Electromagnetic Theory
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Lecture 25
Conductors – IV (Contd.) & Capacitor - I

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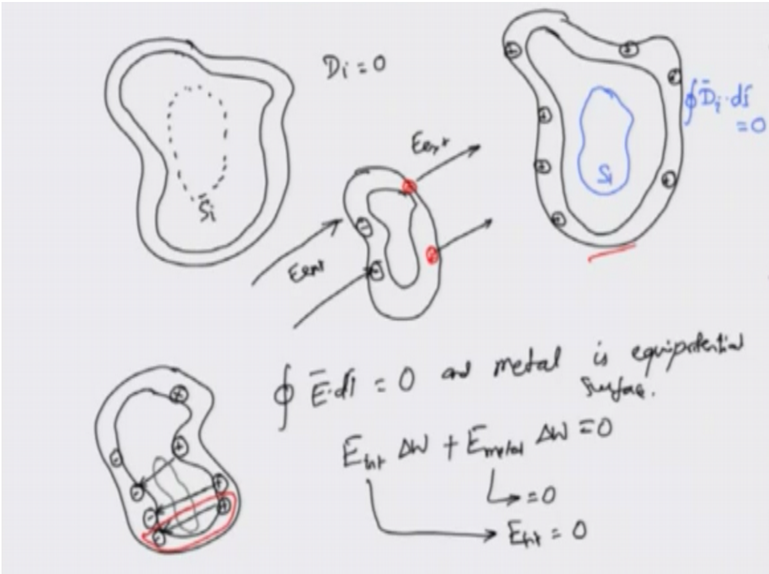

Let us close up conductors by discussing an important aspect of them called enclosures, okay? This is something that you would find in various situations. You have some electric fields that are coming from various sources. It could be the power line electric field, it could be the electric field from your cell phone. It could be the electric field from some other electromagnetic component that you are using and you want to shield this electric field in reaching a certain apparatus.

Well, you have an apparatus which let us say is a medical equipment and you want to shield this medical equipment from any external electric field. How would you do this? Well, you must have also heard about enclosures. So you take a piece of metal, preferably a copper or aluminium and then you surround the apparatus that you want to protect by placing the apparatus in the enclosure formed by a metal.

So if you consider an arbitrary metal enclosure of an arbitrary shape that I have drawn and you can put any apparatus that you want, so there is some apparatus here that I want to protect, now this gets protected because there will not be any electric field inside. Why is there no electric field inside? Obviously if you take, if you now remove the apparatus for a minute, then take the interior surface si because this surface does not enclose any charge, of course, you do not want to place a charge inside assuming that you are not placing a charge inside there are no charges here and consequently there are no Di fields.

Now you imagine what happens when this arbitrary metallic shell is exposed to some charges. It could either be by inducing a charge. So when I induce a charge, these charges would start accumulating on the surface of the conductor. They would accumulate on the surface of the conductor and still there will not be any electric field inside the interior.

It is interesting why that is so, we will discuss that in a moment but going by Gauss's law definitely there will not be any electric field or there won't be any charges enclosed, right? So Gauss's law for this would still tell you that if Di stands for the internal D field then integral of Di dot d s will be equal to zero on the internal surface Si. Suppose you consider the same metallic shell and then expose it to an electric field, an external electric field, call this as E external what would happen?

The external field would simply go or transmit through this enclosure or seem to be transmitted through the enclosure but because of this electric fields there would be charges induced much in the same way as the charges that are induced in the previous charging process. The charges induced are however, that was supposed to be a charge induced, let us write down this, the charge induced is a negative charge here just on the surface of the electric field, on the outer surface, not on the inner surface.

Similarly, there will be charges here which would be induced on the surface they would all be positive charges. So in a way what has happened is, the charges are induced on the outer surface and not on the inner surface. So there is no electric field inside. Now you might ask, all that we have said from Gauss's law was that, if you apply Gauss's law to the case where this was exposed to an external field or some charges were induced is that you can say Di is equal to zero.

But I know that a closed surface integration of a flux density can still give you zero if there are equal amount of D lines coming in and equal amount of D lines coming out. So there must be no divergence, which we understand if there is divergence there must be source of charge, but it could very well happen that they would form continuous tubes or lines of field, circulating or closing up on themselves such that there is no divergence, but how can you say that there is no Di, in other words, what we are claiming is that if I take this arbitrary metallic shell.

Then what we are claiming is, there could be some charges induced on the inside metal or there could be charges on the inside surface of the conductor and there would be an electric field because of this charges on the inner conductor, it could probably happen there. So if there are this situation where our charges are induced on the inside then there will be an electric field coming from positive to negative charge.

So there would be some electric fields that are coming from positive to negative charge. Now if you were to take a Gaussian surface, so let us say this is my Gaussian surface, clearly there would be, the integral of D dot ds will be zero, but there is an electric field or there is a D field inside here. Can this happen? Turns out that this cannot happen, okay? Consider the same scenario that we have shown here.

There are reasons why this will not happen, we will do that. Now, instead of considering the Gaussian surface lying entirely in the interior let me consider a Gaussian surface that lies partially in the interior and partially in the metal. The red color surface that I have shown indicates that the contour is actually lying partially in the interior and partially in the metal.

Let us also assume that all this normal parts of the path will not really contribute much and now if you apply Gauss's law to this one, what you will see is that, or if you apply the integral of E dot dl to this part, you expect that integral of E dot dl to be equal to zero, because this is the potential difference of a point and this is an electrostatic case that we are considering.

So this is equal to zero because metal is suppose to be an equipotential surface. Sorry, what we mean is that this line integral must be equal to zero and metal is equipotential surface. So we take these two facts as our starting points. And now if you apply this line integral equal to the red colored contour that I have shown which is like partly in the interior and partly in the method, you can approximate this integral as E interior times delta w where delta w is the length of the path plus E metal or E conductor into delta w what must be equal to zero.

So clearly delta w is not zero, the path is not zero, the path length. However, E metal is certainly equal to zero. This is because this is the property of a conductor. The property of the conductor is that E metal is equal to zero. Since E metal is zero, it will lead to a conclusion that E int is also equal to zero. If you are not satisfied with this explanation, you think of this in other way. Now if you did not have this zero and internal field was not zero but metal field was definitely zero because metal cannot be having any electric field.

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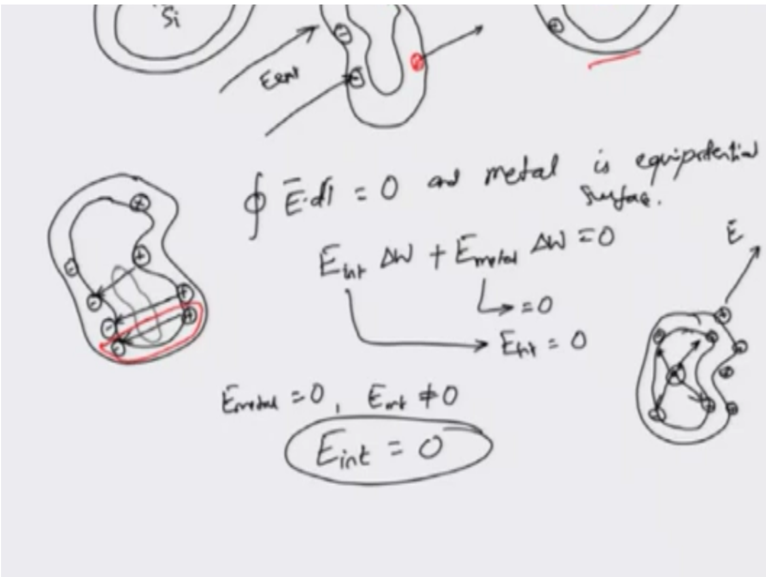

Suppose this happened and E metal was zero but E internal was not zero. That means there is some potential difference between the metal surfaces, but we have just said metal is an equi potential surface. At all points on the metal the potential difference must be zero, that is metal itself must be at a particular potential with respect to some reference or the origin point.

So if the fact that internal electric field is not zero, it simply tells you that the metal itself is not an equipotential surface, now that cannot happen because metal is an equipotential surface, right? So both ways the discussion would show that the condition for internal electric field must be zero. If it is not zero, it will mean that the metal surface is not equipotential and if there is non equipotential then charges would flow from one point to another point, from the higher potential to lower potential.

They essentially move towards each other and neutralize that. So you can start with that condition. But once the charges are neutralized there will not be any electric field inside. So you can have momentarily that is 10 to the power minus 19 seconds of rearrangement time, but for all practical surfaces, that time is so short that we can confidently say that internal electric fields must be zero in that of metallic enclosure.

Now if you deliberately place some charges inside an empty metal shell, what happens is that, suppose this is a charge that I have placed, there will be electric fields. These electric fields will induce charges on the surface and there would be other charges induced on the outer surface as well, such that the electric fields would be because of the internal charge that we have placed.

So we will not discuss this too much here but this condition seems, this condition is not the same condition that we talked about in the last few minutes, right? In the last few minutes we had an empty enclosure. There was nothing of charge that was placed inside, but now we have charges that are deliberately placed inside. Then clearly if you are placing charges then the electric fields cannot be zero inside that of metal enclosure.

So the charges cannot be there, I mean fields cannot be zero inside that of a metallic enclosure when you place a charge inside, okay. This brings us to the end of conductors. We will move on to a next important task of finding what we call as capacitances.

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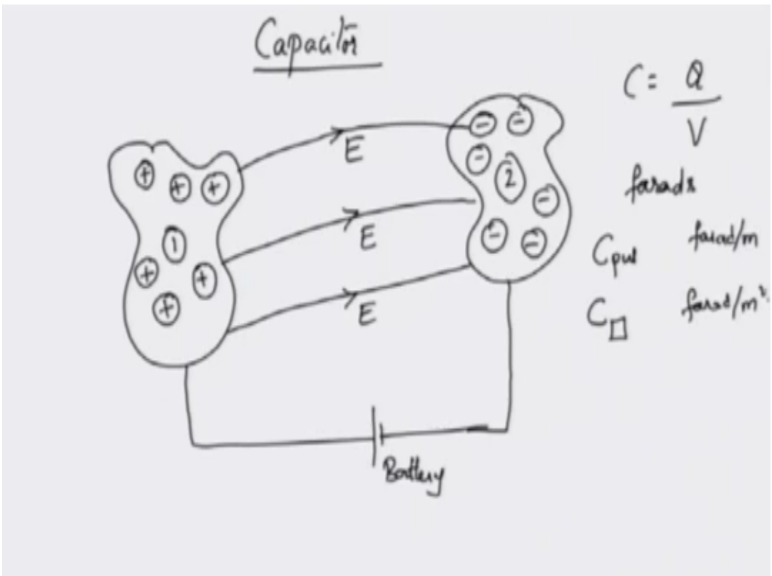

So before that we need to start by defining what a capacitor is and how we go about finding that one. Now amongst many many many applications for electrostatic that we have been studying so far, electrostatic fields, one of the most significant application is to find capacitors. Capacitors pop at various places. You take a solid state device such as a MOSFET or a BJT, you see that there are capacitors associated.

Whenever you have two conductors or two charge layers separated by an insulator there is a capacitor. So capacitors are present in every place where there are two charges or opposite polarities, separated by a insulating layer in between. Of course in many practical transmission systems such as the transmission line or a coaxial cable or a micro strip line the capacitors are quite natural because they are transmission lines.

We will see when we discuss transmission lines that they are modern in terms of circuit quantities of resistance, conductance, inductance and capacitance, so in that case capacitance comes up naturally and we want to establish methods to calculate capacitances. It turns out that although the problem is so fundamental, there is no closed form solution for different kinds of geometries.

There are certain geometries, as simple as a parallel plate capacitor that cannot be solved using any of the techniques that we have developed. In fact, we have to go for numerical technique to kind of find out what the actual capacitance of a structure is. So we will of course not be looking at numerical methods in this and the next class. We will discuss numerical methods shortly afterwards.

Our idea would be to consider situations or geometries of the capacitors and make certain approximations so that we may be able to obtain some closed form expressions. But please note that these closed form expressions are obtained for simple cases only, not for very practical cases. However, the differences will be so small in most cases that one can neglect that, that if you want to get numerically accurate answers you have to employ numerical methods.

Okay, we have given enough introduction about capacitors and we require capacitors. The point about capacitors is that, although we are used to thinking of capacitor as some sort of a parallel plate capacitor or a different kind of capacitor, capacitors are simple geometric functions, in the sense that the geometry of the arrangement of the conductors determine the capacitors In fact capacitor or capacitance of capacitor is nothing but geometric arrangement.

It is actually an arrangement of a geometric arrangement and different geometric arrangements can give you different capacitance values. So we start with two arbitrarily conducting bodies and we charge them with opposite charges, so for example this conducting phase two is charged with all negative charges here and then the conducting body one here is charged with all positive charges.

The charge polarity on both these bodies are different and they are charged to opposite polarity. So this is essentially two bodies that we have. We know that if you take these two charged bodies and place them at some distance apart, you will see that electric field lines are going to be generated from the positive charge and they will terminate on the negative charge.

So there would be seeing lot of electric field lines going from one body to another body. So these are the field lines you would see from one body to another body. Now to generate these charges, you can take the two charges initially uncharged but then apply a battery that will induce charges. Okay, so if you apply a battery that would induce charges. And this application of the battery would cause a potential difference between the two conducting bodies.

However, these are conductors and we have just discussed that conductors are equipotential surface. So keep that in mind. We define capacitance as the amount of charge stored in one of the conductors, because we assume that they both are storing equal amount of charges. So C is equal to the charge Q divided by the potential difference that exist between the two conductors or if you are considering the battery and apply voltage of V how much charge gets stored for every volt that you apply defines the capacitor.

Capacitor is measured in farads. Sometimes we will be looking at capacitor per unit length a concept that is quite popular in transmission lines. In that case you are measuring this as farad per meter, okay? And sometimes especially in VLSI systems you will see that you are looking at capacitance per square. Similarly, we will be looking at resistance per square. So here capacitance per square and this would be farad per meter square or centimeter square or millimeter square depending on the geometry that you are considering.

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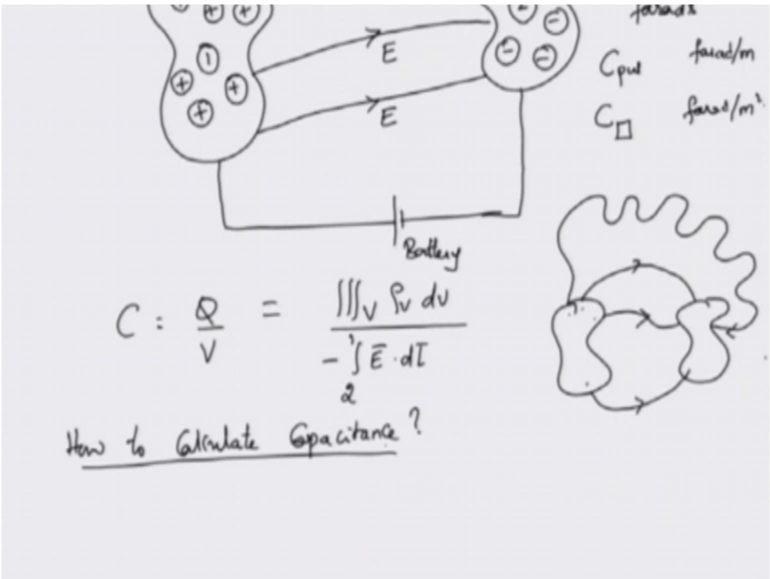

Now the definition of capacitor we have seen. C is equal to Q by V and the amount of charge that gets stored on a given conductor for a given potential difference is the function of how the charges are induced, right? So they must be related to the electric fields that they are going to produce.

So how are charges related to electric fields. Charge stored we know is given by the volume charge density, that is there on the conductor, one of the conductor that you are considering and volume charge density that is integrated throughout the volume of a given conductor. So if it is conductor one, then it would be the charge stored on the conductor one and how do we define the potential difference between the two conductors?

Well, we have already seen that this must be the line integral of the electric field. So you have the line integral of the electric field from conductor 2 to conductor 1, assuming that conductor is at a lower potential and conductor one is at a higher potential. Now here is an important question. What path should I take? The answer to this is that, conductors are equipotential surfaces.

So it does not really matter which path you take and most importantly it does not matter which point you take on the conductor, right? You could for example have two conductor and your path could be this or you can have a path that would be along this way, so some directed path. You could also have a path in the middle. You could have a path that would do all these things and come back and you could have this kind of a path.

Of course, the reason why all these paths work is because at all points in one of the conductors the potential difference is zero. The potential is actually constant. Difference is zero but potential is constant. On all points on the second conductor the same thing, potential is constant. Okay, so the difference in the potential if you want to calculate you can start at any of these points that we have talked about and you can follow any path because in the electrostatic condition the potential difference is independent of the path that you follow.

How do we calculate capacitance or how do we compute capacitance? If you are looking at numerical methods, that would be the questions that you would ask. How do we calculate capacitance? There are two methods to calculate capacitance, at least that we will discuss.

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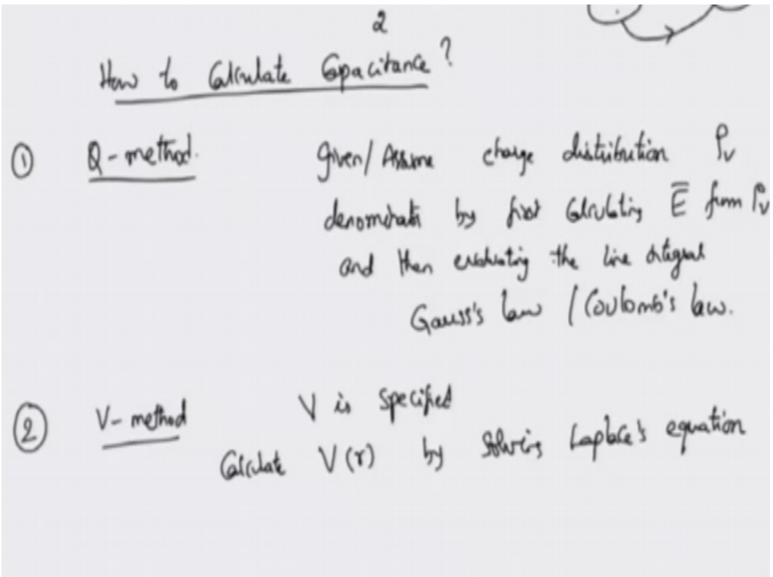

The first method I would call this as Q method or the charge method. In this charge method the idea is that, you start with or you assume given or assume reasonable charge distribution on the conductor. This assumption would mostly be guided by the situation that we have already seen. It could be either a lined charge distribution or it could be a surface charge distribution or it could be a volume charge distribution.

But you have to assume or if the charge distribution is given to you, then no problem you take that particular charge distribution, okay? So from the given charge distribution in general that of the volume charge, let us say rho v, you can evaluate the denominator by first calculating the electric field from given charge distribution rho v and then evaluating the line integral.

So you first calculate the electric field from given charge distribution rho v and then evaluate the line integral, integral of E dot dl between the two conducting parts and you will be able to obtain both the numerator and denominator. And as I said you can either calculate capacitance, capacitance per unit length, or capacitance per unit square. So this step of calculating electric field from the charge distribution requires you to use either Gauss's law or Coulomb's law.

The second method is what we call as the V method. In the V method, you assume that potential are specified, V is specified, say one conductor is held at a particular potential and the other conductor is held at another potential with respect to the origin or a reference or the potential difference is specified. From this, calculate V of r between the regions. Calculate potential between all the points v of r by solving Laplace's equation.

So we know Laplace's equation gives you the potential at all points or all points in the space. You can use that to calculate the potential difference, or the potential function.

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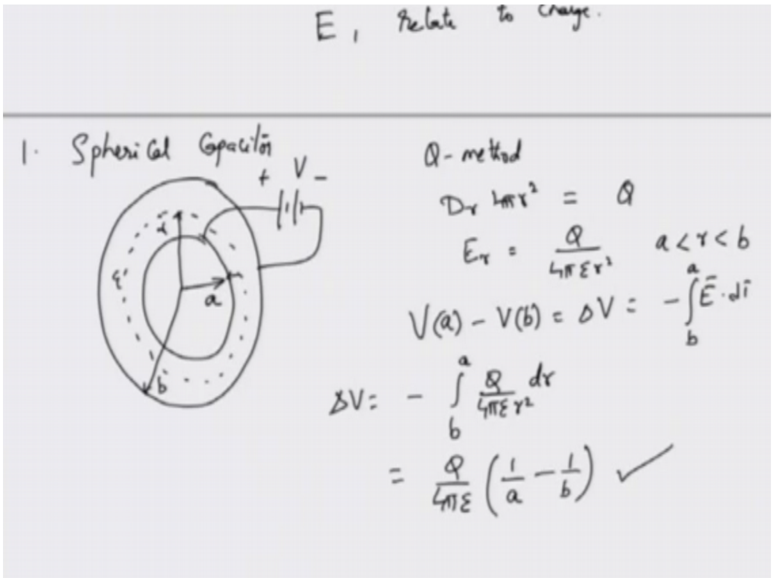

From this calculate electric field. From electric field you relate this to charge distribution and then charge to voltage ratio will give you the capacitance. Okay, we will see both examples in the following. Some of the examples will be simple because we use Gauss's law and where Gauss's law cannot be used, this turns out to be a pretty hard exercise of computing capacitance, if you don't use numerical methods.

Let us start with simple capacitor called spherical capacitor. Here all you have to do is to take a shell of conductor, typically two shells of conductor, let us say, the inner shell has a radius a, the outer shell has a radius b. We assume that b is greater than a. And I will take the inner shell and keep that as positive and take the outer shell and keep that as the negative. That is I take a battery and connect the positive lead to the inner shell and negative lead to the outer shell.

So because of this there will be charges that will be developed because these are conductors and conductors when they are held at different potentials will induce charges. So there are charges that are produced and this charges would form a surface layer of charge of appropriate density. Now if you use Q method, the charge density can be assumed to be spherical.

You know, spherical is symmetric charge density that you can assume. From there, you can calculate what is the electric field. Because of the symmetry it is easy to use Gauss's law. So to any radius r which lies between a and b, any radius r, I can apply Gauss's law. What does Gauss's law tell you? Dr multiplied by 4 pi r square which is the surface area of the sphere of radius r multiplied by the radial component of D because of symmetry there will be only the radial component of D.

This must be equal to the total charge enclosed. Now, total charge enclosed is on the surface of the charge. So what is the total charge enclosed? That is let us call that as some Q, does not really matter how much charge is enclosed on the surface a, so let us call that as Q. From here I know what is the electric field Er. Er is given by Q by 4 pi epsilon r square. What is epsilon?

Epsilon is the material that fills this particular thing. So material that is filling this two medium. So I calculate what is the electric field here. This electric field will be valid from a to b, that is in the region between the two concentric shells that will be valid. What would be the potential difference?

The potential difference would actually be, the potential difference between the inner and the outer conductor call this as some delta v and we know that this is obtained as the line integral, E dot dl from two to… sorry, not two to one, two here is b and this one is a with a minus sign up here. Or we could reverse the integral limits as well. Now I know that I can choose any path.

Let me choose the path which is convenient to me, the electric field is radially decaying. So I will choose the radial path. So I come from r to a. This is the path that I will choose. Radially, I will come from sphere of radius r to sphere of radius a. Radius a is the one where we have kept one metal shell, okay? So if you evaluate this, you are going to see that delta v is equal to minus integral b to a, electric field is q by 4 pi epsilon r square and the line integral along dr r hat therefore this would essentially be dr.

So if you integrate this and substitute the appropriate integral limits you will see that this will be q y 4 pi epsilon, 1 by a minus 1 by b. This kind of makes sense because the inner shell was kept at higher potential and 1 by a is greater than 1 by b. So this is the potential difference that exists between the two shells.

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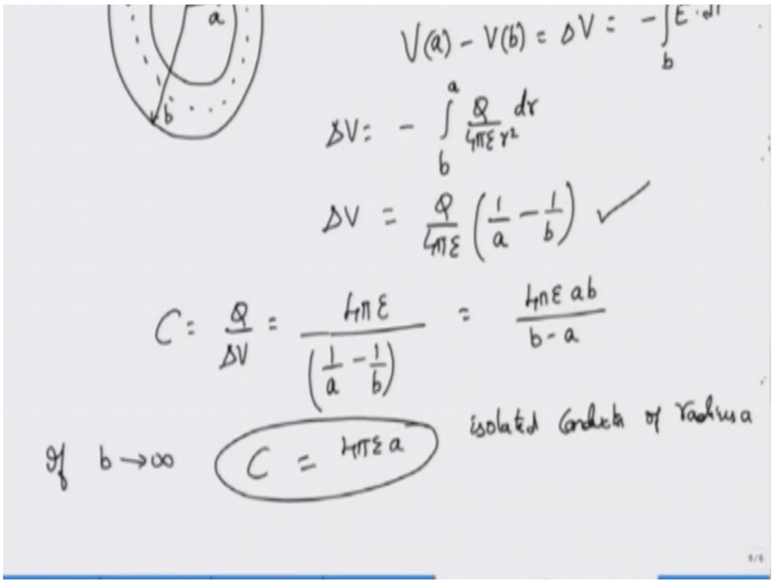

Now I know charge, I know the potential difference, the ratio of these two should give me the capacitance. So C is equal to Q by potential difference delta v. So delta v is equal to Q by 4 pi epsilon times 1 by a minus 1 by b. So you bring this 4 pi epsilon guy to the numerator and what you see is the capacitor given by 4 pi epsilon divided by 1 by a minus 1 by b, okay?

You can simplify this by multiplying by a b, so you get 4 pi epsilon a b by b minus a. This makes sense because b is greater than a, so minus a is a positive quantity. What happens if b goes towards infinity. That is if I take the second shell and start moving the shell away from the shell of the inner shell of radius a.

If I start moving the value of b towards infinity, what happens is that b minus a becomes almost b, b cancels on the numerator and denominator and you get capacitance as 4 pi epsilon a. This is the capacitance of an isolated conductor of radius a, okay? So this is the Q method for finding the capacitance of this spherical capacitor.

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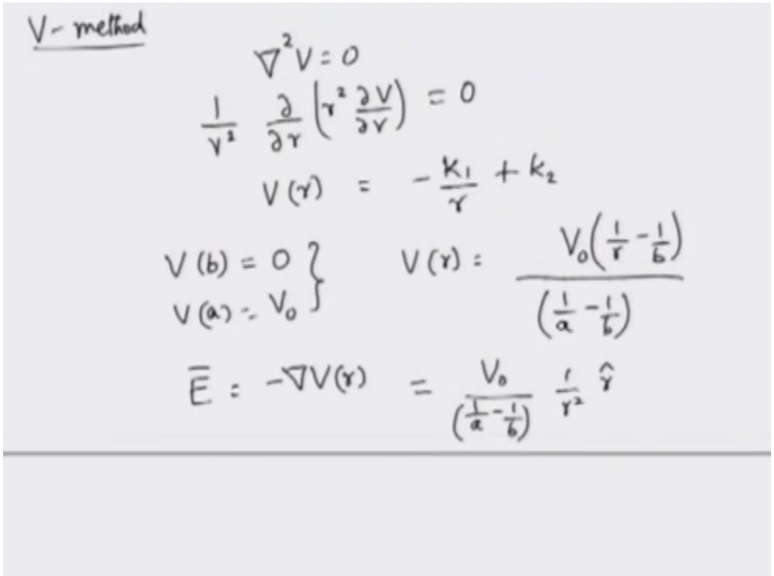

Now let us try to apply V method here. We will discuss rigorous solutions of Poiseuille's and Laplace's equations later, so what we are going to discuss is a very much that is required for finding the capacitances in these simplified structures. So for the V method, I need to solve del square v equal to zero.

Of course, I need to solve this in spherical co-ordinate system and since v is a function of r only, it is reasonable to expect that we can only use the terms corresponding to v of r. So if I do that one and looking at the Laplacian in spherical co-ordinates from textbook or from mathematical handbooks, I get that this is 1 by r square, del del r of r square del v by del r.

The terms corresponding to theta and phi are removed because they do not really help me in finding this one because v is only function of r. So if you solve this equation and convert all the partials to total differential because v is a function of only r, we will see that v of r will be equal to minus some constant, minus k1 by r plus k2, okay? Now you can evaluate this k1 and k2 constants by applying the appropriate boundary condition.

I know that at boundary b the potential is kept zero, that is the potential difference between the two is v and the inner shell is at a potential v with respect to the other shell. So v of b is zero, v of a is some applied potential v0, okay? So if you apply these two boundary conditions to this v of r, you can show that v of r can be written as v 1 by r minus 1 by v in the numerator divided by or v0, 1 by a minus 1 by b, okay?

Now the next step would be to actually find the electric field and we know that electric field is given by minus gradient of the potential, again going to the gradient expression for the spherical terms you will see that this would be 1 by r times del by del r, and if you differentiate this potential v of r with respect to r and solve for the gradient, you will see that this is given by v0 by 1 by a minus 1 by b, times 1 by r square, r hat. So the electric field is radial and it is going as r square. It is going as 1 by r square and it is entirely in the radial direction.

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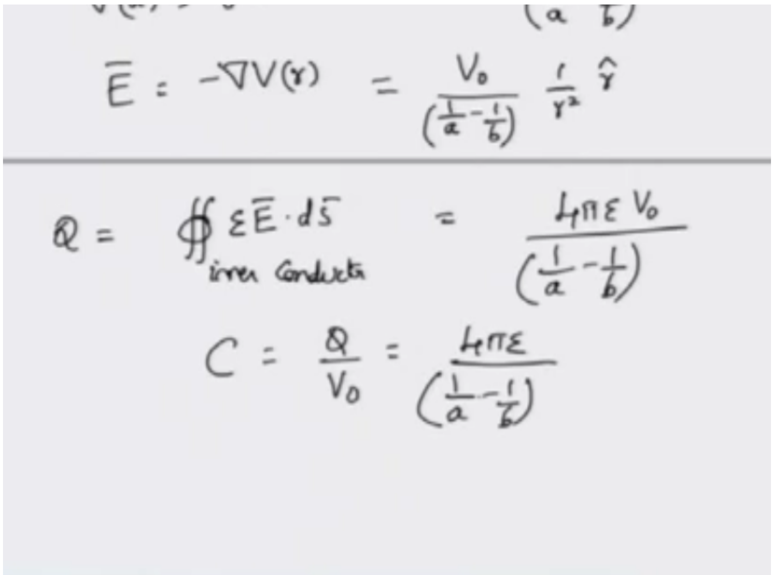

What is the total charge enclosed by the inner shell? The total charge enclosed can be obtained by epsilon E dot ds of the closed surface of the inner conductor, of the inner shell, if you evaluate d dot ds you are going to get the total charge enclosed. So you can see that what it would be, the surface element will be a square sin theta d theta d phi and you can see that the total charge Q will be equal to 4 pi epsilon v0 by 1 by a minus 1 by b giving you the capacitance C as Q by v0 as 4 pi epsilon by 1 by a minus 1 by b, the same as the earlier method.