Electromagnetic Theory
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Lecture – 11
Spherical Co-ordinate System & Potential-II

Now we will discuss very briefly about the Spherical Co-ordinate System because it will be useful later on for us. So let us go to Spherical Co-ordinate System now. Okay, unfortunately for Spherical Co-ordinate System we do not have a two dimensional analogy that we could use in a similar to Cartesian and Cylindrical Co-ordinate System. We had a two dimensional analogy in those cases.

And then we built up from two dimensional to three dimensions. Unfortunately, we will not be able to do that with Spherical Co-ordinate System so let us jump straight away ahead to the three dimensional spherical coordinate system.

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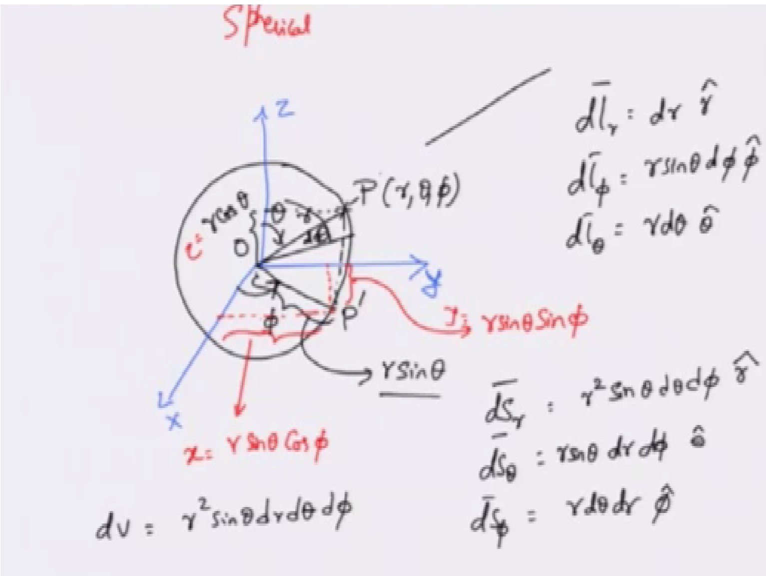

So this is the Spherical Co-ordinate System that I am going to talk about. So assume that I have a sphere, okay and for the sake of it I also will define the x and y co-ordinate system that is I will define the Cartesian co-ordinate system here—sorry this is y and this is x and this is the z axis okay. The sphere has a certain radius; we will denote that radius by r, okay. Now I am looking at this particular point P on the sphere okay of this constant radius r.

So imagine that there is a sphere and there is a constant radius r and this will be given by three values or three co-ordinates just as we described point in three dimensional rectangular

Cartesian co-ordinate system by three values x, y and z three co-ordinates. Similarly, we will have to give three coordinate for defining the point in Spherical Co-ordinate System. Okay. And these points are r theta and 5. What are the r theta and 5? r is the radius of the sphere.

So when you are looking at a particular point you can say that there is a point here and this is a unit sphere assume that this is my hand is a unit sphere and where on the surface of the sphere is my point P located will be given by the radial distance or the radius of the sphere from the origin. Okay. This origin is the same or origin for both spherical as well as for the rectangular Cartesian co-ordinate system.

For the rectangular Cartesian coordinate system, you can imagine that there is a y axis here okay and there is a z axis from the top and there is a x axis coming off from here, okay so you can imagine that one. Now, I have defined where I am located on the spherical on the sphere on the surface of the sphere but I still need to give two other points I have to pin point the point P, right? So one point, would actually be interesting.

I have the z axis over here okay. This is a sphere, I am sorry I did not bring an orange or an apple to show you the sphere, but you find an apple if its available or an orange better if it is available with you and you can try this and waving experiments on that. So you have a P here you have the z axis which is pointing upwards okay and my point is located at somewhere over here. How do I define this point?

Well, all I have to do is I can now consider a line, drawn from the center of the origin, cutting through the point P. Okay. I have the z axis and then there is a point that is cutting through the point P. Line that is cutting through the point P. And I measure the angle between the two lines, okay. So this angle is what I denote by theta and this angle is called as Elevation angle. What are the limits on this elevation angle?

Well, you can be located right on the z axis so which means that angle theta is 0, you could be on the x, y plane where the angle is pi by 2; you could be located at the bottom like earth south pole, okay. So you could be located the bottom and theta will be equal to pi. It does not make any difference whether I am at the elevation of this point P or the opposite I mean or a different point which is at the same angle theta. Okay.

So these both are given by the same value of theta. So I have the angle made between the two lines which is the z axis and the line that is passing through the point P and I call this angle measured from the z axis as theta. Am I done? Unfortunately, no I still need to give one more co-ordinate right; I need to give the value of 5. Now here is where it gets little different, you need to drop a line okay.

On the x, y plane, you need to drop a perpendicular from the point P on to the x, y plane and then joint the origin to this line. So when you draw and drop this one you are going to draw a line from the origin to the point P prime which is exactly perpendicularly above or you know is situated by dropping a perpendicular from the point P on to the x, y plane, okay. Now here this point P prime I know how to specify, right.

All I have to specify is to give the angle 5. Okay. So I have to give the angle 5. Now here is a question. You imagine taking an orange, locate a point in the upper hand sphere and then try to insert a needle or a compass through the point until you reach the half way x, y plane, okay you can get the x, y plane by just cutting off the orange and then you insert the perpendicular or you will drop the perpendicular from the point such that is falls on the x, y plane.

Will that point have-- at what distance will the point b from the origin, what is the radius of that distance you can imagine drawing a circle around that and then what is the radius of that circle. It turns out that this radius will be not equal to r but it will be equal to r sign theta. Okay. Similarly, if you drop a perpendicular from P to z axis okay. So if you drop a perpendicular from P to z axis what you would find is that this length will be equal to r cos theta. Okay.

So this length will be equal to rcos Theta. In fact, this is all giving you the conversion formulas. So if this length is rsin theta let's find the x and y components here by looking at x -- projections on to the x and y axis this length you know from point P prime back on to the x axis has the value of rsin theta which is the length of radius or the radius times cos phi, whereas the y value is given by rsin theta and sin phi. Okay.

So in fact these are the conversion formulas that we were looking for, okay. Z is r cos theta, y is equal to rsin theta phi, x is equal to rsin theta cos phi, okay. Now comes the line integrals, the surface and the volume integrals. What is the line integral along the radial direction, the line integral or the line along the radial direction will be obtained by considering one sphere and you imagine another sphere which is whose radius is increasing from r to r+dr.

So the difference if you radial along will be equal to dr okay along r hat. Now r hat is the radius vector directed in the radius on the sphere okay so along the radius of a sphere. Okay. What will be the line segment along phi? We know that in order to go the line segment from point P prime you will have to go to another point Q prime, okay so if you have a point P and a point Q.

You will now have to go from P prime to Q prime that would be movement of r sin theta along the r length of d phi, right. So that will be equal to rsin theta d phi along the phi axis. What will be the movement along theta? Well, you have a P here now if you take one more point and then move you have moved the distance of or you have moved a angle of d phi okay, sorry d theta.

So it is d theta is the angle that you have moved but the arc length that you have moved will be equal to r d theta along the direction theta. Okay. What about the surface areas, the vector surface areas are ds along let's say r will be equal to you need to move a certain direction along theta and a certain direction along phi and these are the movements that you know how to move along that becomes r along theta rd theta along theta and rsin theta along d phi.

So will give you r2 sin theta, d theta d phi and it will be directed along r hat. Okay. Similarly, ds theta the surface element along the theta direction will be given by you need to move along the dr by a distance of dr and you need to move rsin theta along d phi so you will get and dr along theta. What will be ds phi? ds phi will be rd theta, d phi along phi. Sorry this is rd theta dr along phi. Finally, what is the volume element?

Volume element is r2 Sin theta dr d theta d phi, okay. This is the Spherical Co-ordinate System. The conversion formula from the unit's vectors of x, y, z to the spherical unit vectors r theta and phi, I am not going to drive that and it is in the little bit of a tediousness in driving that one you can look up the textbook if you want to get the answers, okay. So you can find out the conversion formulas from the textbook and practice a couple of problems.

So that you know how to convert from rectangular to cylindrical, cylindrical to rectangular, cylindrical to spherical distribution, okay. Okay, after taking this brief digression on the Spherical Co-ordinate System we are now ready to discuss the point charge.

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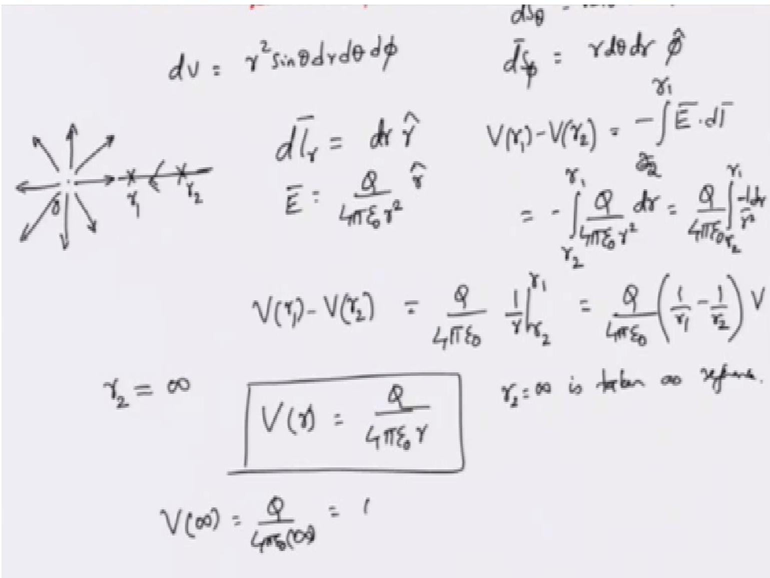

The field of a point charge will be directed radially away from r. It will be directed radially away from r and along a particular path let me choose two points r2 and r1. Okay. I want to find out what is the potential difference between the two points. If I take this particular path. I am choosing this path because it is easy for me the electric fields are radially directed away and the path is also along the radial direction. Okay.

So the path itself is given by dlr s given dr, r hat, okay, do not include a -dr over here okay the -dr will come on its own, so this is dlr is equal to dr r hat and then electric field is given by Q by 4 pi epsilon 0 r2 in the direction r, okay. So the potential difference V of r -V of r2 is obtained by moving from point r2 to r1 E dot dl, right? Now we are fortunate that E and dl we have chosen to be along the same direction parallel so r dot r will be equal to 1.

And then this integral becomes -r2 to r1 and there is dr here so you have Q by 4 pi epsilon 0 r2 and this is dr. Q by 4 pi epsilon is the constant so I can pull this out Q by pi epsilon is the constant I have integral of -1 by r2 dr from r2 to r1 okay. The integral of -1 by r2 is or in the integral of 1 by r2 is -1 by r right so therefore you can there is a minus sign here so this integral will become 1 by r.

So the potential difference that you are going to get V of r1 –V of r2 is given by 4 pi epsilon 0 1 by r between r1 and r2 which will be Q by 4 pi epsilon 0 1 by r 1 -1 by r2. This is in volts. Okay. So the potential difference is in volts. Well, this is a fairly interesting expression because I can imagine going back or coming from r2 is equal to infinity. Imagine that I want to find out the potential at a point r1.

But not from moving the test charge from r2 to r1 where r2 is close to r1; r2 I am going to push it away as for as infinity and I will come from infinity to point r1 in the same direction of the radial thing that we have taken okay. So I will come back from infinity to point r1 and I will try to find out what is the potential at infinity as or in other word as r2 tends to be very – r2 becomes very, very large 1 by r2 will be equal to 0.

So the potential at any point r which is r measured from the origin is given by Q by 4 pi epsilon 0 into r okay where r2 is equal to infinity is taken as reference. Okay. You can see, what is that if I take V of infinity r is equal to infinity this expression will give me Q by 4 pi epsilon 0 infinity. Okay. This will be equal to volts. So in other words what I have done is to set r2 is equal to infinity as a reference for me to measure voltages.

It is like sea level and the mountain except that the sea level pushed away to the edge of the universe, to the end of the point infinity so that all the other potentials at any other points in the space can now be called as absolute potentials, okay. And it is interesting to note that the potential goes as 1 by r okay. You can kind of start suspecting from relationship between V and E although we have already obtained that relationship here.

You can try to think of inverse kind of a relationship. Okay. V was obtained as integral of the electric field okay so you would suspect that electric field will be differential of voltage and we will get to that one, okay. So we have set r2 is equal to infinity as our reference point, okay. Let us know consider some other types of charge distribution electric field distribution and try to find out what will be the potential because of those charge distribution and line distribution. Before that let us do a simple example okay.

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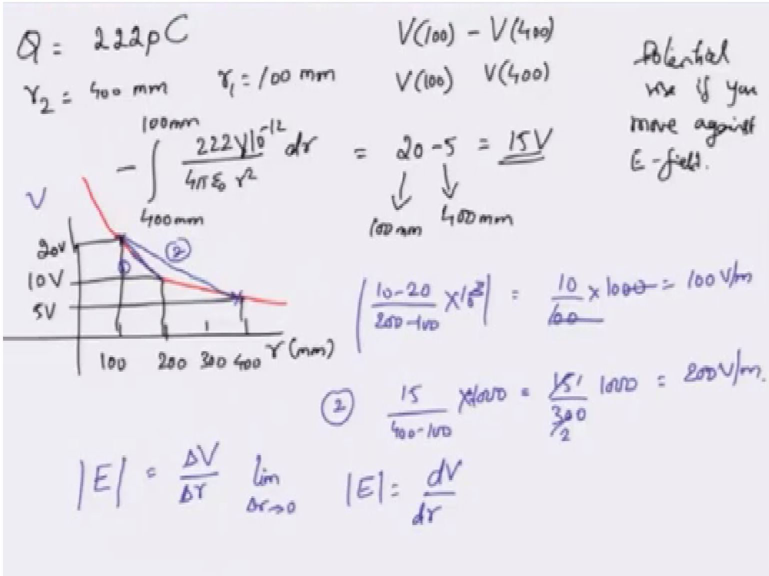

So let us do a simple example, assume that I have a point charge which is 222pC Coulombs so I have a charge which his 222pC Coulombs and I want to consider movement from r2 which is equal to 400 millimeters to r1 which is at 100 millimeters and the field that we are considering is a point charge field okay. So I want to find out what is the potential difference at 100 or potential difference between 100 millimeter and 400 millimeter points okay or the radius 100 and 400.

And I also want to find out what is V of 100 itself and what is V of 400 itself. Now you can do it in two ways one you can write down that integral you know, an integrate from 400 to 100, so you can do this one. You have 222pC Coulombs divided by 4 pi epsilon 0 r2 let me remove the coulombs from here so you have 22pC Coulombs which is 10 power -12 and then integrating along so moving along r2 to r1 from 400 mm to 100 mm okay.

So this sorry—this is 100mm okay and the potential difference you can find out you know you can integrate this expression or you can use the expression that we just derived and we will see that this will be equal to 20 - 5 which is equal to 15 volts, okay. So as you move from point which is at 400mm from the point charge to the point 100 millimeter from the point charge okay so you will actually experience a total potential rise of 15 volts, okay.

So there is a potential rise if you move against the field, if you move against the electric field you are going to see a potential rise okay. So there is a potential rise of about 15 volts you can of course from this expression itself find out that the potential at 100 is 20 and the potential at 400 is 5 volt, okay so this is the potential at 400mm this is the potential at 100mm. okay.

In fact, if you can sketch the potential okay you do couple of other calculation like find out what is the potential at 100, 200, 300, 400 and so on at 400 we already know what is the answer 400 is 5 volt and at 100 we know what is the answer at 100 we have 20 volt okay. If you were to find out what is the potential at 200 millimeter from the origin you are going to find out that this will be about 10 volt okay. And the potential is actually going as one by r okay the potential is going as 1 by r.

What will be the average slope if you take the two points as 200mm and 100mm, what will be the slope of this curve here? The slope of the curve will be 10 - 20, the absolute value of the slope will be 10 - 20 divided by 200 - 100mm into the power 3 let us put that on top this will be equal to 10/100 into 1000. So this is 100 volt per meter. If you try to find out what is the potential?

I mean what is the slope of the potential curve from point 400 to point at 100mm the slope of this curve will be different compared to the slope of this curve, so this simply indicates that if you want to find the slope and make it a good approximation to the continuous curve your intervals be better small okay. So the slope of this curve will be so if we call this as 1 angle we call as 2 the slope of the curve 2 will be a total change of 15 volt.

And the change in terms of this one is 400 – 100. So this is into 10—into 1000, this 300 so you get 15 by 300 this 1, this is 2 so you get a thousand here so this will be about 200 volt per meter. The important point is that you can actually find the potential at different points, this is the potential at different points as a function of radial distance from the point okay. Indeed, what I wanted to highlight was you can actually find out the electric field okay at any point, by looking at the slope of the potential curve, okay.

So if I want to find the potential or the magnitude of the potential at any point r I can find out what is the slope of the potential curve or the potential at that particular point okay. Of course this gets better and better if I take the limit of delta r going to 0, so in that case I get the electric field magnitude to be dv by dr okay. Thus, if I find out the maximum slope that will be the value of the electric field.

And against the direction of the maximum value of the slope will be the direction of the electric field. Okay. So we have seen this already and we have seen this particular charge distribution. “Professor to Student conversation starts” I made a small mistake my student corrected here, this is not 200 volt per meter, this is 50 volt per meter yeah it was obvious I did not see the division properly okay. “Professor to Student conversation ends”

So this numerical value is 50 volt per meter. Okay.

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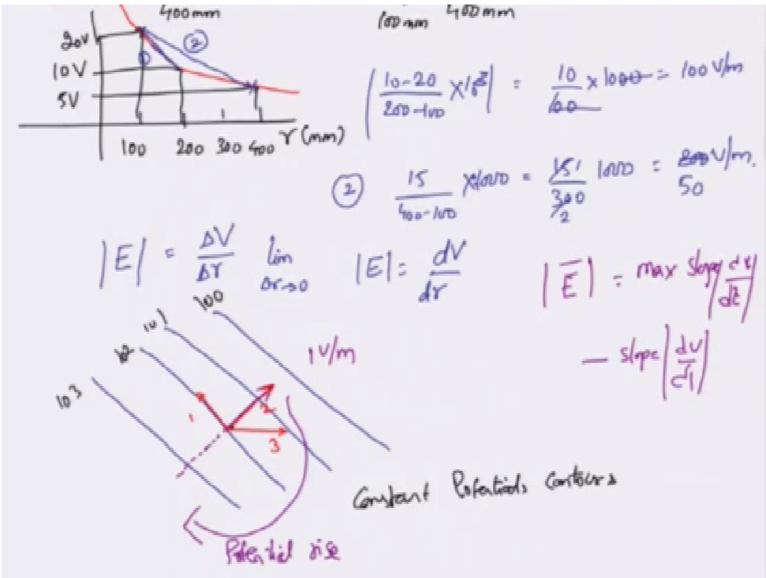

But coming back to the point that I was mentioning the maximum value of the slope will be the value of the electric field at a given point and the direction of the electric field at that point will be exactly opposite to the change of the slope, that is let’s say these are the constant potential surfaces or what we call as potential contours okay constant potential surfaces, or constant potential contours that I am - I have measured on a pieces of plane by some measurement method which we will not have to go in here.

So we have constant potential contours here which are measured let’s say this is 103 volts, this is 102, this is 101 and this is 100 volts, okay. I want to find out the electric field at this particular point. How do I find the electric field at this point? I need to find where the slope would change which direction should I hunt? So I actually try to find the slope along this one I try to find the slope along this direction okay.

I will also try find this slope along this particular direction. Along line which is going backward see here, let’s call this as line1, line2 and line3 along line1 there is no change in the slope because the potential is essentially constant in that line, right? So the electric field magnitude will not be affected over here.

The electric field magnitude will actually be perpendicular to that, remember when the force is perpendicular to the object the object does not move. Similarly, when the electric field and the line segments are in the – are perpendicular to each then the electric field then the electric field—then the work done will be equal to 0 or the potential will be constant, okay. So along line1 there is no potential change, along line 3.

There is some potential change you know as you go around there will be some potential change which you can decompose into potential change along say x axis and potential change along y axis. But very interestingly the slope will actually maximum as you go along the line2 right. So if you take a short distance between these two lines the potential has change by 1 volt so the electric field will be directed along this line2, okay.

So the electric field will be directed along line2 with a value of 1 volt per meter, okay. It will be 1 volt per meter. And actually the potential will be changing if you going in the opposite direction, right. So if you look at the potential on the same line here you will see that the potential change here is 1 volt per meter; the direction of the electric field will be negative of the maximum slope.

So if this is the maximum slope, and then the electric field will have to be directed in this way because as you go against the electric field the potential must rise, right. So if you go against the potential field the potential must rise, okay. So keep in mind these two points. Electric field value will actually be equal to the maximum value of the slope dv by whatever direction dv by dl for example.

And the direction of the electric field will be against or minus of slope dv by dl okay. So if you remember this and also remember that the-- when you go from when you against the electric field you are going to see a potential rise then you can actually write down the electric field and potential contours. So we are going to do that one.

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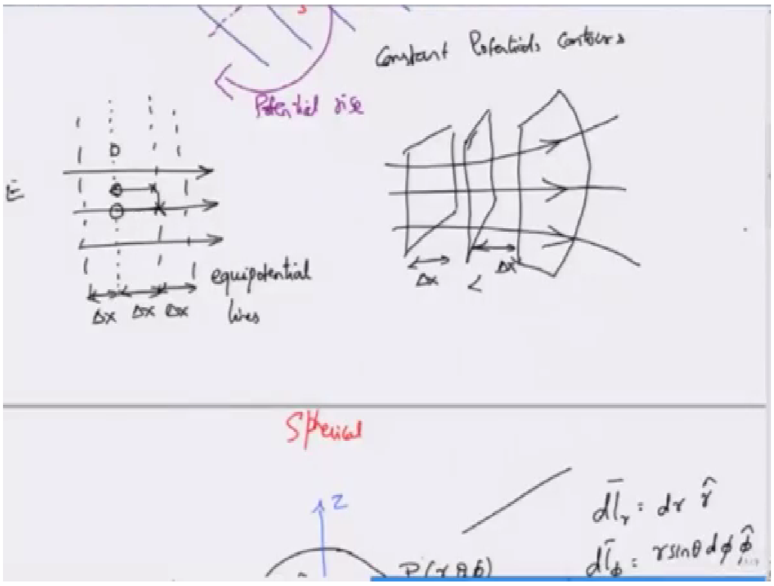

Consider uniform electric field so I have electric field lines which are going horizontal to the right and these are all uniform in electric field, okay. What will happen to the potentials, if you go opposite to the electrics field from some point which I am marked as here x to the points which I marked as 0 then potential will be some value, okay we are not bothered what the potential values is.

If you take any parallel line to the same direction the potential here will be exactly the same because we are assuming that the electric field here is uniform, okay. So in fact the potential will be constant along all this and all the points along this line okay. These lines are called as Equipotential lines okay. These are called as Equipotential lines or constant- they are called a Equipotential lines or they are called as constant potential contours okay or potential surfaces.

Actually they would be surfaces in a plane way so if you take this one they would be surfaces in the plane way with the electric field all pointing uniformly like this, this is the electric field this is the surface that you are looking at, okay. Now suppose the electric field is diverging it is not uniform in this plane it is not uniform, okay. What will happen to the surfaces of constant potential is that you are going to get the surface thus which are all going to –

My drawing may not be very accurate but I am hoping that you are able to see a bulge in the surface here, okay. So as you go from uniform to non-uniform field the constant potentials would also be different. Moreover, a very interesting phenomenon's happens. If the electric field is uniform, then the spacing between the Equipotential lines is constant so let us say this is delta x and this will be the same delta x for the given potential difference.

However, for the non-uniform electric field the spacing between the Equipotential planes-- so if I assume that there is another plain over here okay this spacing delta x will be smaller than this spacing between this two, okay. So this will be smaller this particular spacing delta x and delta x prime for example, okay. Because the field is actually getting weaker here you need to larger distance to get to the same potential difference than you would have obtained if you are moved in strong electric field.

See the strong electric field you move a certain distance delta x you get a certain potential. However, if you move in weaker electric field – the distance you will not get it so you have to move a larger distance in the weaker field in order to get the same potential difference and that’s how the potential contours or the potential surfaces will also start changing. Okay they would also start spreading apart.

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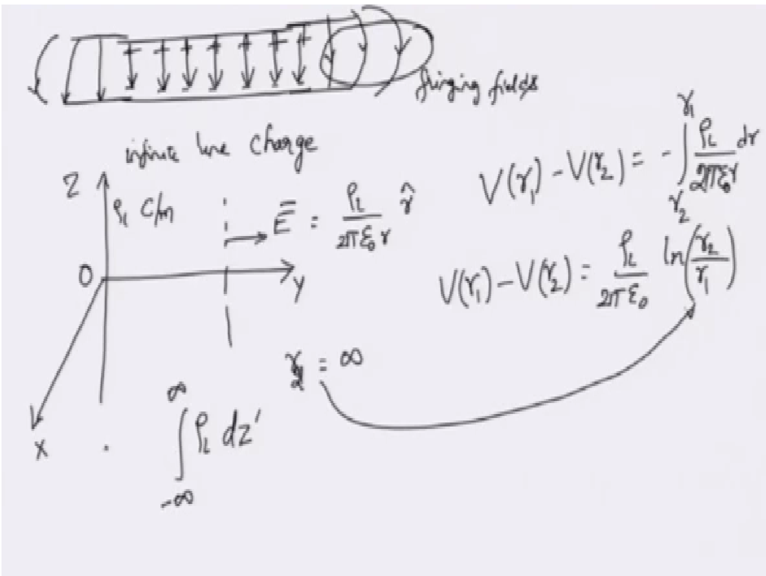

Why are these field lines important? You will see sometimes later that there are structures which are called as Capacitors, okay. And one of the very interesting aspect of the Capacitor is to draw the field line service to computer the field lines. How do I computer the field lines of a Capacitors assuming that the large densities is all uniform and the dimensions are very large what a Capacitor does is to induce charges of opposite polarities on its plates, correct?

So the charges are induced on opposite polarity of the plates and the electric field lines begins from positive charge and terminate on the negative charge as long as you are inside near center okay the field lines will all would be uniform. However, if you start approaching the ends okay, so as you start approaching the ends you are going to see that electric field lines will start to bend and the separation between the two also starts to increase.

This is something that you will find in the micro strip transmission line for example. These field which are at the edges are called as fringing fields and calculation of fringing fields is one of the most complicated problems in Capacitor calculations, okay. So you in order to fringing field you need to understand how to draw the field lines and in this cases the best method for obtaining the fringing field.

Or at least very good approximation method for obtaining the fringing fields is to go graphical because analytically you cannot solve this problem easily okay. In fact, it is very difficult problem to solve analytically. However, by graphical method such as drawing this field lines and spacing them apart so that the Equipotential lines are properly matched you can actually estimate the capacitors of a transmission.

For micro strip transformation line, okay by taking the fringing fields into account. There are elaborate rules of how to obtain these fringing fields we will not worry about those in this particular class for now. Okay. So those rules are not going to worry about that. Okay before we move on to the next one I want to consider the infinite line charge, okay. So we consider the infinite line charge and then try to see whether I can find the potentials for this charge.

So I can actually find the potential for this charge, so I can actually find the potential for this charge, the charge has-- the line charge has a density of row l coulomb per meter at any given radius we already know that the electric field lines are all radially outward and they are given by row l by 2 pi epsilon 0 r, r hat, this r hat is in the cylindrical coordinate.

Imagine that there were cylinder and electric field lines are all at any point r from the origin their values is given row l by 2 pi epsilon 0 into r. Now if I take path very similar to what I did for the spherical charge, okay if I take the same kind of a thing I can actually try to find out what is the potential difference between two points. Let us say which is V of r1 minus V of r2 and I will be obtaining this then by going against the field from r2 to r1.

I can substitute for the field here row l by 2 pi epsilon 0 r, okay. What should be the direction? Well, the same thing that we did in the last for the spherical charge distribution, line segment will also be directed along r and it is given by r-- it is given by dr r hat. So this line integral becomes a simple scalar integral okay. So if you now try to find out what this is you will see that the potential difference V of r1 minus V of r2 is actually given by row l 2 pi epsilon 0 ln the natural log of r1 by r2 with r1 being greater than r2.

Oh sorry! I went from r2 to r1 right, sorry, so this becomes r2 to r1 okay, r2 is the larger of the distance so I went from r2 to r1 and this will be the potential difference. Now very quickly, can I use sorry r2 is equal to infinity as my reference? Can I do that one, unfortunately no, why? Because if you try substituting r2 is equal to infinity in this expression you will see that it becomes quickly ln of infinity, right.

Of course I cannot use r2 is equal to 0 as my reference because in that case also I will not be getting physically correct answers, okay. So what it seems to indicate is that for an infinite line charge I cannot use infinite as a point of reference. For a point charge, I could use this as a point of reference infinite as a point of reference but for a line charge which is infinite I cannot use that one.

The reason why this happens is because the electric field of a point charge goes as 1 by r square, it decreases 1 by r square and an integral of 1 by r square will give you a finite value at infinity. Whereas the field of the line charge and the field of a (()) (32:39) charge or a plane charge we are going to talk about that in the next class will go as 1 by r or constant. Therefore, at infinite you cannot use them.

Because at infinite the charge the integral values sorry the values of these expressions are not going to be 0. The primary culprit being, the slow variation of the electric field, okay. These conditions will be become later very important when we talk about radiation okay until that point let us not worry about it. But what we have to do meanwhile if you want to solve the problem of putting a reference use to pick any other point apart from infinite as my reference.

And then start finding the potentials from that point okay. So this is not the infinite as the reference point but any other point or any other point in the at any other point as the reference and all the potentials would be calibrated with respect to that point, in fact this is practically what happens if you take a Coaxial cable we are going to discuss that shortly if you take a

Coaxial cable it will have an inner and an outer conductor okay which one can approximate as one line and another line okay.

Outer line is where we connect a voltage supply I mean connect the ground and then we say that that is the point of 0 potential, okay and the inner line will be charged and there will be an electric field between the two, we are going to calculate that one. This is an example of long line charges, and inability to use infinity as the point of reference okay. So you can of course object to this entire in saying that in practice how may I am going to realize finite line in such you will be right.

This problem of idealization has happened only because we consider an infinite line charge. In fact, here is a nice puzzle, what is the total charge contain by the infinite charge if the line charge density is uniform if you now go from minus infinity to plus infinity row ldz what will be the value of the total charge in this contain in the linear charge density you try to think about a little bit on this total charge, okay.