Electromagnetic Theory
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Lecture - 24
Conductors - IV

In the last class we started discussing conductors and conductors are mainly used to conduct electricity because conductors as opposed to dielectrics have a lot of free electrons as we discussed in the last class, conductors have an abundance of free electrons because their conduction band overlaps with that of the valence band.

Most conductors such as aluminium, copper, silver they have so much of free electrons, free electrons in the sense that they are very loosely attached to the lattice, a small amount of electric field that we could apply would then enough force or it would accelerate the charges enough such that these charges would start moving and constitute a current.

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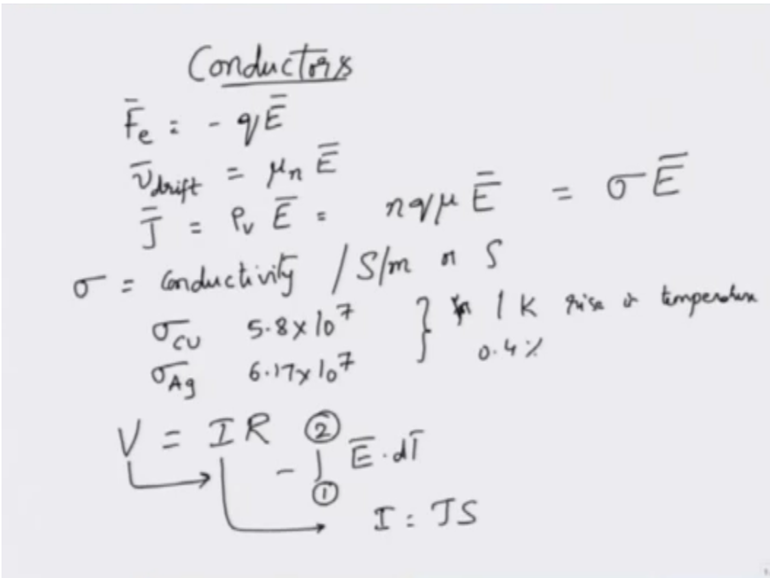

The force on each electron inside that of a conductor is given by the electric field acting on that conductor at that point and it is given by minus q E where E is the electric field and Q us the charge on the electron and this minus sign indicates that the electrons would actually move opposite to the electric field. So while the electric field for example is along the x axis then the movement of the electrons to that electric field would be opposite to that of the x axis.

However, a positive charge such as a hole in a semi conductor would move along the direction of the electric field. So conventionally the direction of the electric field lines would represent the actual direction of a positively charged particle, since electrons are negative they will move opposite. It turns out that the velocity with which these electrons move is called as a drift velocity is actually proportional to the electric field.

So the drift velocity is related to the electric field by a quantity called mobility. Mobility tells us how mobile a charge is, in the sense that how easy is that charge would start to move when an electric field is applied. Compared to holes which you would read in the semi-conductor device physics courses.

The mobility of electrons is much higher. In fact, the difference between the mobilities of holes and electrons is what makes the speed of the devices such as MOSFETs be capped off, in other words there is a limit in the speed because the holes moves at a much slower speed compared to the electrons. And one can actually use this drift velocity, substitute this in the expression for the current density.

We know that the current density is related to the electric field as the velocity, charge density rho v times electric field and you can see that this charge density at any point will be given by n q mu. I am writing this for the case of an electron and I am not really using the minus sign. You just have to remember that electron move against the field, however their current would actually be considered in the direction opposite to the electron movement.

So the conventional current flows from left to right as the electrons would move from right to left. So I am not writing the minus sign, but you have to mentally understand that quantities would be negative wherever appropriate. So when you are considering electron flow you have to use the appropriate minus signs here. So the current density J is related to the electric field by this expression, okay?

Where the charge density is given by, this is rho multiplied by v, so this is rho v, so n is the number of electrons per unit volume, q is the charge. Therefore, this corresponds to charge per unit volume and mu being the mobility of electron would tell how quickly or how fast this electron is moving, okay?

In common language, this n q mu is given by a quantity or is represented by a quantity called sigma and sigma stands for conductivity. And in olden days it was measured in 1 by ohms or sometimes called as mos. The modern units of measurement of conductivity is that of a Siemens, okay? So conductivity is measured in Siemens but these are sometimes also measured in terms of Siemens per meter.

So conductivity or conductance is measured in Siemens and sometimes they are also measured in Siemens per meter when you are considering the appropriate quantities over here or Siemens let us just say and S stands for Siemens. If you look at typical values of the conductivity of say copper, copper has conductivity of about 5.8 multiplied by 10 to the power 7 and that of the silver is around 6.17 multiplied by 10 to the power 7.

Note that these conductivities although they are quite high, they are not really infinite, right? So if you consider a metal or a perfect conductor as that having a value of sigma going to infinity, these are not perfect conductors, there is some amount of resistivity amongst these conductors. Resistivity is roughly inversely proportional to the conductivity. So because sigma is not infinity, there is some amount of resistance.

So if you were to take a capacitor which we will be discussing very shortly and you know the capacitor plates are made up of this conductors or finite conductivity what would happen if the voltage is applied because the resistivity is not zero, there would be some amount of current flow that is happening inside the conductor itself leading to loses. These conductive loses become very important especially at high frequencies as we will see when we discuss transmission lines.

More over the conductivity that we have represented over here or given the values over here are not really constants. They are dependent on temperature and for every one-degree Kelvin or one Kelvin rise in the temperature for every one Kelvin rise in temperature you will see that the conductivities would change by about 0.4 percent. In fact, this can be used as one of the temperature measurement, a very sensitive temperature measurement instruments.

Now let us develop one very important relationship between the voltage and the current of a conductor. Before we go further let me remind you that J is equal to sigma E is a relationship that holds in most materials although calculation of sigma is not really done by just giving n q mu but requires quantum mechanics to properly give you the values. This equation or the result J equals sigma E looks so much like V equals I R or I equal G V that we call this J equals sigma E as Ohms law, okay?

The proper or the more popular form of Ohm's law is that of the relation between voltage and current. We already know what is voltage, how do we represent voltage or how do we relate voltage to electric field. We remember that voltage was the potential difference between any two points and this potential difference was the line integral of the electric field along any path.

In the electrostatic case that we are considering, this line integral and hence the potential difference was independent of the path. But V was related to the line integral of electric fields. So it is given by some point 1 of the conductor to point 2 of the conductor and electric field. So this is how the voltage or the potential difference between two points is related. We also know how to relate current to J because current is J multiplied by S, I am not writing the integral relationship.

But the essential idea is that you take a surface which has to be opened, not closed. So you take a surface and how are the J field lines coming out of the surface and integrate those J over the surface because it could happen that the surface you are considering will be at an angle with respect to the J field lines. So in that case what will happen is I will be equal to integral of J dot ds.

However, if J and S are aligned perpendicularly in the sense that surface is perpendicular to the J field lines, then the current through that open surface will be equal to J multiplied by S.

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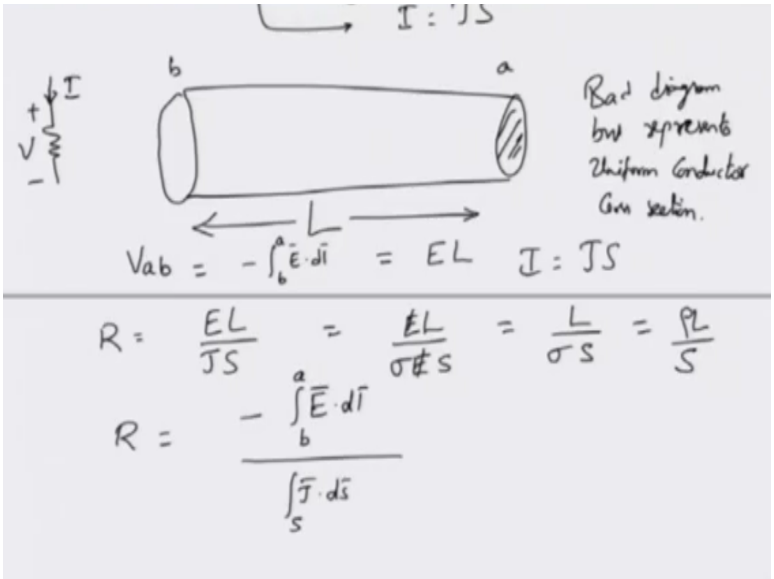

Consider the scenario in which I am considering a piece of conductor of certain length L, of course this must be a uniform conductor but the drawing might not really represent this. So I can say this is bad diagram but represents uniform conductor, conductor of uniform cross section. So how do we calculate the voltage difference between the two.

So if you label these points as some point b and point a, the voltage difference between the points a and b is simply the line integral of the electric fields, that is there inside this conducting material, okay? From point b to point a, so integrate the electric field inside this one assuming dl points long this line from b to a and whatever you do, you are going to get the corresponding potential difference.

So if the electric field and the dl are aligned properly and if the electric happens to be uniform over the piece of length we have considered then you can simply replace this by E multiplied by l. Whether b is at higher potential or a is at higher potential you have to determine appropriately. For example, if this is a situation in which b has a higher potential then the potential difference from b to a would be called as potential drop or voltage drop.

Otherwise a to be would be called, or b to a if b is smaller and a is larger we will call it as potential rise. In a typical circuit element of a resistor that we consider, we assume a potential drop and a current that is flowing in. So if you assume that the current I is flowing in and the potential drop is V, the relationship between V and I is the resistance or the ratio of V by I is the resistance.

So the potential difference or the potential drop let us assume is given by E multiplied by L and the current I will be J multiplied by S, where S is the cross section of this open surface that you are looking at. So if the J field lines are there, then the current through this piece of material will be J multiplied by S.

Let us also assume that these quantities are uniform. Now resistance as we have just described is given by the potential difference which is E L divided by the current through the conductor. The current through the conductor is J S. However, I also know that J is related to electric field, what is it? J is Sigma times E from the field Ohm's law. So from the field ohm's law we have J equals sigma E.

I can substitute that inside here to get sigma E multiplied by S in the denominator, cancelling the electric field from numerator and denominator gives me L by sigma S. This is a relationship that you must have seen quite earlier. This is the expression for the resistance of a wire which has a surface area of S, is made up of conductivity sigma, that is made up of conductor with a conductivity sigma and has a total length of L, a uniform wire of cross section is conductivity sigma having a length L will have a resistance of L by sigma S.

Now sigma is one by rho and rho is called resistivity. So you can rewrite this equation as rho L by S and this will give you the resistance r. Of course this is the resistance for a piece of conductor that we considered where everything was uniform. However, if you want to go to the general relationship between electric field and the current density and the resistance, you have to consider the potential difference between the two parts.

So line integral of the electric field between the two parts, divided by the current that is coming out of the surface. So this is the definition of resistance. One can actually think of this as the definition of resistance and what is the important point about this definition is that the electric field could be non uniform as well as J could be non uniform and in fact one can actually develop a point form of resistance.

So instead of calling a resistance as of a piece, one can also say resistance at every point, that leads to the concept of non uniform and specially varying resistances, for in order to get this non uniform and specially varying resistances, you need the electric field and the J fields to be varying or they be non uniform as well.

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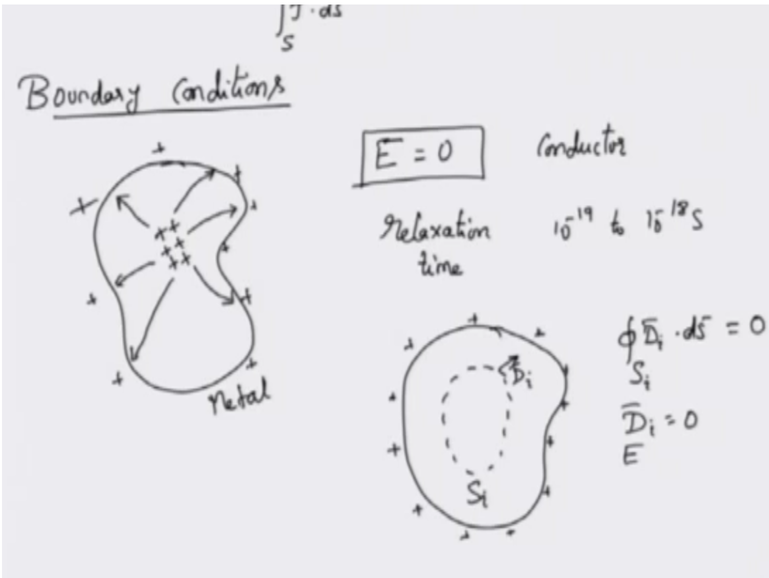

So let us close the discussion on conductors by looking at the boundary conditions. For now, let us look at the boundary condition for D field and E field. We will not touch the boundary condition for the J field because I want to keep the boundary condition for current when we have also discussed the magnetic fields, okay? So right now I don't want to clutter our boundary conditions by taking about J fields.

Although for a very simplified case, you can see that J field has to be continuous across the surface. Now before we talk about boundary conditions here is something we want to ask. Let us take a piece of metal or a piece of conductor, metal of course being a conductor and let us place some charges inside. Let us place some charges in the body of the conductor. What do you expect to happen?

Now what happens is that, these charges which are placed inside the conducting body would rearrange themselves or migrate in such a way that the charges would actually appear on the surface. Of course as we have already discussed, it is not necessary that these are the same charges that would appear on the surface. So we discussed this fact that when you take the current and the electrons are flowing.

It is not necessarily that the first electron would be the one that is actually coming out of a particular piece of conductor that you are considering, right? What would happen is electrons would go to the next lattice, occupy the next available position, from that position it would kick out an electron and this process would happen. Something that we talked about in the analogy of traffic car, cars moving in a traffic.

So similarly here what would happen is when we say that the charges get distributed it is not necessarily that the original charges get distributed. The charges would go inside the lattice into the conductor but they would essentially come out in the sense that they would actually be appearing on the surface, not necessarily the original ones, but the rearranged charges. From the point of view of an observer standing outside the conductor what it simply means is that, you take some charges, put it inside a metal.

You might probably want to dig a small hole through a drill and then place some charges inside and seal the whole. But what you would find is that these charges would not stay inside but rather distribute amongst, I mean distribute themselves in such a way that they would appear on the surface and contribute to the surface charge density. So no charge which is placed inside the conductor body would remain there.

And all the charges would appear on the surface, okay? In such a way that these charges don't just appear randomly, these charges would appear in such a way that there is no electric field inside the conductor. This is a very very crucial result that you need to remember. Even a moderately conducting material would have very negligible amount of electric field. So electric field inside this is a property of the conductor.

One cannot really define this or derive this condition. It is essentially the nature of a conducting material. You might of course ask, how much time would it take for these charges which are placed inside to come or be visible on the surface or they get distributed on to the surface. And this time is what is called as relaxation time and relaxation time for charges is around 10 to the power minus 19 to 10 the power minus 18 seconds.

This relaxation time is so short that in a matter of less than attoseconds that is 10 to the power minus 18 seconds, the charges would all be moved and would be placed on the surface, again remember it is not the original charges which are moving because you can calculate and see that the velocity required will be very high.

It is the effective charge that appears on the outside. Now when such a thing happens, if you go back to the metal, there are certain charges available on the surface. I am indicating only a few charges, but you should imagine that there is actually a layer of charges here. So there is a surface charge layer that is surrounding in the center, they are just below the conducting surface. Of course they cannot just leave the conductor.

There is a very interesting reason why they cannot leave but something that we will not be talking about it. Suffice to say that the conductors are, although we are drawing them as attaching on the outside of the surface they are not leaving the surface. They are just below the conducting surface, but they are essentially surface charges. They are occupying very small distances but they are just below the conducting surfaces.

Now if you ask what is the electric field inside, obviously the electric field inside is zero. What is the D field inside? D field inside is also zero. Why? Consider this interior surface that I have drawn. Obviously this surface does not enclose any charge. So if you want to ask for what is the internal D field from Gauss's law you see that D i dot ds over the interior surface S i will be equal to zero and this implies that D i will also be equal to zero.

We also have seen that the E field is zero, okay?

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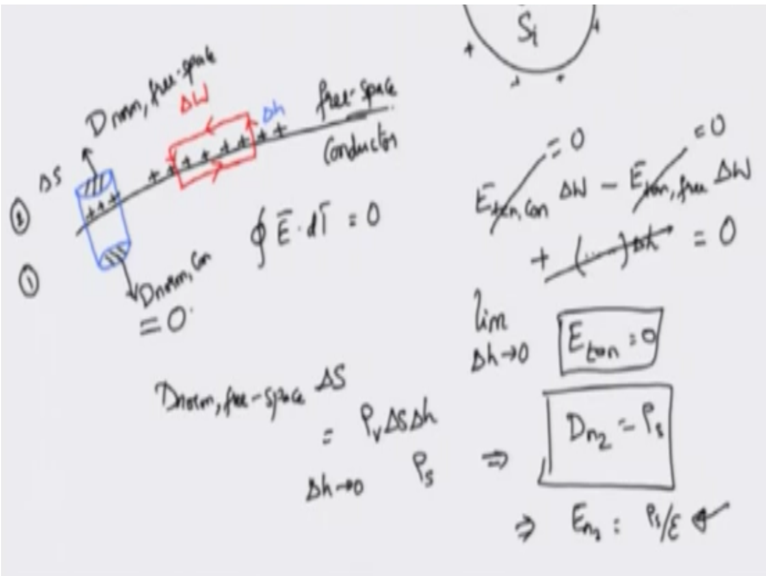

Now we are good to go with boundary condition. Consider free space or a dielectric and then consider one conductor here. We know that charges if at all they are placed, they would be available on the surface of the conductor forming a surface layer of charges. Now to obtain a boundary condition for D and E we follow the procedure that we adopted in the last class for dielectrics.

So you first imagine that there is a path here which has a certain width and a certain height. The path is traverse in a particular direction, direction satisfying the right hand rule. The path has a height delta h and a width delta w. Now apply the electric field the line integral around this path. So if you apply the line integral around this path, what you find is integral of E dot dl which must be equal to zero.

The line integral for this path that we have shown here would be E tangential in the conductor times delta w minus E tangential outside the conductor. So let us call this as free space tangential electric field, times delta w plus some terms that would be multiplied by delta h, that is the normal components of electric field multiplied by delta h, this entire thing will be equal to zero. As before take the limit of delta h going to zero.

If you take the limit of delta h going to zero, then this terms with delta h cancel and you are left with tangential electric field inside the conductor, delta w minus tangential electric field with a free space delta w, that should be equal to zero. Of course there cannot be any tangential electric field inside the material. Inside the conductor, why because, if there was any tangential electric field then it would start to move the charges.

So charges movement is not considered electrostatic and there will not be any tangential electric field. Or this tangential field would move the charges in such a way that after a very short while, that is relaxation time, the charges are there in the equilibrium. So the tangential electric field inside the conductor will be equal to zero which simply means that tangential electric field just outside would also be equal to zero.

So the electric field tangential component both inside the conductor as well as on off side the conductor will be equal to zero and this is the continuity for the tangential electric field. Now to obtain the continuity condition for the normal D field, we imagine writing a box. So you can think of this as a kind of shape that we normally take. So I still have some surface charges and now what happens?

Now I have two surfaces. The top surface and the bottom surface. The bottom surface is in the direction opposite to the top surface, that is the normals are not in the same direction, they are in the opposite direction. Outside let us call this D field as D normal because that is the only component that is necessary to consider in this particular scenario of the Pepsi can thing.

So D normal and in the free space, so D normal free space and the D field in the conductor will be D normal in the conductor. However, we have just shown that D field inside metal will actually be equal to zero or a conductor will be equal to zero, this means that the normal D component inside the conductor will be equal to zero. So this is actually equal to zero.

So if you leave this component out, what you have is, D normal free space, multiplied by whatever the surface area of this top surface that is there, that is let us say delta S, this must be equal to the total charge that I have here, so that must be equal to volume charge density rho v delta s delta h that would be the total charge that is contained and of course, as delta h goes to zero this becomes rho s and delta s on both side will cancel with each other.

This implies that the normal component in the free space and since this is the only component that would be available because the normal component inside the conductor is zero that can dispense of with all the free space ideas and said dn2 where two stands for free space, one stands for conductor, so dn2 will be equal to rho s. So this is the relationship for the normal component of the D field and the tangential component of the D field is simply zero.

So E tangential is equal to zero. So from this you can also find out what would be the normal component of the electric field. The normal component of the electric field will be rho s times whatever the epsilon of the free spaces. If it is not free space dielectric, if it is a different dielectric, then you need to replace that rho s by epsilon because d is epsilon times electric field.

So the normal component d will be equal to rho s and normal component of the electric field En2 will be rho s by epsilon. So this simply implies that En2 is equal to rho s by epsilon, where epsilon is that of the dielectric that surrounds the conductors, okay? Alright, now we summarize what we have learned about the conductor.

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For a conductor let me write down these 3 points, conductors have no internal feeds, that is no electric fields inside. E is equal to zero. They also don't have D field. So essentially they do not have any field inside that of the material. The fields that is D fields or the electric fields are always normal to conductor. The fields are always normal to the conductor. If there is any external field that is applied to the metal.

It will induce charges on the conductor surface, constituting a surface layer of charges rho s. Most importantly, because there is no electric field it means that the potential of a given conductor is actually constant and we call this as equipotential surface. We say that a conductor of an arbitrary shape is actually an example of an equipotential surface. So please note that these conductors that we have considered have not electric fields or D fields.

There are no fields inside. We also have seen that the fields are normal to the conductor. They also induce charges, or charges are induced on the conductor surface when they are immersed in external electric field and these conductors are equipotential surfaces.