Electromagnetic Theory
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Lecture No 34
Biot Savart law & its application

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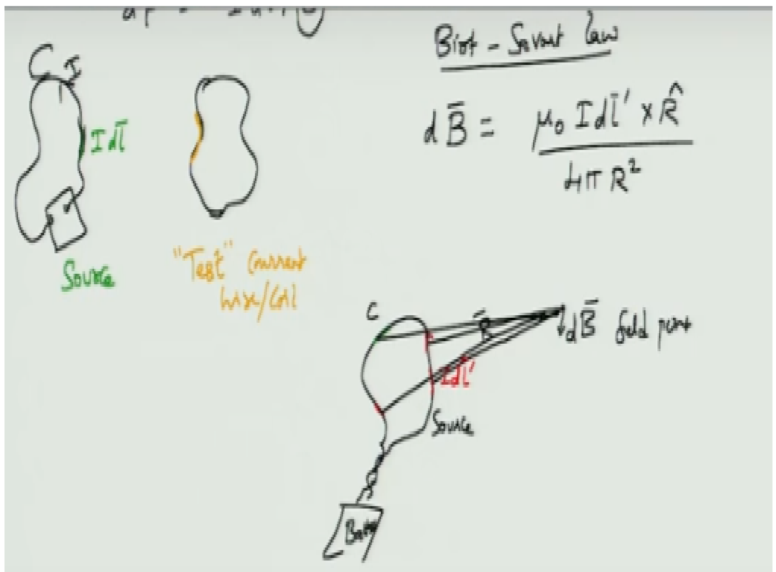

We will come back to the force of one conductor on to the another conductor sometime later after we have looked at Biot Savart law and Ampere's law. So, we will move away from considering the forces but the essential point of the last few minutes of this module was that you have a wire, the wire would get deflected where there is a magnetic field. Alternatively, the wire itself generates a magnetic field, which will not be given by Biot Savart law.

So, if magnetic field deflecting the current was one of the earliest experiments that was performed and then the fact that current itself can deflect a magnetic field is another experiment that was performed and in both cases, we presume that there is a magnetic field, which is the result of current being carried in certain conductor and there would be interactions between these fields.

So, the actual mechanism by which magnetic fields are generated is quite complicated and you really need to invoke quantum mechanics to really understand the generation of the field, which is something that we are not going to do in this course. So, our objective now from this few modules would be to calculate the magnetic field B using certain laws which are experimentally tested and verified and to apply those laws for some practical scenarios.

So, we want to find the magnetic field B because there is some current carrying wire placed somewhere and this current carrying wire is actually generating the magnetic field. So how do I calculate that one? Based on certain experiments it was found out that the magnetic field at any point in the space nearby a current carrying wire is actually given by Mu 0 I dl prime. Now, using the prime to indicate that, this the source current or the source current element * R hat / 4 Pi R square.

So, probably a picture would be of good help here. What we are saying is that, I have some circuit. So I wanted to write a twisted pair to indicate that this is the circuit that is carrying current. And on this circuit, I am considering a very small element I dl prime. It has a certain orientation. This circuit has an orientation and it is carrying a steady current of I. Of course here I am going to connect this one to some battery and I am looking at the magnetic field at some point over here in space or the B field in some space because we are going to soon see that B is not magnetic field conventionally called, will come to that one.

So, this expression for the vector B is actually given by you know the amount of, the infinitesimal amount of the vector B because of the current element, this is called as current element of value I dl prime of the vector element I dl prime is actually inversely proportional to the distance between the two. But, the magnetic field will be perpendicular two vectors. One vector is dl prime, which is going in the direction of the curve or the circuit and other vector is the vector that joins the two points.

This is the source point or the current point and this is the field point. So, the vector would actually be dl * R. And therefore this vector would be in the direction that would be perpendicular to the plane that contains both I dl as well as R. R is the vector distance from the source point I dl prime or I dl prime the current element to the field point, where I am looking at. Of course you would rightly say that this is nonsense because I cannot just isolate a piece of conducting wire.

I mean, I cannot do that one. As I said if you try to isolate a piece of conductor there must be the charges accumulating at the nodes and that is clearly not going to happen, so the current has to flow in a loop. Therefore, the law that we have written in this way should actually be modified such that it applies to the entire current through the loop. So how do I do that? You simply integrate this one.

So if this is I dl prime, you take one more piece and then calculate what is the field because of that? You take one more piece. So I am taking this piece over here I calculating the infinitesimal contribution of this piece or if this current element on the magnetic field and similarly, I will complete the circuit by going back from one point to another point and all points I would have actually calculated the contributions.

After calculating the contributions, I would sum them up but in a limit of small dl the summation would be replaced by an integral.

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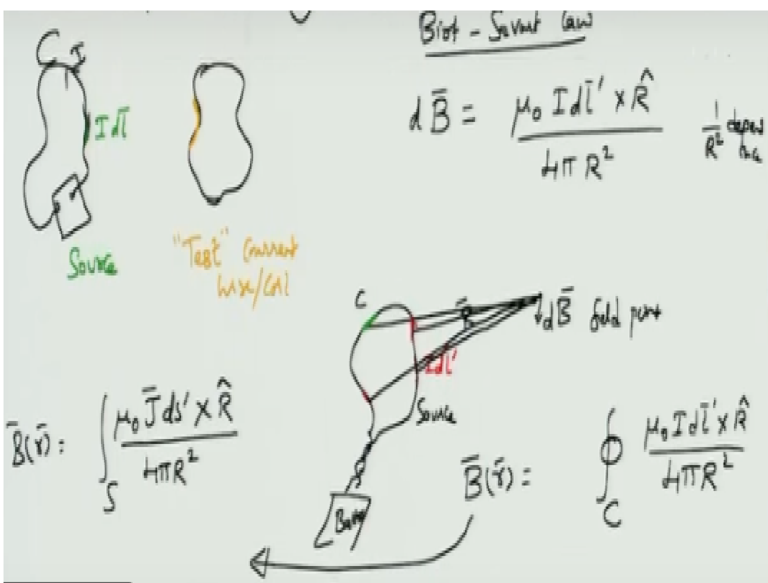

So, what do I have? The magnetic field B at the field point is given by the closed line integral around the circuit C of this quantity. Now, there is a different version of the equation, when you are considering the region of space in that case, the magnetic field region of space in the current that is described by the current density vector. Then you have to integrate over the current density vector.

Because integral of the current density over the surface will give me the corresponding current. The only catch here is that the surface must be open surface you cannot close a surface. Whereas the closed circuit must be because of the closed circuit C. And also note a small difference, we write down traditionally the line element, vector part of the line element is denoted with by making the line element dl prime as a vector.

Whereas J itself is a vector therefore we only have to consider the scalar form for the surface. So, these two laws are called as Biot Savart law. Actually they are derivable from another more general law called as Ampere's law. We will discuss Ampere's law later in the next module. So for this module, the objective would be to try to use this equation that we have just developed and these are called as Biot Savart law to apply in different conditions to find out the magnetic fields.

So, that is what we would like to do now. Before we go there, there are certain things that we need to mention here. So, if you go back to that dB is equal to Mu 0 I dl prime * R hat / 4 Pie R square you will notice two things. One there is a 1 / R square dependence. This 1 / R square dependence seem to come everywhere. You know we saw this 1 / R square dependence in gravity, newton's law of gravitation.

We also saw 1 / R square dependence in Coulomb's law. Now we are seeing this 1 / R square dependence in Biot Savart law. So, there is some interesting thing that is happening because of this 1 / R square. So, that is what we are seeing here as well. So clearly the field will be stronger if you are closer to the current element and if you move away from the current element you would be going as 1 / R square.

The other thing that you have to see here is that, the vector dB is directed in the plane that is perpendicular to both I dl, the current element and the unit vector along the direction from the current element to the point where you are evaluating the magnetic field. This is that cross product that we discussed. There is a third thing that you have to see here, there is a quantity called Mu 0. This is called as Mu 0 or Mu naught. And this quantity Mu 0 or Mu naught is called the permeability of free space.

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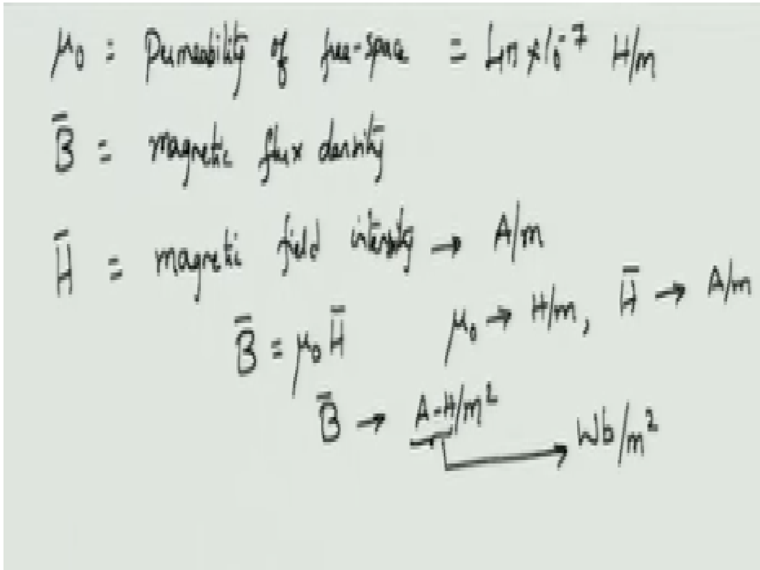

I am assuming that there is no other magnetic material and we are doing everything outside of the magnetic material. So, this is called permeability of free space. So, just like we had permittivity of free space denoted by Epsilon 0. And it was given by some quantity which is approximately 9 * 10 to the power – 12 Farad per metre. We have permeability of free space, which is by definition 4 Pi into 10 to the power -7 Henry per meter.

Again you would suspect that just like Epsilon 0 was related to the capacitance, you have Mu 0 which would be related to the inductance and you will be right. So, when we (()) (09:10) that inductance is measured in Henrys and this Mu 0 is measured in henry per metre. What about the vector B itself? B, here is where things get little interesting because this vector B is called as magnetic field in most physics text books and most physicists call this as magnetic field.

And short form for us would also be calling as magnetic field B itself. In engineering literature, this is called as magnetic induction or magnetic flux density. Just like the vector D is electric flux density and tells you how much of flux lines are coming out per surface area. Here, you have magnetic flux density indicating how many magnetic field lines would come out in a given surface area.

So, this magnetic flux density is also measured in some units. We will discuss that one here. This is actually related to the measurement of another quantity that we will be introducing later and discussing much of that. This H is called as magnetic field intensity or magnetic field for short form. In most engineering literature, this is called as magnetic field intensity or magnetic field itself and this is measured in ampere/meter.

And in free space, these vectors B and H are related to each other by a simple rule, just like D and E was related by epsilon zero in free space in material outside, in free space B and H are related as B is equal to mu zero into H. Now mu zero is measured in henry/meter and H is measured in or there is magnetic field or magnetic field intensity is measured in ampere/meter, making the measurement of B as ampere henry/meter square.

Thank god, at least there is upper meter square, here indicating some sort of a density vector, which is what we have called B as magnetic flux density. And instead of writing ampere henry/meter square every time, we call this ampere per henry as weber and we measure B in terms of weber/meter square.

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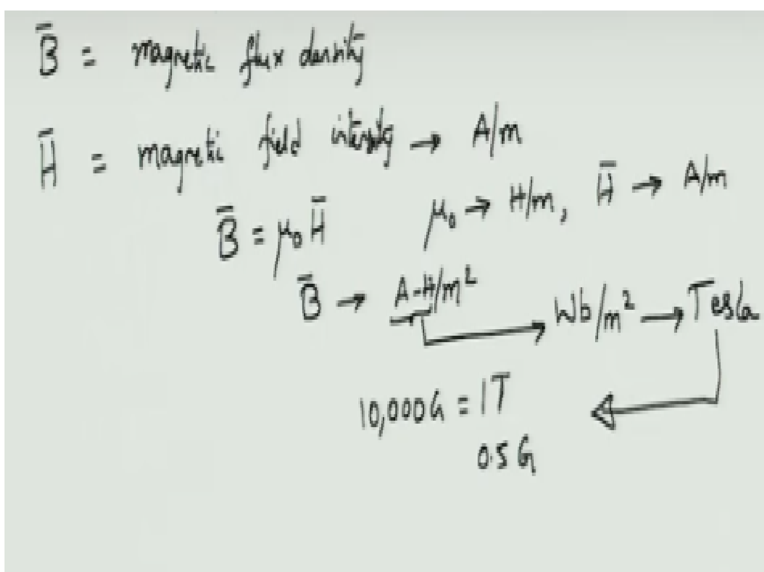

Now instead of writing weber/meter square every time, we denote this by a unit called Tesla, in honor of Tesla, the great, one of the great inventor in the last century. And in older literature, B field was also measured in gauss. Gauss is very nice measurement because the earth's magnetic field density is around 0.5 gauss. So, Tesla is related to gauss, in the sense that 10000 gauss is equal to one Tesla.

So, you have to understand all these different units of measurement units for magnetic field. It is unfortunate that there are so many units. But, that is the way of life here in electrical magnetic field. There is no one set of universal units that are adopted by all people. The mostly commonly or widely adopted units are SI units. In SI units, B is measured either in Tesla or weber/meter square.

I prefer writing this as weber/meter square because it kind of reminds me that this is density vector rather than writing this as Tesla.

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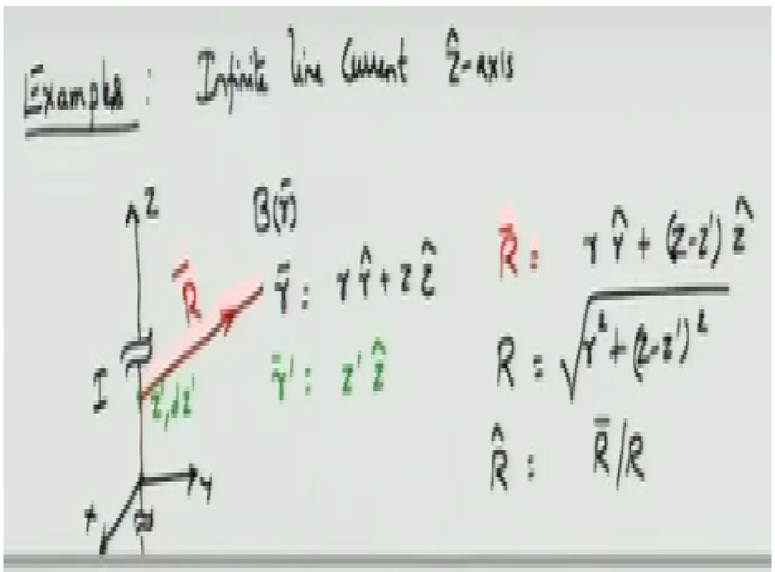

So, we have discussed sufficient enough that we can jump right into the examples. So, let us consider some simple examples and one complex example. This first example is that of an infinite line or infinite line current placed around the z axis. So, this was the line current that we introduced, that we discussed when introducing the magnetic field concept. So, we are going to discuss and find out what could be the magnetic field because of this infinite line current.

So, we have the z axis and the current is actually carried along, the wire is carrying a current of I along the z axis. And we need to find out the magnetic field at some other point over here. Now without thinking too much about symmetry of the problem, which is what we did like when we applied Coulomb's law, we did not really think anything too much about symmetry initially. So we will not do that one here.

So, we will not think about symmetry, we will not talk about the problem, we will just apply Biot-Savart's Law and see what the resulting field would be like. So, the magnetic field is required at this point r and in terms of that it would be nice for us to work in cylindrical coordinate for this case because there is a line which is going along the z axis. So, we are just going to use that fact and say that we will be working with cylindrical coordinate.

So, on the cylindrical coordinates, how can I define the vector r here? The vector r will be, small r, which is the radial distance along r and z, z prime. z is the height at which this point is located. And I am going to consider a small line segment here, which is at a height z and has a height z prime and has a length of dz prime. So the current element here will be given by I dz prime. And this will be located at r prime that is the source point is located at z prime, z hat.

So, clearly the vector R, which is from the current element, directed from the current element to the field point is vector R. And this vector R is given by r minus r prime and it will be equal to r, r hat plus z minus z prime into z hat. What is the magnitude of this vector? The magnitude of this vector is r square plus z minus z prime square under root. And I also require the unit vector R. Unit vector R is given by the vector R divided by its magnitude R.

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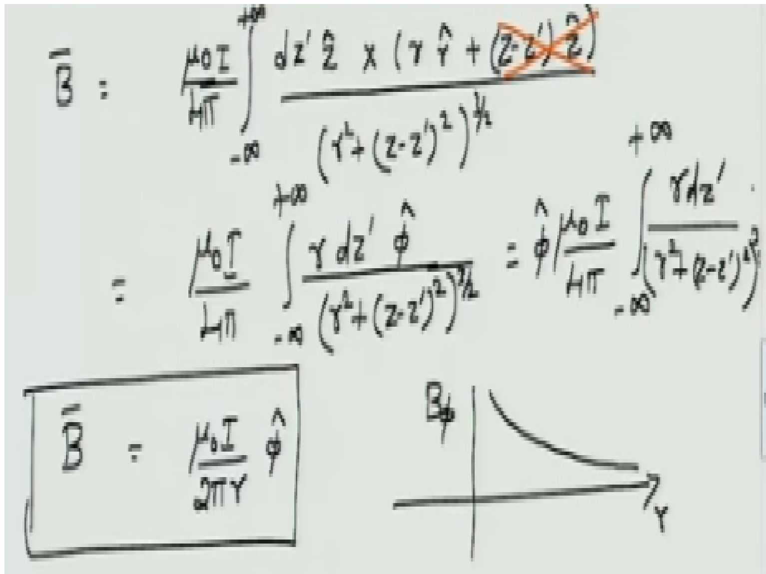

Now we are ready to apply Biot-Savart law. The magnetic field B is given by mu zero/4pi is a constant (()) (16:09) that one from the integral. And I consider the integral from minus infinity to plus infinity because that is where the current element is going. I is also constant, so I am going to remove I. And integration is with respect to z axis, so I have the z prime. But, this is a vector element now. So, this is a vector element now.

So, the vector element is along z axis, so Idz prime, I being a constant I am moving this outside. dz prime, z hat cross the unit vector along r direction. The unit vector along r direction is, that is the unit vector from source to the field point is r, r cap plus z minus z cap into z hat divided by r, there is an r square because of this one and then there is a r because of this, so eventually it becomes r square plus z minus z prime square to the power 3/2. This is something that you already know. You have encountered this earlier also, very familiar to us.

Now look at what is inside here. Inside you actually have a formidable integrant at least it seems that way. To begin with, you have a cross product inside and you have to see what happens to the individual cross product, which direction they would be pointing along. First, we can immediately rule out the second integral here, because z hat cross z hat would be equal to zero from the cross product rule, the second and so, when you take z hat cross this one that could turn out to be zero and therefore this can be removed from the integral.

So, I am now left with only z cross and r. So in which direction should it pass point? This is in the cylindrical coordinates. So, for the cylindrical coordinates, you are looking at z, which is vertically upwards and r, which is this way, so the screw must essentially rotate along phi direction. So, you have the z axis, you have the radial distance r, so you rotate the screw, it could be moving along the phi direction.

So, this becomes z hat cross r, which is along the phi direction and the resulting integrant becomes mu zero I/4pi integral from minus infinity to plus infinity. There is dz prime, there is r here because this r is coming from this one and the vector element is directed along phi axis, at the phi direction. And we have r square plus z minus z prime square to the power 3/2. Now, if you go up and down along the z axis, your vector phi would not change with respect to z.

So, you could go up and down along the z axis, but phi would always be directed along the same direction. In other words, phi is a constant with respect to integration and can be moved out of the integral. r is also a constant in this case, because you are looking at a particular point. Therefore, r is also constant. Only z prime is changing. So, you actually can write this as mu zero I, this will be the direction of phi, divided by 4pi.

And you are left with this integral minus infinity to plus infinity, rdz prime divided by r square plus z minus z prime square to the power 3/2. And we have seen this integral many times. So, the way to solve this integral would be to take r is equal to z minus z prime into tan theta or cot theta and then change the limits of integral, which becomes minus phi by two to plus phi by two and here because of this, it would be r cube.

There will be a r here, there would be a r square because of the differential dz prime coming out. So, you have seen this sufficiently enough that I can write down the final answer without really showing you the steps. I hope that you can fill in the steps, when you look at how we calculated that of a line charge, we encountered the same integral in the electric static case. So, you could apply this knowledge.

And do a simple calculation to show that this integral turns out to be one two by r and the two in the numerator cancels with one of the two in the denominator and you get the magnetic flux density B in the direction of phi and varying only as one by r and not a one by r square. Now this varying of one by r for an infinite line charge should remind you of the variation of the electric field, also as one by r with respect to the infinite line charge density.

So, it is essentially the same sort of behavior, when you have infinite line charge or an infinite current involved. So, if you want to sketch this one as a function of r, so you will see that initially at r is equal to zero, obviously this vector B will blow up. The vector is actually along the phi direction and then its magnitude keeps on decreasing as one by r.

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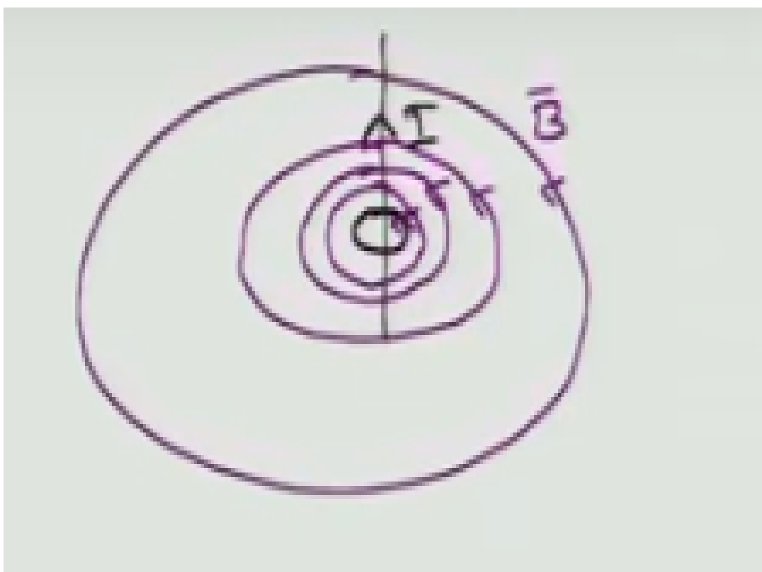

Now it is interesting that you can actually draw this B phi and you will see that if this is the current that is being carried by the infinitely long conductor, then the B fields would actually be located, let me use a different colour here, the B fields would be circulating this current. So, the B fields would actually be circulating these lines. So, this is the B field for infinitely line, infinitely long conductor carrying a steady current of I amperes.

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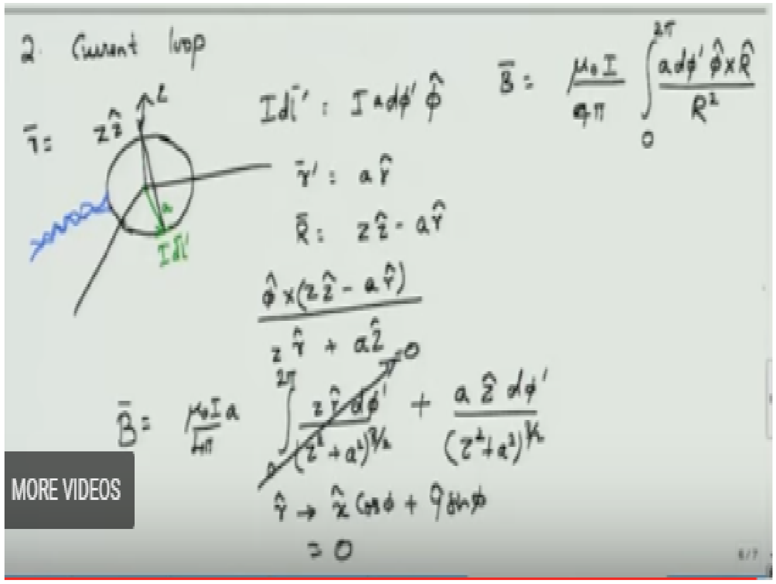

Now as a second example, which is important, we will look at current loop. This becomes very important because this current loop can be used as an antenna and believed to find out what is the magnetic field of this. Of course, in an antenna you are really looking for time varying currents. We want to convey information. Therefore, there is time varying currents there. But here we will introduce you to the static electric field and that is of some importance as well.

So, how would the current loop be there? So, let us assume that there is a current loop in the x y plane. The z direction is perpendicular to this. And again I have to make some space for my current. So, I have to have some space. This is the way in which there will be an incoming and outgoing current. So, this is the current loop that I am considering. The current loop is kept in x y plane, in the horizontal plane, has a radius of a.

So, the radius of this one is a. And at any point here, I need to consider the current element IdL prime. Now, in which direction the current element would be pointing? The current would be pointing along and I know that IdL prime would be pointing in the direction of phi. And what would be the line element at this point? The line element is ad phi prime, pointing in the phi direction.

This is the current. Now, where do I want the field? Now I will be very happy to find the field at a height H, above the loop. I am, I will be not H, let us say at height z, above the loop. Therefore, the r vector, the field point will be given by the z, z hat. And I need to construct a vector, which will take me from the line element to the field point. So, I have r as z, z prime. And what could be the r prime, the source point? The source point will be the vector, which is ar hat.

This is the source vector. This is your direction of r hat vector. So, this is the direction of r hat vector. And you would see that this is ar and therefore the vector r is given by r, which is basically zz prime minus ar hat. And the magnetic field will be given by mu zero I, is going to be anyway constant, divided by 4pi and in the integral I will have to go from zero to 2pi. The line element is along the phi direction cross R hat divided by R square.

So, you can see what would be the phi hat cross zz hat minus ar hat is. So, if you see this one, you are going to see that this will be phi hat cross z. So, phi hat cross z will be along r direction and phi hat cross r will be along minus z direction. So, there is already a minus sign therefore that becomes a plus. So, this becomes zr hat plus az hat. So, this integral will become, sorry this cross (()) (25:35) will become zr hat plus az hat.

You can substitute that one into the expression for the B field. So, B will become mu zero I/4pi, a also is a constant, you can take this a out and then you have left with the integral zr hat divided by this is z square plus a square to the power 3/2 and integration with respect to d phi prime plus az hat integration with respect to d phi prime divided by z square plus a square to the power 3/2. And this integral with respect to r actually goes to zero.

This integral goes to zero. One simple way of thinking about why this should go to zero is because if you look at r hat, r hat can be written as x hat cos phi and y hat sin phi. And integration of cos phi and sin phi over the entire interval of zero to 2pi, this will be equal to zero. Ok. We will continue this in the next module.