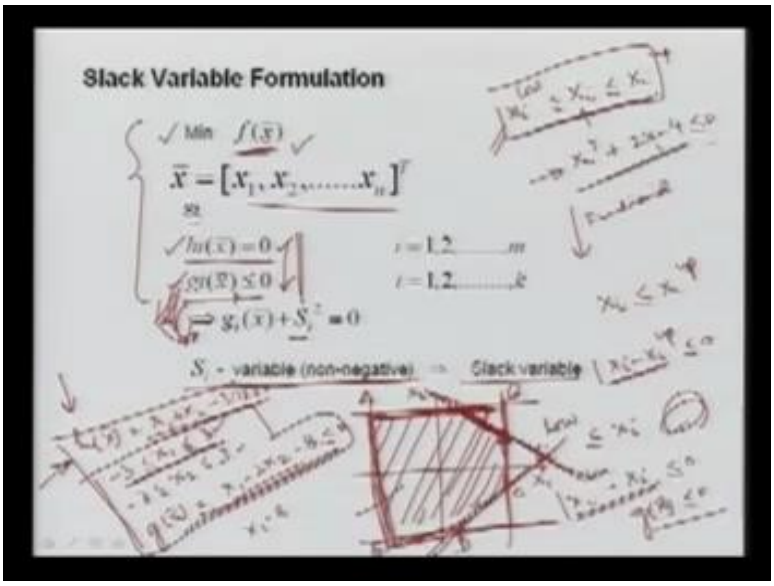
Power System Operations and Control
Prof. S. N. Singh
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Module – 4
Lecture – 4

Welcome to lecture number four of module four. In previous lecture, that is lecture number three, we discussed the some of the non-linear optimization program methods and I discussed to certain techniques. We can go for some more non-linear optimization technique, then I will also discuss some of the linear programming application optimization techniques. So, those are very widely used in power system operation and control analysis purpose.

(Refer Slide Time: 00:50)



So, in the critical conditions, again here just our objective is to minimize or optimize the function f x x bar here again shown, it is a vector of the variables that is a n any numbers subjected to here, the various constraints. One is your h i that is called equality constraint and here g i is inequality constraints. Again, the constraints here I did not defined that in the previous lecture. So, the constraints we can say whether the constraints are the functional constraints or parametric constraints.

For example, if you are putting some constraints on x variable here, x I itself here, then it is you can have some limit, you can say Si upper limit and here I can say Si lower limit.

So, this is basically x here is the parameter and the parametric, so this constraints is known as parametric constraints, whereas if you are suppose writing some constraints like your X i square plus 2 x minus 4 here, i again less than 0. So, this is your function of x, and then it is called your functional constraints, here this is inequality, so it is called functional inequality constraint. So, we can have the two types of constraints, one is your functional constraints and other is your parametric constraints.

In the functional constraints, again we can go for whether it is equality or inequality. So, this hi is nothing but your functional equality constraints, equality means here equal to 0 and g i is your inequality constraint. That includes both your functional as well as your parametric inequalities. One thing also I like to remind you because this x here, if you are writing in this form, so this is upper and lower bound of your state variable or that is a your parameter that x i that we are going to choose. Optimum value of x in a such a fashion that we can get the minimum value or you can say optimum value of function f x.

So, these constraints can be changed into the two inequality constraints, for example, if you are writing here x i is less than or equal to your x i upper, what we can do? We can go for this one, I can say x i minus x i upper is less than equal to 0 and this just like your g x and that function. Similarly, if you are writing for another side here, I can say S i lower is your less than or equal to your x i and I can simply write here, we can send this here. So, it is I can say x your lower minus your x i is less than 0 and this is your another function. So, these functional constraints that is an upper and lower bounds if you are having. So, that can be broken into the two constraints, two inequality constraints, here one is this, another is this, and then we can solve.

So, this includes all the constraints including your functional as well as the parametric constraints and that can be retained in this function, that is your I can say g x is less than 0 in this function. So, here we can have this basically shows the standard expression for any optimal programming technique. Now, again sometimes some people say which constraints are again I discussed binding constraints or non-binding constraints, how we can go for this?

For example, you can see if you are having here let us suppose just I want to say, so simple example on the binding if I will say your function here this constraints h x, here that is the vector of the two variables. If I can write here x 1 plus x 2 here minus 3.75 is equal to 0 and same time I am saying this x 1 here is the inequality constraints bounded by minus 3 here plus 3 and your x 2 here, I can say this is also minus 3 to 3. Another, we have an inequality constraint g x that I can say x 1 minus 2 x 2 minus 8 here is less than 0. So, if you draw here, the region because we have to search the minimum in that region these are the constraints.

So, in that constraints, we have to such resolution that the minimum that we are going to get. So, for example, here for this case, if you draw here let us suppose we are having the variable x 1 this side, x 2 this side, so what will happen? Now, the value of x 2 is 3, we will be having this zone, x 2 is minus 3, here that to this zone, X 1 passing through and X 2 passed through here, so for this what we are getting? We are getting for this and this we are getting this inequality constraints and surrounded by this region. Sometimes, we are also having other constraints; here I can say this is inequality constraints. That is your x 1 minus x 2 minus 8 is less than 0.

So, we can again draw another function here that constraints, we can go this slope is negative on the h 2. So, we can say first here this is a constraints I can say here one here that is linear equation here. Similarly, we will get another linear equation corresponding to this, let us suppose your x 2 is 0, then we are getting for equality first take as a 0 here. So, 0 means x is equal x 1 is equal to 8 means if x 2 is 0, we are getting some value here. If your are getting x 1 is 0, then x 2 value is your minus 4, so you are getting here. So, we are getting some sort of constraints here, so we are here getting some constraints like this, so what is happening?

These constraints inequalities constraints are showing the regions mean we are getting this, then this, then this, then this. So, this is basically your region of the inequality constraints. So, your solution will lie in this one if you are not having these constraints that are equality constraints, but if you are putting the equality constraints, means suppose you are equality constraints are written here H this is a constraints. So, your solution will lie here in this line itself because equality constraints are always binding that must satisfy, here I said this may be not equal to 0, it may be less than that.

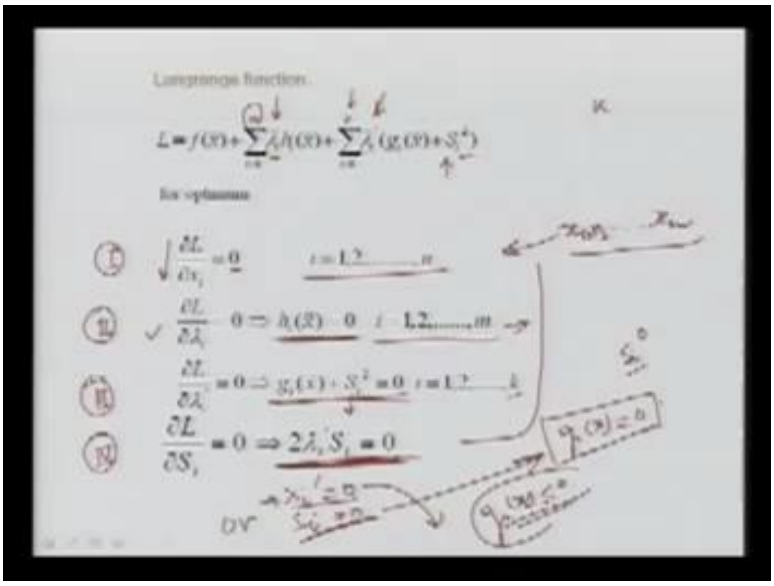
So, this is the regions, so we are getting some regions for the inequalities and equality and the solution here must be on this line. So, from the inequalities, we had this region I can say a b, then we move to c we move to d we move to e and again a. So, this was the region enclosed by three inequalities constraints, so this was known as the feasible region this region. Now, putting these constraints it is the binding constraints, so the solution will be here on this line. Again if your other constraints are binding, let us suppose your this constraints are binding x 1 is 3. So, solution optimal will be here only because this is the binding, so the intersection of this will be over the binding and this will be solution.

Sometimes what happens, your solutions may be not binding and it may be somewhere else suppose this is also binding somewhere here. So, we can say this solution is in feasible, means it is not inside, it is not on this line solution is somewhere else, and your function is going somewhere else. So, that is called in feasible solution, so coming to the your again this extended problem. So, this is the extended notation of any optimization problem that is the we can optimize function f f subjected to the constraints, and again the constraints can be a equality and inequality constraints.

So, in this condition, we saw we add the quality constraints as well as inequality constraints of objective function, here, what we are going to do? In the slack variable, formulation, we convert first the inequality constraints to inequality constraints by adding some slack variables, what happens? Since this function is less than 0, we are trying to add some functions so that we can make a change, so what happens? Here, g i x bar here plus S i a square we are adding equal to 0, why S square because we want that S to be positive in all the case. If it is not S square in some case it may be negative.

So, to avoid this confusion that is we add some Si is non-negative and this is known as a slack variable and it is always positive, so now what happens? This is now changed to this one, and then we can have our extended equation can be again reformulated.

(Refer Slide Time: 10:17)



Then, we can have the multiplier for the equality here this lambda I for all the edges means x may be M in number. So, it is addition of r lambda 1 h 1 plus lambda 2 x 2 and so on so forth. Similarly, here if you are having k inequality constraints, so we are having the Lagrange multiplier lambda I prime. Here, g i x plus x I, so number of inequality constraints again we are going to add that number of flag variables S i. So, the number of flag variables, now S i will be K in number that is the KEL inequality constraints. So, in that flag variable this approach this slack variable formation again our this condition for optima first condition that our this argumented objective function or Lagrange's function l must be derivate with its state variable.

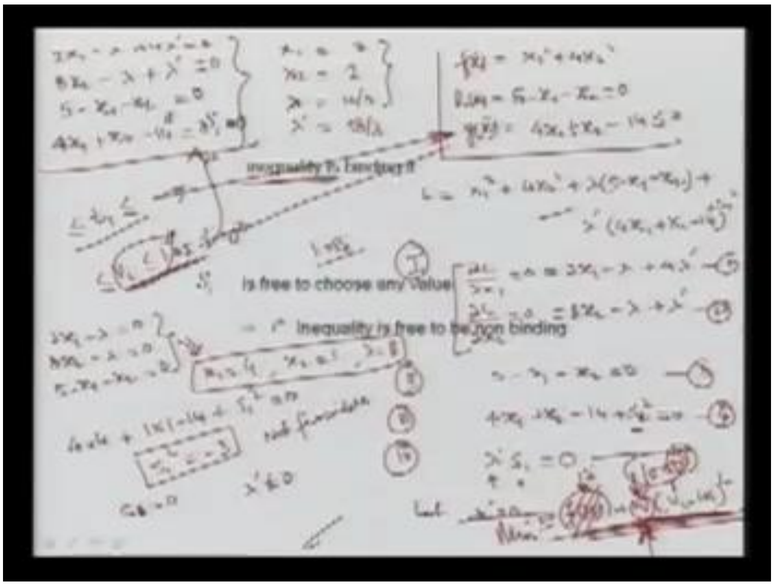
We can say parameter that is x i, and then we have to put 0, so this is true for all the x i, it will be n in number. Already, we are considered that x is a variable of n variables means it is your x 1 to your x n means x comma x 2 so on so forth. So, we have the variables the function is of n variable. Another here the condition here is similar that we can derive here this is nothing but your equality constraints, means we have to derivative this function with Lagrange multiplier of equality constraints. So, here we are getting nothing, but, we are getting the same expression of equality constraints and since we are having n in number equality constraints, so this equation will be m in number.

Another that we are going to third condition here that is we are going to derivate this function with respective to this Lagrange multiplier corresponding to the inequality constraints that is made to equality by adding the slack variable. So, we are just difference in this means we are getting this function again and it is in number here. So, we are getting k equation, another here we are going to derivate that is the fourth condition. In this case is that we are going to derivate this with respect to S i, and then we are getting what happen here the 2 S i lambda I that will be equal to 0 and we are getting this equation.

So, now we are getting these equations and based on that we can derive and we can solve and we can get again the optimal solution, now here if Si is 0, what happens? Now, you can see if this S i is 0 means your g i x is equal to and once it is 0, it becomes equality constraints without adding the slack variable means it is the binding constraints. So, we have to say this equal inequality constraints are binding if S i equal to 0. Now, there is a possibility from this equation there is two condition one is your lambda i prime is equal to 0 or Si is equal to 0. So, if Si equal to 0, then we are getting this constraints and we can say our inequality constraints is binding again the binding, which I explained in the previous one.

Binding means equation will lie on; that means, that much satisfy in all the case otherwise it will a region. Now, if your lambda I prime is 0 means S i is not 0. So, we can say g i x is less than 0 means here this constraint is not binding or we can say this i th inequality constraints is free to be non binding. To see this, let us see another example how we are going to solve and how we are going to get this, so what normally we do? Normally, taking one condition here lambda equal to 0 and then we can solve all these. Then we can check whether it is satisfying or not if it is valid then we have go for another one, and then we can again solve it.

(Refer Slide Time 14:46)



To understand this, let us see here again I already explain here, let us see this one example again that which we discussed in the previous lecture that here our objective function is your x 1 is square plus your 4 x 2 is square. If you remember, it was a f x that is to be minimized and subjected to our constraints equality here x 1 minus your x 2 is equal to 0, this is nothing but your s x. We had the g x function is remember this was nothing but your 4 x 1 plus x 2 minus 14 less than equal to 0. So, this is your extended formulation that we solved it using this condition. Similarly, here I am going to solve this problem using the slack variable formulation approach.

Using the first criteria that we have to write the complete this l matrix that is segmented objective function here l that I can say x 1 is square plus 4 x 2 is square plus here we are having only one. Then, we are going to lambda here 5 minus x 1 minus x 2 plus we are having only one here. So, we will add 1 and that is your lambda prime I can say 4 x 1 plus x 2 minus 14 here. Now, we have to apply first and first one is your del l upon del x 1 is equal to 0 and del l upon del s 2 is equal to 0. If we derivative this, what we are going to get here it is a 2 x 1, this is 0, here it is a minus lambda here plus 4 lambda prime.

For this we are going to here 8 x 2 minus lambda and here plus lambda prime. Now, this is your condition number first, which I wrote in the just if you remember here this one is your first condition.

Now, second we it is nothing but your equality constraint and we are just going to be, sorry here since this edition we have to make here that with your s 1 s square we are going to add because we are trying to make its equality constraint. This is less than that we are making s square added her and that differentiation with x 1 x 2, it will be 0 so that equation will remain the same. Now, second here the condition here just we had to write 5 minus x 1 minus x 2 is equal to 0. So, we are having equation 1, equation 2, equation 3 and now we are going to derivate that is the third condition. Here, we are going to derivate with the lambda prime and this is nothing but we are going to get here 4 x 1 plus x 2 minus 14 plus your xi x 1 here s square equal to 0 that is your equation 4.

The fourth equation that we are going to derivate with the S i means we are going to get lambda prime into S 1 is equal to 0, now how to solve this? To solve this, we have to again go for taking one value here 0 that we have to take whether this lambda is 0 or not or we can take the first take lambda equal to lambda prime equal to 0, means here you can say either this is 0 or this is 0, then only we can get this expression that is 5. So, let your lambda prime is 0 if your lambda prime is 0, now we can solve this means we have four equations now put this value.

So, we are getting two x 1 minus lambda is equal to 0, we are getting 8 x to minus lambda equal to 0 and for this we are getting as it is five minus x 1 minus x 2 is equal to 0. So, from here we are having three equations and we can solve it and that after solving this if we solve here we are going to get the value here x 1 is equal to means I will get x 1 is equal to 4 x 2 is equal to 1 and lambda is equal to your this, if you solve this, you will get this value. Now, put this value in your equation number 4, so putting this we are getting 4 into 4 plus 1 into 1 minus 14 plus s 1 square is equal to 0, what we are getting?

Now, we are going to get this s 1 is square is equal to here 16 plus 17 means 3 means we are getting minus 3 and then it is not feasible because here S as I said S is always positive, we cannot get any imaginary term. So, here this s 1 this is not feasible you can say not feasible. It means here lambda dash is equal to not 0 means we are going to have this S i is equal to 0. So, if you are going to put this Si equal to 0 S 1 here we have taken S 1 because only one constraint and then we had to again solve here, so what we are going to get? If this is not 0 means this is 0 means lambda not is not 0.

So, we had the equation first, now we can write is 2 x 1 minus lambda plus 4 lambda prime equal to 0. Here, again your 8 x 2 minus lambda plus your lambda prime is equal to 0, your equality constraint 5 minus x 1 minus x 2 equal to 0. Now, here this S 1 is 0 in this case, so we are going to get four x 2 minus plus x 3 minus 14 equal to 0. So, now we have the four variables, variables are x 1, x 2 lambda and lambda prime. We are having four equations four unknown and then we can solve uniquely and if you will solve you will get this x 1 equal to your 3 x 2 equal to 2 lambda here is 10 by 3 and your this slack S prime here will be your 58 by 3.

So, this shows that this result is similar to your condition means you are getting same optimal value here and there is no change. So, now this is same as the condition and you can say the formulae is slightly different. So, both are equally applicable for the solving the non-linear optimization problem, I mean both equality and inequality constraints. Now, let us see that again sometimes if we called hard inequality constraints and sometimes it is called soft inequality constraints. This inequality constraints here in the power systems we are having the two type inequality constraints like hard inequality constraints which cannot be violated and that is like transformer types.

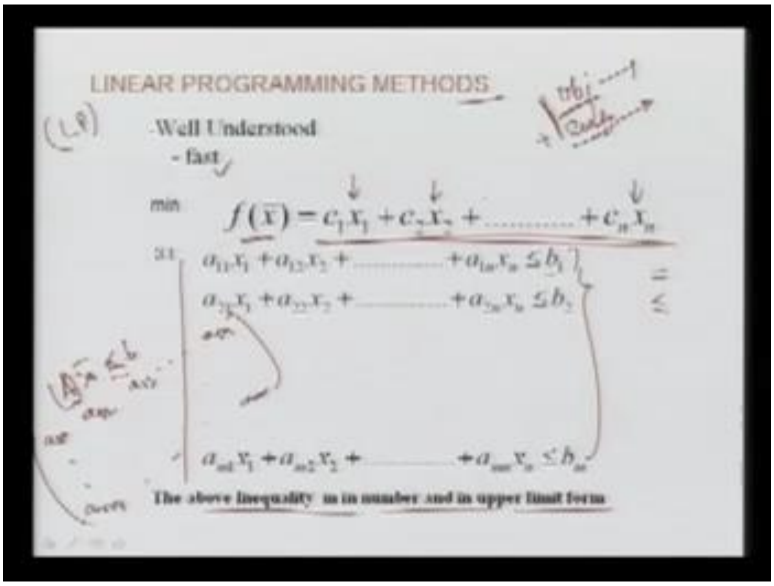
We can change, we can exceed otherwise transform will burn, and we also having some soft inequalities, if there is slack variations in the inequality that is not harmful and small violation is allow. For example, if I am putting the typing of the transformer one should be less than this typing value. So, it is called hard inequality must satisfy for example, voltage if I can say voltage at any, it is less than 1.05 per unit if it is voltage is 1.05 hardly matters. So, this is called your soft inequality and this is called your hard inequality. So, the fast handling the soft inequality, normally we use the penalty function methods means, and again what is that penalty function methods.

The penalty function methods are slightly here we in objective function, here in f x we add that a penalty function we can say W, and then we just for this equation what we do we write this we can go for V I minus 1.05 term here. So, what happens this is the penalty factor based on that is slack variation then objective will be slightly different. So, this is handled, this inequality in this fashion. So, instead of going foe the g x in this fashion here or we can say g x we used here, we can go for the penalty function for this one for the soft inequality.

For the hard inequality, we have to consider the g x and we have to be meet we have to meet that one very strictly and that is why it is called hard inequality. So, in power system both are allowed, so if you are going for the soft, then you can adding the objective function with the some penalty function W. Again, this weight factor is very important if you are using very high value of weight factor what will happen? You are going to minimize this, then your optimization problem will shift from here to here, it will try to be this one first, and then it is this one. So, if you are using very low value of this one, then what will happen? If your low value of this one causes violation of these constraints.

So, normally some optimal some value between the higher and lower side value this weight factors are decided and these are added because again you have to see what is the value of this, what is the unit of this? Suppose, you are adding something thousands here in five, so always it will try to minimize thousand rather than five because it has no meaning, it will ignore this optimization this completely. So, this weight factor or penalty factor is also very important and this factor value is basically chosen very accurately.

(Refer Slide Time 25:44)



Now, come to other programming methods that is your linear programming approach for solving some optimization problem.

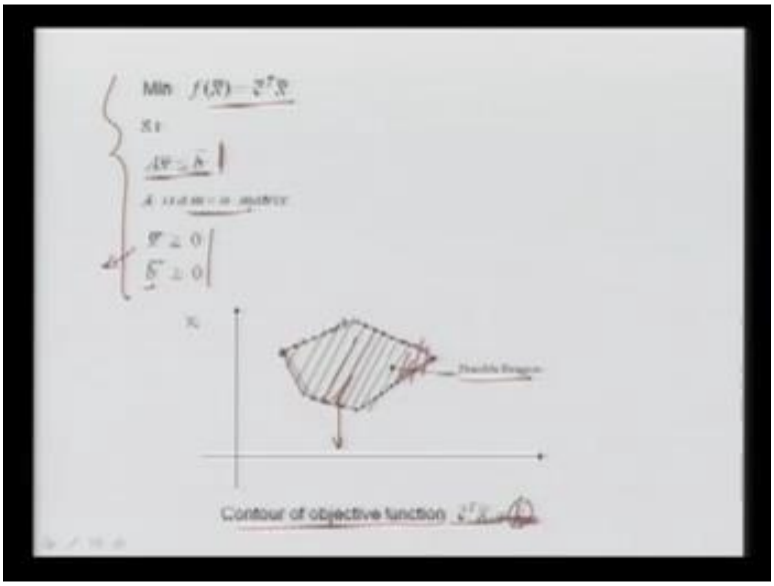
As I said for the optimization program, this method is very fast compared to your non-linear programming approach because non-linear programming approach every time just you are solving some set of differential equations. You are sometimes solving the inverse of that because solving the whole set of equations every time you are investing in each iteration. So, that requires much reduced time, so the linear programming approach are very fast, but, sometimes converges problem you get problem some problematic give some solution. Also, in the actual sense in our power system, we do not have the problems of linear in nature.

Our power system is a non-linear system, then how you can assume that we can have the linear system. If you are having the linear, your means linear programming methods can be only applied if your objective and your constraints both are here linear. So, you have to have your objective linear and your constraints must also be linear if any one of this in it is non-linear, and then you have to use your non-linear approach. Now, since we are not having the linear constraints very few, mostly we are having functional constraints their non-linear highly non-linear you can say objective functions are also almost non-linear, then, what we do? We normally linearise the problem at the operating points.

Then, we move and solve the LP programming, so it is called LP methods they are very popular. So, the LP we have to linearise first then you apply this linear programming methods to optimize your problem and then you can move ahead. Again to understand this, let us slightly go into the linear programming formulations, and then we will see some methods to solve the optimization problem. Here, in this case here your objective function as I said, it will be linear, means it is a linear combination of your state variable that is your x 1 to x 2 here again this x. So, we are having some coefficients some cross functions associated with all this your state variables that is a x 1 to x n and that we are adding here.

So, this your objective function subjected to your sets of equality and non your inequality constraints, what here? I am doing here if it is equal to 0 or b 1 b 1 to b n value may be 0, so then it is your equality constraints. So, here it can be equal or it can be less than this, we can write in the general formulation in this fashion. Your problem may be combined of this, so above inequalities m in number and in upper form here this is upper form.

(Refer Slide Time 28:49)

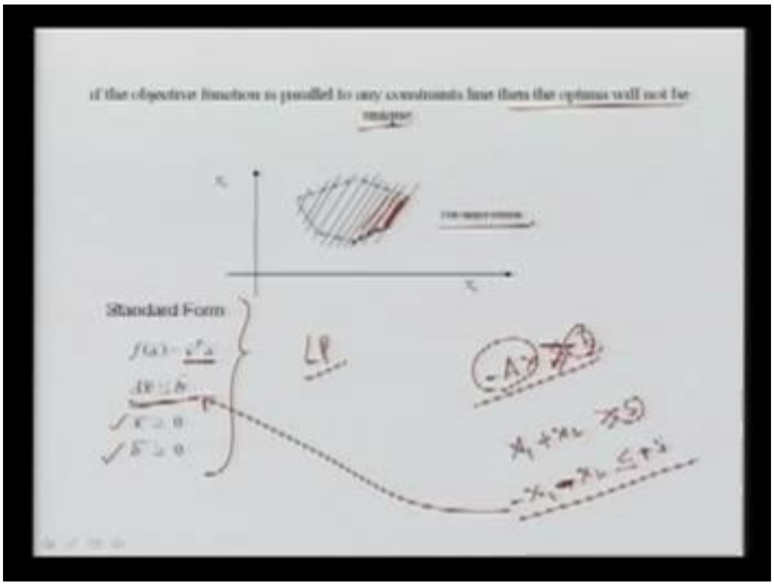


So, what do? We can write this minimization function here that is c t the transpose of the cross function with the state variable subjected to a x. Again, we have formulated this here this complete matrix, we can write here a x less than or equal to b and a is a matrix that is a coefficient a 1 1 a 1 2 means we are getting this value is your a 1 1 a 1 2 a 1 3 to your a 1 n and here again a n 1 to here a nn. So, we are having here m we have taken, so m n we are taking this and this we can we have that you can see here. So, a is your m cross n matrix, in this case also all the x here, they are greater than 0 and at the same time the b we also assuming b is greater than 0.

So, this is your standard formulation of this, now all these inequalities here they are nothing but they are the giving some regions, and based on that we can have the solution should lie in this feasible region that is called your feasible region. So, this is nothing but this is a contour of objective function that is equal to one in this region. This is your this parallel line showing your contour of objective functions that is for k and for different value, what will be this value, when it will be forgiven value k? We are having always lines parallel, now if your objective function is parallel to any of the constraints line, then optimal will be not unique means for example, here your this constraint is parallel to this what happens then you are you will not have the unique solution.

However, in the previous case here you can see if it is not parallel, so what we are getting? We are getting for any value here, this value is solution of the optimal solution or this is your optimal solution, again depending upon whether you are minimizing or maximizing the function.

(Refer Slide Time 31:00)

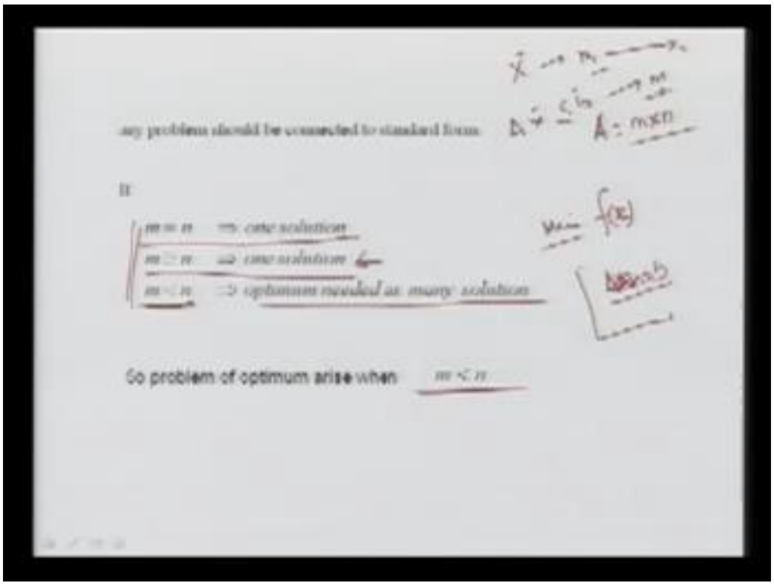


So, this here if it is a any of your objective function here contour these are the objective function if it is parallel to the any of constraints here this is was constraints are here. So, it was parallel, so it is called it will not have the unique optimal. So, in standard normally the LP programming techniques I approach we write here f x is the cast function multiplied by state variable subjected to x is less than or equal to b and x will be always positive and b is also positive will represent in terms of this. No doubt here let us suppose you are having x is your b and if it is a b is negative, what we can do? We can multiply by this negative, we can multiply by this and we can change in this session.

So, this value will become the positive, and then we can write again here means we can convert this inequality from this side to that side and this is always true. So, it is very well true that any here the function that is for example, I will show you. Let us suppose your x 1 plus x 2 here that is your less than I can say greater than 5, what I can do? This is greater than 5, so this can be converted here, this is your x 1 plus your x 2, again I am going to for minus here and then it is this minus let us this minus I, so this minus I want to convert into positive.

So, I have multiplied by the minus, and then it will be positive, but this sign will also reversed if you are multiplying by minus. So, this is your function and again you can say you are having this term, some case it is not possible, then you have to add some flag variables and other things just we will discuss later on.

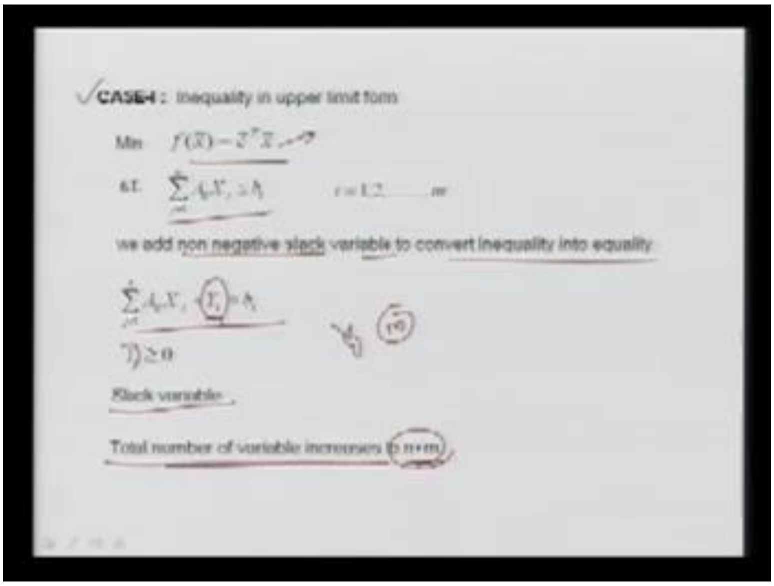
(Refer Slide Time 32:54)



So, if you are this any problem should be connected means converted to any extended form now we are having the different possibility. You see your a matrix was your m cross n matrix means you are having the x factor is your x 1 2 x n you are this a x here that is less than b here this equation was your m in number. Now, if m is equal to n then we will have one unique solution, if you are having m greater than n, then you will also have one solution, but if you are having m less than n, in this case we normally go for optimization and some will be of course will be in this case.

If m is more than n, some will be dependent on another equation and then we can have the rank of n. So, if m is less than n then optimum is needed because we are going to have many solutions. In this case, if m is less than n, so there is a possibility that we have a infinite number of solutions and then we can have only further optimization. If we are having equal to this, then your function here that we are going to minimize here, it will be f x and we have to only solve here a x that is a x is equal to b, and then you can solve it. So, the problem arise only when m is less than n, so, we can again let us see the various methods.

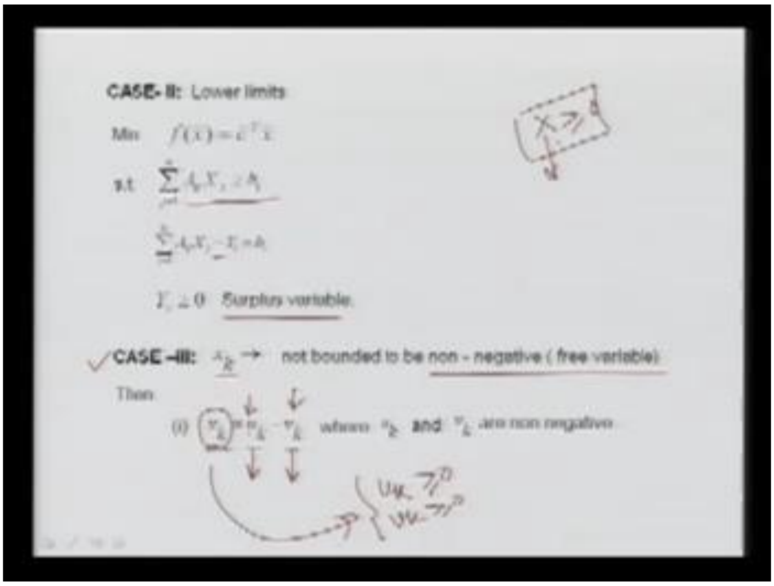
(Refer Slide Time 34:29)



Let in the case one the inequality in the upper form as I said here this we are having the inequality and the objective function of this. So, we can add a non negative flag variable to convert the inequality constraints to equality. We want to convert in terms of here equality constraint we want to solve. So, the extended form which I showed here you that here normally we try to go for equality constraint here. In that case we see what this number of equations here is and number of variable. I will come to that point again how it will be converted then it will be clearer. Now, this is then we can add here negative flag variable here, and we can make this y i is added.

This value is positive always, and then we can have this equality constraint, and this y I is called slack variable, and now we are having total number of variables are now increase to n plus m. It means we have all this n equation here n variables we are adding. So, and m here variables we are adding this y y is m in number, so the total variables that is now it is going to be x and y it is your n plus m variables.

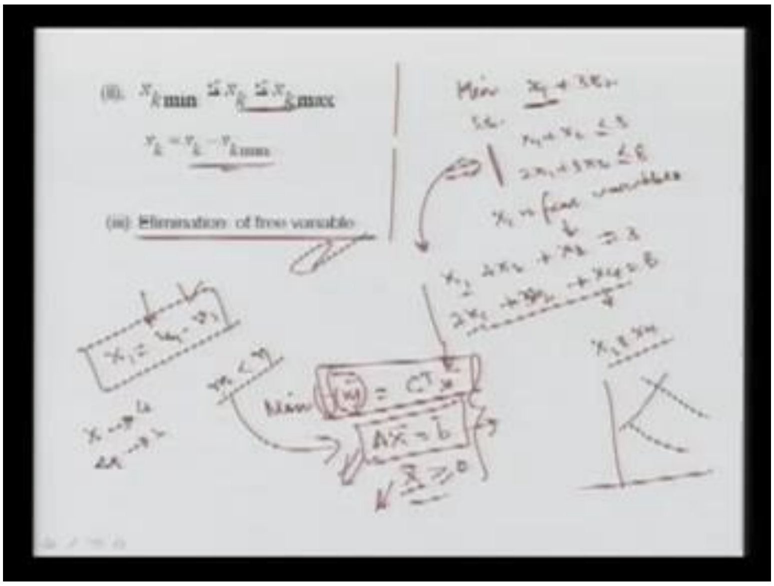
(Refer Slide Time: 35:51)



If it is in lower form as I said if it is in the fashion, what will you do? Because we are taking b i is positive you must remember, so here what we can do? We can subtract this because this is a surplus and then it is y i is called surplus variable. In the case three, what we can do? If this is your x k not bounded to be non negative or free variable means if it is not bounded. It means x k may can be a negative or positive as I said x should be greater than or equal to 0, but I can say it is a non binding x can be any value there is no binding here. Here, we are putting the binding some constraints we are putting if it is a free variable, then this x k can be changed as another variable for any k th variable u minus v and these 2 will be your non negative value.

If it is more than this it will be negative if this is u is more than v, then it will be positive. So, it can be free variable and then what we are going to do we are just creating this free variable into this two variable. Now, here I can say your u k must be greater than 0 and your v k also is greater than 0. So, one free variable now we are just going to have the two variables means again we are increasing the number of variables accordingly.

(Refer Slide Time 37:18)



So, in standard form here as I said this is your equality constraint, now we can change in this fashion because we can this equation here, we can this x k minus x minimum we can go this side and then we can convert and you can get it. So, elimination of the free variables there is several variables extra, so we can eliminate and then we can use some optimization program. So, now let us see some examples for this few variables, let us suppose we have a function that is a minimization of your x 1 plus your 3 x 2 and subjected to the constraints that is here x 1 plus your x 2 is less than or equal to 3 and your 2 x 1 plus your 3 3 x 2 is your less than 8.

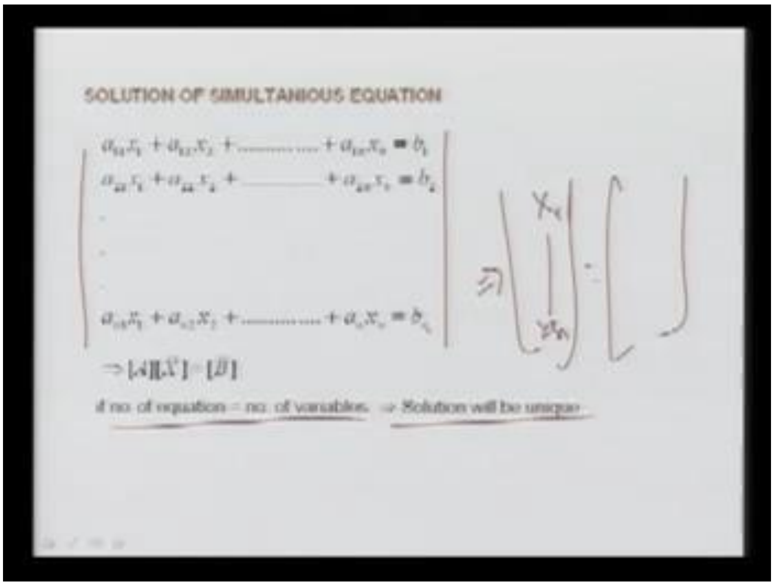
Now, it is said here x 1 is free variable is x 1 is free variable free variable, so what we have to do? We have to convert the whole problem into that we can have the total number of variables we can exceed. So, in this normally we add some more constraints means I can say this here we have to go for in that fashion means we have to add first slack variables, what I am going to do? Here, this x 1 plus your x 2 plus we are adding another variable x 3 that is equal to 3 means we have to convert this inequality to equality constraints by adding some extra variables. Now, similarly, here your 2 x 1 plus your x 3 x 2 plus your x 4 is equal to your 8.

So, now here we have added two more variables as I said that can be converted, so here x 3 and x 4 are the slack variables and now the total number of variables now increased to 4. To see that we are having some free variables free variables that I am going to say that your let us suppose x 1 is your free variable. So, x 1 here I can say your u 1 minus v 1, so in that we can replace this one here x 1 x 2 and then we can solve, so now what we can do? You can say again in terms of different fashion we can replace. So, here we can put the x one here we can put x 1 here and now our whole problem can be formulated that your f x is your c transpose x prime. Then, here a x plus equal to b and here this x variables are greater than 0, so this your standard form.

Then, we can solve without any problem, now, some times when we can eliminate that variable that we should be free there is no constraints here we are putting x. So, here we have to eliminate x 1 we have to put these values and then we can solve it. So, this is your standard problem and then to this here just see how we can go to solve it because this is the reason this is the equality constraints some equality constraints here different are coming. So, we are going to solve, and we are going to get the values, and after getting that value, we will see which value will be the optimum why I am saying this? Here, you know this your x will be now several variables in this case, how much it is now 4 your equation is only 2.

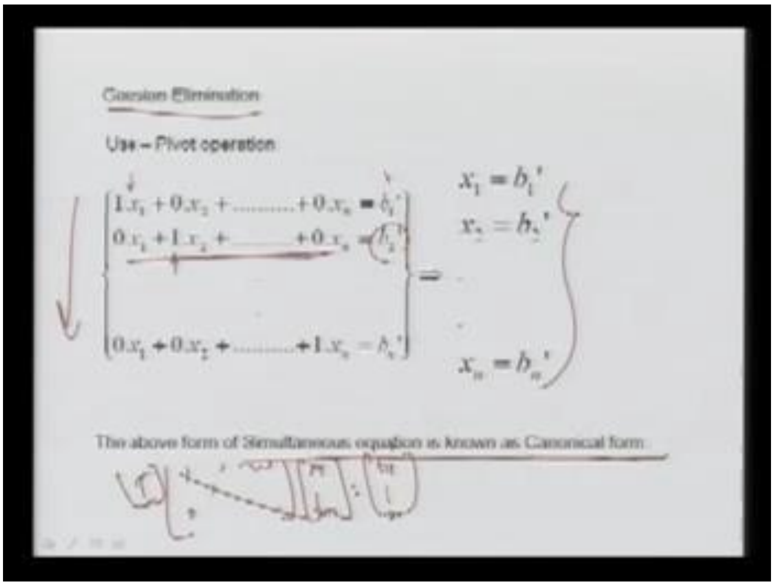
It means I can say here this is a condition, when m is less than n, it shows that we are having the infinite number of solutions. Now, in that we are searching which one is your minimum or maximum value at per this one? So, we have to take the state factors those will the cost function minimum. So, as I said if you are having m is less than n then only it is possible that we can have the optimum solution and then we can say. So, here we are going to see this, so we have to solve first this means we have to get the various solution, and then at we will see which one is minimum. Some methods are there like simplex method, it will take care along with the solution of this so that you can say which are the variables are basic variables which are not basic variable.

(Refer Slide Time 41:50)



So, the solution of the simultaneous equations here now you are having here equations and then let us sees this how much. So, if number of equations is equal to number of variable solution will be unique here. So, number of here n equations n variables, so unique solution will get means your x 1 to your x n will get one solution here.

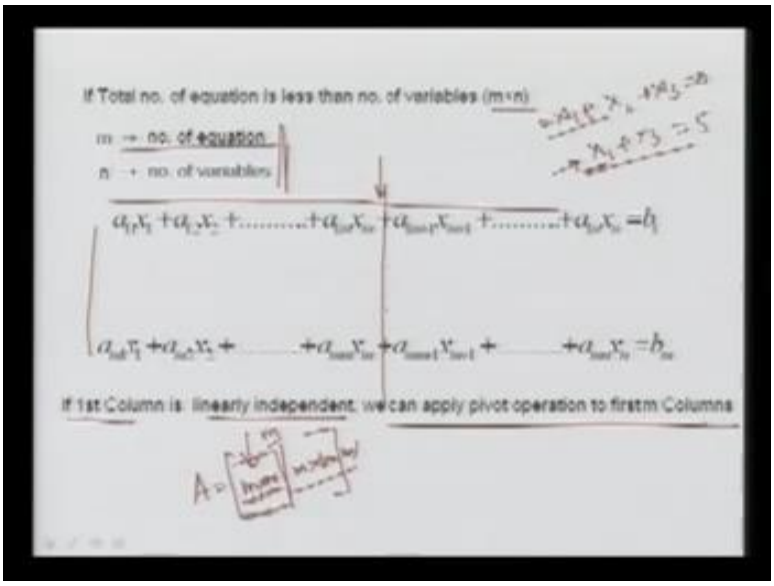
(Refer Slide Time 42:14)



We can use the various methods and normally the methods are called the Gaussian elimination method. That basically use the pivotal method normally what we do we put in the canonical form means we can have this here unity and then we can go for the simplifying the equation. We can get the one that x 1 here and other coefficients will be 0, then we will go for x 2 another will be 0. It means we have to do some sort of multiplication subtraction addition so that we can get this type of means we have to see only this side automatically this value will be coming accordingly.

So, we can replace that standard form here this equation here in terms of here, and then you can see from here what we are getting your x 1 will be your b 1 x 2 will be your b 2 prime and x n will be b n and this is your solution. So, this is called use of pivot operation, and then we can above form of simultaneous equation is known as the canonical form, because here you can see what you are getting unity, others are 0 0. Here, your x 1 to your x n, and then you are getting here b one prime to b n prime, so this is your canonical form means I matrix.

(Refer Slide Time 43:31)

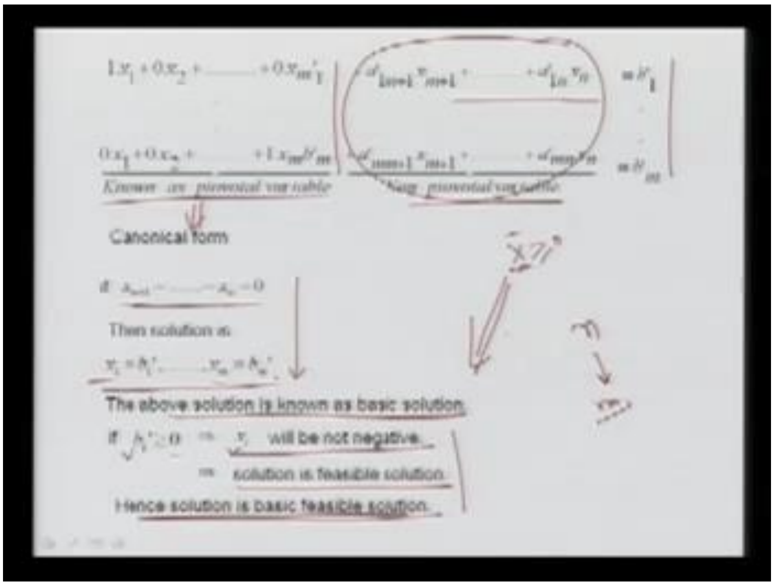


If total number of equations is less than number of variables m in this case, then how will you solve, to solve this again what normally do we have to select the m variables because your number of equations are m. So, we are trying that since your number of equations are m, so we are trying to form this whole matrix this.

Now, your matrix is now can be divided into here m cross m remaining here m cross m minus n is the remaining is coming here. So, we can divide this matrix means here you can say we have the matrix that is a square matrix a 1 1 a 1 2 a 1 m here a m 1 a m 1 two a m m and then we can write this. So, the first column is linearly independent, we can apply the pivot operation to the first m column. For example, here this is 0, then we cannot apply the pivot because here if you are dividing by 0, it is not defined. For example, if you are having your x 2 plus x 3 is equal to 0 you have x 1 plus x 3 is equal to 5.

You have to choose this equation first because here this x 1 is 0 her, so you cannot apply the pivot because the duration of 0 is undefined. So, you have to just change here and there you have to take this x one here and then so on s forth or you can change x 1 variable here and there. So, we can choose, so that is why it is said if a first column is linearly independent, then we can apply the pivot to the first m column because we want that this matrix should have the rank m, otherwise we cannot go for here the canonical form. It means it must have the rank m of this portion, then we can take those coefficient and variables, and then we can go for the pivot operation.

(Refer Slide Time 45:33)



So, if we are going for the pivotal, and then these are known as the pivotal variable you can say we have done the same pivoting operations means x 1 x 2 x 3 and x m. Remaining, just we have change in this and we can say it is a non pivotal variables.

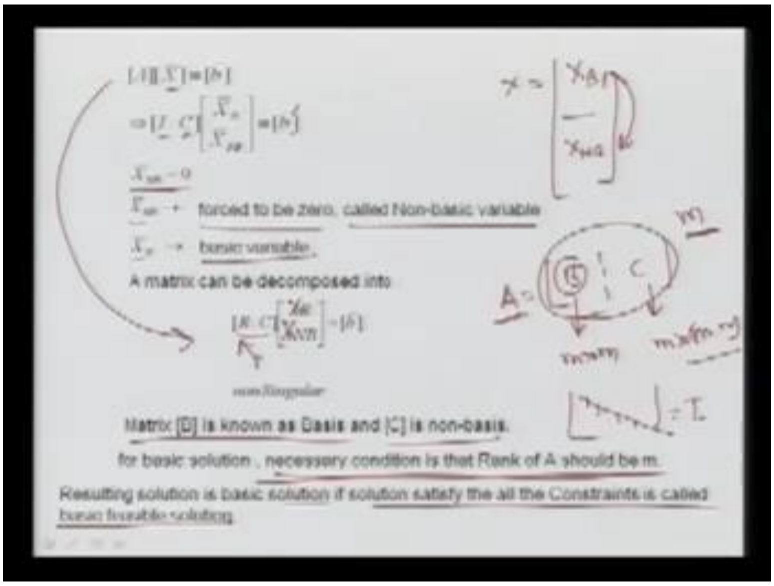
So, it is in your canonical form, now what we can do if you put all these values to 0 means x m to x n 0, then we are getting x 1 is equal to this x 2 equal to this. So, this is one solution means x 1 to x m the values just regards from here, remaining we put 0 because this we are put in 0 because our x is also greater than or equal to 0 means it is a

0 is one variable. So, this is one solution as I said if your number of equations are less than number of variables, you will have infinite solutions number of solution are infinite. So, this is one solution and this solution is known as the basic solution basic solution means we have short listed that the out here n variables.

We have just chosen these m variables so that we can have the pivotal operation we can form in the canonical form as I have shown here. Then, we can remain we can go and another side and then we put the non pivotal variables 0, pivotal variables we can get here and that solution is called as basic solution this one solution. So, if b i is greater than 0, x i will be non negative and that because our condition is x is greater than this vector is greater than 0. Then solution is called feasible solution, but there may be possibility that this is not so then it is called non feasible solution basic solution.

So, the angle solution is basic, then it is called basic solution feasible because we are getting this value this positive means x i is your non negative or positive that is satisfying this. So, this is your feasible solution as I said s many solutions will be there someone will be non feasible and other may be your feasible solution.

(Refer Slide Time 47:45)



So, what we can do? We can go for this is your identity matrix remaining is c converted matrix and we just converted here in this form let be prime. So, we put this x b that is a non basic variables means x is divided means your x here it is nothing but x basic variables, and then others are your x non basic variables. So, non basic variables we put

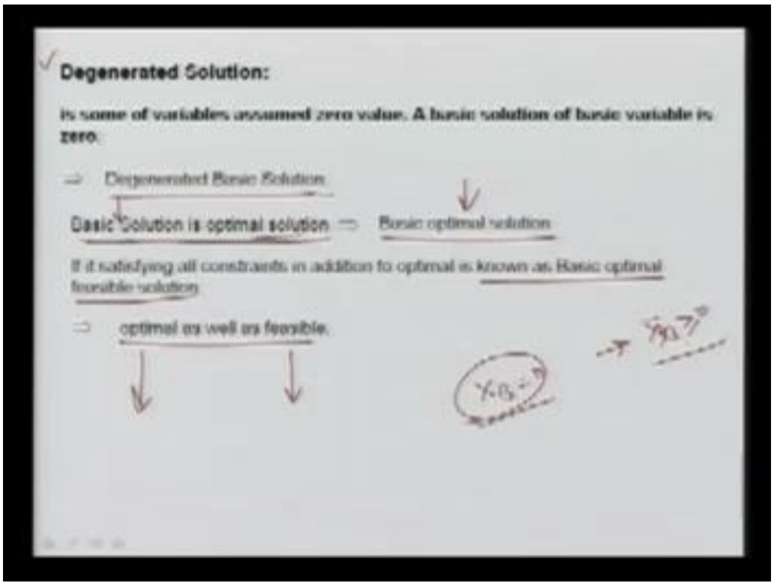
0 basic variables, we get the values from here. So, this x n b here put 0, and we will get the force to 0 and called non-basic variables and then x be remaining that we get the solution from here they are called basic solution.

Even though without doing any iteration, without doing any manipulation in the first equation can be converted into this form. Here it is your x b and x n b, so we can simply this matrix without doing any pivot we can write the b and here another is your c matrix this b is your m cross m and this your matrix here m cross m minus n matrix. This B must be non singular, otherwise you will get not you will not get the basic solution you cannot invert it here you cannot go for the pivotal operation. So, this matrix b is known as the basic and c is called the non basic matrix.

So, the matrix which you are going to choose where you are going to operate the pivotal operation you want to make the canonical or identity matrix here this 1, 1, 1, 1, this is called identity matrix similarly, elements are 0, then it is called your basic. For the basic solution, the necessary condition is the rank of A should be m, means the total here the rank of A should be m in complete. So, here the b may not be m, but we have to search we have to change here and there we have to choose the variable, here accordingly in such a fashion that the B matrix should also have the rank m. So, if the rank of a is less than m, so of course, you will not have the basic solution here.

So, the resulting solution is the basic solution if the solution satisfies all the constraints is called the basic feasible solution. So, all the constraints we have to satisfy then that solution is called your basic solution.

(Refer Slide Time 50:16)

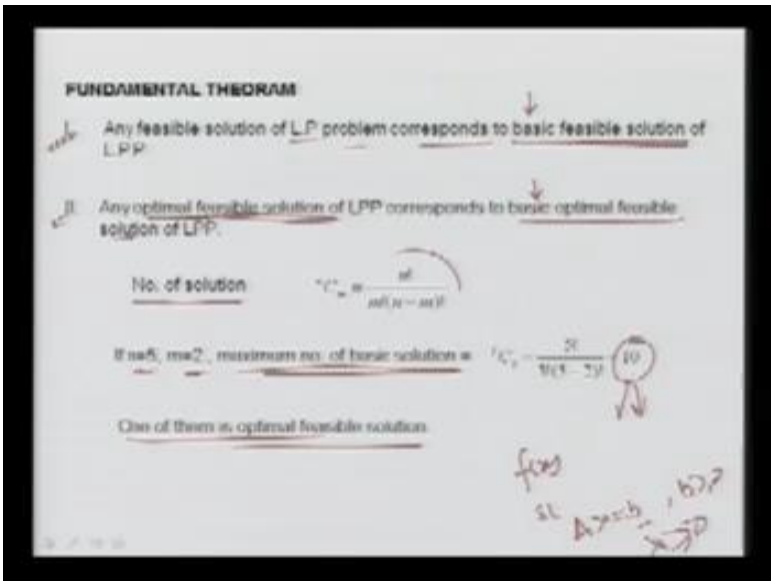


Sometimes it is a degenerative solutions, what is the people called the degenerated solution is the some of the variables. Assume 0 value, the basic solution of the basic variable is 0 means some of the basic variable even the suppose it is coming 0 means here you are just x b is 0, it is valid because your basic solution as I said x b is greater than equal to zero it is satisfying this. So, if it is 0, then it is called degenerated basic solution because if you see here as I said means here we are getting the solution here, this is a x m means it is your basic solution. Here, some of this value may be 0 here we are forced to 0 here we are getting 0.

So, in that case it is called some degenerated solution, and it is called degenerated basic solution. Now, the basic solution is optimal solution in that case this basic solution is the optimal solution means it will give your basic optimal solution. If your solution which you are getting is optimal one, then it is called your basic optimal solution.

If it is satisfying all the constraints in addition to the optimal is known as the basic optimal feasible solution. So, the optimal as well as feasible solution we want for your complete problem. So, for that we have to solve and we have to see some algorithms are there, there are some fundamental theorems are there those are used to see whether the solution is feasible and optimal feasible. It means all combinations we can say feasible non feasible optimal non optimal feasible this non optimal feasible or non feasible, of course there will in optimality there.

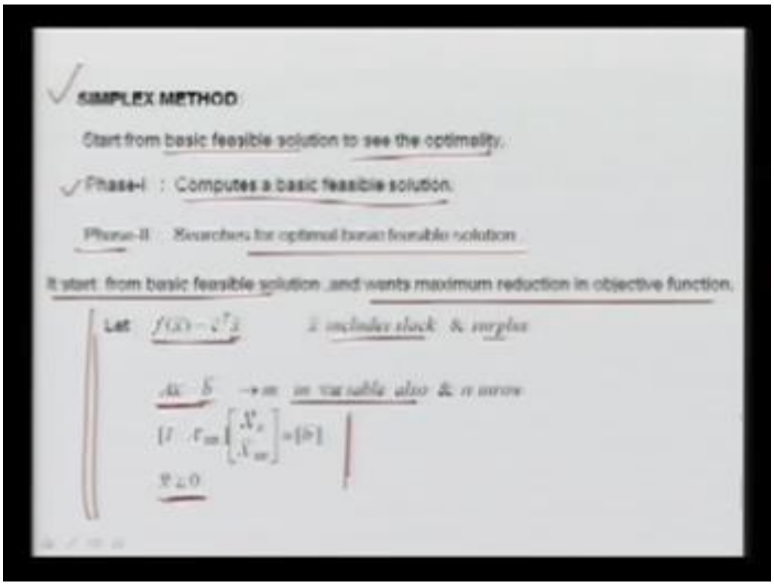
(Refer Slide Time 52:13)



So, let us see some fundamental theorem in this LP problem, so first fundamental theorem is any feasible solution of LP that is a linear programming problem correspond to the basic feasible solution. So, any feasible solution that is you are getting that will be your basic feasible solution. Second is your any optimal feasible solution will be your basic optimal feasible solution here just we have add the basic. So, the basic solution just we are getting it will be the optimal basic feasible solution. So, the number of solution as I said the number of solution that is a possible here the n c m means you are going to get this much solution. If your number of here n equals to 5 m equal to 2 the maximum number of basic solution will be your 10.

So, from this one will be optimal, so the basic solution that we are going to get it is your ten in number although the solution is infinite. So, basic we will be only your ten solutions, so in that ten that one will be optimal. We have to find that optimal value so that we can say our linear programming our standard here that is f x is minimum subjected to constraint that is our constraints is equal to b and here x is greater than B and of course here B is also greater than equal to 0.

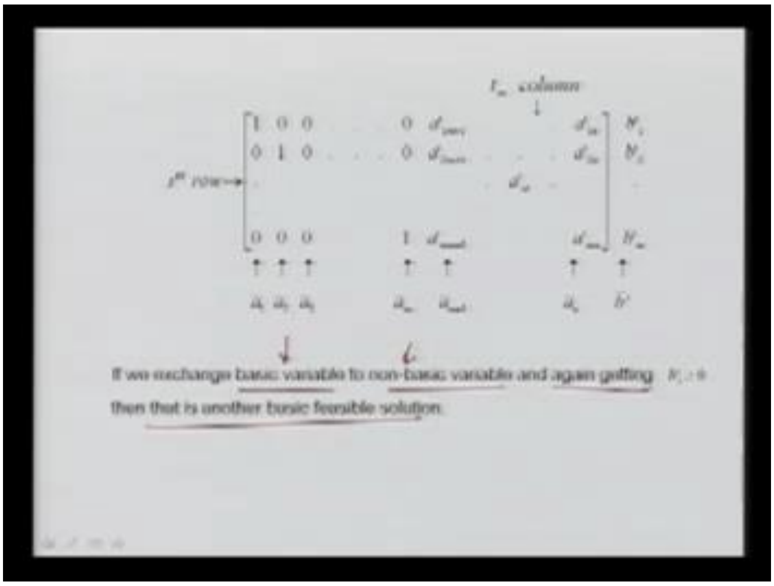
(Refer Slide Time 53:35)



So, this we have to search and the various methods are there first one is very popular that is called simplex method. This method is start from the basic feasible solution means we have to get the basic feasible solution first and then we have to see the optimality. For example, that in phase one it will computing the basic feasible solution, then in phase two it is searching for the optimal basic solution. So, it is said that it is start from the basic feasible solution and wants to maximize the reduction in the objective function at the same time. So, this is your standard problem that is a function that to be minimized that includes here x prime all the including slack and surplus, means we are making that here equality constraints.

Then, we are having the constraints here, so we are having m in m rows and now n variables and we have to solve this. So, we have to solve the basic solution here and then we have to see the optimality at the same time.

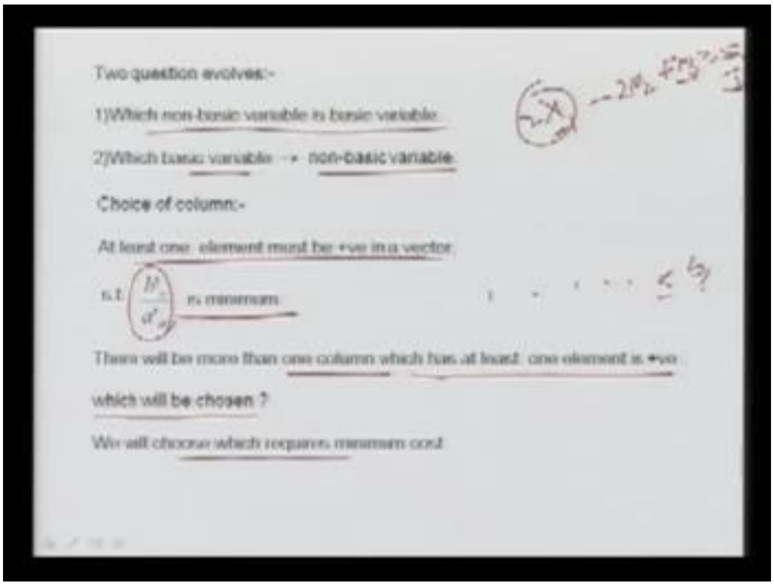
(Refer Slide Time 54:33)



What we do in this changing from the basic to non basic always we extend the variable the basic variable to non basic variable. Again, getting back to another basic feasible solution means we have to keep on same time we have to see which whether it is optimal or not. So, it is not possible that we have to solve one basic solution, then search you solve one then see the optimal. So, you are suppose for in a simple problem as I said in the variables here it is only this your number of variables five equation is two you are getting ten basic solutions. If your number of variables are hundred and here this two you will get tremendous variable

So, it is not possible to every time to solve this and then you can search for optimal. So, there are some techniques that even though while changing your basic solution here, and there from basic to non basic variables we can check whether it will be suitable or not for our optimal value.

(Refer Slide Time 55:27)

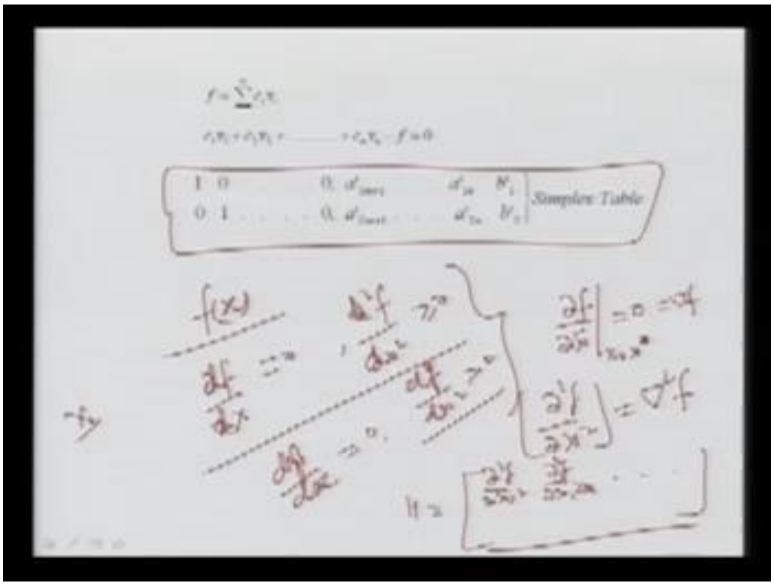


So, that is why here the question arise that which non basic variable should be the basic and which basic variable should be non basic. So, we have to choose in that fashion to this simplex method gives some idea, so at least one element must be positive in a vector subjected to this will be the minimum. So, we have to choose in that vector in that here the various vector that equal to here b s upon A and this will be giving your minimum. For example, here you are having x 1 minus x 2 plus x 3 here is greater than 5, let us go to your two, so which one will be the basic? I am telling you have to see here this is a positive this is positive, so you have to see that here two other positive.

Now, which one will be the basic variable you have to divide by 2 here how much getting this is less it is one, so this will be your basic variable. So, there will be more number of one column which will be at least one element positive as I said several columns, now which will be chosen and we will choose which require the minimum cost. So, we have to choose according to the value which will give the minimum cost, and then we can go for the simplex table and then this simplex programming give idea that we have to go for the minimization of your objective function.

Now, I have to now recap is this module because we saw in the previous two lectures we saw the we also saw the curves and unit commitment problem, even though I just briefly introduced the optimization that is economically problem. Then, I discussed the various optimization techniques that is non-linear, and also I get some idea about this non-linear programming prose.

(Refer Slide Time 57:18)



In this optimization algorithm, just I said if you are function here is f x is a single variable then the necessary condition for your this optimal here this del f your del x is equal to be 0. You have to obtain solve this you have to get the x, and then you have to see double differentiation of your del x square you should be greater than 0. So, this is the necessary condition for the sufficient condition, it should be minimum global minimum here we have to go for the d f upon d x is equal to 0 and your d f double differentiation d x square is greater than 0. So, this is for single variable x only one variable here there is a no need to go for partial because its one variable. So, I can say it is a direct derivative, we can use no partial because other variables are not there.

However, if you are going for the multi variable system, in that case we have to again this necessary condition will be your del f here del x bar will be your 0. So, this will be the 0 at x equal to x prime x star that is your optimal value, and we can solve it and that it also called the grade of f is equal to 0. Another here that is your del f here upon del x square this should be positive same a different matrix. So, here this matrix basically it is A matrix called h matrix is called and it is also denoted by here grade of f, and then we have to see.

Basically, this matrix x is nothing, but, here del f a square del x 1 a square del f a square our del x 1 del x 2 and so on so forth. So, this is the h square matrix and then it must be positive definite. Now, for the sufficient conditions this must be your not same definite it must be your positive definite variable, and then we can solve and we can say our global optimality. So, here in this module we saw the various power flow techniques and various optimization techniques are used to control the power flow to achieve certain objective, may be minimum cost minimum emission and other variables are well.

Thank you.