Power System Operations and Control
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Module - 4
Lecture - 3

So, welcome to lecture number three. In this lecture I will discuss about the various optimization methods, techniques to solve the power system problems.

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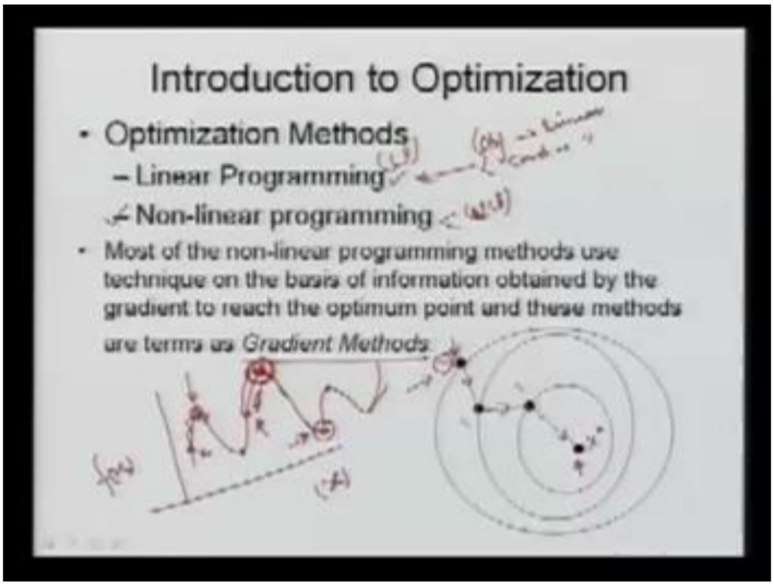

To begin with, just I will try to introduce the optimization what is the optimization every person everybody in this life always try to optimize even though in terms of your daily routine works. For example, if you are traveling from one point to another point, always you find the minimum distance, and then you try to cover means you try to optimize your travel subjected to your various constraints. Similarly, suppose you are having some money in your pocket always you try to optimize that money so that you can utilize that resource very efficiently and optimize its way. So, in the power system also in always we try to use the optimization method to achieve our various objectives.

So, there are various optimization methods existing and they can be categorized based on again several ways based on whether they are linear or non-linear problems. So, we can linearize that the linear programming optimization methods or we can go for your non-linear programming methods. So, in these linear programming methods our objective function where we are going to optimize it must be linear and linear and your constants that are constants must also be linear. So, once you have formulation your objective as well as the constraints if they are there. So, it must be linear then we can say our optimization is your linear programming approach, we have to apply and that is called LP method sometime very powerful and it is very fast.

Another is your non-linear of programming in which at least any of either constraint or objectives or both will be non-linear function then it is called your NLP method, and then we have to use the NLP method for optimization of this method. Most of non-linear programming method use techniques on the basis of information obtained by the gradient to reach the optimal point and these methods are termed as the gradient methods. So, gradient methods are very very popular and they use some sort of information to move and that information basically based on the gradient to reach the optimal point.

For example, in this figure you can say if you are starting here with some x not again you must also know for getting the optimal solution, always we start with some initial guess, and normally that guess should be very close to your optimal value. Otherwise, there may be possibility that your optimization method may fail to get this optimal solution it may give some local solution it may give some optimal solution and so on so forth. So, here thus we use the gradient method, we start with the some initial point x naught, it moves to another point your x 1 here. Then, it calculates another gradient, and then it will move to x 2 and then finally, it is coming to your x star that is your optimal point.

So, it is based on your gradient method and it is moving like this and reaching your optimal value here. Now, to again to understand this, to go in the detail in the non-linear, let us suppose we are having a function. Now, question, why we start with the initial guess? That is very close to that in any optimization methods there may be some various optimal points. Some may be your global, some may be your local, for example, let us suppose this is your function, now you can see this is one maximum this is another maxima and this is the highest one, so there may be the various local optimal points.

So, here we can say the local optima and another is your global means that is one highest among other let us suppose again here we are having them, so what is happening? We have the several peaks, so these are the maxima value, these are your the minima values. So, in any this is function let our function of x, let us go for one variable, this function is going in this way, so what happens here? We are having only one global optima, this maxima and only one global here in minima. So, if your objective is to minimize this function, then we are optimization that we have to give global minima here.

If your objective is to optimize for the maxima, then your global maxima will be here, there are several local minima's and maxima's and therefore, there may be possibility you may stuck at some global local point optimization problem. That is why we start very close for example, if you are starting from here let us suppose your x naught is here, you are moving with the some direction here, and here there may be possibility you can land up here, and you will get here the solution. You are not coming here and this is giving your local optimal solution, so for this we have to start somewhere here very close to your global point and then we can reach here at this optimal point.

So, this selection of initial guess is very very important and it also depends upon method to methods some methods give some direction some ideas to check it, but mostly if you are giving here your starting point very close to your optimal value. Let us say very easy that you will get a minimum time and you can get your global optimization. So, this is important of this x naught, another requirement for the optimization is that we should have the function of the continuous because most of the classical methods I am talking here about the non-linear programming approaches. Most of the classical methods the method that which are conventional methods, I can say Newton's method, I will discuss later.

They require some derivative information, if your function is not differentiable at any point, means it has some discontinuities. Then, we cannot apply the NLP the conventional NLP, then we have to go for some realistic or we can go for some non conventional application of ai technique. We can apply like genetic algorithms some related dynamic say other programming and so on so forth. So, it is your let us see now gradient conventional method.

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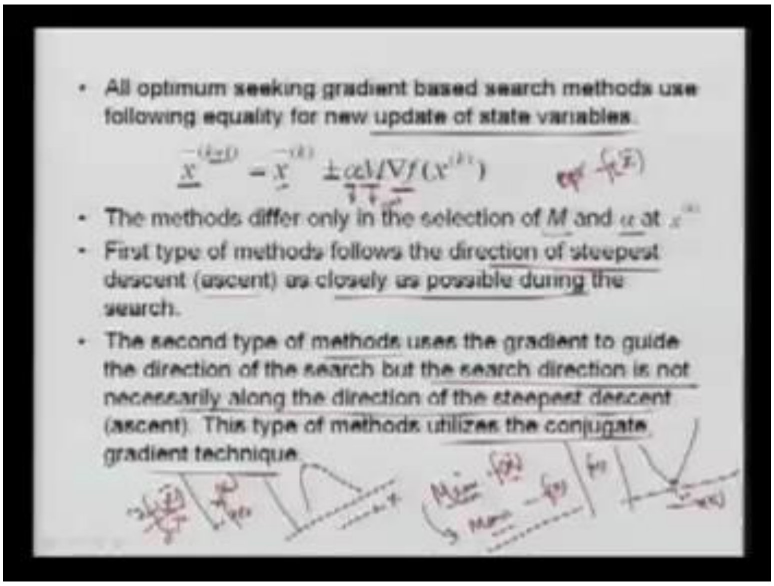

I am just discussing at optimum seeking gradient based search methods using the following quantity for the new update of the state variable. If your x is your value here, the bar is showing the vector because your function may be of the several variable it is not only x 1, it is our x, it may be x 1, x 2 x n. So, we can write your update value of x that is the states that we are obtaining so that we can get our function f x here that should have the optimal value here. Again, it is maximum minima, it depends again because again the optima the maximization and minimization are reciprocal. It means if you are just for example, you are going to say minimization of x function f x, it will be similar to maximization of function minus f x.

It means if you are multiplied by minus one your function transfers from minima to maxima. So, it is immaterial means whether you are going to minimize or maximize only simple here minus sign is changed and you can solve by using any conventional methods. So, most of the method basically go for the minimum suppose you want to maximize then you multiplies that function with the minus one and then you can minimize and you can apply to the any method. It is true, for example, I will show you, let us suppose a function here, we have here let us suppose minima, so we have this minimum value this is your f x, this is your x for a single variable I am talking. The point here it is your minima and we can get here the minimum value x naught.

Now, if you want to maximize, then you have to multiply it by minus 1 and you can say what you are getting here the function that is minus f x. You are getting this function and this is your x, so this curve it is just inverted, so it is minus x I am talking of here, so this value will be like this and you can get it. So, this means minima and maxima should not confuse, normally we that is why we call the optimization, we never say minimization or maximization because both are can say complementary to each other. So, here I was talking about the updated value of the state variable that is the x I am talking x is a vector that is k plus 1 means at k at plus 1 iteration, k is a iteration count we are in any iteration k plus 1.

Then, we can use the value of x in the previous iteration that is the k th iteration plus minus some alpha multiplied by m into your change here del f at x k what is this let us say first basically this alpha is a scalar quantity, m here is of matrix. That is your n cross matrix and this value here is the differentiation, this value x k is nothing but this differentiation of this function f x this is vector, so it is a here x at your x k th iteration. So, this is your partial derivative of function f with respect to x at variable. So, we can write it in this general form one most of method, they differ only in terms of selection of m and alpha at x k naught.

Now, we have the two type of methods based on this the first type of method follows the direction of the steepest descent that is the ascent we call and a closely as possible during the search. We use the direction of the steepest descent, we will see what is the steepest descent, and it will follow that path means this given the direction here in the second type of method it use the gradient to guide the direction of search. The search direction is not necessarily along with the direction of steepest steepest descent, means it will give the gradient will give the guidance of direction, but it is not the steepest steepest descent. This method basically utilizes the conjugate gradient technique, and we will see later on, now let us sees what this is? Your steepest descent method.

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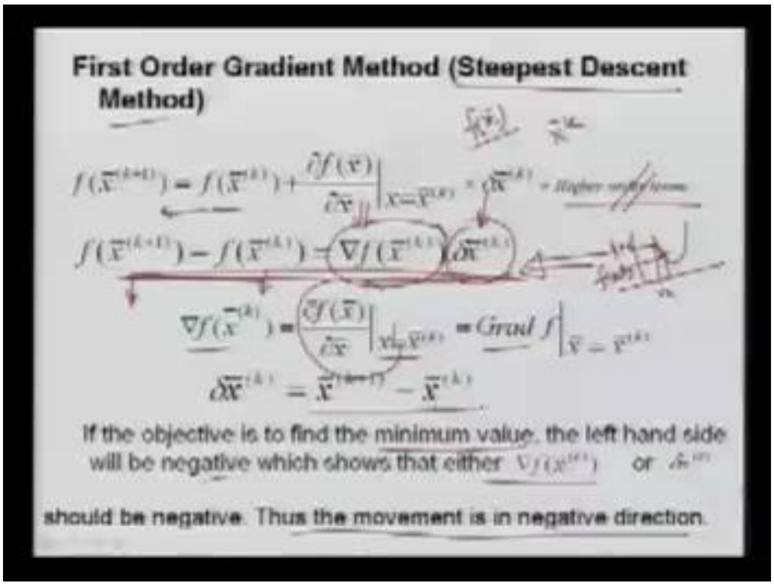

It is also called the first order gradient method in this method, let us we have again the function f x here we want to optimize this function; now this function we can expand using the let us say expansion term. That we can write this f x k plus 1 is equal to your x k, here that is 0 naught value plus we can differentiate this x plus higher order term. So, based on the tailor series expansion any fractional function at the point x plus k this function at k bar we can write here in this way. Now, we can ignore the higher order terms here and we can simplify this, so we can take this term this side we can move. So, we can write here that we can get this differentiation term we can write this one this x k is nothing but your changes here x k plus 1 minus x k.

This is your partial derivative at x is equal to x plus k sometimes called the gradient. So, you can say now you are getting this change in f here we are getting this function now if your objective is to find the minimum value. The left hand side of this expression here side will be negative we want to minimum value, what will happen? This value will be less than this value, means we are keep on minimizing you are moving in that direction. For example, here this is your function, so you are here at x k, now here this is function is your f k here, now you are moving here in this direction your f x k plus 1 will be less than your x k. Now, you can say this whole quantity will be negative means this will be lesser than this value.

It means whole this will be negative which shows that either for the negative this either we should have this negative or we should have this negative two possibilities. Either we should have this negative or this negative to have this negative because we want to minimize this thus the moment is in the negative direction here x k plus 1 if you are incrementing keep on moving, so this value will be in the negative direction.

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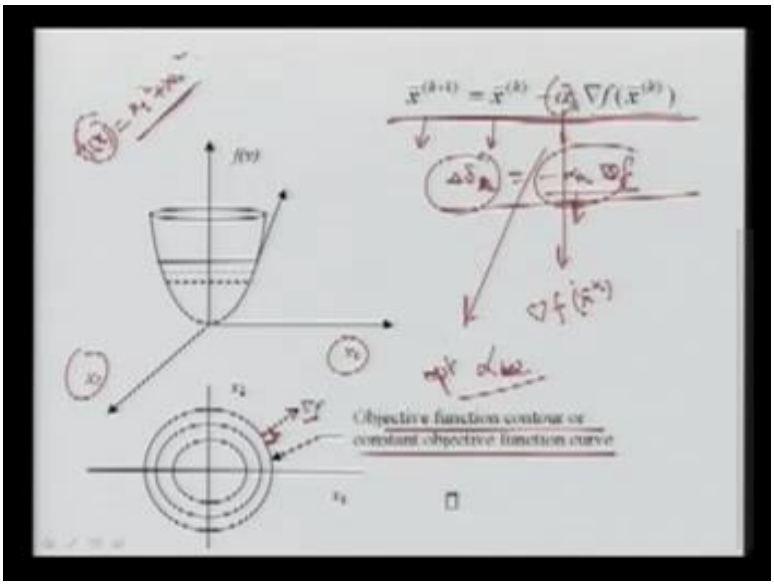

Now, I can say here your x k plus 1 here will be equal to your initial x k minus alpha k with the grad of f s k, but it is shown it is showing that this value is nothing but your delta this x k that will be equal to your minus alpha k grad of f. So, if this is negative, you can see it is here we are going to write alpha some magnitude we are adding. It means your iterations that is we are keep on adding with the some constant, and then we are getting this another important feature that here we are achieving this alpha k is the step length and it is a positive scalar value at the k th expression. At the same time, the direction of here f x k will be perpendicular to the contour of the function f x as shown here you can say this grad of f this is a contour this is your function let us suppose.

So, the contour of this one this is moving like this, it is in two variable function x 1 and x 2, it is moving and then it is your like this value is achieving means this function is nothing but I can say f x here it is your x 1 square plus your x 2 square. So, here you can say this function is a pi r square and it is a circle the equation.

Here, the contour will be the circles you can say this is the circle we are having. So, at any point this grade of f will be perpendicular to this value as I said. So, the objective function contour this is called objective function contour or constant objective function curve here means for fix values here this is a equation of square term. So, this is the contour this is a function x y the variables two variables are there and it will be the perpendicular of that one. The value of alpha this one, sorry we are talking the value of this alpha k here is very critical, and with small value of alpha k the solution time required will be larger where as a large value of alpha k may result in divergence of solution.

Hence, the optimal value of alpha k is required, so here alpha k we have to take the optimal value and that we have to decide it optimally, so the optimal choice of alpha t come into the picture.

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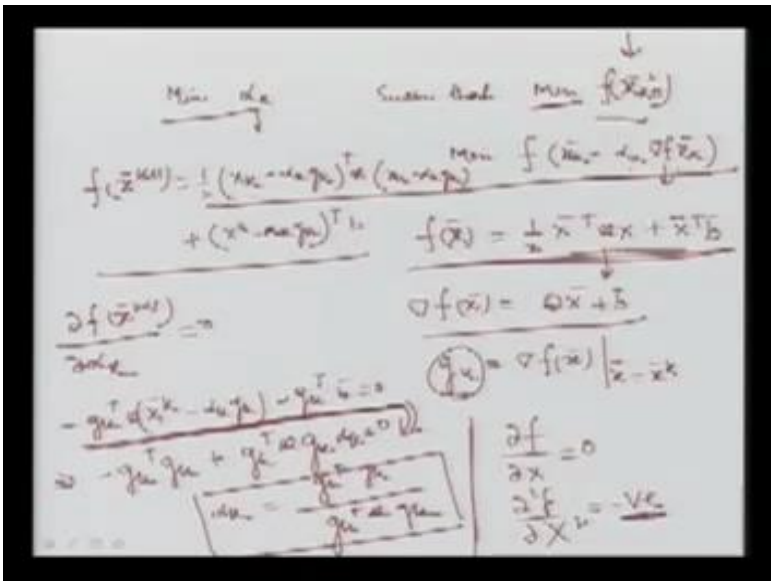

So, let us see the optimal choice of alpha k, and then we have to min minimize this alpha k such that the condition is that such that here your minimum f x k plus 1 here this is the vector or we can say the minimum here. Now, I can replace this with your minimum of your f that I can write here x k minus alpha k here the grad of f here x I can say k this is vector and this is your waveform. So, we have to minimize this subjected that we can get the minimum of this value because we are going to minimize this.

So, here let us take a quadratic function here f, now I can take f x bar it is your quadratic and I am adding this function is your x transpose some here q x plus some your x transpose t that is b. So, this is basically nothing but your quadratic function thus we want to calculate what will be the optimal value of alpha for this objective function bar term denotes. Again, the x is having the various variables d is the vector curve representing the linear term here it was b and this is your quadratic term and q is your basically the coefficient matrix for this quadratic curve. Now, I want to calculate the grad this grad, I want to calculate this f x here it will be nothing but I can write this q x bar plus your b this is again it is also vector.

So, we are getting the grad of f this, so this is nothing but let us write here this g k it is nothing but I can write the grad of f of your x bar at x is equal to x here k bar that is we are trying to write. Now, our objective is to minimize this function here or you can say minimize this function here and we can use this value here. So, I can write now this function f here that is your x k plus 1. I can write using the Taylor series expansion it is nothing, but simply I am putting that value here in terms of this function we are putting. So, we are getting this your simply here x k and now I can write it is half of your xk minus alpha k here g k that is the transpose into q.

Here, your again x k minus alpha k g k here this is a quadratic term plus here I can write x k minus your alpha k g k here it is your transpose b. Now, what I did just I replaced this term here, you can say this was your alpha grad just value I wrote here is the g k at k value. Then, we can write in this fashion now to have minimum of we know this delta f this x k plus 1 here with the differentiation of alpha k must be 0. For the sample expression, we know to be the minima always the function is here is for the minima the necessary condition that this should be zero already we have this you know it very well. Again, if you are going for the double differentiation here if it is a negative, then that value will be the maxima value at which value we are getting negative.

It means that we will give maxima if it is a positive, it will give your minima at 0 again we cannot say anything and we can go for further. So, here you can differentiate this and then we are getting here that is I can say minus g k because this is 0. Here, we are going for the g k that is your transpose here your q x k here in bracket minus alpha k g k this term means this, and we are differencing this, we will get minus here g k transpose b prime and this equal to 0.

If you will solve this what we are going to get if you are going to replace this x k etcetera you will get here that is minus g k transpose g k plus here g k transpose q g k, and then we are getting alpha k is equal to 0. I can say alpha k is nothing but your g k transpose g k divided by g k transpose q into g k only which I skip from this expression to this expression vary directly what we can do if you will expand this. While expansion, you will find some of the term will become 0, for example here I just want to explain this term this you can remember this g k t transpose this g k t, this multiplied by this and so on, I am writing here.

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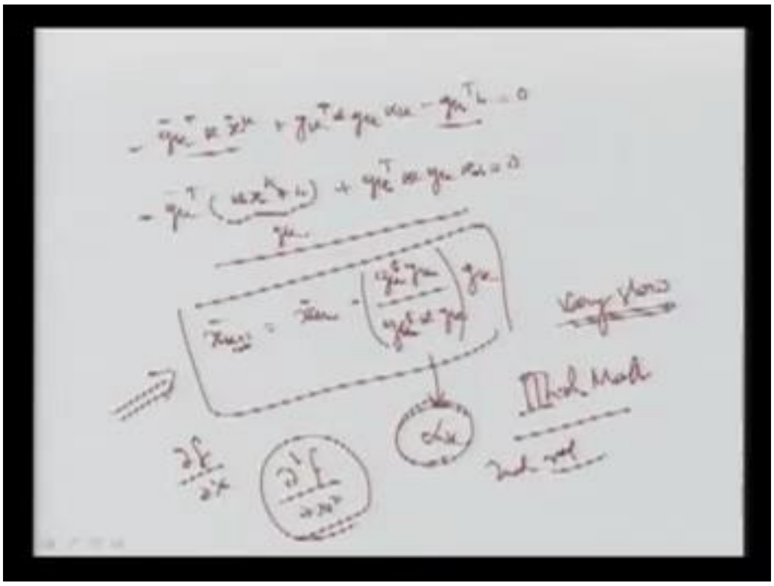

We can write here that g k transpose minus here it is your q, it will be x k minus here minus minus will be plus g k transpose it is your q g k here alpha k minus g k transpose beta is equal to 0. Now, you can see from this value from this value and this value you combine together, now we can write from here we are getting g k transpose here q x k minus beta plus here your g k transpose q g k and alpha equal to 0. This value what is this value this value is nothing but your g k if you remember here you can see here this value is the g k. So, I can write simply here this is if this was basically this is negative value.

If this was negative and this was negative, so it is negative and this will be positive. So, this was your g k and then based on that I wrote this expression here you can see this one.

Then, I can say alpha k here, now this update of state vector using the steepest descent technique for the quadratic function will be given by this expression that is your x k plus 1. Here, sometimes I am writing x k at lower or upper basically x k is the x variable at k plus 1'th iteration, here I can write here x k minus that is your alpha k g k transpose g k divided by your g k transpose q g k and then we are multiplying by here g k. So, this is for your quadratic function as I have explained in the previous case, we can use in this fashion the main disadvantage of steepest descent method is that it is very slow this method is very slow.

It gives your value and then we can get this value is you can get the optimal value. So, here the alpha k is keep on changing means every time we are calculating the grad of f and based that alpha is calculated that is the step length. So, another method that is called the second order and I can write here second method that is second method I am discussing it is your Newton method it is called and it is called second order gradient method. The Newton Raphson method proposes the property of quadratic conversions as we saw in the load flow we know that in load flow that is Newton Raphson method is very fast because it is giving quadratic conversions.

The optimality of function is obtained in finite number of steps the main condition of the method is to have the initial guess very close to the optimal point. Otherwise, there will be possibility of diverse solution already I explained that point that we should have the initial guess very close to your optimal value. Otherwise, you will get the diverse solution the main condition of this method to have initial guess very close as I said since the second derivative of the function is required, therefore it must adjust must exist that one. It means it is using the second order gradient second order. So, it means here del f upon del x is existing we should also have del f upon this function should also exist if it is not existing then this method will not be applied now to see this method again I can just write the function f x here.

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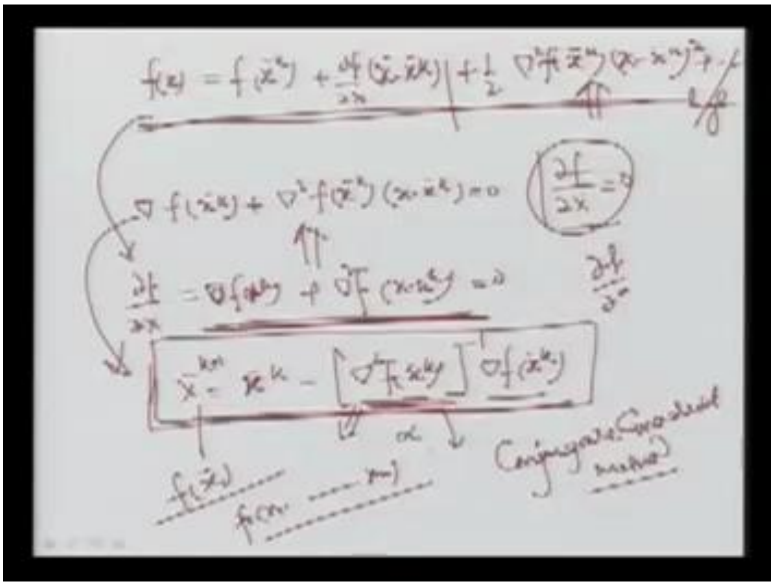

We can write at any point at any k th iteration I can write it here f x k value here plus we can differentiate here del x or I can write the grad. Here, it is your this value is your x minus x k here plus half of your either say double differentiation of this function here and here it is x k I am talking and here your x minus x k here that is square plus other high order term. So, here we are going for higher order term normally here in the Newton Raphson method, we ignore the order that is more than two here it is a square. So, we are going up to this portion in the first order we took up to here and remaining we ignored.

Here, we are talking this term as well we saw this the first order necessary condition for optimality that I said here this del f here over the x should be 0, you already know it. Now, based on this the equation here, and the equation this equation and this equation now we can get here this value as I said 0. So, what we are going to get we can define this grad of f this x k plus your grad of this f. Here, f x k here multiplied by your x minus x k here that will be equal to 0, using this expression we can get this one very well what happen now you are having this function ignore that one.

Now, you differentiate this function and put it 0 means from here you can differentiate this f here by del x is equal to this differentiation will be 0 plus here no you have to differentiate this function and then here you can say del f x k here.

Then, you are differentiating this function that is double f here x minus x k here and that is grad of f and then you are putting 0. So, you are getting this expression that is this expression because here to the double your differentiating, now from here this is 0 or from this equation. We can write your x bar will be your x bar k minus, this is your term this del f x k here inverse of this and this your grad of f x k. From here, what we are getting we can expand this. So, x we can get in terms of this value and we are getting this now you can see with the previous this is the general expression which I wrote I say this is your alpha. Then, we are using some grad, so in this case the alpha is your double differentiation of your function with at the value of x k and the inverse value is existing.

So, here we are going for, so in this condition, we must have the value here that that is why I said the double differentiation must exist if it is not existing this is say 0 inverse of this will be infinite and you cannot solve it. So, this is Newton method and here alpha is also keep on changing this value is kept on changing in every iteration. So, I can say simply here for any iteration x plus 1 we can write this expression. This method is very fast because it provides the quadratic convergence, but it has some disadvantages that it requires additional memory to keep this is a matrix Hessian matrix inverse it is existing if it is a vector, so it will be matrix.

The additional CPU time it has some more for every time you are inviting this and inversion of Hessian matrix is every time must exist. This is also called Hessian matrix another method is called your conjugate gradient method conjugate conjugate gradient method. So, this method is superior to both previous method means your first order and second order. In this method, the search is made along the certain sets of direction to ensure the optimality of the function is achieved in certain number of iteration Fledger power method is most popular minimization of unconstraint non-linear functions. Another method is gradient projection method, these are the various method and I am not going to detail about that method.

Normally, they use some sort of the similar type of technique, now this is the case when your optimization problem is not having constraints means you are having simple objective function f here that is your x and having the different value. It means you are having f that is your x 1 to x n the function of n variable. You are going to minimize for one variable it is very well clear that we can differentiate, and then we can go for double differentiation, and then we can get it, but if you are going for the multiple variable.

This this function its value is of course, true, but we have to go for these methods and we can calculate and it is iterative process. So, for the constraints optimization then we have to use another one and normally this Lagrange's multiplier method is very popular that we normally use.

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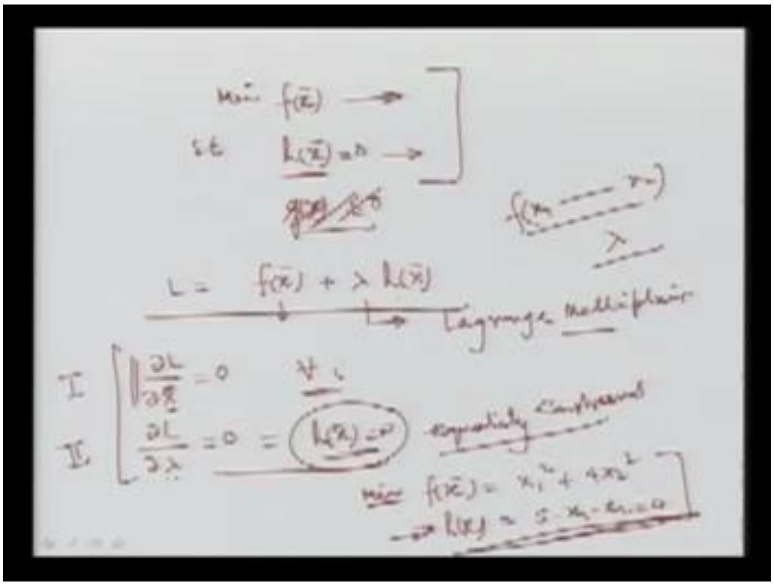

So, if your objective function here is f x is there and you have some function here that is the equality constants 0 means here you are going to minimize this function that is objective function subject to your constant. We are talking about the constraints here on that is the equality constraints, we can have the constraints on the limiting value of x value here, but we are not having any inequality. The function inequality we are not using functional means we can have here let us suppose your g x here and that should be less than your 0. So, this is a function of x, so we are not including here then here is your your objective function this is your equality constraint and then what we do normally we go for Lagrange's function method and we use another function called is called l.

We use this f x here, and then we add some lambda into h x, so this is also called augmented function or sometimes called another Lagrange's function and this alpha is called Lagrange's multiplier. This is known as the Lagrange's function equation and also sometimes called the augmented objective function because this is your objective function is augmented by your equality constant.

This alpha is called your Lagrange's multiplier Lagrange's multiplier now here then for objective for this your minimum value again, thus we are going to have this as per this optimization. This del l differentiated with x variable must be 0 and also this your this l with respect to your lambda must be 0, now here this is a x bar I am writing, so this objective function if it is having x of n variable. So, we are going to have here for every basically I for means every x y it will be suppose you are having your function x 1 to x n, then this will be your number of n plus 1, so you are going to have n plus 1 equation.

Then, you can have variables now how much x to x n here and plus lambda then you can solve it very easily. So, this is basically nothing, but I can say it is your x is 0, you can differentiate this is your 0. We are getting this differentiation with this differentiation equal to 0 means you are getting this 0 because this is independent alpha. This is partial derivative, so this is nothing but again you are equality constant equality constraints. Now, we can solve to see one example just I will show you one example here let us take a function that is your function objective function is here I can say f x 2 variable I am talking.

Let us suppose your x 1 square plus 4 x 2 square and your equality here at f x here I can say 5 minus x 1 minus x 2 is equal to 0 and I want to solve this I want to here maximize minimize this function here. So, the minimization of this function at I am trying to minimize subjected to this constraint no doubt if this constraint is not there minimum will be certainly you can see the x 1 and x 2 will be 0. Due to this, if you are putting x 1 and x 2 0, this is not five is equal to not 0. So, it is not satisfying, so we have to get the minimum value subjected to these equality constraints here. To solve this, we have to again use this procedure and we have to use these condition condition number one condition number two that is for objective function.

So, it is your augmented objective function must be differentiated with the state variables. We will get here the number of equation is equal to number of state variables plus another equation we are getting here. So, now we are having n number n plus 1 equations and n plus 1 known, means n here plus lambda another one and we can solve uniquely you will get the another solution correct. So, just let us see how to minimize the function if you are having these equality constraints. We have to apply the first order necessary condition and that condition. As I said here this your differentiation of l with respect to your x 1 and your del n with the x 2 because we are having two state variable x 1 and x 2.

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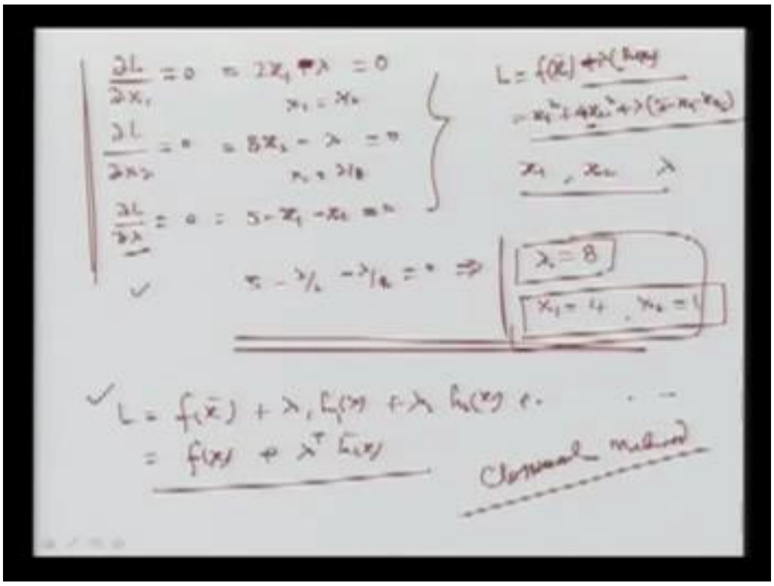

It means your function is a of two variable function, so it must be 0 here, it must be 0. So, if you are differentiating this function with respect to x one you can see here you are getting 2 x for the f x here and I can say here we are getting that will be equal to your 2 x 1. Now, the second term here if you are going for this will be 0, now another term we are just differentiating with the lambda means here what I am going to do means your l is nothing, but your f x. I can say here f x bar minus let us take here I am just going to take plus let us take plus now because I am defined with this. Here, it is your lambda h x means we can have this x 1 square plus four x 2 square plus lambda and that is your five minus x 1 minus x 2.

So, we are differentiating this function with the x 1, so this was your 2 x 1, this was 0 now here with the lambda as it is. So, I can say plus lambda this is 0 and I can say x one is differentiated. So, we are getting minus here and that is equal to 0 if you are differentiating with this two, so this is 0, this term here it is 8. Now, I can write 8 x 2 minus here the lambda here the x 2 and I can say here the 0 another term we are getting here that is I can say del l upon del lambda and this is nothing but this is whole 0. We are getting 5 minus x 1 minus x 2 is equal to 0.

Now, we are having three unknowns that is your x 1 x 2 and the lambda and we are having three equations, we can solve it uniquely. It means you can see here I can say x one from here I can say x 1 will be your lambda by 2 here your x 2 is equal to lambda by eight and we can put here. So, we can set 5 minus your lambda by 2 minus your lambda by 8 is equal to 0. If you will solve here, you will get lambda is is equal to your 8 value you can see lambda is equal to 8, it is unity here 4, and then it is equal to 0. So, lambda you are getting 8 from this value you can say. Now, your x 1 will be your 4 and your x 2 will be your unity, so this is your optimal value and then we can solve I, so this is using your Lagrange's multiplier method.

In this method, what we are doing you can see very well that we are not considering the inequality constraints means first the methods we saw with the help of without constraints now we are solving if you are having equality constraints. Then, we can use Lagrange's multiplier methods here very easily another method that I am going to discuss that is if you are having some inequality constraint even though to this generalize means you are having several constraints what will happen in this function. So, it is your l it is nothing, but your f x plus I can say lambda 1 into h x h 1 x plus lambda 2 into h 2 x and plus and so on. So, in general form I can say f x here plus your lambda, I can say transpose your h x value and then we can get this.

So, here we can have the several equality constraints, and then we can form in this fashion, and then we can again apply the same procedure that we can go for the first order necessary condition. Then, we can go for this one, and then we can solve and we can get the optimal value here for this one now we can also. So, this method basically is also called the classical method, now let us include some inequality constraints.

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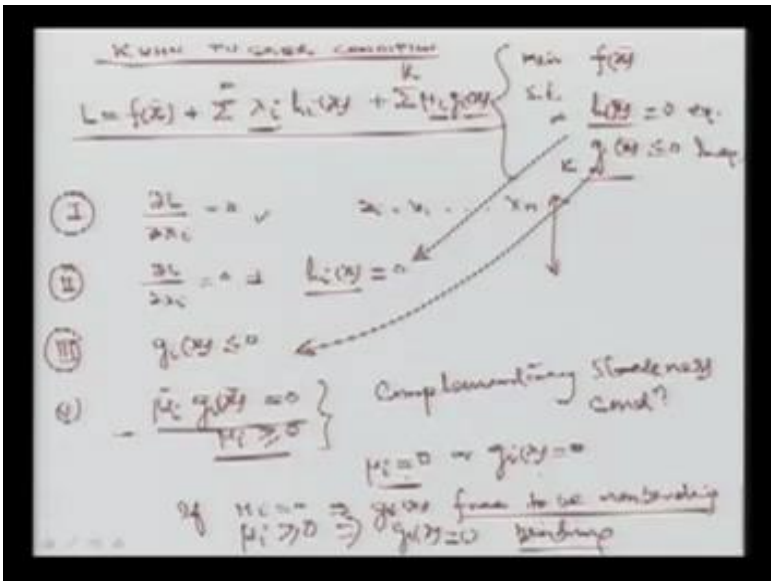

It means we are having equality as well as inequality means our objective function here is your f x, then it is we are going to minimize here, then we are having some equality constant h x is equal to 0. We have some inequality constant that is here equal to zero again the function of several variables. So, these are the constraints here it is your equality this is your inequality now for this the very there are so many methods. In the various methods, one is called the Kuhn Tucker condition it is called Kuhn Tucker condition Kuhn Tucker condition method. These methods that we can use your equality as well as inequality constraints, it can handle very well.

So, this approach is similar to Lagrange's method and here what we can do we use the Lagrange's augmented function. Then, we use this f x here plus here the summation of here several, let us suppose we have several function of this and several functions of this. So, I can say here lambda I into hi x you can add all the function plus here you are going for the mu I and the g i x means we are having let us suppose this is your m equality and we are having the k inequality. So, here it is up to k here up to m means we have to add all this equality with your Lagrange's multiplier lambda. Then, we use another variable that is mu related to your inequality constraint, and this is your augmented objective function.

So, in this kuhn tucker condition is that first condition is that we have the several condition for kuhn tucker first one is that your del l upon your del xi is equal to 0.

It means for all the state variables this is the partial derivative of this function augmented function objective function must be 0. Here, it is for all xi is equal to your x 1 to your x n second condition is that your again that is called del l upon lambda i is equal to 0 and this shows that your hi x is equal to 0, this is nothing but your equality constraint previously your classical method. So, these two condition are as it is with your Lagrange's multiplier method or you can say classical method, the third condition here that is added here it is called your g i that is x should be less than 0. The fourth condition is called that is your mu I g i x here it should be equal to 0 and where mu I is your greater than or equal to 0.

So, these are the four condition based on that we go for, basically you can see this is your first condition is primary condition second one is your equality constraints as it is this condition is your equality constraints as it is we are adding one extra here condition. This condition is called your complementary slackness condition, it is called complementary complementary slackness condition in this condition what is happening it is said that this here mu I into g i x is equal to 0. It means here the possibility is that either mu I is 0 or your g i x is 0 or both are 0 means possibility that this is equal to 0 or this is equal to 0 or here we are having both are equal to 0, then we can get this condition.

So, if mu is 0 means your g i x is not 0, and then it will call g i is free to binding means if mu I is 0, then this indicate that g i x is free to be not binding is free to be non binding. It means this constraint is non binding means this is 0, so what happens? The whole function is 0, so this function sis even though there is no need to consider means it is a non binding constraint. Now, if it is not, so if your mu I is greater than 0 because we have the condition too either it will be equal to 0 or more than 0 if this is more than 0, then what is happened then g i in this condition g i must be 0.

Then gi here x must be 0 and then it is your binding constant and it is just like it is binding and it is sometimes called active constraints to see this, let us take same example with some extra addition of that one.

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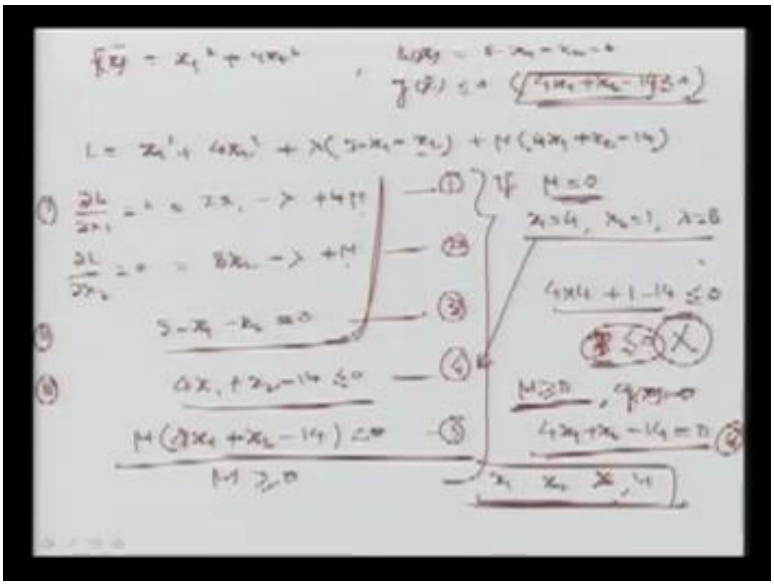

It means again our objective function here is your f x here are two variable same function I am using x 1 square plus 4 x 2 square your hx I am taking only one that is your I can say 5 minus x 1 minus x 2 is equal to 0. We are having your g x is related with a function that is less than 0 and here I have defined your four x 1 plus your x 2 minus 14 it is less than or equal to 0, so we are having this function is g x. It means your g x is this value means this value is your g x this 14, now we can again solve this l with your x 1 square plus four x 2 square plus lambda 5 minus x 1 minus x 2 here plus your mu. I can say 4 x 1 plus your x 2 minus 14, here we are using this function.

So, this is your augmented Lagrange's function or augmented objective function now the condition first that it is your del l upon del x one and del l upon del x 2 must be 0, two variables will be considered. So, now for this we can get it 2 x 1, this is 0, here it is minus lambda because this is differentiation here this will be 0, here we are getting plus four mu for this other will be 0. For this, we are getting your 8 x 2 minus lambda for this case and here we are getting plus your mu now second condition is your h x that we are getting this 5 minus x 1 minus x 2 is equal to 0.

So, this is your equation number one this is your equation number 2, this is your equation number 3, your third condition that your four x 1 plus your x 2 minus your fourteen is less than 0 equation 4.

Another equation just we are going to have your mu that is multiplied by your 4 x 1 plus your x 2 minus 14 is equal to 0 where mu is greater than or equal to 0, that is your fifth equation. Now, what we can do now we can go for the solving this first to see this first let us take mu is equal to 0 if mu is 0, what happens? Now, this will be not 0, this mu is 0, so we can again solve this value. It means here there is no mu we cannot we do not know this. So, this is a equality which we can solve and this is nothing but if mu is 0, it is our previous case when there was no inequality.

So, we again we can get your x 1 is equal to 4 x 2 is equal to your 1 lambda is equal to 8 if you put this value here for if I am using this if you are putting here let us say what we are getting. So, we are getting this 4 into 4 plus this 1 minus 14, how much this less than 0 16 plus 17, then it is a positive means the 17 is less than 0, which know seventeen minus here 14 it is 3 is not less than, so it is not correct. So, mu cannot be 0 because we saw it here now it means mu is greater than 0, then in that case it is your 4 x 1 plus your x 2 minus 14 is equal to 0 because the g x is 0. Now, what now we have now we have equation number one equation number two equation number three and now we have another equation here that is a I can say 6, so we have this four equations.

Now, our unknowns are known because mu value you have to calculate here. So, we have x 1 we have 2 x 2 we have your lambda and we have mu, so four equation four unknown. Then, we can solve it and we will find the different value, so in this case when we consider that suppose this mu is 0 we found that 3 is less than 0, which is not feasible means this mu is equal to not 0. It means our g i is binding and binding means it is the just equality constraint because equality constraints are always binding than that one. So, just I put mu is not equal to 0 means we have to have this g x is equal to 0 means we have this condition here. It means we have to now solve that is value using this, so now we have the four equations, now I can solve now I can write here.

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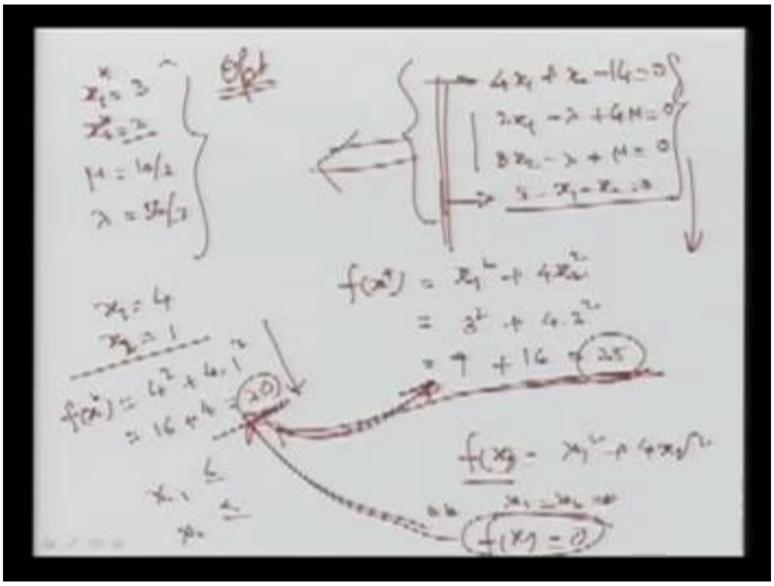

This four x 1 plus your x 2 minus 14 is equal to 0 that is equality constraint, we had your 2 x 1 minus lambda plus 4 mu here is equal to 0. We have this 8 x 2 minus lambda plus mu is equal to 0 and we have this 5 minus x 1 minus x 2 is equal to 0. So, this is your equality constraint this is your inequality constraint and these are your the first order necessary condition from here if you will saw am not going to solve completely. So, we will get here x 1 is equal to 3, x 2 is equal to your 2 mu is equal to here 10 by 3 and lambda is equal to here 56 by 3. So, these are your optimal values, now what will be the optimal function value, then you can use in this value this simply f x here basically these are called the star value means optimal value.

So, f x star is means we have to use this x 1 square plus 4 x 2 x square you can put here the value. So, it is a three square plus four into 2 squares, so how much getting this 9 plus 16 we are getting 25. Now, you can see in the previous example when equality inequality was not there we got your x 1 is equal to your 4 and x 1 is equal to 1, now you can see in these conditions sorry x 2 is equal to 1. Here this f x here optimal value was your 4 square plus 4 into your one square and we are getting 16 plus 4, but we are getting twenty five what does it mean you can say this minimum value here, now it is moving from here because of binding constant.

You are not getting minimum inequality was not there means the inequality is binding and this minimum value is shifted from here 20 to this 25 when there is no equality then what was that function. That function this simply let us suppose this x one this f x without here the x 1 square plus 4 x 2 square this the optimal value here will be your x 1 is equal to your x 2 that will be 0 and here the f x is equal to 0. This means you can say the minimum value here is 0 when we use some constants here the value will be either zero or it will be more than that no doubt about it means if the constants are binding.

These equality constants are all binding, so it is a binding, so it moves from 0 to 20, similarly when we put another inequality constant it may be binding it may not be binding, but it was the binding and we saw that is we are getting the 25. Similarly, we can put the constants on the state variable that is the x 1 and x 2 we can put the certain values that should not be less than that and then we can using that optimizing program and then we can solve it accordingly. So, this is your Kuhn tucker condition to get the optimal value and this is very good method that we can get here no doubt the number of equations if you are your number of variables is more.

Your number of constants is more your number of equations is keeping on increasing and it is not possible to solve this equation by simple elimination method. So, you have to go for the some sort of technique that may be iterative techniques. You can use the gauss siedel method or any method to arrive the values here from here. So, numbers of equations are more increasing then you have to use some sort of techniques of to solve these values and then we can get the optimal value of this objective function and this state variable accordingly.

So, this is your Kuhn Tucker technique, so we saw the non-linear of course, we are starting with the without constants then we saw the constraints with equality then we saw equality as well as inequality. Then, we discuss the Kuhn Tucker method, another method is called the slack variable formation and that I will discuss in the next lecture.

Thank you.