

Power System Operations and Control
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Module -2
Equipment and Stability Constraints in System Operation
Lecture 6

Now, let us see the lecture number 6 of module 2. In the previous lecture; that is lecture number five, we saw the various definitions of the various type of stabilities, and there difference of classification, again with respect to the disturbance means, magnitude of disturbance time frame, and again the quantities of interest. In this lecture I will discuss about transient stability of the power system, and then we will see what are the ways that we can analyze.

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TRANSIENT STABILITY ANALYSIS
(Classical approach)

Assumptions

1. Mechanical input to generator remains constant. (Governor system action neglected)
2. Machine damping and AVR action neglected. Synchronous Machine modeled as constant voltage source behind transient reactance (X_d')
3. Network transients neglected. Thus static model of network can be used.
4. Loads represented as constant impedance/admittance.
5. Mechanical angle of each machine rotor coincides with electrical phase of voltage behind transient reactance.

Analysis utilizes

- Static equations for networks. ✓
- Dynamic equations for machines. ✓

Handwritten diagrams: A circle with 'pm' and an arrow pointing to it, and a circuit diagram showing a voltage source E_f in series with a reactance jX_d' connected to a load Y_L .

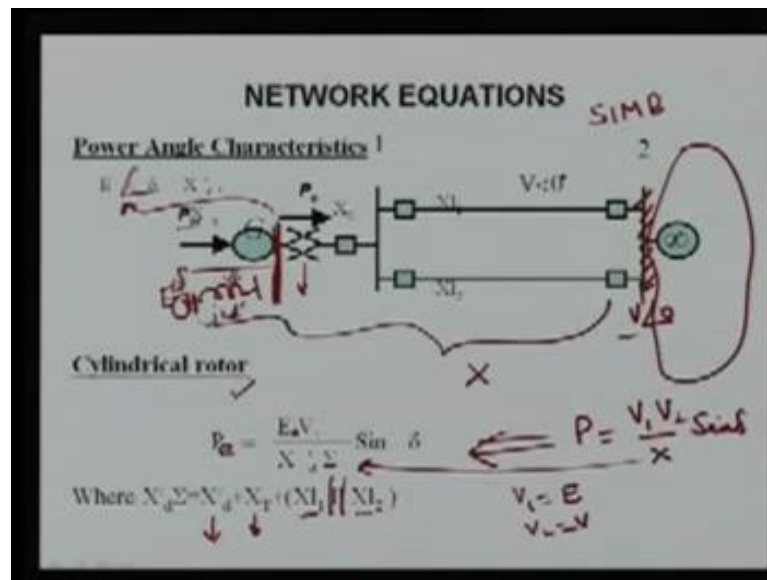
First one is a classical approach for the transient stability analysis, and the various assumptions are made in the classical approach. First one is the mechanical input to the generator remains constant. Means governing system action is neglected. Means if your system here this, whatever the generators here we are giving the mechanical power, and that is coming through your turbine system, again turbine system then governing system. So, this we are assuming for that analysis period, this pm is almost constant, and that assumption is valid. Because here the transient stability our analysis is concerned up to 5 to 10 seconds. So, by that here the governor action or prime over action is almost

negligible, because the time constant of this is very high. So, we can ignore this, means your mechanical power input to the generators are remain constant. Second assumption here is the mechanical damping; that is machine damping, and the AVR action is also neglected means the damping of the machine that is D , and the automatic voltage regulator action is also neglected.

The synchronous machine model as constant voltage source, behind a transient reactance. Means here it is your generator. So, we model this generator with a some reactance here and this is your terminal voltage. So, this reactance is nothing, but your X_d' . Again as in the previous lectures we have seen the synchronous alternators may of two kind, two types; one is your cylindrical rotors, another is the salient type of rotor of machine, where X_d and X_c are different, however in the synchronous machine it is a X_s ; that is constant and it is irrespective of the D and Q X_s components. So, if it is a salient then it is X_d' , and if it is your cylindrical rotor machine then it will be X_s . So, it is your some, the constant voltage source E_f behind this voltage behind this terminal voltage this t , and this is your sub transient reactance of this alternator than we can model in this way. So, the machine can be represented by the classical; that is also called the classical representation of the machine model. Also we know in the power system, all that components they have some dynamics, and they have some their equations and their dynamics are there. For your example your network dynamics, protection system dynamics, your governor dynamics, your mechanical inputs dynamics.

So, all are having the dynamics, but in this classical approach, we also ignore that network transients. Means those are neglected, thus the static model of network is consider. Only we have to go for the static model in this classical approach. Load also may have some dynamics; for example, your induction machine load, but also they are changing with the change of voltage, as well as changing in your reactive power requirement. But in this classical approach, we model load at the simple impedance or admittance form. The mechanical angle of each machine rotor coincides with electrical phase of voltage behind transient reactance. So, these are the five assumptions are made in the classical analysis of the transient stability. So, analysis utilizes the static equation of network, and the dynamic equation of machine. As I said, here the network, this we have to take static model, and others we have to take here the dynamics of the machine, here that is included.

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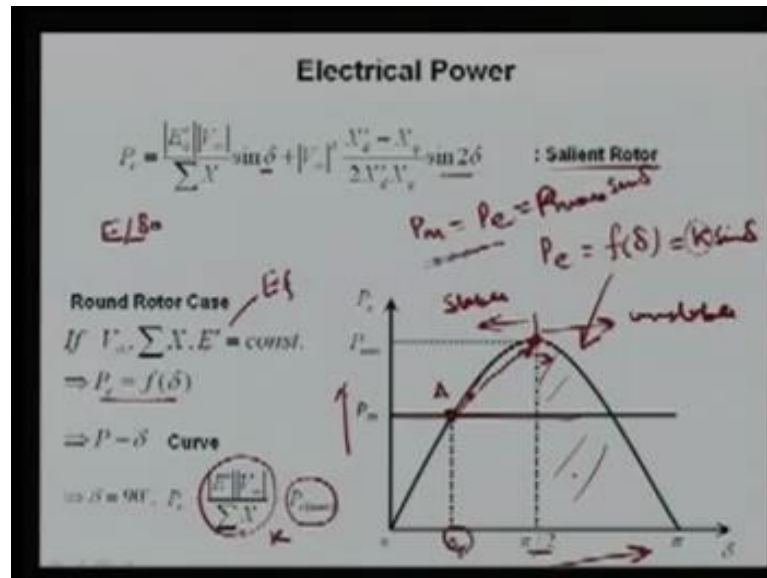
Let us see a generator G here that is connected with the transformer having impedance X_d , and here these blocks are basically the circuit breakers; means we can trip this portion, we can trip this transmission lines as well, and here we have the infinite system. Normally, most of the analysis people go for either single machine infinite bus system; that is called SIMB single machine infinite bus machine system. So, this is basically your Infinite bus, where normally this voltage angle is zero. Means again what is the infinite bus, why it is called infinite, whether infinite in terms of its location, infinite in terms of its, what is infinite term why. So, infinite bus is a bus, which is if you are injecting power, or you are extracting power from that bus. Normally they should not be any change in the voltage and the frequency of the system of that bus, so this is called infinite. Means you can draw effectively infinite power, which is basically not possible, but if you are drawing large power, and there is no change in the voltage and the frequency of that system, then we can say that bus where we are drawing the power, is known as your infinite bus. So, here basically this is another system, and where this bus is located, so you can draw more power here at this bus. Means here practically the voltage an angle is constant, what about the power you are taking.

So, we had the system; one generator, the transformer, and then we have the two lines; line X is here that is impedance is $X1$ one, and another is here your $X1$ two. Now, this machine as I said in the classical model, we can represent this machine as E and then we can have a reactance here, and this is your voltage source; that is your X_d prime . So,

now, your this bus, this system we can represent here with this one. Now, this is a transformer. So, this P_m we have assumed constant, but this electrical power P is changing and it depends upon the system here the angle etcetera. Now, for the cylindrical rotor, we can write the P . If you remembered already we have defined is P for any machines here is $V_1 V_2 \sin \delta$ upon X and this δ is the difference between the angle of the two buses. And that bus is the element with X_s is considered. For this case we are just considering here one bus here, and another we are talking here. So, the Impedance in this bus, is let us suppose now here what will be this X .

Now this X component just we have to calculate. So, we have to write here now our V_1 in this equation is nothing, but your E , because we have taken here $E \cos \delta$ of this bus. So, your this V_1 is E , and your V_2 is nothing, but your V , and X is the total reactance of this whole system, including your this as well. So, this is your complete. So, we can write now this can be simplified this P ; that is P_e is equal to E multiplied by your voltage; that is infinite bus voltage divided by the X_d' prime sum. Means the total X here as I said here is X and the $\sin \delta$ basically angle here the zero and the internal here angle is your δ . Now, here this X_d' prime will be nothing, but the reactance of your generator; that is X_d' prime, the reactance of your transformer, here this is this. And these two lines basically this is in series with this, and the impedance parallel effect impedance is series with these three. So, these two are in parallel. So, it is a parallel combination of the reactance of these two lines this X_{l1} and X_{l2} thus it is a parallel. So, it is a total X_d' prime will be this, and then we can write electrical power which is flowing out from this system. Here P_e can be related here, and that is basically in terms of you cylindrical rotor machine.

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To see the same system, if your machine is a salient rotor type of synchronous machine, then we have also derived this. Here this is your Eq prime, here now here Eq and the Vd both are the different one. Means here this Eq prime multiplied by your V infinite plus some of the reactance's in that line, and again here we have another component here. So, here this is delta. It is a 2 delta, and already in the previous lecture I have shown that. So, if your round rotor case, if your V infinitely. This X and E prime are constant. E prime is nothing, but in the previous case it was E, if are constant, then we are write this Pe is nothing, but is a function of your simply delta, or you can say sine delta. And if you draw then you will find here a curve of your sinusoidal here. Means we are getting some constant k sine delta. Means if your delta is changing, the power that will be keep on changing, this Pe. So, this is called your P delta curve, and at the 90 degree pi by 2. This k term will maximum and it is called Pe; that is Pe maximum. Means when it is your degree is 90 degree then we can have the P maximum, means maximum power that we can flow. So, the area which is just this side it is your stable zone, this and outside this is your unstable. Now, always what will be your operating point. Let us suppose your mechanical power X Pm is this axis. So, your electrical always we know for their steady operation, this Pm should be equal to your Pe. Then here we can have your k; that is k term is here. This is nothing, but your k; that is Pmax we can say Pmax sine delta. So, this is a curve of sine delta.

So, this will satisfy at a point, and that is basically the intersection of this curve, this and this curve. So, this point A is your operation of point, and the angle that is a torque angle or machine angle that will be your delta naught here. So, your machine which was here this I wrote E angle here, now it will be delta naught, if your system is without any disturbance and this is steady state it is operating with this, your previous case, this your transmission like this. Now, and this is your delta angle where it is operating. So, the weekend can go up to slowly if you are slowly and slowly we can increase, without disturbing the system slowly power is increased, then we can go up to this point, and this is normally called your steady state stability limit, but always we operate our power system, must be low this steady state that is Pmax value, because always you know there is a dynamics, always there is a change in the system. So, if you are operating here, there may be possibility your system may be your unstable zone, because this area is unstable and this side beyond this pi by 2, and then your system will be collapse, and it is not possible to operate

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Power Transfer on a line

Bus-1 $V_1 \angle \delta_1$ $\bar{Z} = R + jX$ Bus-2 $V_2 \angle \delta_2$

$$P_0 = \frac{1}{R^2 + X^2} [RV_1^2 - RV_1 V_2 \cos \delta + V_1 V_2 X \sin \delta]$$

If $R \approx 0$

$$P_0 \approx \frac{V_1 V_2}{X} \sin \delta$$

$$P_g = \left[\bar{V}_i \cdot (\bar{V}_i - \bar{V}_j)^* \right] / Z^*$$

$$P = \frac{V_1 V_2}{X} \sin(\delta_1 - \delta_2)$$

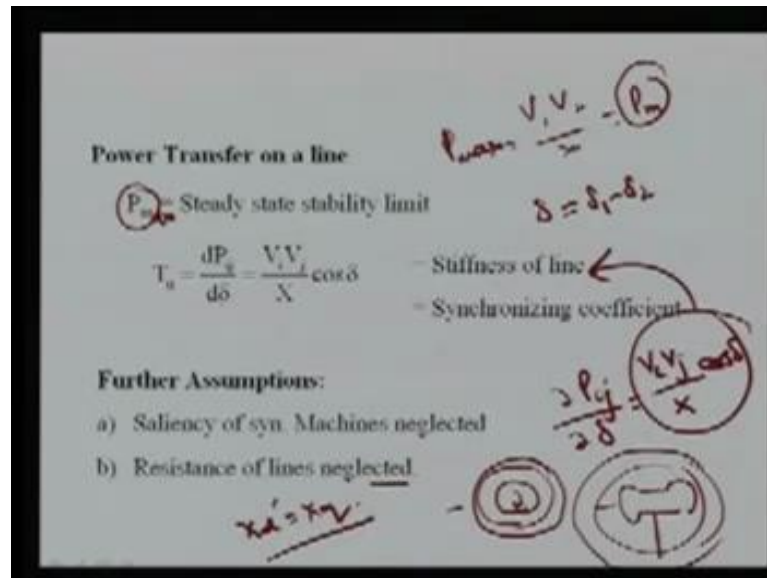
$R \approx 0$

Let us see the power transfer on a transmission line. So, far we had seen the power transfer including your machine, transformer, and the transmission line. Here let us we have a bus i and the bus j, and we have the angle here $V_j \delta_{ij}$, and here the voltage magnitude is V_i and angle is δ_i , and this Z of this line is R plus jX . So, we can write here the P_{ij} in this line with this formula. This is nothing, here this P_{ij} is nothing, but your V_i ; that is complex into your i j that is your V_i minus V_j conjugate divided by your

$i Z'$ prime and here basically the real of this, and if you simply you will get this expression. So, basically it is a z square. If your R is zero, means your line is loss less, means normally R is very small. So, we can ignore compared to the X then we can get whole expression R is zero we will get this and this is zero. So, we can get this X X will be cancelled and we will get the P_{ij} is approximately $V_i V_j \sin \delta$ divided by X reactance of this line $\sin \delta$, or we can say this $P_m \sin \delta$. So, for the line as of full system this formula is equally valid. Only you have to remember in this case, that this δ is the angle between the two buses, where these two voltages.

So, that is why here I write this P between two nodes. Node means two buses. Here this voltage of one bus, voltage of another bus divided by X between these two buses. Means there may be several for example, here one is this, there may be another line here there may be some transformer here and then another bus here. So, if you are talking this is V_1 and V_2 . So, that this Whole will be the total of this line one line going transformer. And the angle here is the difference between angles of these two buses. So, here X is the total X and then it is your $\sin$ basically δ_2 minus δ_1 , or basically or basically here δ_1 minus δ_2 what is the direction. Means if your power is your calculating the power showing P in this one. So, it is δ_1 minus δ_2 as I explained in the previous lectures, the real power will flow from higher angle to lower angle. So, this expression is equally valid, and this is the case when R is very small and that can be taken as zero, then this full expression is valid. If R is very high, then we can apply this one, especially for the distributed system you can write this. You have to take this expression which is the original expression.

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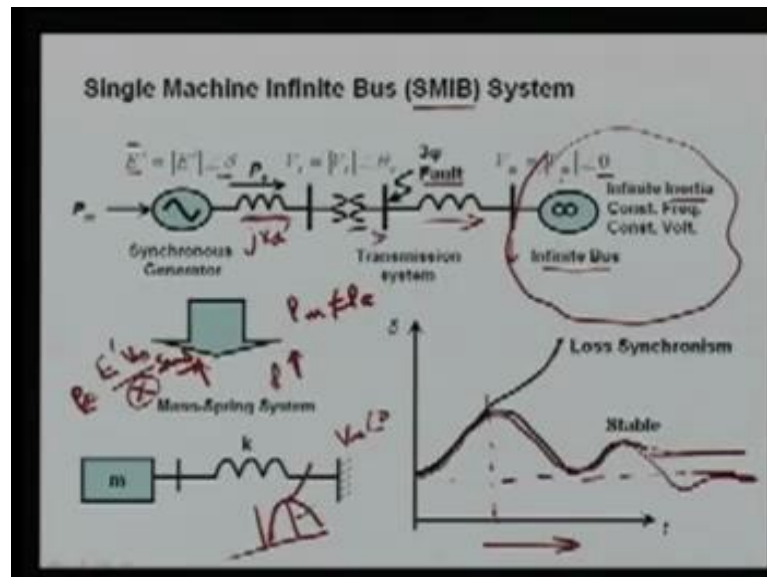


Now, let us see another one. This steady state stability limit that is P_m ; that is your $V_1 V_2$ upon X . It is sometimes called V_m sometimes P_{max} , so it is given in the different book here. Normally this P_m I will denote the mechanical power. So, it is better to write the P_{max} . So, it is the steady state stability limit. If you differentiate the P_{ij} in the previous case, if you differentiating this, that we have if you are differentiating that the P_{ij} with reference to the δ here that is again here δ , I have just written it is nothing the δ between the bus 1 and bus 2 is your δ . So, always you must be very much careful what is this δ . So, if you are differentiating this, then the voltage here is a $V_i V_j$ and here you X , it is your $\cos \delta$. And this coefficient is called the stiffness of line or it is called the synchronizing coefficient of the line. So, this term gets your idea that what is the stiffness, constant of the line that is very much important, how much your system is stable.

Also in the classical approach, we may need more assumptions, and that is basically the saliencies of synchronous machines are neglected, and resistance of lines are also neglected. In the classical approach, as I said your two types of machines may be there, here it is a stator, and here is your rotor which is rotating. So, if it is there the constant air gap, then it is called in rotor, and if you are having some poles, and we have the different air gap; that is rotating in the machine here. So, here we have the different air gap, here we have different air gap and this called the saliency machine. So, we can ignore means here we can write this $X_d' = X_q'$ then we have to ignore saliency we have

to assume, but both quantities are very close, but normally it is not so easy, it is very different. And also the resistances of the lines it is not only lines, even though of the transformers we have to ignore, and the resistance of your synchronous machine is also ignore.

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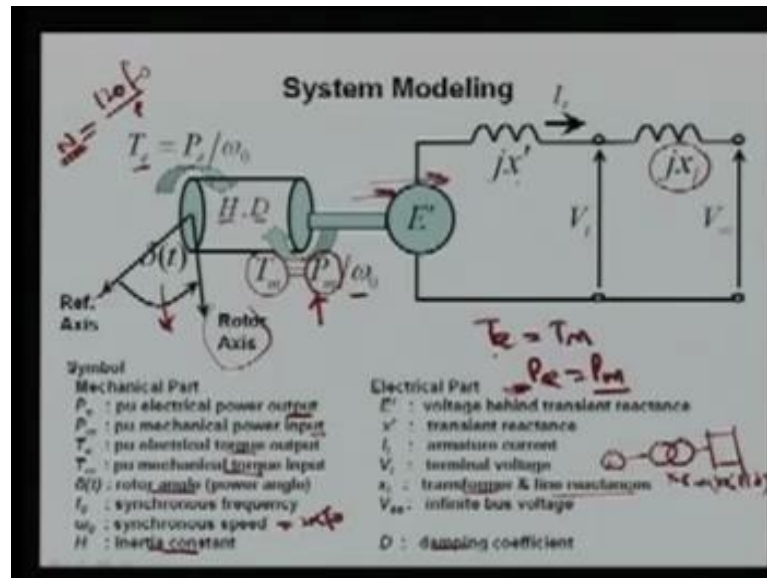
Now, let us see the single machine infinite bus; this SMIB the single machine infinite bus system, where again the previous case we use here a generator G. This is your machine is represented by its equivalent impedance; that is nothing, but our jx_d or X_d prime here. You have to take the sub synchronous transient reactance, and this machine is the internal here the prime that is equal to magnitude this is complex quantity. So, the magnitude with the angle δ with reference to the infinite bus. So, the Infinite bus basically denoted by, it has the infinite inertia, system inertia; whole this system basically. This is a system where this bus is there, so it has the infinite inertia, it has a constant frequency, and it has a constant voltage. So, these three criteria basically gives you the criteria for the infinite bus, and the voltage here as I said the voltage is V infinite, here I have written angle zero, and with the reference to this angle we have taken this angle δ . This is your transformer this is your terminal voltage of generator.

This is another bus, where this is a line there and it has some impedance. Now, this machine here is nothing, but your spring mass system. Here there is some inductance it was A the total this we can denote whole this machine. This is your infinite bus where

this V infinite angle zero, and this is another mass. So, this is just like a masses spring system, and then it will have some, and then we can analyze dynamics of the system, as you know the dynamics of the system can be retained and solve the differential equation. Now, there is a two type of concept, if there is a three phase. Let us suppose at this bus as we shown here. If there is three phase fault, there will be this machine delta will be changing, why. This electrical power due to this fault, more power will be flown as i said the power here is your nothing, but your $E' \sin \delta$ that is infinite the total X completely $X \sin \delta$. What will happen now this is in steady state, and this P_m is almost constant. We have assumed this now this three phase fault is there, what will happen. This X will be change, and this X will be reducing.

So, means more power is flowing. So, your P_m is not equal to P_e during this three phases fault. So, this to make it what will happen this P_e this delta will keep on increasing, because this value is larger, if it maintain the P_e constant this delta, you can say this delta will increase, and now you can say delta is increasing. So, over time period during the fault, this delta again can follow the different trajectory. One condition you can see if delta is falling like this, and finally, it is stable. No doubt, there is a possibility at this point, this fault is clear. If the fault is persisting, and this fault is not severe, so delta will increase and then it will be stable, but if this fault is clear. So, then delta again may come to its initial value, may be, again depends upon the, whether fault is removed or not. If fault is removed, then it may come to its original value, and the system is called to be stable, but this delta can increase like this, till this fault is again cleared, the system is this delta is keep on increasing and it losing their stability, because here if you see, if this is your delta and this delta is keep on increasing, then system is unstable and system is unstable then it is called there is a loss of synchronism. So, it is losing synchronism operation, and then we can say system is unstable.

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To see the system modeling, again this whole the rotating mass, here it has some inertia constant. This rotating mass we have the synchronous generators. The systems also we are have some damping. Damping means we have some resistances, means there is some damping factor as well, and let us say it is D, and here this your Tm this mechanical torque that is mechanical power divided by this normal frequency that is omega not. And this omega naught is nothing, but it is equal to your 2 pi f not, and f naught is your synchronous frequency. This synchronous frequency is decided as I said; it is 120 f upon p; that is how much you are rotating. This rotating is p this is giving your frequency based on the rotation. So, number of poles of the machine. So, this is a rotating mass, and again the two forces; one is Tm, another is your Te. So, your electrical power which is coming through this one going that side, it is an electric power and the input we are giving in terms of mechanical power. So, at the balance as I said here again this Te must be equal to your Tm, or we can say this Pe will be equal to your Pm. Means your electrical power will be equal to your mechanical power, and then it is called steady state. But during the steady state it may not be equal it will try to equalize this Te.

As I said this Pm we have achieving the constant in this transient stability analysis, so only parameter which is changing your Pe. So, various in this system modeling that will be used, this Pe in per unit electrical power output that is power which is coming out, Pm is mechanical power input that Te is electrical torque, Tm is mechanical torque input again, delta is the rotor angle. And this angle is the angle between the two here your

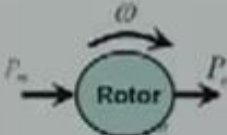
reference axis, and here your rotor axis. So, this is the angle between the reference where you are measuring all these angles, and this is the angle between your, and this is called your torque angle, all load, angle between the two axis here. Your H is the inertia constant of machine; whole system basically here we are talking. If all machines are coming into 1, one combined machine then it is inertia constant of complete system, or if you are analyzing one machine with one machine, then it is inertia constant of one machine.

So, D is your damping coefficient as I said. Here this is your V infinite is here infinite voltage. This is terminal of this generator. This is the X_{prime} ; that is a transient reactance of synchronous machine, and here is E is behind the transient reactance. So, what about the power that is here coming out, again this will be equal to your P_m in the steady state. This X_l this include your the transformer and the line reactance's, as I said we had one transformer and then we had the two lines, again the same example and here it was E_f . So, this is your X_t plus and X_{l1} plus parallel to X_{t2} parallel. So, it is X is the combine of the transformer as well as the two transmission lines, those are in parallel.

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Swing Equation

Under normal operating conditions, the relative position of the rotor axis and the resultant magnetic field axis is fixed. The angle between the two is known as the power angle or torque angle. During any disturbance, rotor will decelerate or accelerate with respect to the synchronizing rotating air gap field, and a relative motion begins. The equation describing this relative motion is known as the swing equation. If, after this oscillatory period, the rotor locks back into synchronous speed, the generator will maintain its stability. If the disturbance does not involve any net change in power, the rotor returns to its original position. If the disturbance is created by a change in generation, load or in network conditions, the rotor comes to a new operating power angle relative to the synchronously revolving field. Swing equation can be expressed in terms of a 2nd order nonlinear differential equation as



$$\frac{H}{\pi f_s} \frac{d^2 \delta}{dt^2} = P_m - P_e(\delta) - D \frac{d\delta}{dt}$$

$$M = \frac{H}{\pi f_s} \quad \omega = \frac{d\delta}{dt} \Rightarrow M \frac{d\omega}{dt} = P_m - P_e - P_D$$

$\frac{d\delta}{dt} = \omega$

Neglect damping power term ($P_D = D \frac{d\delta}{dt} = 0$)

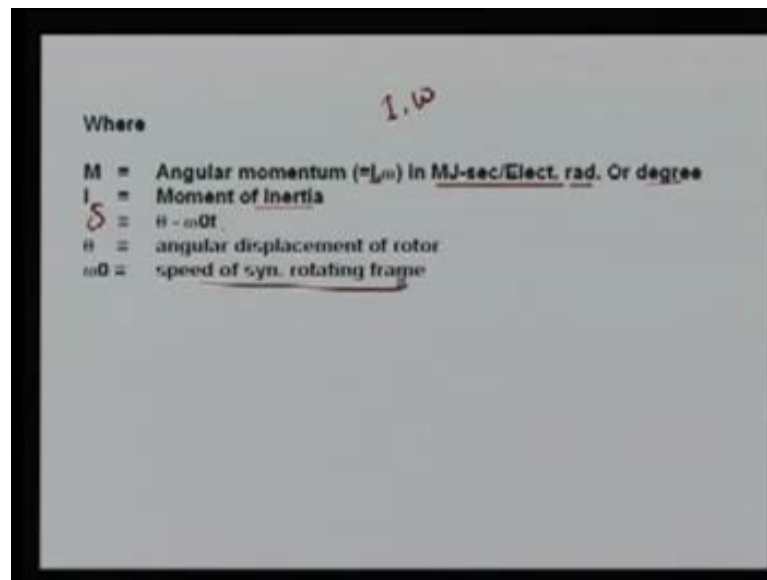
$M \frac{d\omega}{dt} = P_m - P_e$

So, for the transient analysis, normally we use the swing equation. Let us what is the swing equation. Under the normal operating condition, the relative position of rotor axis, and the resultant magnetic field axis is checks. So, the rotating axis of the rotor, and the magnetic axis is field, is fixed. The angle between these two is known as the power angle

or the torque angle. During any disturbance, rotor will accelerate or decelerate. There by what will happen, there will be change in the rotor angle. So, with acceleration and the deceleration, with respect to the synchronizing rotor air gap mmf and a relative motion became basically. The equation describing this relative motion is known as swing equation. Means during any disturbance, rotor will decelerate or accelerate. Again depending upon the P_m and T_m balance; means whether the T_m is more. Means input is more than the electrical output, then machine will accelerate. If your input is less than your output then it will be decelerate. Then accelerate with respect to the synchronizing rotating air gap mf, and our relative motion begins, and the equation describing this relative motion is known as swing equation.

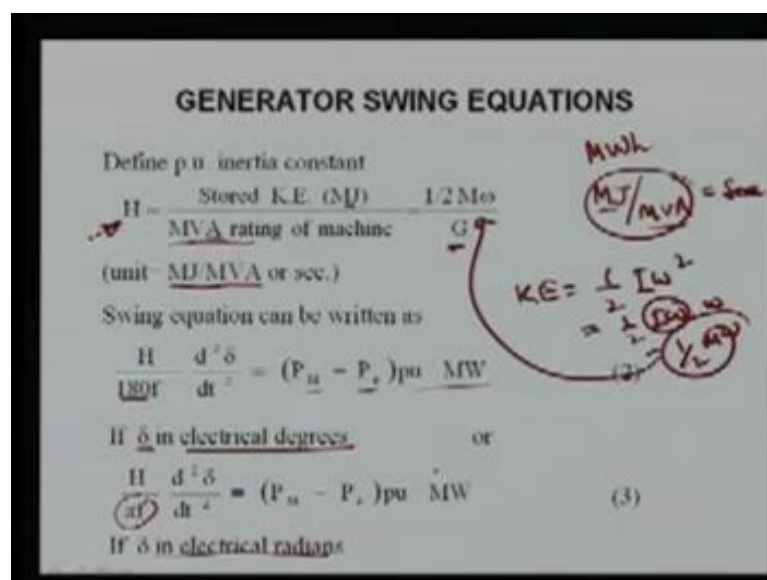
If after this oscillatory period the rotor locks back into the synchronous speed, the generator will maintain its stability. If the disturbance. There will not involve any net change in power. The rotor returns to its original position. If the disturbance is created by a change in generation, load, or network condition, or network topology. The rotor comes to a new operating angle, relatively synchronously revolving field. Swing equation can be expressed in terms of the second ordered non-linear differential equation as here H over πf naught second derivative of δ with respect to time; that will be equal to P_m minus P_e δ and that is a function of your δ minus this damping constant and then we have D δ upon dt . Basically this D δ upon dt is nothing, but your changing speed. So, if we neglect this term this speed E ; the damping effect as I said in the classical analysis we always neglect this. So, this can be neglected. So, we will have the equation here like this. Now you can see earlier I was using this H , and I now I am using you know. So, this M and H they are defined they are related, as this M angular momentum is equal to H upon πf not, and this ω is nothing, but this δ δ upon dt . So, whole this equation can be replaced and can be retained in terms of ω .

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So, this M is angular momentum, and this is defined as inertia i of whole system multiplied by angular speed, and its unit is your mega joule second per electrical radian or degree. i is your moment of inertia, here you δ is θ minus $\omega_0 t$, and θ is angular displacement of rotor, and ω_0 is the speed of the synchronous rotating frame.

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Now, let us again I use this H and how we have to get the information of this H . Now let us define the per unit inertia constant H ; that is H is defined at the stored kinetic energy

So, knowing this M , knowing your G and ω ; that is a synchronous speed we can get H , or knowing H we can get M vice versa. So, the swing equation can be written as in terms of H . Again if you are using δ in electrical degrees, then it is one Atf here double differentiation of δ with respect to time, and here you power minus, this mechanical power minus your electrical power in per unit. So, here we are talking just about the per unit, it is divided by the base unit. So, it is per unit always, because this is H in per unit, this unit at all in per unit. So, it is not in megawatt. So, you have to write the base, but you have to use, and base is of course, is a MVA rating of the machine. So, simply we have divided here. So, if you are using the δ in electrical radians, then here it is a πf and then we can write this one.

GENERATOR SWING EQUATIONS (Contd.)

Typical Values of H

3-10 MJ/MVA	Steam turbine gen.
2-4 MJ/MVA	Hydro gen.

Average Values

Syn. Motor	2.0 MJ/MVA
Ind. Motor	0.5 MJ/MVA
Syn. Condensers - large	1.25 MJ/MVA
- small	1.00 MJ/MVA

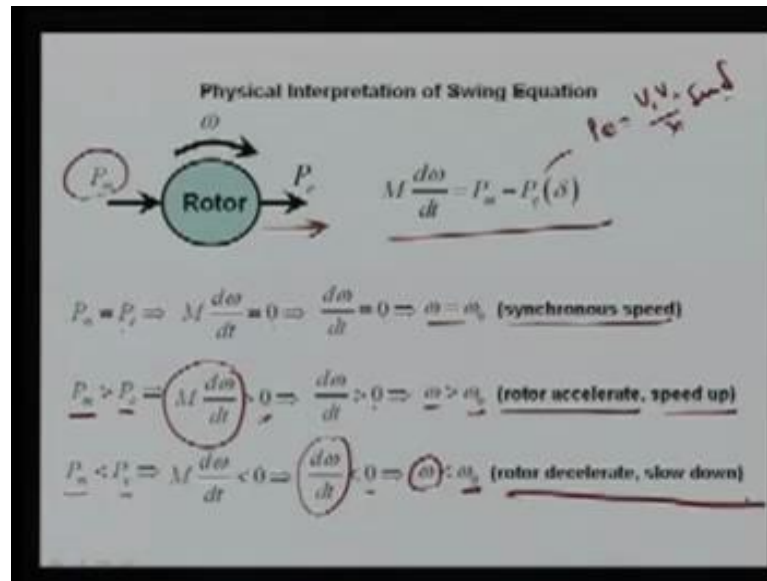
Handwritten notes and diagrams:

- A dashed circle with a squiggle inside, connected by a dashed line to the '3-10 MJ/MVA' value.
- A diagram of a generator (circle with a squiggle) connected to a motor (rectangle) and a condenser (circle with a cross), with arrows indicating power flow.
- A diagram of a generator (circle with a squiggle) connected to a motor (rectangle).
- A diagram of a generator (circle with a squiggle) connected to a motor (rectangle) and a condenser (circle with a cross), with arrows indicating power flow.
- A diagram of a generator (circle with a squiggle) connected to a motor (rectangle) and a condenser (circle with a cross), with arrows indicating power flow.

Now, the various types of machines are used; synchronous machines I am talking, and then the inertia constant is different for the different type of machine, and this inertia constant H here it is basically 3 to 10 second for steam turbine generator system. It is from two to four for a hydro generator system. Normally if H is large then we say this machine is large, if it is a small, then machine is small. You can see the Induction motor, this inertia constant is 0.5 second means its size is very small. So, this inertia constant gives information about the size of the machine. Why this steam turbine generator system is having high inertia, because you know here we have this generator, that generator is coupled with the various types of turbines, different stages of turbines. So, they also high speed and rotating mass are there. So, the total H of the system is more compared to the hydro, where one the hydro turbines are there, and then it is coupled with your turbine system here. So, it has a lesser time constant H , means inertia constant.

So, it is 2 to 4 second however the steam turbine depending upon the size, if it is very large then H very high, and if it is small then it is again small. Again, in terms of synchronous motor, because we can use the synchronous motors as well. So, the average value of H , it may two again 2 second. Synchronous condensers, if it is large then it may be average value is 1.25, and if it small then it is one. What is the synchronous condenser. To understand here the synchronous condenser are nothing, but they are some sought of alternators, but mechanical power output, this electrical power output is zero. Means if you are using here this generator, and there is no here P_e is zero. You are rotating to provide the reactive power only Q , then it is known as synchronous condenser. So, you can either generate, or absorb reactive power by this synchronous alternator. We know this by changing the excitation system we can generate or we can absorb the reactive power. So, it is acting as some variable capacitor along with the inductor. See one zone it is giving, another it is also absorbing. So, it can do, and we are not taking any real power load then it is called synchronous condenser.

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Now, let us see the physical interpretation of swing equation; that is here. Means we can derive this swing equation in terms of change in the speed with respect to time, the derivative of speed with respect to time this M value this P_m minus P_e , and that P_e is a function of δ . Again this P_e is nothing, but P_e ; that is approximate the $V_1 V_2$ upon X sine δ . Normally these voltages are constant, we assume that constant, so it is a function of sine δ , and it is a non-linear function. So, this differential equation is a non-linear differential equation. So, here it is your rotor, and rotor is rotated by the mechanical input, and then we are taking the electrical output, and this machine is rotating with a speed of ω . Now, if this P_m is equal to P_e what will happen. This component will be equal to zero, means if P_m is equal to P_e , then this component here $M \frac{d\omega}{dt}$ will be zero and this. So, that $\frac{d\omega}{dt}$ is zero; means your synchronous machine is running at the synchronous speed; neither it is accelerating nor it is decelerating, so, it is a steady state operation.

Let us take if mechanical power is more, means here this more than this output, what will happen. This machine will rotate, and accelerate, why it is. So, because always we know this energy is conserved, when this power is coming here, it is going less. So, that power will be store in this machine itself. And how an alternating, or you can say rotating mass will store the energy, and that is no doubt in terms of kinetic energy means that energy will be stored in the kinetic energy, so the kinetic energy will increase. Therefore, the speed of the machine will increase, so this is same concept. So, if the P_m is more than

Pe, than this component will be more than zero. What does it mean. Your rate of change of speed is more, means your this omega will be now more than your synchronous speed, and then this rotor will accelerate, or you can say it will be speeding up. The reverse is also true. Means your mechanical power is less than electrical power, means you are taking more power, than your input, what will happen. The energy which is here in this mass will be taken out, and then means kinetic energy will be reducing; therefore, the speed of system will be reducing, and then we can say this change in omega with respect to t will be less than zero, or the rotating here speed will be less than its normal or can say frequent synchronous speed of the system, and the rotor will decelerate, or it will be slowing down.

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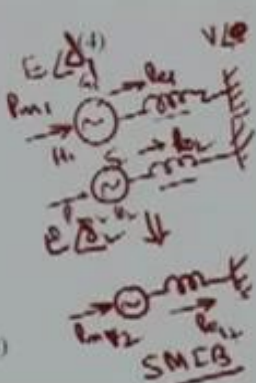
SWING EQUATIONS FOR TWO COHERENT MACHINES

$$\frac{H_{12}}{\pi f} \frac{d^2 \delta_{12}}{dt^2} = P_{M12} - P_{E12}$$

Where $H_{12} = \frac{H_1 H_2}{H_1 + H_2}$

$$P_{M12} = \frac{P_{M1} H_2 + P_{M2} H_1}{H_1 + H_2}$$

$$P_{E12} = \frac{P_{E1} H_2 + P_{E2} H_1}{H_1 + H_2}$$

$$\delta_{12} = \delta_1 - \delta_2 \quad (\text{in elect. radians})$$


So, the swing equation of the two coherent machines. So, far we have used H. here it is H or M whatever you are talking about. So, if it is one machine here, and then it is connected with your Infinite bus here. Now, if there are two machines; means here you having two machines, and then you want to include and combine together, then we can say this two coherent machine can be combined, and their inertia constant can be also added together. So, here it is your Pm one, another machine here it is your Pm two, and this another here, this is synchronous machine. So, if we are having two coherent machines; like here machine one, I can say G one. Here it is your G two, and it is connected by a, its equivalent here internal impedance. Here again internal of this machine, and this is your infinite bus; means voltage angle is constant. So, these two

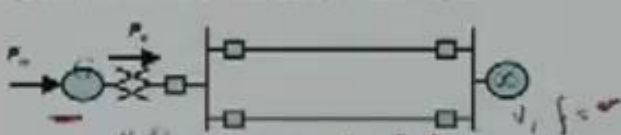
machines can be combined together, and then again we can basically this, we can write a single machine here, with the some here equivalent and this machine is like this. So, here this is your, now this Pm equivalent, and again the electrical power which is going the P equivalent here. So, I can say P 1 two, here Pm 1 2 I can write.

So, we can represent these two machines here like this one here, if both are in coherent means they are oscillating and rotating in the same speed. So, if inertia constant this here H one, here is S two, and the electrical power which is feeding is P 1 and here is Pe two, then we can write this H 1 two, will be nothing, but H 1 S 2 over H 1 plus S 2 we can have this one expression for here. And this your Pm 1 2 can be related at the Pm 1 S 2 minus Pm two, means Pm 2 here multiplied by this H one. So, this Pm 1 multiplied by H 2 minus Pm 2 multiplied by H 1 divided by H 1 plus S two. Similarly the Pe 1 2 just combine here, will be the P 1 here multiplied by this S 2 minus here Pe 2 multiplied by H 1 and then divided by here and the angle here. If this is the angle E 1 angle delta one, here this is E angle delta two. So, the angle here the difference will be your delta 1 minus delta 2 with reference to zero, and then we can go for equivalent. So, this is again why we are going for if we are having two machines, then you can again go for, this is nothing, but your single machine infinite bus system SMIB, and then you can go for classical approach and then you can analyze it. So, for analysis for the transient stability for this single machine infinite bus system, we can use the equal area criteria, and that is very popularly used.

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Equal Area Criterion

Equal area criterion is a quick prediction of stability. This method is based on the graphical interpretation of the energy stored in the rotating masses as an aid to determine if the machine maintains its stability after a disturbance. The method is only applicable to a one-machine system connected to an infinite bus or a two-machine system.



Rotating equation:
$$\frac{H}{\omega_s} \frac{d^2\delta}{dt^2} = P_m - P_e = P_m - P_s \sin\delta$$

Where P_s is the accelerating power, from the above equation, we can derive that

$$\frac{d\delta}{dt} = \sqrt{\frac{2H}{P_s}} \int (P_m - P_s \sin\delta) d\delta$$

The equation gives the relative speed of the machine with respect to the synchronously revolving reference frame. For stability, this speed must become zero at some time after this disturbance. Therefore, we have for the stability criterion

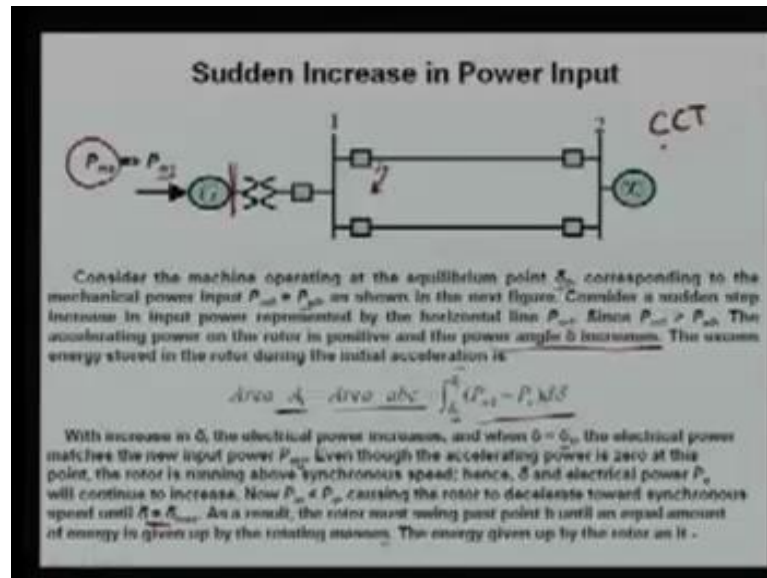
$$\int_{\delta_1}^{\delta_2} (P_m - P_s \sin\delta) d\delta = 0$$

$\omega = \frac{d\delta}{dt} = \omega_s \sin\delta$

So, the equal area criteria, is a quick prediction of stability. It is a very quickly you can analyze, whether your system is stable or not, following the fault. This method is based on the graphical interpretation of the energy stored in the rotating mass, as an aid to determine if the system machine maintains its stability after a disturbance. The method is only applicable to one machine system connected to an infinite bus or two machines system. Means if the two machines basically what is one machine infinite bus system, you can see again the same example which I explained earlier one. Here, one generating system E_f . Here we have the infinite bus, means this is also a machine, which have the high inertia. Means large inertia, infinite inertia, and where the voltage and the frequency are the constant. So, it is only applicable for the two machines systems means this is two machines, or other words you can say it is the single machine infinite bus system. So, it is only limited to that, it is not possible to go for the multiple machine system. If the machines are several and connected by the different transmission lines, then we have to go for another analysis.

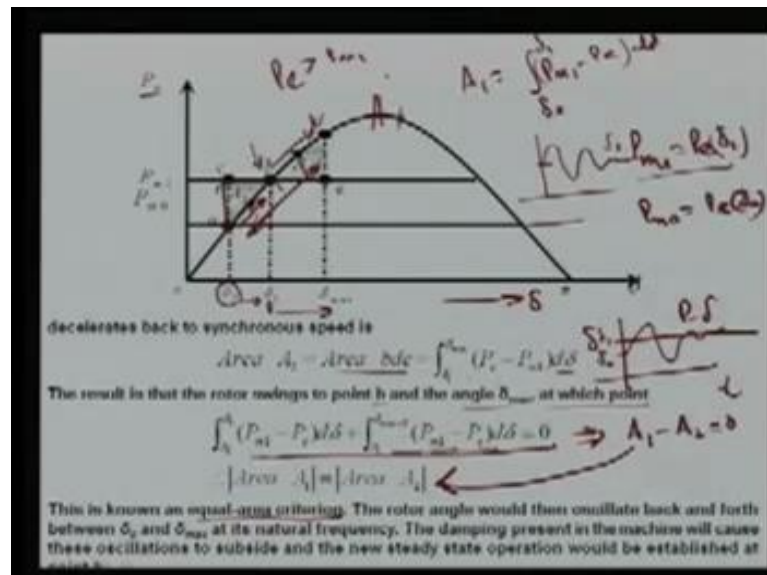
We have to solve the equations the non-linear equations by help of several differential equation solution methods. So, the swing equation here as again I can write; that is written by this one, and this P_m minus P_e is nothing, but it is P_a it is called the accelerating power. So, whether P_a is zero or it is positive or negative, it decides whether your machine accelerating, your machine is decelerating, or it is in steady state operation. So, this equation, now you can see, this equation can be written as in this form. Means here from this equation you can see I can write this $d\omega$ over dt , is some constant integration of P_m minus P_e angle δ , we can difference. We can derive this equation no doubt very easily. We have to write the two differential equation and finally, or you can differentiate this equation. Here you will find this equation without any problem. So, this equation gives the relative speed, what is this. This is your relative speed of the machine, with respect to synchronously revolving frame reference, for stability this speed must be zero, at some time after this disturbance, therefore, we have for the stability criteria this must be zero, means we can have this δ from δ_0 to δ_{max} where it was operating to the δ , if we are integrating for that period then it will be zero. To understand this it is very easy.

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Let us consider the sudden increase in power input, means here this is your system, this is infinite bus, where we have suddenly. Earlier it was operating at the P_m not, and then we have increase this P_m one. Suddenly we have increase in power input. Consider the machine operating at the equilibrium point δ_0 , what is the δ naught

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You can see here, this is your δ_0 . This is your δ variation, this is your P_e , this is nothing, but your P_e δ curve of the combined component system, including you are here this infinite bus, here another terminal, here your generating terminal, here your

generator, and this is your transformer included everything. So, it is operating here as I said, here it is operating, because mechanical power P_m naught is this axis. So, the operating point will be your A, where it is intersection of this P delta curve with this your P_m naught curve that is constant. So, it is a constant we have assumed. So, this delta naught is your initial operating angle, and corresponding to this mechanical power P_m naught that will be not equal to your P_e naught; means your electrical power is equal to your mechanical power in the steady state, as shown in the figure, consider a sudden step. Sudden step here, you can see suddenly it is increased here, to $P_m 1$. Now, what will happen, now its stable, this is P_m , so your B point is the post increased stable point, this B point, because at that point here this $P_m 1$ will be equal to P_e , now it is I can say delta one, where it is equal.

So, at the Initial it was P_m not, it was equal to your P_e delta not. So, this B point is your another post increased the power stable. So, just we have seen the sudden change means; since $P_m 1$ is greater than $P_e 1$. The accelerating power on the rotor is positive, because we have increased the P_m , means at that point your P_e was P_e not, so once you are increasing your machine will accelerate, because if P_m is more than P_e as in previous slide I showed you that it will be increasing, and it will be accelerating, so your power delta angle delta will increasing. Means here your this delta, here it will be increasing like this. So, it is increasing. The axis energy is stored in the rotor during the initial accelerating is here area A one; that is that is area ABCD; means here this area, this energy is that is ABC, that area you can say area A 1 is the stored energy of this machine; that is ABC and it is basically integration of delta naught to delta 1, means here is a delta not, means this curve minus this curve if you are integrating from here to here, means your area A 1 will be nothing, but integrating from zero naught here, to delta 1. Here this is a constant curve. So, this is your $P_m 1$ minus this curve; that is P_e , and again it is a delta.

So, this area is just this area here it is written as this one. With the increase in the delta the electrical power will increase, and when this delta is equal to delta one, the electrical power of machine matches the new power that is P_m one, even though the accelerating power is zero at this point, the rotor is running above the synchronous speed. Hence the delta and the electrical power $E E$ will continue to increase, and now this $P_m 1$ will be less than and causing the rotor to decelerate the synchronous speed until again this delta

is δ_{max} . As a result the rotor must swing fast point B, until an equal amount of energy is given by the rotating mass. The energy given by the rotor as it is. Now what happens, now thus from here it has to go here, and your rotor is now accelerating. Once it is accelerating here, it has reached, but your rotor is accelerating at this point B, it is satisfied this, but it is your machine is accelerating. So, what will happen it will try to accelerate and it may go up to this point d. And here, again now speed of the machine will be synchronous speed.

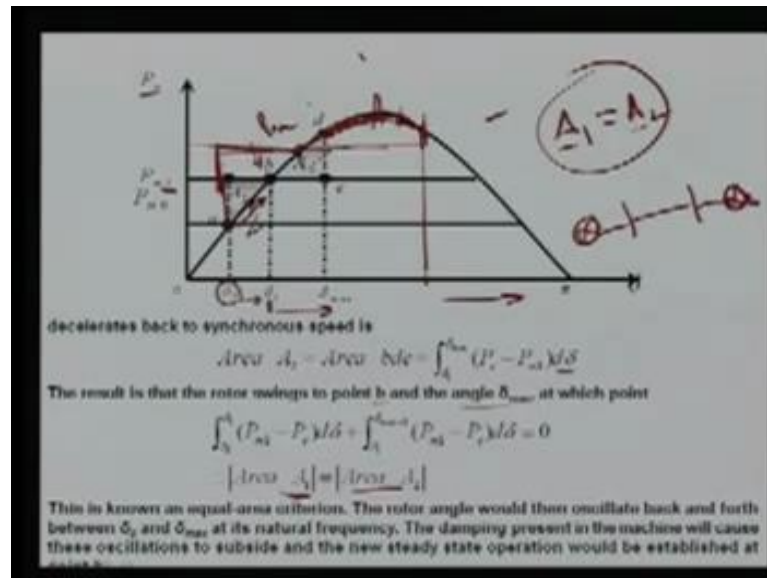
Here it was synchronous speed it started accelerating, due to this mechanical power is more, and here it is, it saw that P_m is P_e , then it will be retarding, but the δ will be keep on increasing and it will be increasing up to d, and again it will be, at this point it will see that your P_e is more than P_m one. So, it will again retard it back. So, this is basically it will be going back and the system damping will try to stabilize at certain point b. So, the deceleration here back to the at the synchronous here area A two; that is the area BDE will be nothing, but the integration from the δ_1 to δ_{max} . Here at this δ_{max} what is happening, the machine is not accelerating, machine speed has. It is rotating at the synchronous speed, but their electrical power is more. So, it is integration between this and this. So, it is P_m minus P_m one. It is this curve minus this curve, if you are integrating from here to here. This δ_1 to δ_{max} ; that is P_e minus P_m one. If you are integrating with δ then you can see the total.

The resultant is the total swing is to point b; the angle δ_{max} at which the point, you can add these two areas; that is δ_2 to δ_1 ; that is P_m 1 minus P_e δ , the pervious area plus here from this area, if you are, means from the previous equation here basically. From this equation if you can write this equation in here. Means, it 0 naught to 0 1 P_m 1 minus e. Here again δ_1 to δ_{max} , here the same equation, what is happening. now you can see this is nothing, but your minus area one. So, I can write this is nothing, but area A 1 minus area 2 is equal to 0, this is the area which we have taken. So, now what is happening from here we can say, the area 1 is equal to the area two, and this is the case of the stability, if the system is stable that must be satisfied. So, if the area here that is accelerating energy, kinetic energy, here and is equal to area 2 then it is known as system is stable, if area 1 is equal to area two, and this is known as equal area criteria. The rotor angle would done oscillate back and forth between angle here.

So, basically your rotor will be oscillating between these two points. It is not going beyond that range it is here again less than this, and its natural frequency. The damping frequency in the machine will cause these oscillations to subside, and the new steady state oscillations would be established at the point b after certain time. So, the machine is here basically if you see it is oscillating your delta here, from here it is oscillating and finally, it is new delta 1 it will be stabilized. So, if you are plotting with respect to time what will happen. Here this is let us suppose your time, this is delta. Means you are machine here operating initially at delta not, when the disturbance has taken up. Means just you have to increase of the sudden input of the generating system, what will happen, here it will be oscillating, and the finally, it will be steady state at the lambda naught delta naught. So, this is basically, and this damping is system damping, because there is some damping in the system, and it will be stabilized and the system will be stable. But there is possibility if there is again excessive change.

This point d has gone beyond this then machine will not come back, and then it is set the machine is system is unstable. So, in this lecture we saw that transient stability analysis, using the classical approach we made certain assumptions, but it is very advantageous if you can form the single machine infinite bus system, and then you can see the behavior of that machine with reference to certain fault in the system. It is quickly giving the idea with your system is stable or not without integrating the system. Means without solving the differential equation here, with the help of the two areas we can know, whether system is stable or not.

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In this case we saw that area one, here is equal to your area 2 for the stable system, but if this area is more than your area two, the system is again this area kinetic, this area basically always gives information that whether you are at the verge of the stability, or whether your system is very strong in the sense of the stability. So, this equal area criteria is used to know that whether how much you can load your system; for example, in this case you can see your system is operating here. Now, it is always advantageous to know that how much you can load in the system. Means in this case it is a Pm one, there is a possibility you can go up to Pm two. Here it is you Pm 2 this point, and then there is a sudden change here your what happen, your system may go somewhere here, and then you are somewhere angle here. So, what now what will happen now this area is the different area, then what you have area here. So, with the help of we can know what is the maximum loading, that suddenly we can put on the generator, without losing the stability of the system.

And this classical approach as I said it is limited to the single machine infinite bus, or the two machines here, this is another machine, and this is another machine then we can analyze, and then we can know using your equal area; means your area one; that is your energy storage here; that is accelerating. So, energy that you are giving that is increase, that energy in terms of accelerating and here is a decelerating. So, in this lecture we have seen, that only just I have taken only one case, when there is a sudden change in the input to the power. In the next lecture, we will see that how much we can load, and at the

same time we can also see if there will be any fault in that system here. There may be some fault here at this line at the end, and if it is tripped out, then your whole your reactance the topology of the system will change.

There may possibility to do this fault this circuit breaker is tripped, and then we will see what will happen during the fault, what will happen the after the fault, and what will be the system stability criteria. We can also determine what will be the critical clearing angle. Means at what time you will clear your fault; otherwise you system will unstable, and then we will see with the help of equal area criteria, and then you can determine your critical clearing time; normally it is called CCT. So, to know their stability of the system; one approach here is the CCT, means what is your critical clearing time. This also gives the stability majors, at the same time some another approached is used; that is called energy module functions, how energy margin you have, and that is also used to access system stability in event of the transient faults; transient disturbances, or I can say large disturbances and for that you have to analyze your transient stability analysis.

Thank you.