del dot curl of A is equal to zero and because of this reason, I can denote the vector del dot B equal to zero by defining B as del cross A. So we can write down B as del cross A and A is called the magnetic vector potential. It would be interesting to find out what are the units of A. What are the units of A?

Think about that a little bit. B is measured in weber per meter square and for the curl operation, what is that operation involving? So curl of A will have something like del Ax by del y, del Ay by del x and appropriately del Az by del x and so on, right? So if you find out what is the unit that we need to measure will have to measured Ax or Ay or Az will have to be measured as weber per meter square multiplied by meter because the change in y or change in x will have to be measured in meters.

So small change in y, small change in x as measured in meters. So there for the units of A is weber per meter. So I know that B is equal to del cross A. I also know that B is equal to mu zero H for the free space which means what we are going to consider for now.

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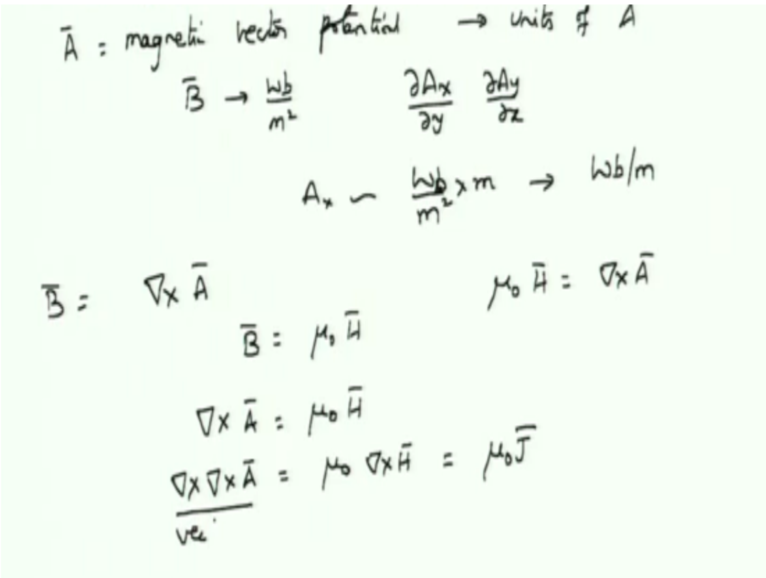

So if I say substitute for B here, what I get is mu zero H is equal to del cross A. Now let me consider this equation in a reverse way. Del cross A is equal to mu zero H and then take the curl of this left hand side and the right hand side, so I get mu zero curl of H. This curl of H we already know can be easily written down. This is mu zero multiplied by J. But what is this curl or curl of A. This is called as vector.

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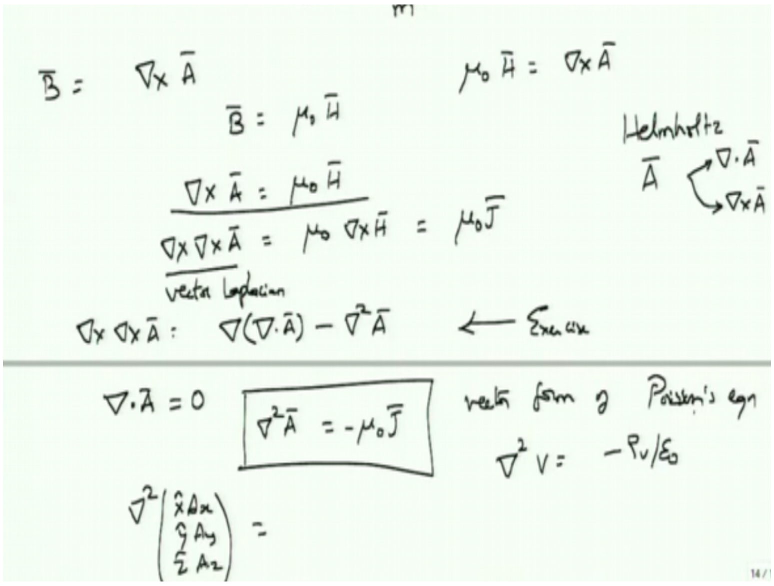

This will actually define what is called as Vector Laplacian. You can show this that curl or curl of A is actually given by gradient of del dot A, divergence of A minus del square A, okay? You can show that this is the vector identity. I will leave this as an exercise to you to show that this is an identity that holds alright. Now, we can simplify this expression by choosing an appropriate value for del dot A.

There is a theorem called Helmholtz theorem which tells us that any vector field A is uniquely determined if we give both divergence of the field as well as the curl of the field. The curl of the field we have already given. Curl of the field is mu zero multiplied by H. So we have already defined that there is already a relationship for curl of A. So if we specify delta A and we are free to choose whatever the specification that we want in this case, then we have completely specified this vector potential A.

So let me choose the simplest one that is possible. I will choose this del dot A to be equal to zero. I could have of course chosen del dot A is equal to any constant as well, but just as electric scalar potential does not by itself mean anything but only the potential differences mean. Similarly, magnetic vector potential does not by itself mean, but the differential potential is what matters.

So I couldn't choose delta at A is equal to zero or I can choose it to be any constant and the gradient of a constant would also be equal to zero. So I have del dot A equal to zero as chosen which is what I am going to do now. So this left hand side will become minus del square of A equals mu zero J. So mention to put the minus sign to the right hand side, so that we can exploit a know relationship over here.

This expression which I have written here is the vector form of Poisson's equation, correct? If you do not see it, look at this. Del square v is equal to minus rho v by epsilon zero. This is the scalar form of Poisson's equation because v is a scalar. Here it is actually a vector Laplacian because this A itself has 3 components. It has x hat Ax, y hat Ay and z hat Az and this del square operator has to operate on each of these three components.

Of course this del square has such a simple expressions or relationships only in the Cartesian co-ordinate systems, these expressions are quite complicated in other co-ordinate systems. So for other co-ordinate systems you have to go back to the defining expression for del cross del cross A and work with those expressions.

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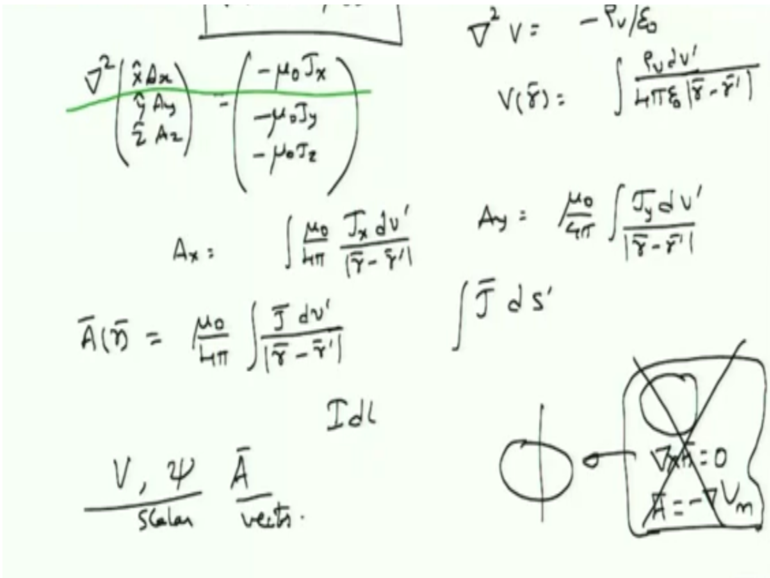

So for Cartesian co-ordinate system this del square can be applied linearly to Ax, Ay and Az and the correspond terms on the right hand side would be minus mu zero Jx minus mou Jy minus mu zero Jz. So taken with respect to each of these wave forms it would be a Poisson' s equation and we know how to solve this Poisson's equation. At least formally I know what to write down for Ax. Ax will be integral of mu zero by 4 pi.

See, for del square v equals minus rho v by epsilon zero, the expression for V was that integral of rho v dv prime which is the volume charge integration divided by 4 pi epsilon zero r minus r prime magnitude, correct? So this was something that, you know this epsilon zero is sitting in the denominator but here mu zero is sitting in the numerator therefore you write down this mu zero in the numerator and everything else will be the same.

So you have Jx and in most general case it would be a volume integral. So divided by r minus r prime magnitude. Similarly, what would be Ay, Ay will be mu zero by 4 pi, I can push this mu zero by 4 pi out of the integral and I get Jy dv prime divided by r minus r prime magnitude and similarly I can write down for Az as well.

So the vector potential A at the field point r is given by mu zero by 4 pi integral of J dv prime where J is the point to point variation of the vector density, the current density J, current density, and r minus r prime is the magnitude of the vector that joins the source and the field points. In case J is defined only as a surface then you replace that integral over the surface. In case J is defined as a filament of current, then you need to replace this one by I dl or J zero dl something like that.

So why did we actually introduce this vector potential. We will be employing vector potential very soon to find out the fields of a dipole and the fields of a dipole are important because that is how we are going to introduce the magnetization vector which is model for how the magnetic matter would behave when they are subjected to external magnetic fields. To calculate those, to simplify those calculations we use the vector potential A.

It is possible to not introduce the vector potential. However, in most cases especially in the radiation problems it becomes easier to find out the fields, the magnetic fields if you know the vector potential. For that reason, we are introducing vector potential. There is another potential that we could have introduced. See, we have del cross H is equal to some current enclosed only when the path enclose the current.

If we consider a path that does not enclose the current so you have the current going like this and this is one path which enclosed the current and this is another path which does not enclose the current, so clearly del cross H will be equal to zero here. I can write down H as some gradient of Vm, Vm standing for magnetic scalar potential. We can do this but magnetic scalar potential is not very useful except in very very special cases.

So we will not be introducing the scalar potential for magneto static fields, okay? In any case this scalar potential is actually multi-valued. Magneto static scalar potential is multi-valued. I am dealing with that when there are currents here or the path encloses current is not possible. So for this reason we are not introducing the magneto static scalar potential although the textbook for this course talks about it. This is not really important as well.

So for us we have two potential. The scalar potential V or its general form psi and the magnetic vector potential A. This is scalar and this is vector. We are going to use this to find out the fields of a magnetic dipole and to model magnetization in the next module.