Electromagnetic Theory
Prof. Pradeep Kumar K
Department of Electrical Engineering
Indian Institute of Technology – Kanpur

Lecture-12
Vector Analysis- III and Electric Potential - III

In the class, we talked about potentials and we also find out the potential difference between 2 points r1 and r2, because of a uniform infinite line charge, okay. So we said that for the uniform infinite line charge, you cannot use infinity as the point of reference, so we have to use some other point in finite value of r as a point of reference for 0 volts and measure all the other voltages at other points with respect to voltage at that 0, which is not at infinity, okay.

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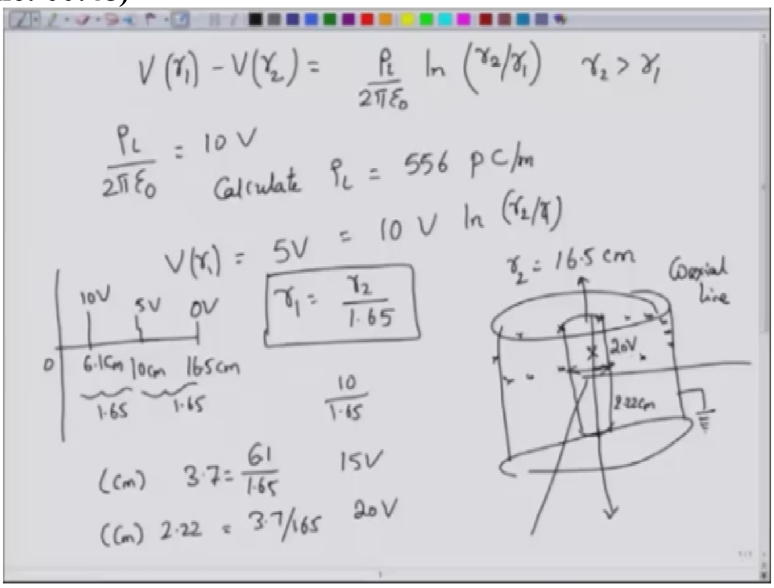

So you hopefully you remember this expression v of the potential difference between the 2 points r1 and r2, if you move some r2 to r1 was given by this expression in which I am assuming that r2 is greater than r1, okay. Let me say this rho L /2 pi epsilon 0, is equal to some 10 volts, okay. r2/r1, of course has no dimensions because it corresponds to the ratio of the 2 distances or 2 radial distances.

So you have the potential difference being given by this expression. For simplicity, let us consider this rho L/ 2 pi, epsilon 0, to be 10 volts, okay. From this expression, you can calculate, what will be the value of rho L and it will turn out to be 556 pico coulomb/meter; pico standing for 10 to the power -12, so it will be 556 pico coulombs/meter. Now let us say, I want to write down or I want to find the radial distances, which all correspond to the same potential with a potential unit of 5 volts.

That is, I want to find out the constant potential contours, which all defer by 5 volts, okay. So my question is; what should be the ratio of the radii r2 and r1, in order to obtain that 5 volt potential differences, which of course depends on which part r2 or which radial distance r2, I will be selecting as the reference of the 0 potential, okay. If I substitute this 10 into this expression and then say that with certain value of r2.

I am going to consider that as 0, then the voltage at this particular point r1 should be equal to 5 volt, right. So at r2, whatever value of r2 that I choose that will be the reference at that point the voltage will be equal to 0, so if I move along from r2 to r1, what should be the value of r1 in relation to r2; such that the potential there will be equal to 5 volts, okay and remember that potential rise will happen only when I move against the electric field.

So I have, v of r1 is equal to 5 volt, I can substitute here this into 10 volt and then there is a log of r2/r1. You can solve this expression and you will see that r1 will be equal to R2/1.65, okay. So the ratio of r1 and r2 should be such that, r1=r2/1.65. So, because there is 1.65, let me arbitrary choose r2=16.5 cm as the point of 0 voltage, okay. So, I have the infinite line charge over here and along this radial distance or some 16.5 cm.

I am going to choose this as the point of 0 potential, okay. In fact, it will not be up and down, but it would rather be along a circle. So at all the points, which are radially at 16.5 cm, the potential will be constant and the constant potential contour will actually be a circle in this case, okay, having a particular value of radius. So if I choose this 16.5 cm as the 0 voltage potential contour.

What will now be r1? such that the potential there should be equal to 5 volt, it turns out that if r2=16.5 cm and if I want to obtain 5 volts, then r1 must be equal to 16.5/1.65, which is 10 cm, okay. I am calibrating everything in terms of cm, this is the infinite line charge, okay this is at 0, so at some point, 16.5 cm radial distance away from the line charge, the potential is taken to be 0.

And if I now take r1 is equal to 10 cm, that is if I go 10 cm from the origin, the potential here will be 5 volts, okay. Because this will be r1=r2/1.65 and r2 is 16.5, which I have taken as the arbitrary point of 0 volt. So at r1=10 cm from the origin, the potential is 5 volt. You can again try to find out, where the potential gets incremented by another 5 volts, that is, I want to find out; what could be the value of r, where I get 10 volts and that value will be 10cm/1.65, right.

So, because these ratios that you are looking, at the ratio between these 2 points is 1.65. Similarly, the ratio between these 2 must also be 1.65 and the value of R from the origin, where the potential will be 10 volts is given by approximately 6.1 cm, okay. So it could be around 6.1 cm. If you continue doing this at different values, you are going to get different potentials.

And the potential keeps on rising and the potential goes as the logarithmic value, okay. For example, at 6.1/1.65, which is approximately 3.7 cm, the potential here will be equal to 15 volts. The potential becomes 20 volts at 3.7/1.65, which is around 2.22 cm. The potential becomes 20 volts at 2.22 cm from the origin, okay and at 16.5, it is 0, all right. Suppose I consider a metal line or a metal surface, okay, which I am going to cover at 16.5.

I cannot show you the metal surface over here, I am showing something like this, so this is the centre, where I have the central line, okay, the infinite line charge and I am going to consider a metallic surface around the 16.5, okay. At all these points around the surface of this metal, which is located with the radius of 16.5, the potential value will be equal and the potential value will be equal to 0 volt, okay.

Because that is what I have taken as the reference and we know that a metal can be used for grounding purpose. So the metal is grounded in the sense that the potential at that surface, on the surface will be equal to 0 volts. However, and fortunately that does not change anything about the problem in the sense that, the (potential at the centre of the line or sorry) the potential at that 2.22 cm from the origin, if I am going to put one more metal here.

The potential is here is still will be equal to 20 volts, okay. Such an arrangement of 2 metallic surfaces, which are concentric with respect to each other is known as a coaxial line, okay. Of course, what I have not told you is that coaxial lines actually have some amount of thickness here, okay for both the inner and outer radius, but for our purposes, for a very simplified understanding, this kind of a picture is sufficient.

You take infinite line charge or an essentially a long line charge and then neglect all the fringing effects that is happening at the end. Okay and for this line charge, you now take 2 metal surfaces around the 2 metal surfaces, you place them, one at 2.22 cm and one at 16.5 cm, take the 16.5cm as the reference and assumed that this 2.22 cm thickness that we have is so small, that the charge distribution around this 2.22 cm cylinder can be approximated by a line charge density at the centre of the cylinder okay.

This is an approximation that we are making, but this is an excellent approximation for practical coaxial cables. Okay, for a practical coaxial cable, one can to the first approximation, think of; replace an inner conductor by a line of uniform line charge density and the surface will essentially be the grounded or the 0 reference potential okay. All other points inside the surface, inside the coaxial cable, the potentials will be as, we have already calculated okay.

Now if I ask you that, for the same coaxial line situation or for the same coaxial line problem, that outer at 16.5, I have the 0 potential located by grounding that coaxial cable.

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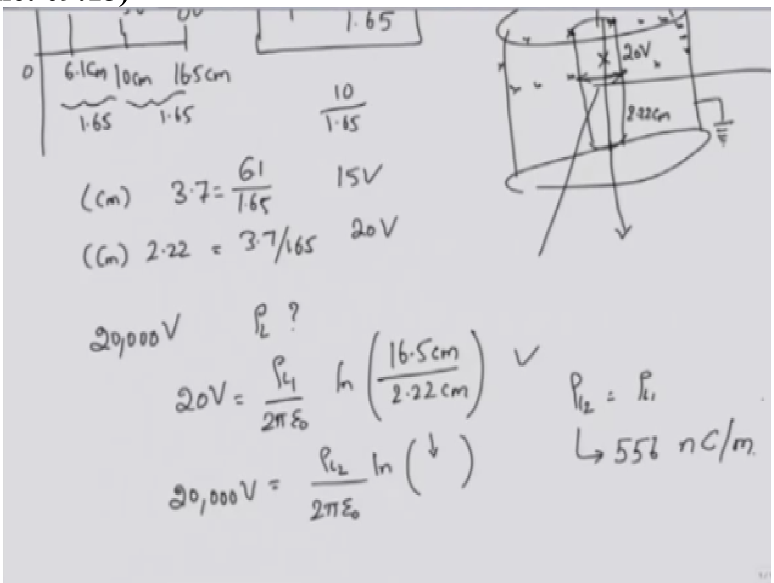

But the centre conductor potential is actually a 20,000 voltage, which is about 1000 times higher than what I have shown in the problem. Can you now guess what will be the line charge density in order to present potential? The line charge density would obviously not be the same as 556 pico coulomb/meter. For a 556 pico coulombs/meter with this configuration of 16.5 outer radius and 2.2 inner radius coaxial line.

The central conductor will be at a potential of 20 volts. Now that we have increase from 20 to 20,000, what will be the new value of rho L. You can easily calculate by going back to these rho L/2 pi epsilon 0 expression or you can also calculate it by observing that you have increase the potential by a factor of 1000, the charge should also essentially increase by a factor of 1000.

You can show that, for 20 volts at 16.5 cm to 2.22 cm coaxial line radius, the line charge density and the medium is all related according to these expressions. If you now say that the potential has to be 20,000 volts inside the conductor rho L2 will be the new line charge density, that would be 2 pi epsilon 0 ln, this particular thing has not change right. This is still the same 16.5 and 2.22 cm.

So you can divide one by the other and show that, rho L2 will be equal to rho L1. In other words, the new line charge density that we are looking for in this 20, 000-volt scenario is, 556 Nano coulomb/meter okay. Here again you can see the appearance of Pico coulombs, Nano coulombs, the amount of charges that are there or charge density is actually very small because Pico coulombs and Nano coulombs are what you actually see in practice.

We have seen that one coulomb is actually a very large quantity okay. Here are some of the other field distributions that I am going to show you, I hope that you can follow these new field distributions okay.

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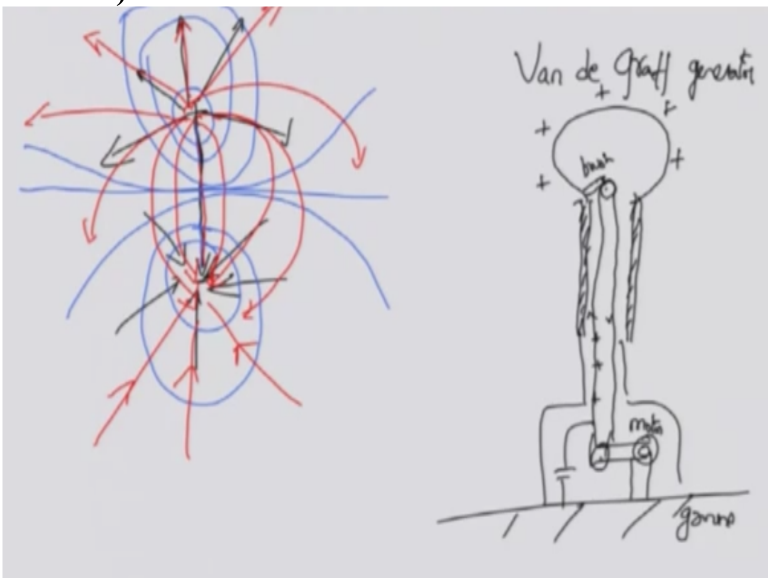

Suppose I have a positive charge here, a point charge and the negative charge that is situated here. We have not really calculated the electric field because of this configuration okay. We will calculate the electric field shortly later, but for now you can kind of imagine that if I don't have the negative charges, then the electric filed lines would have all be located radially away right; there would all be radially away.

However, the charges for the negative point charge would all be radial, but they would all be pointing inverts, right. Now that we have a positive and the negative charge, what happens is that? The electric field line will begin from the positive charge, okay so it will begin from the positive charge and then end on the negative charge and away from the positive charge of the vertical direction, here the charge is still radial.

Whereas for here, the charge would actually be not be radial. You can show that the constant potentials would also change, the potentials would now be constant potentials will all be now contours, right, they would all be spheres of a particular value. However, they themselves will have to change because of fields get weaker down here, the spheres are not exactly nice, the centres are not situated at the charges themselves.

So these are the constant potential lines, okay. How about the field lines? The field lines must be perpendicular to the field, perpendicular to the potentials, okay. So there is one potential which is in the centre, okay. Like this, these are the potential contours, the field line would of course all be parallel to the contours, okay; the field lines are all like this, sorry; the field lines are all for the positive charge, they are all going, for negative charge they are all going inside.

So outside, the field lines are all still radially directed away and then they start to bend towards this one. We will actually derive the electric field and I will give you a mat lab way of solving or finding or sketching this particular electric field some time later. What would happen if you had both positive charges, then the charges would not be meeting together, because both are positive charges, the charges are, they are actually going to repel against each other.

Therefore, the contour lines would all shift, the contour lines would be like this, the fields would all shift away from each other, right. The fields would be away from each other, okay. Now there is one very interesting application of producing very high voltages called Van de Graff generator, okay. A Van De Graff generator can produce high voltage charges by high voltage applications, okay. How does the Van De Graff generator look like?

It actually is very simple conceptually, there is an insulating piece covering up a metal and then there is a big shell, okay, of a large radius, that is made up of the metal; the metal plates are also brought together to connect them to a flat surface, you know on a ground and then there is an insulating material made up of rubber or something, which is put on a pulley on a conveyer belt kind of a thing.

And then made to rotate around or made to go around this belt, okay. They are connected here like the belt and these are energised by applying a potential, okay. These are energised by applying a potential and a motor that you can think of, will actually perform the operation of carrying the belt around the loop, okay and there is a brush here, which will transfer the electric charges that are generated at the base, they will transfer that to the metal shell, okay.

So the metal shell gets charged and all the charges that are sitting at the bottom will be transported away. From the contact of the brush; this is the brush; with the insulating material out here will transfer this. This is essentially what the old type of experiments that we used to do; we take a glass rod and then rub it over a dry cloth, bring it very close to comb which has been; you take a comb and then you comb your hair and then bring the two together.

They would repel, they would attract. These are essentially the same kind of experiments, except that these are generated to generate. I mean these are designed to generate a large amount of charges, okay, for a large amount of voltages, they can handle voltages around 50 to 100 kilo volts, here, okay, here. So what would be the charge configuration or the field lines around this Van De Graff generator.

We have a shell which is all covered up on the surface by positive charges and we know that these positive charges are going to generate field lines, which all will be radially directed away, right.

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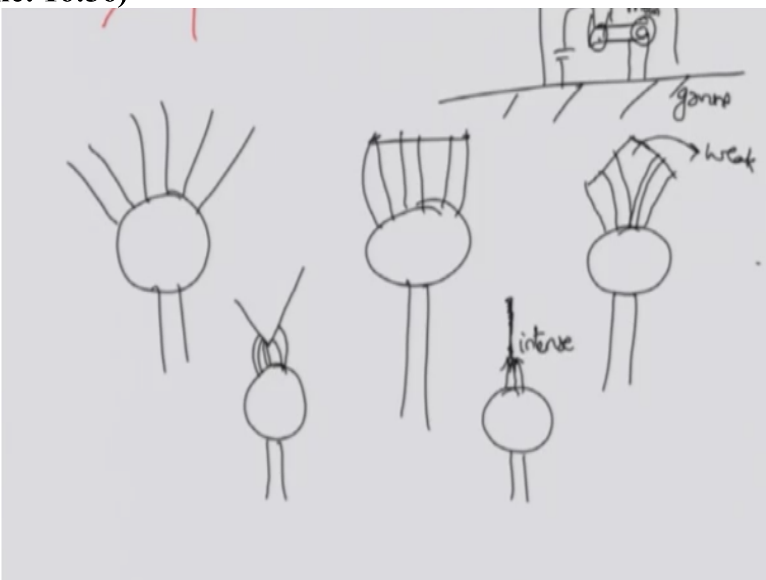

So the field lines, if you have to look at, because on the top of the shell, you are going to see that the field lines, there is a brush; I am just showing that this is a metal shell and there is an insulating material here, the charges are all located with the positive sign here, I am assuming that these are positive, so the field lines are all going to be radially away like this, okay. So the field lines are all radially away this way.

Now if I bring a metal plate, I bring a flat metal plate up here, what will happen to the field lines? Well, because on the metal in the static electric field the potential will all be constant and the metal; because there are free electrons in the metal, metal gets charged to the opposite polarity, so some of the field lines will converge on to the metal, okay. So the sum of the field lines will converge on to the metal and you can see that the fields are now bending, okay.

Suppose on the same metal shell, the Van De Graff generator, I put a corner; okay metal plate with the corner like this. What would happen? Will the fields get attracted to the corner or the fields will not get attracted to the corner? Remember if this metal gets charged, right that becomes an equipotential surface, then the field lines must always be perpendicular to the equipotential lines, right; so that is what we saw in the previous case.

You had a point charge, all the field lines were radially directed away, but the constant potentials were all spheres and on a surface of a sphere, if this is an electric fields and this is a contour of the constant potential, they are orthogonal to each other, okay. So this was something which probably I missed telling you in the last class, but essential thing is that, electric field lines and the constant potential lines or the constant potential contours are all always orthogonal to each other.

That was evident in the pictures that we drew in the last field line configuration, okay. So with metal plate brought near the shell, what will happen to the field lines? The field lines have to perpendicularly land on the metal plate, right. So therefore, and they cannot do this by a straight line, right. So because of the perpendicular, because of the curved nature of this plate which have introduced because of the corner.

The field lines would all land perpendicularly on the metal surfaces, but none of them would get attracted to the corner. So the field here is weak actually, okay. On the other hand, instead of corner, I put a wedge, okay; If I put a wedge, then what happens is that, the field lines are all going to be landing on the wedge, okay. So the field lines get attracted to the wedge, however the field lines avoid the corners.

What happens if on this metal shell I place a metallic edge? Okay I place a metallic edge, what happens to the field? Well this is a very very special case of a wedge in which you take a wedge, okay and then you bring them together, what happens is that at the base, the field line would all converge and the field at the edge will be very very intense, okay. So the field at the edge will be very intense. The field here is intense.

So far we have talked about point charges, we also talked about line charges, I did not give you the expression for the potential at any given point, where I have a collection of charges.
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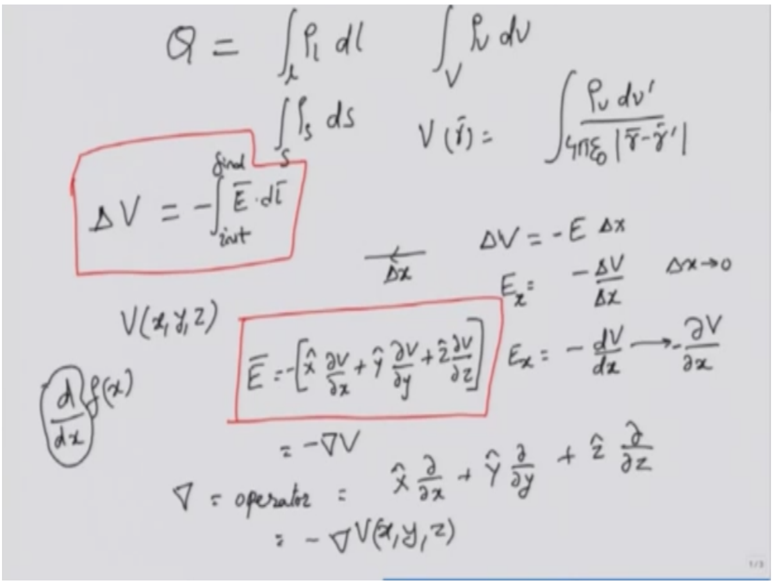

These are not infinite charges that I am talking about or the charge density which are uniform but extend all the way to infinity. These are collection of charges that I am looking at, okay. So for these collection of charges, what could be the expression for the total charge? total charge will be Q; I know is the total charge if it is the line integral or a line charge then the total charge will be rho l, dl, it would be integral or the surface.

If it is the surface line charge or a surface charge density, it would be an integral over the volume, if it is a volume charge density. This is a volume charge density, a line charge density and the surface charge density, so the potential at any point in space, v of r is simply obtained at the superposition of the potentials by considering them to be point charges, essentially saying that the summation becomes integral.

And you replace the charge Q, which was there in the point charge potential calculation by the appropriate integral, okay. So the appropriate integral with the more general case will be the volume charge integral divided by the integral has to go not only on the sources but there is also radial distance involved, remember there is a radial distance from the source point to the field point.

So this would be the expression for potential assuming that have already chosen a point of constant potential, okay. So this is the expression for the potential of a continuous charge distribution, okay. Well, we originally said that the potential difference between 2 points, delta v is actually the negative of the line integral, right, some point, from the initial point to the final point okay. Is there a way? I can turn this equation around.

So that I can express electric field in terms of the potential function? Is there any advantage of doing such a conversion or asking for such a relation? Yes, if I perform this, if I invert this relationship from the potential to electric field and express the electric field in terms of potential function, then I can calculate electric field by making practical measurements of the potential, right.

So normally what happens in electromagnetic problems is that, I do not know what is the electric field or what is the charge distribution? In fact, those are the quantities that one has to calculate them. What can be measured? Is the potential because of this charge distribution or any other potential because of the charge distribution. So I can make the measurements on the potentials easily and from there I should be able construct the electrical field.

If I can find the recipe for doing that, then my life become so simple, right. I do measurements and then maybe I interpolate them and then I can find out what is the electric field, okay. This operation of going from potential to electric field involves introducing a new operator called gradient. What is the gradient? Imagine a surface, imagine the surface of the hill or just imagine a hill, okay.

Now if you ever climbed a hill, then you know that at point you stand, okay assuming that all the parts are accessible to you. At any point you stand, in normally if you want to climb to the top. You are going to find where the slope is actually changing at its maximum, right. So if I have this particular smooth curve here and if I am standing at this particular point, I will try to find out where the potential is; sorry where the slope is maximum.

Because imagine at this point, I will be standing, I will move horizontally like this but the slope is not changing if I move along these line, right, the slope does not change. If I move downwards the slope changes, but the slope change is not what I am looking for, right, I mean I am looking for to climb up and there is a direction. If I actually draw many many such points here.

I will find out that only along this particular direction towards the up direction, where the potential is changing, I can, if I follow that path, okay, If the path is accessible to me, if I follow that path, then I will reach the top very easily, okay. So this is something that you might have observed in practice but you might not have seen that there is a mathematical way to capture that phenomenon.

And this mathematical way to capture that phenomenon is to give this value or to give this quantity of the derivative along the particular direction or sometimes called as directional derivative, (()) (24:52) and this name is gradient, okay. To introduce new to the gradient, let us go back to one dimension. This is the distance delta, x that I am considering, okay. I move along this one.

And let us say that the electric field is directed along the same direction that is opposite direction but along parallel to this delta x, okay. What will be the total change in the potential here? The total change in the potential will be delta v and if you look at this, the delta v will be equal to electric field into delta x. There is a minus sign here, because minus, if you are moving against the field.

So the electric field can be written as –del v/del x, we have seen this earlier, right. We have seen this –del v/del r and then del r was, taken to 0, but here I am trying to make it looking more precise, so what field actually I have got, the field that I have actually obtained by doing this operation by moving along delta x is actually a x component of the field, okay and in the limit of delta x going to 0.

The electric field at this point is actually given by –dv/dx, okay. However, in general the potential could be a function of all three components, v would be a function of x, y and z, in that case I have to replace this dv/dx by the corresponding partial derivative, okay, by the corresponding partial derivative – del v/del x, so I can actually build up other components of the electric field by following this idea, right.

So I have the total electric field given as partial derivative. So this would be x hat, there is a minus sign which is common to all of them, - x hat del v/ del x + y hat del v/ del y + z hat del v/ del z, okay. So this particular expression in which I have managed to invert the integral relationship, so this was the integral relationship that I had which would give me the potential difference given the electric field.

In this expression I know what is the potential difference or the potential function, from there I am extracting the electric field. The quantity that is sitting here in the right hand side of this one is more commonly expressed as – gradient v, where gradient is an operator, gradient is an operator which is defined as x hat del / del x + y hat del / del y + z hat del/ del z. Why am I saying that del is an operator? Because del by itself.

If I write like this have no meaning, this is like saying, hey what is the meaning of d/ dx, right. If I write down this d/dx, unless I attached this one with some function of x, there is no pointing just writing d/ dx. Similarly, if I just give you del operator, there is no pointing that one, in fact in the d/ dx, yx, fx, example, d/dx is called as the linear differential operator, okay.

Similarly, you have directional dependent, that is, the different partial derivative along x will give you the x component, partial derivative along y will give you the electric field for the y component and del/ del z, that is partial derivative along the z, will give you the z component.

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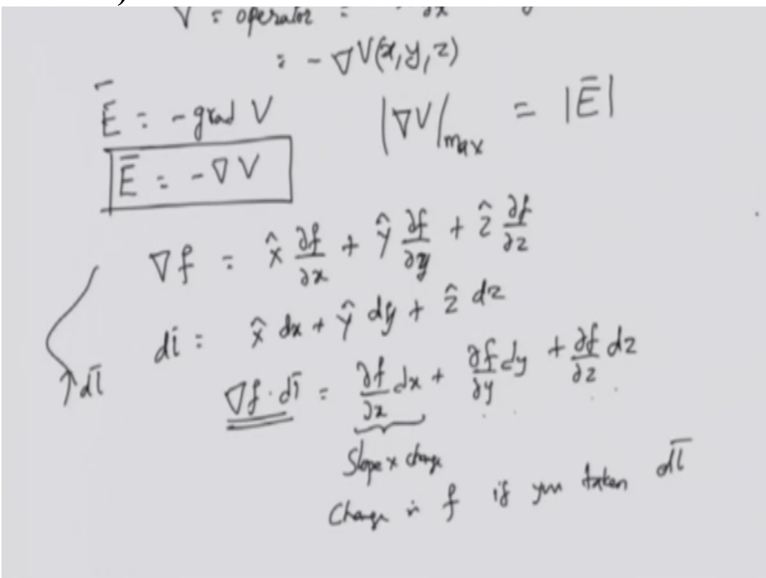

The operator does not make sense until I attached the potential function to this one and why am I taking negative? Again it has to do with potential rise, so you go against the field, you are increasing the potential, okay. So this operation is called gradient and the electric field is given as; in some text books you actually use the word grad, what is important to notice the grad and del are essentially one in the same.

Grad is more descriptive perhaps but the commonly used description is to use a del operator, okay. So the electric field which is the vector quantity is given by the negative of the gradient of a scalar quantity, okay. So you have to be careful here, you have on this right hand side of this expression, potential function v, which is the scalar and I am applying some operative to this potential functions so as to convert that potential into a vector quantity, okay.

This is the operation varying the input sort of to this gradient is a scalar and then the result will be a vector. We will see how interesting our life has become now, because I have the potential v, correct, which is the scalar function, which is easily measurable, from there I am actually able to determine the vector quantity electric field, okay. In fact, this kind of method can be used in practice to map out the electric field lines of a given configuration, okay.

So this sometimes quite widely used, okay. What does gradient of v max tell us, the gradient of potential at its max will give me the magnitude of the electric field at any given point, okay. It will give the magnitude of any given point. In fact, this is very general, for example, let us assume that there is some scalar function f and I calculate the scalar function, the gradient of this scalar function, what will be the result?

This will be x hat del f/ del x + y hat del f/ del y + z hat del f/ del z, okay. If I now consider some path, okay, say the path dl is given by, can be decomposed into x hat dx + y hat dy + z hat dz and then I ask you what is the component of del f on this line dl, you can obtain that one by taking the dot product of the gradient with respect to the line segment and you are going to get del f/ del x, dx + del f/ del y dy+ del f/ del z dz.

What is this quantity del f/ del x dx, now del f/ del x gives you the slope of the function f as they move along the x axis. Therefore, this is basically slope into change along the x axis. Similarly slope into change along the y axis, slope into change the z axis, therefore what is quantity giving you is the total change in the function f, is you had taken a directional path or a directional line along dl, so if you go in a given direction dl.

You can find out what would be the change in the function value. The maximum change will give you the maximum change in the function value and this is what you are going to use if you want to climb a hill or go down the valley, okay. So, we will stop the potential at this point, we will now introduce another important electromagnetic quantity and get to the Maxwell's first equation by a via Gauss's law.