

**Power System Operations and Control**  
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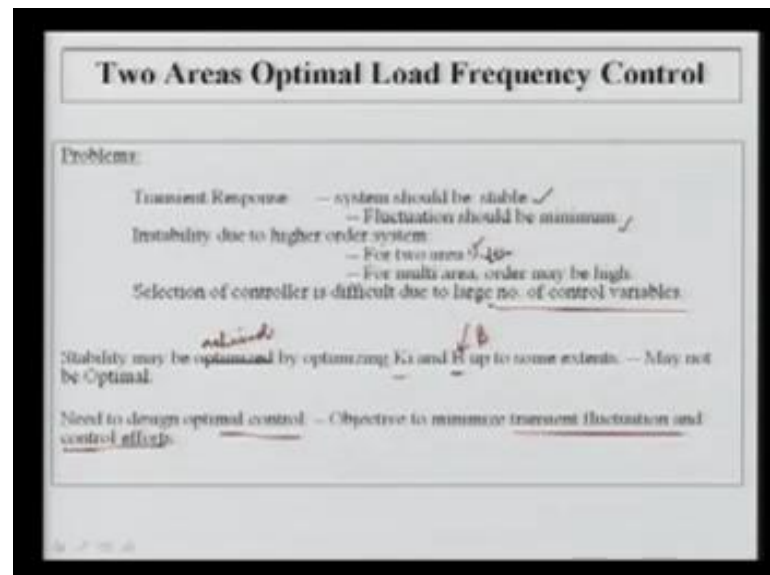
**Module – 3**

**Lecture – 6**

Welcome to lecture number 6 of module 3; that is module state, frequency and voltage control. In the previous lecture, we saw that once we are going for the multi area load frequency control, the number of state variables becoming very high, means the major objective for this control area that we must minimize the steady state errors. And then we had the integral controllers. For the 2 area we saw that we decided the  $K_{i1}$  and  $K_{i2}$  and based on that, we tried to reduce the error. But, that sometimes the selection of  $K_i$ 's is not optimal and the system may end up, with unstable control.

So, if you want to reduce the error sometimes your system may be unstable, that is the major hurdle. So, what we have to do. We have to design the optimal load frequency control of the AGC. Again, I will be discussing only 2 area case. So, similarly, we can go for the multi area case as well.

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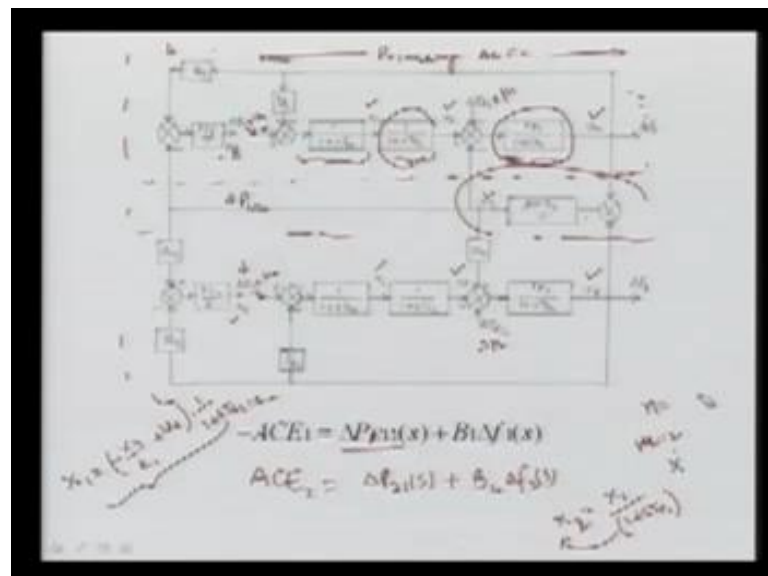
So, the major problems are that the transient response in the designing of optimal control. First one is that the transient response of the system that, we require the system should be stable. The fluctuations should be minimum means the over shoot must be less

and also the instability due to the higher order system. We will see for the 2 area system we are having 90 states. If you are going for more suppose, you are going for the 3 area system the order will be again very high.

If you are going for higher area then, selection of controller is difficult due to the large number of control variables and sometimes the controller that, you are designing may not be optimal so, our concern here that; we can design some optimal load frequency control. And we will see for that 2 area systems. The stability may be here, what we are doing this stability may be achieved by optimizing here it is not optimized. Basically, it is achieved by optimizing  $K_i$  and the bias up to some extent but, again here, the controller may not be optimal means we may have some optimal solution.

So, the stability no doubt we can include by choosing proper  $K_i$  and bias value that is  $b$  and  $k$ . Here  $b$  is nothing but, the betas. We can improve the stability but, we cannot say whatever, the value you are just choosing it is a optimal value. So, the need to design the optimal control and objective is to minimize the transient fluctuations and it should have the minimum control effort. So, these are the basic criteria based on that we have to decide the optimal load frequency control that is also called AGC.

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Now, let us see the 2 area control system. We defined here if you in the laplace domain this is your area 1. Here, this hole this is your area 1 and here this is your 2 and it is connected by your tie line. This is your area 2 and this is your tie line you can say the

model. Now, what to achieve the optimal control in the previous case we saw that these inputs that is change in  $p_c 1$  and the  $p_c 2$  were directly given here as input. So, the area control error where your change in you are the this power that is here  $p_1 2$  please remember.

So, here it is a not  $p_c$ . It is  $p_1 2$  plus  $b_1$  that is bias factor multiplied by the frequency of area 1. Similarly, we had this ACE why I am writing negative. It depends upon the value of  $k_i$ . If you are writing here minus  $K_i$  then it will be the positive value. So, it hardly matters. So, a  $c_2$  will be nothing, but, you change in  $p_2 1$  s plus here  $\beta_2$  change in your  $f_2$  s so, this where the control area signals and they were directly given here and then our loops were operating.

So, this is your up to this here it is your primary ALFC that is called primary ALFC loop. This is your called secondary. This here this loop is your secondary ALFC loop.

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Step 1:  
Conventional feedback loops are opened.

Step 2:  
Express the system model in to state space form

$$\dot{X} = AX + BU + FP \quad (n \times n, n \times m, n \times k)$$

State Vector,  $X = [x_1, x_2, \dots, x_n]^T, n = 9$  ( $x_1 \dots x_9$ )

Input Vector,  $U = [u_1, u_2, \dots, u_m]^T, m = 2$  ( $u_1, u_2$ )

✓ Disturbance Vector,  $P = [p_1, p_2, \dots, p_k]^T, k = 2$  ( $p_1, p_2$ )

Now, what we have to do first 1 is that we have to a step for the optimal control that I will state here. The conventional feedback loop should be open means; what we did. Here this loop we have opened. Now, it is not directly connected. So, we have just opened the loop and then express the system model into the state space form means; we have to represent that  $\dot{x}$  is equal to  $AX$  matrix plus  $BU$  plus  $FP$  u So, here a is we know that it is your state transition matrix. So, here we can express the system into the

state space form. That is  $\dot{x}$  must be equal to your  $AX$  plus  $BU$  plus  $Fp$  here  $A$  is your state transition matrix  $B$  is your input matrix and  $f$  is your disturbance matrix.

The states that is  $x$  here is known as the state factor and it is consisting of all the states from  $x_1$  to  $x_n$  and in this case we will see we are having ninety states. For example:  $y_1$  to  $y_9$  states you can say the now we are starting from here. So, this is your state 1 that is  $x_1$  here is state  $x_2$ . Now, why  $x_2$  because you can say there is AST 1 so, there is some differentiation of that  $x$  state, because we are writing here  $\dot{x}$  here there  $s$  laplace is there.

So, here we are going to another state. Now, this  $x_2$  here again we are having STP means; here  $x_3$  state. Similarly, here we can write  $x_4$   $x_5$  here  $x_6$  and then we have another state here corresponding to this input to this. Here we are having  $x_7$  here we are having  $x_8$  and here we are having  $x_9$ , because again here 1 upon  $s$  here from this. So, in total for the 2 area system we are having 9 states means;  $n$  is equal to 9. So, here I have written the states are now, 9 in number and they are from  $x_1$  to here  $x_9$  and already I have shown you the input vectors; what are the inputs where, this we are changing the generation.

The inputs are can be your  $m$  in number but, in this case  $m$  is equal to 2 means; only we are having  $u_1$  and your  $u_2$  what are those you can see here again. Input are nothing but, these are your inputs means; the inputs that we are talking this input in this case. Here what are the inputs that are coming here that is your  $p_c$ . That is here I can say it is your  $u_2$  and here the  $p_c$  1 is your  $u_1$ . Those are coming here as input. So, the  $u_1$  and  $u_2$  are the inputs. Here they are coming here. They are basically it is written something away.

So,  $u_1$  to this  $v_1$  this is your  $u_2$ . These are your 2 inputs that we are considering. So, your  $u$  vector here means that is  $m$  is 2 in this case. Now, another 1 is your disturbance vector that is  $p$ . It is your again it can be  $k$  in number, but, in this again we are having 2 disturbances means what are those, I can write here  $p_1$  to  $p_2$  and these are nothing but, the change in the loading and that is nothing but, here it is your  $p_{l1}$  and your change in  $p_{l2}$  are the disturbance factor.

So, whole this system here the states we cannot represent in terms of state space representation including the input factors  $u_1$  and  $u_2$ . We are having the disturbance factor here that is your  $p_1$  and here we are having is equal to  $p_2$  and we can write

completely the state space form. Here the main objective is why we are trying to do that normally for the optimal control they input is created by the linear. Here  $u_1$  and  $u_2$  are created by the linear combination of all the states means that is full feedback. Here what we are getting.

We are taking only 1 states here, we are taking only 1 state, but, in the optimal control design we have to take the linear combination of all the input state means; from starting from  $x_1 \times x_2 \times x_3 \times x_4 \times x_5 \times x_6 \times x_7 \times x_8$  and  $x_9$ . So,  $u_1$  will be combination of all with they different scaling factor means different  $k$  that is  $k_s$ . So, we have to now write the state space form and then we will see how we are going to. So, if you are taking input as the that is from all the state then it is called full feedback. It is not if you are connecting this 1 as  $x_2 u_1$  then it is not a full feedback.

So, we have to write now the state space representation of this model and we have to consider let us take first 1 here for writing for this state  $x_1$ . How we can write here we know that this is a transfer function of your governor. This is a transfer function of a turbine. This is the power system transfer function. So, what we are going to do. We are going to write what means; your  $x_1$  will be nothing, but, the input which is coming to this transfer function. That is nothing, but, here what is your bet  $x_3$  is the state which is coming here means we are getting minus  $x_3$  by  $r_1$ ; why minus because, here the minus sign is there plus your input is  $u_1$ .

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Handwritten mathematical equations for a power system model, showing state equations (1) through (4) and a feedback equation (5). The equations are:

$$(1 + sT_{G1})x_1 = \Delta P_{11} - \frac{x_3}{R_1}$$

$$\dot{x}_1 = -\frac{1}{T_{G1}}x_1 - \frac{x_3}{T_{G1}R_1} + \frac{U_1}{T_{G1}} \quad (1)$$

$$\dot{x}_2 = -\frac{1}{T_{T1}}x_2 + \frac{x_1}{T_{T1}} \quad (2)$$

$$\dot{x}_3 = -\frac{1}{T_{P1}}x_3 + \frac{K_{P1}}{T_{P1}}x_2 - \frac{K_{P1}}{T_{P1}}x_1 - \frac{K_{P1}}{T_{P1}}P_1 \quad (3)$$

$$\dot{x}_4 = -\frac{1}{T_{G2}}x_4 - \frac{x_3}{T_{G2}R_2} + \frac{U_2}{T_{G2}} \quad (4)$$

$$U_1 = -\frac{1}{r_1}x_3 + \frac{r_2}{r_1}U_2$$

There are additional handwritten notes and arrows indicating relationships between the equations and variables, such as  $\Delta P_{11} = x_1$  and  $\Delta P_{22} = x_2$ .

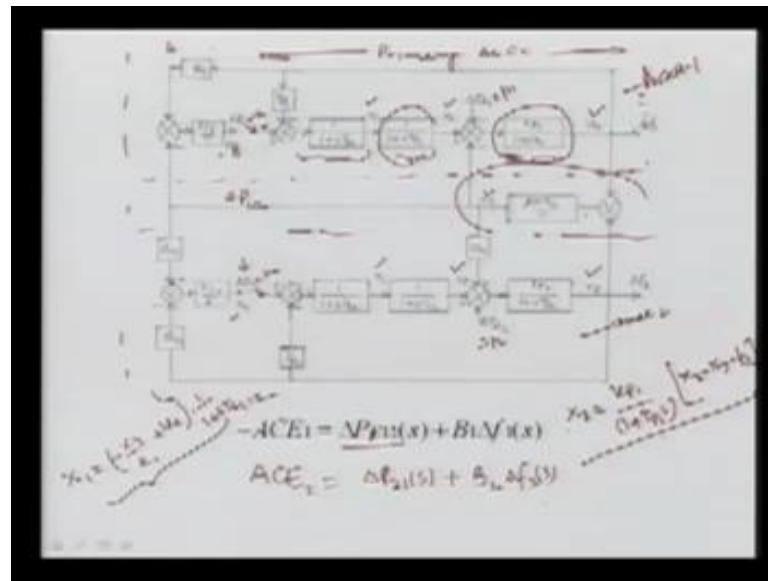
Here that is multiplied by this transfer function and that transfer function is nothing, but, your  $\frac{1}{1 + sT_g}$  from here we can easily write this expression. Here we can write  $\frac{1}{1 + sT_g} \times$  is equal to change in here  $p_c$  that is nothing, but, your  $u_1$  we have taken and minus  $x_3$  upon  $r_1$  from here what we can do. Here now, you can again simplify this by what we can do. We can write this  $\frac{1}{1 + sT_g} \times 1$  is equal to I can write minus  $x_1$  minus  $x_3$  upon  $r_1$  plus  $q_1$ .

So, what is this  $s \times 1$  is  $x$  dot. So, I can write here  $x_1$  dot and this  $\frac{1}{1 + sT_g}$  will be divided. So, I can write here  $x_1$  upon  $\frac{1}{1 + sT_g}$  minus  $x_3$  upon  $r_1$   $\frac{1}{1 + sT_g}$  minus  $u_1$   $u_1$  upon  $\frac{1}{1 + sT_g}$ . So, this is the first state  $x_1$  state space representation and that is retained here you can see this; that is related with the equation 1. Similarly, we can write for equation 2 also equation 2 you can see again how we have related this  $x_2$  with related to your  $x_1$  means; I can write here this  $x_1$  or you can  $x_2$  will be equal to your  $x_1$  state multiplied by this transfer function here and that is nothing, but, your  $\frac{1}{1 + sT_1}$ .

This will be multiplied here and then, we can write we can write here that this state expression 2 means; we denoted that your  $x_2$  here  $\frac{1}{1 + sT_1}$  is equal to your  $x_1$  means we can write sorry this is not dot because  $s$  will be coming here. So, I can write  $s \times x_2$  will be equal to minus  $x_2$  upon  $p_t$   $\frac{1}{1 + sT_1}$  plus  $x_1$  upon  $p_t$   $\frac{1}{1 + sT_1}$  and here this you can be replaced and we can write  $x$  dot and you can see thus we have written this equation 2. Similarly, we can write for  $x_3$  dot and  $x_3$  dot is nothing, but, what we are getting we are getting from here.

Now, this is it is not doubt it is very important. Here we can see how many inputs are coming to this transfer function. That is your power system transfer function here including your load release etcetera. For this we can write here this  $x_3$  will be all the inputs here algebraic sum multiplied by this transfer function.

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So, I can write here, I can write a very easy way that your  $x_3$  will be nothing, but, this transfer function I can write this  $k p 1$ . Here  $1 + k p 1 s$  and then multiplication of all here that is input state is your  $x_2$  that is positive minus  $x_7$  that is going here which is  $x_7$  and minus  $p 1$ . So, here we have to write in this fashion. So, we are getting  $x_3$  state in terms of  $x_2$   $x_7$  and the disturbance vector of  $p 1$  why we are writing  $1$  because we have considered this is your area  $1$ . This is your area  $2$ .

So, the  $2$  denotes area  $2$  and  $1$  that is subscript denotes that  $1$ . So, what we can do. We can again multiply here and then we can write simply this  $x_3$  dot or in other words simply I can say we can this equation for  $x_3$ . That  $x_3$  dot will be equal to minus  $1$  upon  $t p 1$   $x_3$  you can say always this  $x_2$  is here it is  $1 + s t$  is there. So,  $1$  term will be with negative of this plus  $k p 1$  upon  $t p 1$   $x_2$  minus  $x_7$  was there. So, this coefficient will be arising plus minus  $k p 1$  upon  $t p 1$  and it is equation  $3$ .

Similarly, we can write the equation  $x_4$  and this is similar to your equation  $1$  why you can see here from this figure this  $x_4$  is just exactly similar to your  $x_1$  here. This  $x_4$  is the similar to  $x_4$ , only the difference is that here the parameters are  $t g 2 r 2$  and your  $f 2$  or you can say a state  $x$  is coming. So, I can simply say that we can from the equation  $1$  we can write equation  $4$ ; that is your  $x$  dot  $4$  will be equal to minus  $1$  upon  $t g 2$  here earlier  $t g 1$  was there  $x_1$  was replaced by  $x_4$ . Here  $x_3$  in this case it is  $x_6 t g 2$  into  $t g r 2$  plus here your now input vector is  $2$  divided by  $t g 2$ . So, we are getting equation

number 4. Now, similarly, we can write the state space representation of x 5 and that will be similar to your x 2 and your x 6 here will be similar to your x 3.

So, this is for your area 2 here, this 1 to 3 or for area 1 and we will see x 7 is your tie line bias tie line control. So, we can write this x 5 similar to you're the previous 1 I can say simple you can see this is your x 5, I can even though right from x 2 this x 5 dot will be minus 1 upon t t 2 x 2 is now equal to your x 4 no it is x 5 and plus your x 1 is your nothing, but, your x 3, in this case upon t t 2; now, here x 5. So, it will be x 4 because 3 is for area 1. So, it will be x 4 here.

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The image shows handwritten equations and a block diagram. The equations are:

$$\dot{x}_5 = -\frac{1}{T_{12}}x_5 + \frac{x_4}{T_{12}} \quad (5)$$

$$\dot{x}_6 = -\frac{1}{T_{p2}}x_6 + \frac{K_{p2}}{T_{p2}}x_5 - \frac{K_{p2}}{T_{p2}}\omega_{12}x_7 - \frac{K_{p2}}{T_{p2}}P_2 \quad (6)$$

$$\dot{x}_7 = 2\pi f_{12}x_5 - 2\pi f_{12}x_6 \quad (7)$$

$$\dot{x}_8 = -K_{A1}x_8 - B_1K_{A1}x_5 \quad (8)$$

$$\dot{x}_9 = -K_{A2}\omega_{12}x_7 - B_2K_{A2}x_6 \quad (9)$$

Below the equations is a block diagram showing the state vector  $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix}$  and the input vector  $\begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$ . The diagram shows the relationship between the states and the inputs, with feedback loops and gain blocks. There are also handwritten notes on the right side of the diagram, including  $\dot{x}_7 = 2\pi f_{12}(x_5 - x_6)$  and  $\dot{x}_8 = -\frac{K_{A1}}{s}(x_5 + B_1x_8)$ .

So, you can see here simply this x 5 dot is equal to minus 1 over t t 2 x 5 plus x 4 upon t t 2. So, we are getting equation number 5. Similarly, we can write the x 6. That is related with your x 3 of area 1 and we can replace all this 1 subscript here by 2. So, you can say minus 1 over t p 2 x 6 plus 1 p 2 upon t p 2 x 5 minus k p 2 upon t p 2 a 1 2 into x here another parameter is appearing due to the change in the base minus k p 2 upon t p 2 into p 2. That is another disturbance vector that is occurring in area 2.

So, we are getting this equation number 6. Now, we have to again write this further state 7 in state 7 we can see again here what we are getting you will you saw earlier 1 means x 7 was related. Here it was the 2 pi t 1 2 divided by s and the inputs are coming here you x 3 minus x 4. For example you can see here very easily with this block diagram. Here x 7 is equal to multiplication of the input that is coming here. So, it is positive here it is



negative. So,  $x_3$  minus  $x_6$  multiplied by this here and we are getting  $x_7$ . So, we can write very easily this equation and I can say simply here  $s$  will be coming here and this will be cancelled out.

So, I can write  $\dot{x}_7$  it is your  $2\pi \times 10^3 \times x_3$  minus  $2\pi \times 10^3 \times x_6$  and you can see this equation we have written here you can see; the  $2\pi \times 10^3 \times x_3$  minus  $2\pi \times 10^3 \times x_6$  that is for your tie line model. Now, we are having 2 another 1 that we are going to have some integral controller and that controller are denoted by for area 1 it is  $x_8$  and another state  $x_9$  for your area 2 and then we can similarly, we can write and we know that this  $x_8$  here is related with nothing, but, your  $K_i$  upon  $s$ . Here with the minus sign plus we are getting the state for the 1 case ‘

Here we are getting your  $x_3$  multiplied by  $b_1$  plus here that is your coming  $x_7$  for example, you can see what I am writing. Let us see again this block diagram here. So, this  $x_8$  will be equal to the input which is coming here as well as here. So, what we are getting we are adding here this and this together. Then multiplied by this negative  $K_i$  we are getting the positive. So, this  $x_3$  multiplied by  $b_1$  and this  $p_1 \times 2$  that is  $x_7$ . Here both are added with the negative sign then multiplied by  $k_i$ . So, we are getting that.

So, we can write here for  $\dot{x}_z$  is minus  $K_{i1} \times x_7$  minus  $b_1 K_{i1} \times x_3$  that is for integral controller of area 1. Similarly, we can write this  $\dot{x}_9$  for integral controller of area 2 and having the bias factor  $b_2$  and the integral  $K_{i2}$  that is  $K_{i2} \times a_{12}$  as again it is appearing due to the difference in the bases of area 1 and area 2. If both are equal bases having equal bases then  $a_{12}$  will be minus 1. So, all these means we are having a states from 1 to 9 that can be written as the matrix  $a$  having the state's 1 to 9 plus matrix  $b$  here and inputs are  $e_1$  and  $e_2$  and the disturbance vector here your  $p_1$  and  $p_2$ .

Now, the order of this matrix will be your 9 cross 9; that is state transfer matrix. The order of  $b$  will be your 9 cross 2 and this disturbance vector will be having 9 cross 2, because we are having 90 states and the 2 disturbance here similarly, for this 1. So, we can now from all these 9 equations we can represent the matrix  $a$  as follows and this will be your matrix  $a$ .

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$$A = \begin{bmatrix} \frac{1}{T_m} & 0 & -\frac{1}{T_m R_0} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{T_n} & -\frac{1}{T_n} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{K_{p1}}{T_n} & -\frac{1}{T_n} & 0 & 0 & 0 & \frac{K_{p1}}{T_n} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{T_{o1}} & 0 & -\frac{1}{T_{o1} R_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{T_{r1}} & -\frac{1}{T_{r1}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{K_{p2}}{T_{r1}} & -\frac{1}{T_{r1}} & -\frac{K_{p2}}{T_{r1}} \omega_a & 0 & 0 \\ 0 & 0 & 2\pi T_{11} & 0 & 0 & -2\pi T_{11} & 0 & 0 & 0 \\ 0 & 0 & -B_1 K_1 & 0 & 0 & 0 & -K_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -B_1 K_1 & -K_{11} \omega_a & 0 & 0 \end{bmatrix}$$

So, we are getting this state transition matrix a here that is you can see the beauty of this matrix is that we are getting this matrix most of the elements are 0. So, this matrix is not full. So, you can see in 1 row we are getting 2 elements 2 elements here 3 elements. Here 2 element 2 element 3 elements 2 elements and 2 elements near the m 2 i 1. So, in the most of the cases we are getting 2 elements and the 2 rows only we are getting 3 elements and the remaining here they are 0.

So, now we can count how many elements we are getting in this state transfer matrix. So, this is very sparse highly sparse matrix. For example you can say we are getting 2 here 2 here we are getting 3 here. We are getting 2 here elements 2 3 2 two and 2. So, you can say here we are getting only twenty elements in total out of here eighty 1 elements. So, we are getting 7 t 1 elements 0.

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$$B = \begin{bmatrix} \frac{1}{T_m} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{T_m} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } F = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{K_{ps}}{T_n} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Similarly, if you will see the b max b max is you can say only you are getting 2 elements. Here corresponding to t g 1 and t g 2 that is coming and your f disturbance matrix also we are getting the 2 elements here and other here.

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Step 3:  
Generate control inputs  $U_1$  and  $U_2$  by means of feedback of all the states

$$U = Kx$$

Second form is  $\dot{x} = Ax + Bu$  which does not contain the disturbance vector. For constant disturbance vector  $P$ :

$$0 = A\bar{x} + B\bar{u} + P$$

$$\bar{x} = \bar{x}' + \bar{x}_s \quad (\text{Transient} + \text{Steady state})$$

$$\bar{u} = \bar{u}' + \bar{u}_s \quad (\text{Transient} + \text{Steady state})$$

$$0 = A(\bar{x}' + \bar{x}_s) + B(\bar{u}' + \bar{u}_s) + P$$

$$\bar{x}' = A\bar{x}' + B\bar{u}'$$

$$0 = A\bar{x}_s + B\bar{u}_s + P$$

Now, after we have represented the system by opening the loop we are representing the system into the state space representation then in step 3 to design the optimal controller we have to generate the control inputs  $u_1$  and  $u_2$  by means of a feedback of all the states means; we have to write we have to take the states of all this  $x_1$  to  $x_9$ . Then I can

say this  $u$  vector will be your  $k$  multiplication  $x$  dot you know this  $x$  is 9. So, it will be your  $2 \times 9$  matrix and you can see means your  $u_1$  will be nothing, but,  $k_{11}$  multiplied by  $x_1$ . Here  $k_{12}$  multiplied by  $x_2$  till  $k_{19}$  multiplied by  $x_9$ .

So, we are getting the linear combination of all the states and then we are fitting to the input 1 here and again you can say only the varying quantity is here the gains we are deciding and we have to calculate the gains  $k_{11}$  one to  $k_{19}$  and again here for the second input also we are getting  $k_{21}$   $k_{22}$  and  $k_{29}$  means; we are getting again the 9 here gains and 9 gains here. So, we have to calculate the gains optimally. This now we are having eighteen gains eighteen then we have to calculate.

So, here let us see what is meaning of this input. It means; that whenever we are giving here your that is input that is your  $p_c$  1 that is  $u_1$  it is nothing, but, your change in  $p_c$  1 reference. So, this is basically we are getting the summation of all the states and that is coming with the here gain I can say  $k_{11}$   $k_{11}$  and that is coming here. Let us suppose is your  $x_1$ . Similarly, we are getting  $x_2$ . Here your gain is your  $k_{12}$  and here adding and similarly, we are adding  $x_9$  with the gain  $k_{19}$  and then finally, it is here added.

So, here we are adding all the states with the some constant and then we are fitting to here. So, this is the way that we are designing 1 thing we saw in state space form as we had vector like  $\dot{x}$  is equal to your  $AX$  plus  $BU$  plus  $f_p$ , but, in form always we write  $\dot{x}$  is equal to  $x$  plus  $v$   $u$  means there is no disturbance vector. So, we have to eliminate this vector and then we have to go for the form. Then only we can design the optimal controller.

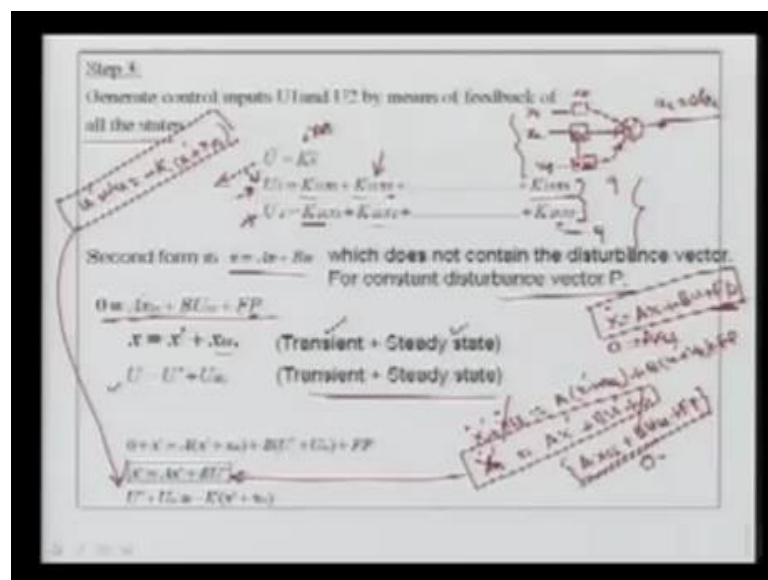
For that we know that the for the constant disturbance vector  $p_i$ ; if the disturbance vector is constant the your  $\dot{x}$  here that is the matrix here  $\dot{x}$  will be equal to 0 means; now, we will get the steady state states that is  $x_{all}$   $x$  factor will be having a steady state and also your input will be having the steady state values. So, for constant disturbance vector this means we have seen some loading exposed once fixed what will happen you are the time change in the states will be 0 because, all the states will attain the steady state condition.

So, here  $x$  this will be 0. Now, it will be your  $AX$  here  $s$   $s$ , but, we have written here means a multiplied by your steady state states plus  $b$  your input states. That is coming from your all the inputs states plus here  $f$  multiplied by here the  $p$  disturbance vector.

Now, we know that  $x$  a steady state during the transient condition means at any state we are getting the this  $x$  is the 2 component 1 is called  $x$  prime that is a transient and another we are getting  $x$  s; what happens when the transient is die out means this component becomes 0. So, we are getting the steady state value.

So, we can say state here can be represented by the 2 terms, 1 is you transient and another is your state value that is a steady state. Similarly, we can write they input vector  $u$  that will be equal to the transient input plus your steady state input and we can delineate into 2 different transient and steady state values. Now, we can put this value here in your condition. We can put here in this expression in this steady space form of our proposed state space representation.

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Now, what we can do, this  $x$  will be represented by your different value just what I am going to do, I am going to put here this  $x$  dot is nothing, but, your  $x$  prime plus  $x$  s s and here I am going to put dot means the time varying quantity. Similarly, I can write here  $a$ . Now,  $x$  prime plus  $x$  s s plus here I am writing  $B U$  prime plus  $u$  s s plus of course,  $x$  plus  $p$  what is happening is if you are taking the laplace here time domain of a steady state, it will be 0 as we know and now, we are going to represent we are using from the previous case means; I can write here  $A X$  prime plus  $B U$  prime.

Now, plus I am writing another factor here that is remaining that is  $A X$  s s plus  $B U$  s s plus your  $F P$  another factor and this is equal to your  $x$  dot prime; what happens. This is

nothing, but, 0 you can see from this expression because during the steady state always we will get this relation because the transient is die out. So, this is equal to 0. So, we are getting x here this is your 0, it will vanish. So, we can get this expression here simply that is  $\dot{x}$  is equal to  $Ax + Bu$ . Here we are writing only for the transient case, means; a steady state is vanished out and then we have to design the controller accordingly.

So, what we decided here in the previous case again from here we can write your  $u$  prime plus  $u$  steady state that will be equal to your minus  $y$ ; I am writing minus, because here we are taking the integral controller with the negative input of area because once it is increasing we have to decrease the frequency rises. We have to decrease the output, if the frequency falls we have to increase the input. So, the negative sign is used and then it is the x matrix that is your  $\dot{x}$  plus  $s$  we know it very well.

So, we can use here this relation again in terms of your transient as well as in a steady state conditions and then; we can for the optimal control system the state and control vector should have the 0 a steady state inputs means; for optimal control the system state and the control vectors should have 0 a steady state value. We do not want that this state should change here and there.

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$$\dot{x} = Ax + Bu + w$$

$$J = \int_0^{\infty} (x^T Q x + u^T R u) dt$$

$$K = -(R + B^T P B)^{-1} B^T P A$$

$$P = -\int_0^{\infty} e^{A t} Q e^{A^T t} dt$$

For optimal control the system states and control vectors should have zero steady state value, so

Step 4: The feedback matrix (K) is to be determined.  $P = \int_0^{\infty} (x^T Q x + u^T R u) dt$

Where Q & R symmetrical matrix and determine through design consideration. K is obtained from the reduced matrix RICCATI equations.

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- $K = -R^{-1} B^T P$
- $A^T P + P A - (P B + Q) (R + B^T P B)^{-1} (B^T P A + Q)$
- $P = -\int_0^{\infty} e^{A t} Q e^{A^T t} dt$
- $A^T P + P A - (P B + Q) (R + B^T P B)^{-1} (B^T P A + Q)$
- $u = -Kx$
- For stable all eigen values of  $A - BK$  should have  $-ve$  real parts.
- $\lambda = -\alpha \pm j\omega$
- $\omega = \sqrt{\alpha^2 - \beta^2}$

So, what we will get here already we defined this. So, for the optimal control the system state and the control vectors should have the 0 a steady state value then, what we can do

means this is 0. This is 0. So, I can write  $u$  prime is equal to minus  $k$  into here again these are the vectors and this  $x$  prime. So, the  $u$  transient will be equal to minus  $k$  time  $x$  transient. Now, in a step 4 now, we have seen this we require this matrix  $k$ .

So, the feedback  $k$  is to be determined and normally what we do; we use this  $p$  i that is 1 performance index here and that is here integration of a performance here performance index. That is your  $x$  prime into your  $q$   $x$  t here prime  $u$   $r$   $u$   $t$   $d$   $t$  where  $r$  and  $q$  are the symmetrical matrices and determining through the design consideration  $k$  is obtained from the reduced riccati equation. So, the normally what we do certain performance index is minimized and that performance index here is called  $p$  i.

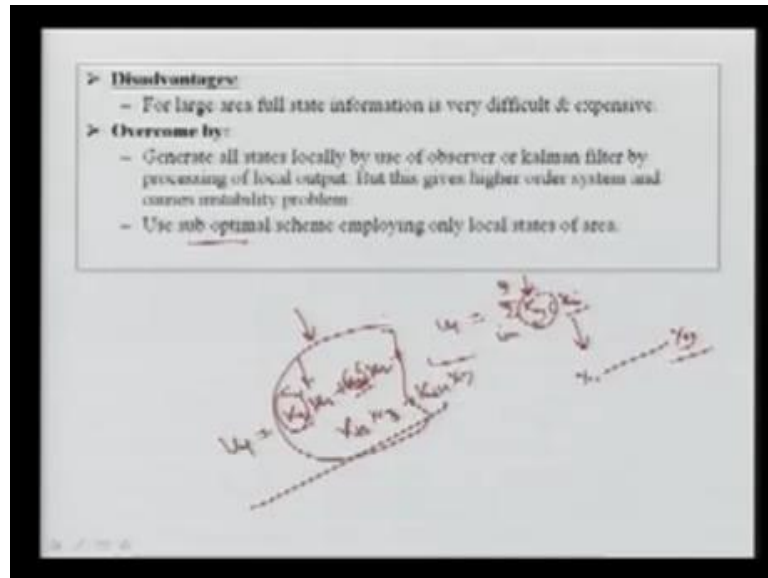
It is not a proportional integral. It is a performance index and that here we integrate from 0 to certain here value. That is we go for infinite basically and then it is your  $x$  prime into  $q$   $x$  transpose plus here and these 2  $q$  and  $r$  matrices are the symmetrical matrix and they are determined through the design considerations. The value of  $k$  is obtained from the reduced matrix of riccati equation and the riccati equation is here is nothing, but, your this is riccati equation.

That is your I can say a transpose now, I can  $s$  plus  $s$  into a minus  $s$   $b$   $r$  inverse  $b$   $t$   $s$  plus your  $q$  is equal to 0 means; knowing the value of normally, we try to minimize this and by that minimization get the  $q$  and  $r$  values. If you are putting the  $q$  and  $r$  matrices here  $b$  matrix is fixed  $a$  is also known to you here. It depends upon again state space, we formed that. Now, from this equation we require the matrix  $s$ 0020. So, means; matrix  $s$  must be determined and then we can relate this what is the  $k$  in this case;  $k$  will be the matrix will be nothing, but, your  $r$  inverse.

Now, your  $b$  transposes. Now, it is your  $s$  means once you are getting  $s$  matrix you can get your  $k$  matrix. So, for all stable cases all the Eigenvalues of a minus  $b$   $k$  should have the negative real part. Now, what is happening you can see we designed here  $x$  prime that is equal to your  $A$   $x$  prime plus if you remember here we got  $B$   $u$  prime. Now, we saw this  $u$  prime is nothing, but, minus  $k$   $x$  prime you remember here already, I have written this value here.

So, if you replace this we are getting a minus  $b$   $k$  matrix here and then it is you  $x$  prime. So, this here this for all the stable here we should have the Eigenvalue of this should have the negative real part. If it is not having real negative part then system will be

unstable. So, we want that stable system. So, we have to take the Eigenvalue here means; the value of  $k$  should be such that we should have this negative Eigen means real part of Eigenvalue should be the negative.



So, what is happening you have to have the information gains we determined of course, no problem, but, this  $x$  state that is  $x_1$  to  $x_9$  to determine it is very difficult sometimes and also it is very expensive sometimes; if you want to obtain the state of the governing system we want the state of turbine system already, we have used you know this very simple model. For the turbine we have used very simple. We have not used the re hitting another thing.



this disadvantage normally, we generate all the state locally by use of observer or kalman filter and then by processing the local output.

So, we can generate this state means; we cannot if then we have to generate this state because we have decided these optimal gains, we have to use this. So, we can generate these using the kalman filter all local observer, but, this gives higher order system and cause instability problem. Now, what is happening we are designing another kalman filter. So, it will again some states will be included and then that may cause the instability of the system.

That is again very dangerous or what, we can do we can use sub optimal scheme implying only the local state of that area means here instead of going for all these case what I can say now your  $u_1$  can be simply I can say  $k_{11}$  another constant. Now, I can say here your  $x_1$  plus  $k_{12} x_2$  and plus here another state here it was  $k_{13} x_3$  you can say  $x_3$ . So, only the 3 states this time considered or utmost we can consider another state I can say  $x_4$ . Here that  $x_7$  which I was using.

So, for 1 area we can use the local signal that is very easy. For example we have the 2 area. Let us suppose, we are the designing the controller and another state let us suppose for Rajasthan and taking the input signal for the generators from very remote place and here taking there will be lot of errors lot of noises. So, it is very difficult as I said and expensive too to get that signal. So, it is possible that we can go for the suboptimal system that is also good because we are having some optimal controller here. Then we have to take the design the system in a such way and we have to choose these gains and we have to take the states locally.

So, if it is difficult to get the optimal controller then we can go for sub optimal control and that is good enough for that. So, what we saw in an area there are too many generators and then we have to decide we have to allocate the load change in that particular area. Suppose, there is a hundred mega watt load increase in 1 of the area then this hundred megawatt must be shared by all the generators and then, we have to decide how to it will be sharing and that basically, is based on some participation factor  $p$  of normally, we called it is not a power factor.

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**Generation Allocation:**

- ✓ Each area consists of many generating units
- Output of each unit must be according to economics
- ✓ System load is always changing
- Economic load dispatch must be done

Individual generator output = base point + pf \*  $\Delta P_{\text{Total}}$

$$pf_i = \frac{\Delta P_{i, \text{area}}}{\Delta P_{\text{Total}}} = \frac{\text{change in generation at } i^{\text{th}} \text{ unit in a area}}{\text{Total change in generation required in that area}}$$

$\sum pf_i = 1.0$

$$P_{i, \text{demand}} = P_{i, \text{base}} + pf_i * \Delta P_{\text{Total}}$$

$$\sum P_{i, \text{demand}} = \sum P_{i, \text{base}} + \Delta P_{\text{Total}}$$

$$\Delta P_{\text{Total}} = P_{\text{area, Total}} - \sum P_{i, \text{base}}$$

*Handwritten notes:*  
 $P_1 = 100$   
 $P_2 = 200$   
 $P_3 = 300$   
 $P_4 = 400$   
 $P_5 = 500$   
 $P_6 = 600$   
 $P_7 = 700$   
 $P_8 = 800$   
 $P_9 = 900$   
 $P_{10} = 1000$   
 $\Delta P_{\text{Total}} = 100$   
 $P_1 = 100$   
 $P_2 = 200$   
 $P_3 = 300$   
 $P_4 = 400$   
 $P_5 = 500$   
 $P_6 = 600$   
 $P_7 = 700$   
 $P_8 = 800$   
 $P_9 = 900$   
 $P_{10} = 1000$

It is participation factor. So, each area consists of many generating units. So, there is output for each unit must be according to economics means we have to set their units. So, that the chief units must generate the full loading then expensive generators should come later on because, always here we have to generate with the minimum cost of generation of electricity and we also note that the system load is always changing. So, if it is always changing the loading should also keep on tracking that and the loading should also change.

So, here the economic load dispatch must be done to meet that load demand in the economic way. So, the individual generator output can be decided at the base point; what is present below that plus some participation factor multiplication by the change in the total load. So, let us suppose this hundred megawatt load is changed. So, if the participation factor of that 1 is 0.1 the 1 multiplied by hundred means it is here it is point. Let us suppose, the p of 1 is 0.1 and you have change in the load here.

Let us suppose 100 megawatt. Then we have to multiply this means the 10 megawatt will be changed from the base loading based loading means; where the generator is loading right now. And now, there is some change means whether increase or decrease then; we have to increase and decrease according to the participation factor. And that participation factor is defined for any highest unit, I will be equal to change in the generation in

highest unit. How much you are going to change divided by the total change in that area total change in the load.

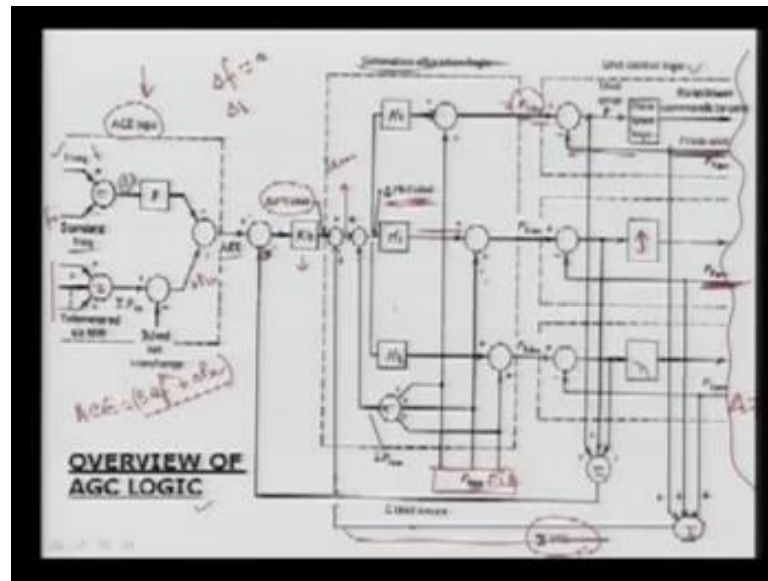
So, that is I have written here change in generation at highest unit in area divided by total change in the generation required in that area always this participation factor must be unity why we can see this. Let us suppose this is  $p_i$  is defined as your change in your  $p_i$  divided by your change in here  $p_{\text{total}}$ . Now, if your sum is sum in this what is happening you are summing this summing this is nothing, but, the generation of all the units that is nothing but, is  $p_{\text{total}}$ . So, we are getting change in  $p_{\text{total}}$  divided by change in  $p_{\text{total}}$  and this is you unity.

So, the total of the participation factor suppose your units are having 2. So, the participation factor 1 is let us suppose 0.1 and other  $p_2$  is let us suppose 0.6. So, your  $p_1 + p_2$  will be automatically, here 1 here 0.7 means; 0.7, means we have to 1 minus here the participation factor this plus this means 0.1 minus 0.6 here. So, we are getting sorry; it is 0.3. So, the participation factor is always related with the summation of the participation factor of all the units that will be equal to unity.

So, I can say this is now, designed with the change in the loading can be written as the  $p_i$  base which is loaded right. Now, plus the participation factor of that here multiplied by the change of the total load in that area, it will sum together means sum of  $i \times \Delta p_i$  means all the units if you are adding the desired change. Here again we are going for the desired base plus here the  $p_{\text{total}}$ . Now, this  $p_{\text{total}}$  is nothing, but, what is that. Here change in  $p_{\text{total}}$  is a total power the new total how much load system and what is right now we are loading.

So, this here the  $p_{\text{new total}}$  means; load that is increased from the base case minus which was the highest means; that is change in the total load and then we can relate with this equation.

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Now, we can again relate this 1 in this overview of AGC logic. So, let us see the overview of AGC logic and that is based on your base point and participation factor method; what we do this is for the single area this figure shows for the single area. We have here the single area and we are this is area 1, I can say and we are having some measurements using that elementary data we are measuring the system frequency here from this area and also we are measuring the tie line power from all the here tie lines.

So, the meter as I have written the telemeter type power flow megawatt that is sum together and now, we are this total sum of the tie line that is flowing from the area 1 to its sum together and then; we are just subtracting. That is the reference value. That is how much is said that will go from that area. So, here that is subtracted. So, whatever we are getting here change in from you can say area to other areas. This is p 1 2 if you remember.

Now, that frequency we are measuring and we are having some standard or rated frequency. That is you can say  $f_{naught}$  and this is your  $f$  we are measuring from the system. If there is any difference here what we are getting change in the frequency and that is multiplied by the bias factor  $b$  and here both are here added together and then finally, it is your area control error that we have defined in the previous case. So, this is ACE logic, I can say means area control error logic.

Here we are not going for the optimal control. It is a normal control and we will see how we are incorporating other base point loading here. So, this is your ACE logic means we are measuring the system frequency and the tie line power flows and then, we are getting the ACE and come to here as you it is very standard and already we have defined this ACE is nothing but, your change in  $f$ . That is multiplied by  $b$  plus your change in  $p$  1 2. Here again we are adding we can minus, because we are using the integrator because change in the frequency we require more power.

So, here I have now what we are doing this is coming here. We want to this is your error. Now, we can also check at the same time. We are measuring the real powers of all the generators that how much error in this your present unit and the desired 1 means; we want that this must be desired. There is some error in the previous state means; previous condition previous time that what is the error from your actual that is the measurement from the unit here and the desired 1.

So, that difference here for all the units here we are having  $n$  units you can say. So, they are adding together here and that is now, here again added together here that error and then this ACE is going. Here why you are adding here this will be your negative sign because we are taking here the positive. So, that will be added again because we want more generation. So, here the  $k$  we have taken positive then it is your negative. If you are taking  $k$  negative then; it will be negative and this will be your positive and then we have to take care.

So, now, we are integrating your using that is an integrator that is a reset controller and finally, here. That is your change in the  $p$  total we require here. Now, this  $p$  total what we are going to do. We are also measuring what is the current loading of the system then; current loading of the system that is the summation of all the  $p_{gi}$ 's here they are coming and finally, it is added together. Then here we are getting that value. That is called you are the  $p$  desired.

Here we are getting some value here and then, this value we are getting the  $p$  desired and this  $p$  desired value is now subtracted from the  $p$  base. As I said the  $p$  base we sum together. Here the  $p$  base we are taking how much we have to set the  $p$  base. Basically, this  $p$  base is nothing but, it comes through economic load dispatch. We know that here is the demand, how much we have to load so, that we can achieve the economic loading.

So, this  $p$  base is basically decided again based on these generators. This  $p_{gi}$  and we can get this  $p$  base for individual.

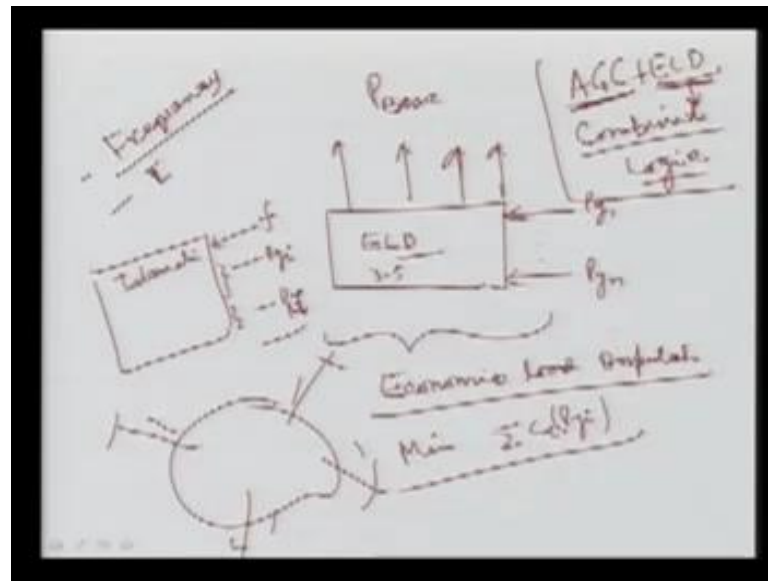
So, this is now subtracted from here. Now, the  $p$  total that we require is nothing, but,  $p_{\text{desire}}$  minus summation of  $p$  base as  $i$  in previous equations we defined. So, now the change, in the total power in that area is distributed over the various generators from 1 to  $n$  here and again; we are using the participation factor means for 1 generator is a  $p_{f1}$  value  $p_{f2}$  value  $p_{f3}$  and so on, so forth. And then it will and then finally, we are getting here the values of 1 here and that value is going to be added with the  $p$  base.

That will give you the  $p_{\text{desire}}$  of that 1 with this change plus the  $p$  base will be giving the desired of 1 and then this desired 1 is compared, with the actual 1 and then; we are giving ways and that is input signal to the corresponding generator; so, that whether we have to increase or decrease accordingly. Similarly, for here for this unit also this coming after the  $p_{f2}$ ; we are adding with the  $p$  base 1. So, 2 here that we are getting the  $p_{\text{desire}}$  unit and then we are comparing the actual here.

This is actual and then, we are giving the are over to change that 1 and this is this area basically, it is called generation allocation logic means how we are allocating the generation. So, it is written here it is generation allocation logic. If you are going at particular unit and we are changing the unit based on the required logic that is called your unit control logic and you can say for each unit we are having 1 logic. So, you can say that is the in dotted line you can say unit 1 unit 2 unit 3 and so on, so forth.

So, we are 3 logics ACE logic. This generation allocation logic and the unit control logic logic, here we have not shown here that is the  $p$  base which we calculate. It is nothing, but, we just solve the economic load dispatch and let us see how it is coming. So, here the  $p$  base the output, I will show here in the next this; I will draw a block means; what we do.

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Here this is your economic load dispatch, what we do. We measure the current this  $p_g$ . Here too your all the inputs are measured telemetered. Since, all the generations we are getting the input that is we are measuring. Now, here we are doing the economic load dispatch. We know the total sum total demand of the generation and based on that we can now, the output of this  $ELD$  will be your that is called  $p_{base}$ . This economic load dispatch is normally done 3 to 5 minutes in every interval and then the  $p_{base}$  is set.

So, this is called your economic load dispatch logic here our intention is to minimize the total cost of the system. The total cost of generation basically we are talking here economic dispatch. So, it is a summation of the cost of here the generation of  $i$ . Here, I can say that is function of real power output and then we have to minimize subjected to the various constraint. That is the power balance equations. So, this is your economic load dispatch logic. So, what we did; we had another unit. That is called the telemetry.

Here telemetry in telemetry, we are measuring the system frequency. That is only 1 input. We are measuring here the power that is  $p_g$  i's of all the generation of that unit and we are measuring here the tie line. That is is going from that area means this is your area. We may have different tie lines here. So, how much net schedule; that means, we can measure this and this and then; we can decide that, I can say  $I_1$  from area  $i_2$  1 and then, we are processing here and we are doing for this logic.

So, this is if you are combining this ELD then, it is called your AGC included, with your economic load dispatch. Here,, combined logic here we had this AGC that is automatic generation control plus economic load dispatch logic we are concluding then; it is called combined logic of AGC and ELD. So, we know that whenever, you are changing why the ELD included here, because economic load dispatch means; you are changing the real power and once you are changing the real power then the load is changed frequency is changed.

So, it is related with complete frequency control and we discuss. Here other than this economic load dispatch there is another is called optimal rate power dispatch. That optimal rate power dispatch is changing voltage or you can say the reactive power generation. Thereby, we are changing voltage and the system losses and the injections are different 1 and then we will say this loop. That is your voltage control loop of the alternator we will discuss, in the our next lecture of the same module.

So, now I can recap what I discussed, in this first 6 lectures that was related to your frequency control. In frequency control frequency control, in this we model the various components those are related for the frequency calculation of the system means; we model governor turbine power system and then; we saw that we require some controller because the frequency error due to the certain disturbances change in the loadings in that area it is not 0. We propose our p i integral controller that is reset controller for 1 area.

Then later on we move that if the areas are interconnected. As we, saw the various advantage and disadvantages for having the interconnections. So, then we model the tie line power model and then; we analyze for the 2 area case having the tie line concept. Then we also moved ahead to design sub optimal controllers. So, that we can have the stable as well as optimal control logic and then we define that 1 as well. Finally, we just realize with the diagram here, with this diagram that how we can achieve this complete over view of a g c logic including your economic load dispatch.

We saw that for any particular area the major objective is your nothing but, here that AGC objective is that the total system frequency are very close to its normal value means; we have to operate, with value and to maintain the correct value of interchange power always we have to be always that, we have to maintain; that means, there will be as I said this change in the frequency error should be 0.



Change in your tie line power flow from any particular area must be 0 and then means; whenever, there is a change we have to meet that load from that area only. So, these are the basic AGC logic. In economic load dispatch logic that we have to reset, because we are going to change once load is changed we are going to share the load why not we have to achieve the economic loading of these criteria these generators and then, we can get the minimum cost of generation.

So, the combined logic of this AGC plus ELD also I discussed and then; so, far discussed only for the frequency control and then, in next lecture we will see the voltage control. We will also model the exciter. We will also see the various loops and we will check about the stability whether, it is stable or not because the gains of exciters are also very important and we will see, in the next lecture.

Thank you.