

Power System Operations and Control
Prof. Dr. S.N. Singh
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Module-3
Frequency and Voltage Control
Lecture-2

Welcome to lecture number 2 of module 3. In previous lecture, that is lecture number 1 we discussed the modeling of governing system. And now, in this lecture I will try to model the turbine as well as the generators. And so that we can see how we can control the frequency of the system and how the various loops are working in this ALFC or you can say load frequency control.

(Refer Slide Time: 00:47)

Turbine Model

Steam Turbine without reheater

$$G_T(s) = \frac{K_T}{(1 + sT_T)} \quad T_T = 0.1 - 0.5 \text{ sec}$$

Hydro Turbine

$$G_H(s) = \frac{K_H(1 - sT_H)}{(1 + sT_H)} \quad T_H = \text{Time delay}$$

Steam turbine with Reheater

Rating of H.P. stage is α p.u.
 Rating of L.P./L.P. stage = $1 - \alpha$

Let T_R = time constant of Reheater ($\approx 4 - 10$ sec)
 K_T = static gain of turbine
 T_T = Turbine time constant

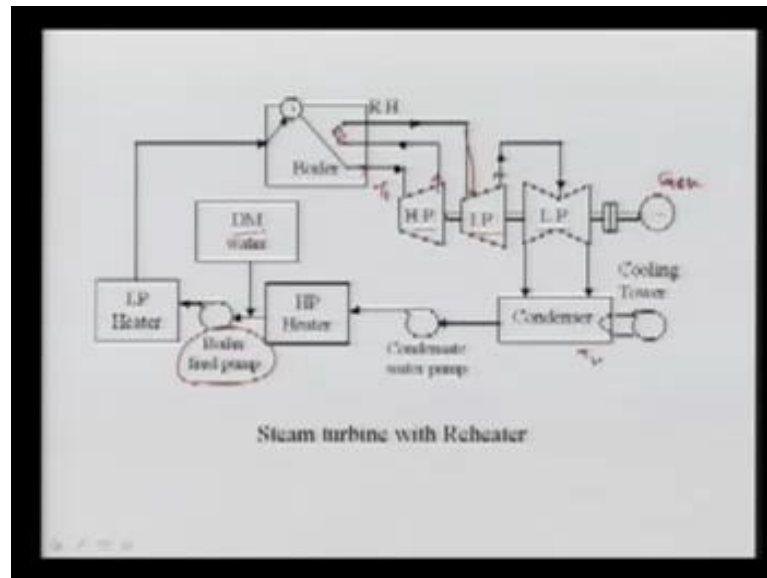
So, to have this turbine model as I discussed we can have the steam turbines or we can have the water turbines. Means the turbines which are used for the hydro power generators that is called your hydro turbines here I have written this. And if we are using the steam as an energy flow medium then it is called steam turbine. Steam turbine can have either re heater or without heater they can work. If it is without re heater then we can run the steam turbine in a very simple transfer function that is a G T G is the transfer function and T denotes the turbine in the S domain. I can say here it is a KT that is a gain after turbine over 1 plus stt and this TT is the time constant of the turbine. And normally

this it is approximately that is a line between 0.1 to 0.5 second. For hydro turbines here the transfer function is slightly different and here I can write the gain of the turbine is KT that is multiplied by $1 - 2STW$ divided by $1 + STW$.

And here TW is the time delay, why this time delay is involved? Because the water is coming from very far end it may be through the ((refer time; 02:06)) and other things and it will be having the time delay as well. But if your steam turbine is having the reheating facility then we have to model that re heaters as well. So, the turbine with the re heater now, what we can do if it is having the re heat facility means certainly your turbines will be having more than one stage. That it will have HP turbine that is a high pressure turbine here I have written this rating of high pressure turbine. Let us it is α I am talking in the per unit system then if a total power rating of the turbine is unity then I can say the rating of IP and LP stages is $1 - \alpha$.

So, the total is 1 per unit if we add α and $1 - \alpha$. You will get unity means if the total rating in the per unit is 1, then IP sp stage is having α per unit then the remaining stages may have IP or and LP it will be $1 - \alpha$. So, the time constants for the re heater it is normally 4 to 40 second, because the steam from this outlet of HP will go to the boiler. And then after re heated it will be coming back to the IP and or LP stages. So, it is a time travel involves and that is normally 4 to 10 seconds. So, static gain again I have used the KT as the static gain of the turbine and it is the time constant of turbine that we have used here you can see. So, using all these then we can have the time constant or we can have the transfer function models.

(Refer Slide Time: 03:51)



Again this how the steam flow which is going even though I explained in the previous lecture that here. You can say your HP turbine that is high pressure turbine here it is your intermediate pressure turbine. And here it is your low pressure turbine and all these are connected on the same shaft. And it is coupled with your generator G here that is called generator. So, once you can see here the steam which is coming from the boiler it is going to the SP and you can see here the expansion process is done. And due to the expansion process this steam will do work and that work will be in form of the rotation of the shaft. And after this is which is coming out that is steam after expansion means it will release the energy which is in the steam. It will going back to the boiler again and here it is basically reheated. And after once it is reheated it is called a re heating processor process.

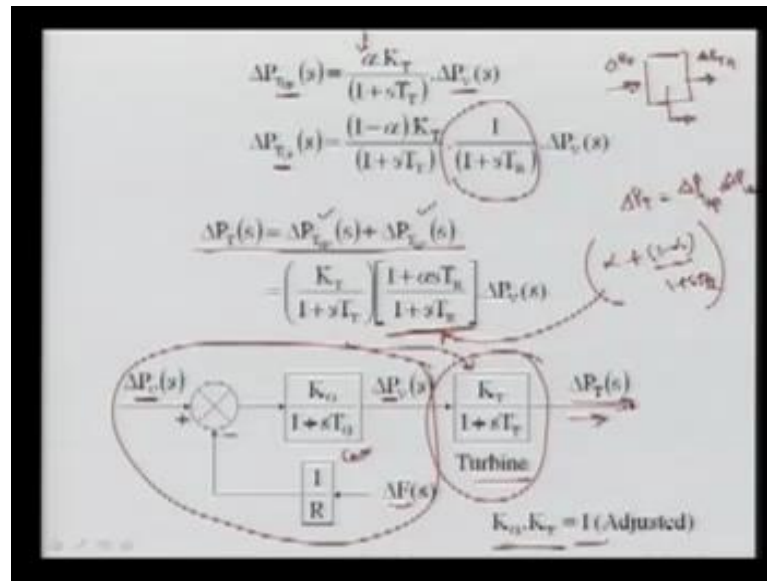
And the finally, it is again here coming to an IP state were again it is doing the work. Again here I want to mention that the output of this SP turbine is reheated, because the energy consumes in SP is very very high and then the steam which is coming out from the SP turbine it is having low pressure and thus low temperature as well. So, here there will be if it is not reheated there will be possibility that this steam will be converted to the water. And here and that will damage the blade and the efficiency will be poor. So, if you are reheating then again we can utilize the remaining energy in terms of here IP and SP turbines. This IP here again the steam is going out and the finally, it is going to the

LP here where expansion process taken place and then after coming out from the LP it is this steam is condensed.

So, we try to reduce the temperature as you know this temperature difference normally gets the work done. So, here if T_2 we can minimize that is the T_2 here it is a T_1 that relates the efficiency of this whole cycle that is a steam cycle here. So, we have the cooling towers and we normally condense this steam we convert it to the water and this water is still having the high temperature. Now, then we are having some pumps once it is condensed then we are sending to hp heaters. Because some it again it is heated, because the energy which is coming out the boiler it is heated here. And then this heater water is passing through the boiler and there will be some leakage here this demineralization water means normally it is called dematerialize there is no mineral is involved otherwise that will damage the blade of the turbine.

So, here thus the leakage etcetera will happen some steam will be going out in the air. So, that will be compensated by the DM water and this water is combine together and then we are using the BFP that is called boiler feed pump. This boiler feed pump again it is passing through the IP heater. Because we there is so many heaters stays, because the energy which is coming out from the boiler that is utilized then we can increase the efficiency to HP heaters. And LP heaters are used and then it is coming to your boiler here there is boiler it is heated and that is a steam. So, this is basically a complete cycle with the reheating facility.

(Refer Slide Time: 07:09)



Now, to, go for the derivation for the transfer function of the turbine with reheating facility. Here I can say that this HP part the power output of this HP turbine that can be related with the simple transfer function that is a KT 1 plus ST . And then it is multiplied by here you can say alpha term and that is coming multiplied by the valve output. Means here this is your turbine this is your changing the valve power. And here that is we are getting change in the pth that is a high pressure turbine. So, this transfer function, because here we are using the only the alpha times of that one so, here we are just multiplying alpha. Now, for the LP and IP units' power here now, 1 minus alpha is multiplied by this transfer function. Because the remaining energy is going there plus here there is a some travel time. This TR that is a reheating times constant tr that is added here and then it is your P .

So, the total power means here the total turbine power is nothing but your change in the power here of your THP stage and plus here change in your power at IP and LP stages. So, here you can see this is this expression. So, the total output of the turbine including our various stages will be the summation of all the power of the stages. So, here your HP stage here it is your LP and combination of your IP stage. That is a intermediate pressure low pressure and high pressure turbines. So, if will add and then you can simplify then you will find that KT 1 plus St is common here we can take. And then we can go for here now, what we are getting? This stage here alpha plus I can say 1 minus alpha here 1 plus STR and this is simplified and will get this figure. So, you can see we are getting a

complex transfer function of the complete turbine which is having the reheating facility. To go for the simplicity let us take it is not having the reheating facility.

So, only we have to ignore this means we have to put α is equal to unity means only HP part is there high pressure turbine is there. So, this part here this part will vanish. So, will have a simple turbine transfer function that is retain here and I have used this simple turbine without reheating facility. So, it will come here this turbine model this is the governor model if you remember which I derived in the lecture number 1. Now, this PV which is coming the here what is happening? This your change in the PC is the change in the differentiating here change in the frequency. That is coming through the drooping characteristic with the negative sign. It is now, coming to your governing and governor is taking action whenever there is change in frequency or change in the power reference this governor will act.

And it will try to change the input to the turbine that is a steam and that is you can say valve power that is a change in PV. And this PV now it is steam power that is coming to the turbine and finally, it is giving the turbine power or you can say rotational power that is where your generator is connected. So, now, this is the model of your governing system this model this is your turbine model. And now this is going to your generator where generator is coupled and we will see the generator model later on. Normally we try to make in such a fashion that the multiplication of K_g and K_T here we normally keep it unity. Means if you governor K_g gain and your here the turbine K_T we normally rejoin in such a fashion that we can make it unity. So, what will happen? Now, the multiplication of this will be unity and we will see in the later part as well.

(Refer Slide Time: 11:22)

Calculation of ΔP_{TSS} $K_T, K_G > 1$

Case-1: Let $\Delta F(s) = 0$

$$\Delta P_T(s) = \frac{1}{(1+sT_{G1})(1+sT_T)} \left[\Delta P_C(s) - \frac{\Delta F(s)}{R} \right]$$

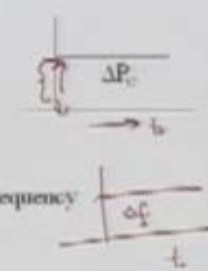
$$\Delta P_{TSS} = \lim_{s \rightarrow 0} s \Delta P_T(s)$$

For step response $\Delta P_C(s) = \frac{\Delta P_C}{s}$

$$\therefore \Delta P_{TSS} = \Delta P_C$$

Case-2: When $\Delta P_C = 0$

$$\Delta F(s) = \frac{\Delta F}{s} \rightarrow \text{step change in frequency}$$

$$\Delta P_T(s) = \frac{1}{(1+sT_{G1})(1+sT_T)} \left[-\frac{\Delta F}{sR} \right]$$


Now, let us see now keeping this K_T and your K_G is unity we can write here this change in the power. Means from here from this expression from this expression we can derive what will be this change in the turbine power by in terms of you can say change in the reference setting and in terms of frequency setting and we can write in the simple way. Because this minus one upon R change in F and that is multiplied by these 2 time constants or you can say transfer functions. So, here you can see that is what we are getting this is the transfer function of governor; this is the transfer function of turbine. That is multiplied by here change in this reference setting minus change in the frequency divided by R that is a regulation or cons slope or you can say drooping characteristic we will see that later on again.

So, we can write this is a complete this input output relation. So, in this case input is your change in frequency and change in your reference power. Now, to see the impact or you can say the change in the steady state turbine output power that is a change here I normally say this change in PTSS the SS denotes the steady state condition. You know this final value theorem. If there is a any change in any of the input then as per final value theorem we can obtain the steady state value by limiting S tends to 0 multiplying S with this your output here that is change in turbine power in S domain. So, here once we are going for this and we are putting here now, we can get the steady state value of the turbine that is a PTSS. First case I am assuming there is no change in the frequency of

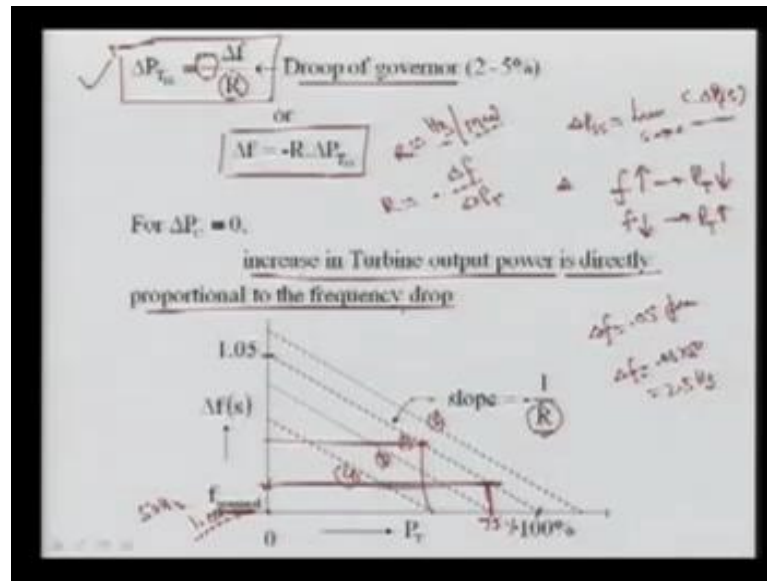
the system. And we want to change the output of the turbine by changing the step response.

Means suddenly here you can see in this here step change here time is change and the differentiating is changed up to this magnitude. So, this is the step input response just I want to know here this change now, in the domain I can say S domain the change in the PCs will be nothing but for this one. We know it is the change in PC that magnitude here this is your magnitude divided by S is your step response. So, after putting this value here and putting your change in FS is 0. So, and putting S multiplied here we can get this steady state value in the turbine output. That is we can see here what we are going to get here we are to change it here what we will get here S and here this is this by s. So, S S will be cancelled and all this S term will be 0's here 0.

So, we are going to get here this expression. So, it shows that whenever you want to go for the steady state increase means there is a step input it will be finally, it will be settled to its power turbine power will be increased. So, when you are willing to increase the power this steady state value will be increased of course, we have the time constant. So, it will may require some times and it will settle down, but the final value will be change in PC. In another case let us suppose I do not want to change the p reference that is a PC means I do not want to give raise or lower command. So, and frequency is changed means suddenly if system frequency changes and again I am talking about the step change in the frequency.

Here again I can say this is your time and change in frequency here that is your change in f. So, this magnitude is changed. So, I want to see how much the turbine output power is going to be changed during the steady state condition here suddenly change means there will be some settling transient and finally, it will be settled. So, here what in this case now this change in the PCs is 0 and we can put it 0. And we can get the expression from here from this expression here we are getting change in the PTS will be 1 over this governing transfer function here your turbine transfer function and multiplied by change in f divided by SR. Because here that change in f is here change in f divided by S and R is already regulation.

(Refer Slide Time: 16:25)



So, again using your final value theorem in S domain we can get this ptss here means we can get again this change in ptss is nothing but limit S tends to 0 f into change in PTS. So, using this we can get this expression final that is with the change in the frequency in this case we did not consider the change in the reference setting. So, now, you can see for change here the 0 means there is no change in the differentiating the increase in turbine output power is directly proportional to the frequency drop what I want to say why it is drop? Means here I am talking this increase here I am talking drop, because the terms we are we are getting here is the negative. Means if your frequency falls down the turbine output must increase to maintain the frequency as we know we require that and this is giving that characteristic.

So, whenever there is a increase in the frequency then turbine power must fall here the change in here is the positive then change in the turbine output is negative so, it is a vice versa. So, in other words I can say if you are changing the frequency I can say your system frequency increases then your turbine power output will reduced. Or this is reducing then turbine power must increase to compensate the remaining energy. And therefore, we can maintain the system frequency. In this expression here you can see the R is appearing and that is 1 over R and that here it is called the droop of the governor. This characteristic is called the droop of governor and normally the value of R is kept between 2 to 5 percent. I will come to that point what is the 2 to 5 percent? Basically the unit of R you can see here it is nothing but you can see the unit of R here it is Hertz per

mega watt why? Here R will go here means we can get here this R is nothing but your change in f divided by change in power of course, it is negative sign.

So, unit is not related with the plus and minus sign, but it related with the variables that is are coming in to the picture. So, this is a frequency this is your power. So, it is hertz per mega watt is the unit in actual value. But in the percentage it is we can write again I come later part you will see how that we can change from per unit? To means suppose one we are taking per unit another will taking actual value or both are we are taking per unit here both are taking actual value and we will see that. So, this here gives a relation that your change in frequency will be negative of the change in the turbine output. So, M power is increasing frequency here it is increasing what will happen? The frequency of the system will be changed accordingly. So, the increase in the turbine power system frequency falls means the power must increase to compensate that.

To see that let us see 1 characteristic here. This characteristic is nothing but you are the droop characteristic and here normally you can say this is the nominal frequency in our case it is 50 Hertz. And this is from 50 hertz to that is we have increased to 1.05 means here it is 1 per unit you can say. So, this is the change in the frequency and here change in the turbine output power and this slope which is showing one upon minus R . So, we have the family of curves this is 1 here we have 2 here we have 3 curves and 4 curves what does it shows that? Here for 100 percent power if this load is removed if this load suddenly comes to 0 your frequency will be increased by 5 percent. So, your frequency will become 1.05 means change will be point here now, the change will be 0.5 per unit. So, this gives that is if you are loading 100 percent your frequency if it is 100 per rated 250 hertz and if you are just changing then again you're this will be there will be change here with the 5 percent. Means now, the load is reduced to 0.

Now, we are going to change in the frequency here is 0.5 multiplied by 50 means there will be change in 25 hertz in the frequency. So, we have the family of curves again they are having the slopes. So, now, suppose you want to have this what will be? If you are your operating that you are maintaining controlling your frequency for 75 percent here of the loading. If you are increasing here then how much you are going to get how much the now, you can see this is the frequency which will be if this value at this point if you are reducing then your how much frequency this increase just you are going for have. Means for 70 percent increase here this will be your frequency increase. If there for 50 percent

here your frequency increase will be this one and for 0 percent you will get this one. So, this relates the loading of the turbine with here the frequency and again it depends upon this characteristic that is a slope here R that is very important.

(Refer Slide Time: 22:12)

$$\begin{aligned}
 R &= 0.05 \text{ pu} \\
 R &= 0.05 \frac{\text{pu Hz}}{\text{pu MW}} = 0.05 \frac{\text{Hz}}{\text{MW}} \\
 &= \frac{0.05 \times 50}{100} \frac{\text{Hz}}{\text{MW}} \\
 &= \frac{2.5}{100} \frac{\text{Hz}}{\text{MW}} \\
 &= 0.025 \frac{\text{Hz}}{\text{MW}}
 \end{aligned}$$

$R = \frac{\Delta f}{\Delta P}$
 $R = \frac{\text{Hz}}{\text{MW}}$
 $R = \frac{\text{Hz}}{\text{pu MW}}$
 $R = \text{pu Hz}$
 \downarrow
 Δf

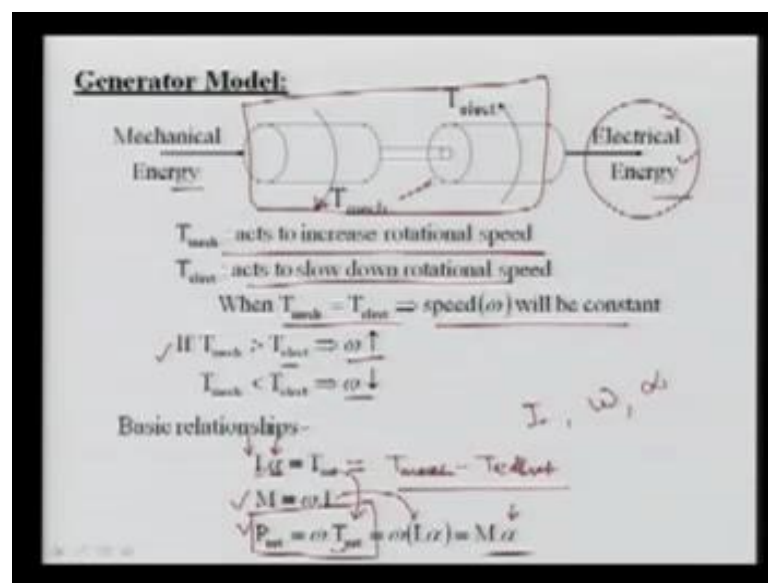
Now, as I discussed that is what this is? Let us suppose R 0.05 per unit as I said the unit of R can be your hertz per mega watt or it can be your hertz per unit mega watt. Means here the mega watt is in per unit. So, just I am writing, because the per unit is very confusion, because per unit of hertz will be different than per unit of mega watt. So, it is it is better write the per unit mega watt means power we are using the per unit quantity. That we can also write here pu hertz over this pu mega watt and this is called completely pu means your change in the frequency in the per unit here and the per unit mega watt it gives you per unit value. Now, for this you are let us supposing your base power is 100 mega watt and your frequency is 50 hertz then R is equal to 0.05 per unit is nothing but we can write here 0.5 per unit hertz divided by per unit mega watt.

Now, we want to calculate in the actual value. So, here 0.05 now, from per unit to actual value we have to multiply by it is the base value and that is 50 Hertz. Now, we are getting here hertz divided by here hertz that is a power 100 and now, we are getting mega watt. So, we are getting here 2.5 divided by 100 here that are hertz per mega watt. So, now, this is the way of converting and finally, you can see this value just we are going to get 0.25 hertz per mega watt. So, this is very important you can say we started

in the per unit 0.5, but this value has actual value is this one means if you are changing 1 mega watt power here there is a change of 0.25 hertz in the system frequency.

So, if you are increasing then it will be reducing as I said we had the relation. That we are going to the change in here is minus or change in turbine power. So, it is with the negative sign. So, when you want to increase the turbine power that is increased system frequency. Here if you are increasing system frequency will increase, but we want that reverse action. So, if the frequency falls down this turbine will automatically try to increase the this output because here the R sign is there. So, this is basically the concept for this R that is a regulation. So, we saw the combine effect of governor and turbine modeling. Now, let us model another very important component in this load frequency control that is generator.

(Refer Slide Time: 25:24)



So, the generator modeling that consists here we can now understand that this is your mechanical power or mechanical energy input through the turbine which is rotating here this is rotating. So, the torque is developed and that is called the T mechanical torque. Now, on the shaft your electrical that is a generator is connected which is giving output in terms of electrical energy. And you know the turbine generator system is nothing but we are from mechanical energy we are converting to the electrical energy. Although in the whole process it is not only the mechanical energy is converted to electrical energy. But the fuel energy that is the coal energy or you can say water energy that is in hydro

power stations that is coming to the turbine. So, that energy is converted to the mechanical energy and from mechanical energy we normally get to this electrical energy.

So, you can see this turbine torque T here that is coming this T mechanical that is going in 1 direction here this. Let us suppose this rotating trying to rotating, but the electrical that is output it will try to oppose and that is a basically the equilibrium condition. If this torque is in the same direction what will happen? This will keep on accelerating and system will be out of step. So, always had the steady state condition when the T mechanical energy is equal to your electrical energy in that case here this T mechanical will be equal to your the T electrical and the speed of this rotating mask will be the constant. If there is a balance imbalance here means if 1 quantity will increase let us suppose the T mechanical is increasing more what does that mean? That your mechanical energy is more than your electrical energy this machine will try to accelerate. Means if your T mechanical is greater than T electrical the speed of rotation body will increase by. Because this energy going output here output energy input energy some energy is here kept as you know the energy cannot be wasted.

So, as per energy conservation law that energy is going from one form to another form means that mechanical energy since it is more we are getting less in energy. So, that energy is mechanical energy is going to convert it into the rotational energy and that is nothing but your kinetic energy and that kinetic energy you know that speed will increase and then speed is increased and that is we have the here. Reverse is also true if we are feeding less mechanical power, less mechanical energy and your electrical output is more then it is a reverse situation. Means you are drawing more power from this whole system this is your complete system where rotating bodies is there. So, if you are getting more then what will happen? Whatever the energy which is stored in the system here that will be coming out. Means the stored kinetic energy is released in the electrical energy and this if it is k_i k is reduced means the change in frequency will be reduced and then frequency will fall.

So, in ((refer time 28:32)) I want to say that T mechanical that is torque acts to increase the rotational speed it will try. Because input you are giving it will try to increase whereas, this T electrical will acts to slow down the rotational speed and thus it will do some work and then electrical energy will be generated and that is in three phase power that is you are going to get. To model this we have to have this rotating body concept,

because there is a some energy that is a input and output. It has some inertia that is a inertia constant I can say I which I have used I is a inertia constant and here it is rotating that is we have the angular speed ω will also use this angular acceleration α , because if there is a any mismatch this there will be changed in the ω and this α is your angular acceleration. And now, another term that is called m . Means angular momentum which is defined as the inertia constant multiplied by your speed that is ω . So, the basic relationship just I want to derive the T_{net} is nothing but T_{net} I can say it is your T mechanical minus T electrical. Electrical means what is coming in and what is coming out that difference is called T_{net} .

So, and that can be related with this here this inertia constant I multiplied by here the acceleration that is a power. So, this torque here divide multiplied by the acceleration and in inertia constant that is related by the T_{net} angular momentum already I derived. Here this M will be equal to your $\omega \cdot i$. Now, this T_{net} as you know this torque if it is multiplied by speed then it will give the power so, the torque here and the power relationship with this expression. Now, if we will put the T_{net} value from here this first equation we can put here. And now, we can use this ω into I here this relation. So, we can derive M into angular acceleration that is α will be nothing but your P_{net} . Here we are talking all the values in actual quantity you must remember this because in the per unit this P_{net} will be equal to T_{net} . Here we are talking this p is in megawatt this is in Newton meter and this is in radiant per second. So, here everything here I am deriving in actual quantities later on will convert to the per unit if we want to calculate in the per unit otherwise you can also us in actual quantities.

(Refer Slide Time: 31:31)

$\sqrt{T_{net} = T_{mech} - T_{elec} = \text{Net accelerating torque} = T_a}$
 $M = \text{Angular momentum of machine}$
 $I = \text{Moment of inertia of machine}$
 $\alpha = \text{rotational acceleration (rad/sec}^2\text{)}$
 $\omega = \text{rotational speed (rad/sec)}$

The difference in Electrical & Mechanical torque will cause to accelerate or decelerate. Speed of machine under acceleration will be

$$\omega = \omega_0 + \alpha t$$

we know that

$$\omega = \frac{d\delta}{dt} \Rightarrow \delta = \int \omega dt$$

$\delta = \text{phase angle of rotating m/c}$

Now, So, as I said the T_{net} is nothing but it is the $T_{mechanical}$ minus $T_{electrical}$ and this is also called net accelerating torque and sometimes people called the T_a . So, T_{net} in different books you will find the different names. So, this is nothing but the difference in the input torque to the output torque. So, that is the net which is inside the rotating machine and that is called net accelerating torque. M is angular momentum of the machine; I as I defined as the moment of inertia of machine; α that is a rotational acceleration in radian per second square and ω is your rotational speed in radian per second. The difference in electrical and mechanical torque will cause to accelerate or decelerate as I said if T_a is positive then machine will accelerate as already I discuss here. That here this concept is the difference here if it is more, then is to be accelerate and if it is a difference is negative then it will try to de accelerate.

So, here the difference in the electrical and mechanical torque will cause to machine accelerate or decelerate. The speed of the machine under acceleration will be that is a ω_0 plus what is your acceleration? Acceleration here the rotation acceleration multiplied by time. So, this will be your the speed of machine under the acceleration. If there is no acceleration then the speed will be your ω_0 ω_0 is the base or you can say constant speed where both are balanced the $T_{mechanical}$ is equal to your $T_{electrical}$. So, this relation is very widely used and will see in the later part of this lecture. We also know that this ω is nothing but this here the phase angle of rotating machine is δ then we can relate this ω with your change rate of change of

angular phase angle that is of rotation machine or in other words I can say this delta is the integration of omega with respect to time that is deceleration.

(Refer Slide Time: 33:55)

$$\Delta\delta = \int (\omega_s + \alpha t) dt - \int \omega_r dt$$

Machine absolute phase angle Phase angle of reference

$$= \omega_s t + \frac{1}{2} \alpha t^2 - \omega_r t$$

$$\Delta\delta = \frac{1}{2} \alpha t^2$$

Deviation from nominal speed

$$\Delta\omega = \frac{d}{dt}(\Delta\delta) = \alpha t$$

$$T_m = L\alpha = L \frac{d}{dt}(\Delta\omega) = L \frac{d^2}{dt^2}(\Delta\delta)$$

$$P_m = P_{mech} - P_{elec} = P_{mech} + \Delta P_m$$

But, $P_{mech} = P_{mech_0} + \Delta P_{mech}$

Handwritten notes on the right:

- $T_m \frac{d(\Delta\delta)}{dt} = T_m \Delta\omega$
- $T_m \Delta\omega = L \alpha t$
- $\Delta\delta = \alpha t^2$
- $\alpha = \frac{d\Delta\omega}{dt}$
- $\Delta\omega = \alpha t$
- $\Delta\delta = \frac{1}{2} \alpha t^2$
- $T_m \Delta\omega = T_m \alpha t$

Now, let us see here. How we can so, I want to calculate what is the change in the angular or you can say phase angle of the rotating mask. Here what I want to derive let us suppose this is your reference value. Means you can talk with the some any reference value and here this is your rotating mask which is rotating. Normally in this rotating system here if it is a reference from here this is rotating both are rotating with the same speed then this angle is fixed and it is you can say locking torque angle. But if this router will try to accelerate more than this delta will try to increase and this is also rotating this is rotating as omega naught or you can say at the synchronous speed. Both are normally in the steady state they are operating at the synchronous speed. So, that this torque will be this delta angle is fixed. But if it is accelerating what will happen? This angle will increase if it is decelerating then this angle will decrease and vice versa.

So, change in this angle I can say this acceleration minus with their initial that is a phase angel of the reference angle. So, change in the delta that is here machine absolute phase angle here that is we are going for this minus with the reference ((refer time 35:12)) how much it has reached? And from this we can subtract then we can get the change. So, change in the phase angle is retained as the integration of omega naught plus omega T and this here we are into dt. And here basically I am sorry this is alpha not omega as I

said we wrote this expression you remember this is your ω is nothing but $\omega_0 + \alpha T$ which is I am using here minus here that is a phase angle of the difference angle here ω_0 or you can say synchronous speed. So, if you are integrating here you will get $\omega_0 t$, because this is a constant.

So, this T term will be appearing here α is constant. So, only in the case it is a constant if it is a change in α then it is a different. So, we have to α is also changing we are assuming this α is constant. So, this α here now T could T^2 upon 2 will be there minus $\omega_0 T$ here that we can get it. So, finally, what is happening? We are getting the change in the phase angle is nothing but $\frac{1}{2} \alpha T^2$. Now, the deviation from the nominal speed what we are trying to do? This change is nothing but it is we can see here $\frac{1}{\Delta t}$ we have to derivate this means change in the angle divided by dt and if you are derivating this you are getting this. So, we are getting the change in speed is nothing but your αT or we can also if you remember I said here ω is nothing but your $\omega_0 + \alpha t$. You will take this side so, $\omega - \omega_0$ that will be equal to your αT and this is nothing but your change in ω .

So, we also get the same thing from that expression and also we can derive from here this expression. So, this is equally valid and this is the equation. Means change in angular speed is nothing but your time multiplied by your acceleration angular acceleration this I am talking. We also had the 1 relation that T_{net} is nothing but your I that is angular here I is your moment of inertia and α is your angular acceleration that we can multiply together so, we are getting this. Now, this α this α is nothing here you can say what is α ? α is ω upon here t . So, I can say here α is nothing but your change in ω over change in time. So, we can write here I into the change in speed over change in the time and we can get this. And it will put here this value ω in this difference here we can get I multiplied by the double differentiation of change in phase angle Δ .

So, we are getting 1 relation here. This means in the simple way I can say here I into d^2 over dt^2 change in phase angle will be equal to your T_{net} . Now, here just we have derived in a change in angle is very good. We want to here see the change that is a T_{net} from your the differentiating means here this we have P_{net} . P_{net} is nothing but your I can say the p mechanical minus your p electrical. Or in similar fashion I can say this

Tnet is nothing but your T mechanical of course, minus T electrical means here this the Tnet is the difference between mechanical and electrical and a power net is your mechanical p mechanical minus p electrical. We know that this during this initial case this I can say it is nothing but your Pnet at the where it is the difference actually we are talking plus some change in Pnet or we can say how much thus we are changing from one value to another value? So, this is your Pnet that is we are change in the Pnet we are getting, but the p mechanical is equal to your p mechanical not which was earlier plus your p mechanical.

(Refer Slide Time: 40:06)

Handwritten derivation on a slide:

$$P_{net} = P_{mech} - P_{elect} \quad (\text{During SS condition})$$

$$\Delta P_{net} = \Delta P_{mech} - \Delta P_{elect}$$

$$\therefore P_{net} = (\omega_b + \Delta \omega) T_{net} = (\omega_b + \Delta \omega) (T_{mech} + \Delta T_{mech})$$

$$= P_{net,ss} + \Delta P_{net} = \omega_b T_{mech,ss} + \Delta \omega T_{mech,ss} + \omega_b \Delta T_{mech} + \Delta \omega \Delta T_{mech}$$

During SS, $P_{net} = T_{net} = 0$

$$\Delta P_{net} = \omega_b \Delta T_{mech} + \Delta \omega T_{mech,ss} + \Delta \omega \Delta T_{mech}$$

From equation,

$$\Delta P_{mech} - \Delta P_{elect} = \omega_b (\Delta T_{mech} - \Delta T_{elect}) \quad \text{--- (1)}$$

$$T_{net} = T_{mech} - T_{elect} = T_{mech,ss} + \Delta T_{mech} - (T_{elect,ss} + \Delta T_{elect})$$

$$= \Delta T_{mech} - \Delta T_{elect}$$

$$\Delta T_{mech} - \Delta T_{elect} = L \frac{d}{dt} (\Delta \omega)$$

Or in other words I can say here we can derive this expression I can use some space here what is happening? As I said your the speed net it is as I said your p mechanical minus p electrical and we know that this p mechanical p mechanical. I can write is equal to your p mechanical that is during the steady state minus sorry plus change in p mechanical. Similarly, we can write this p electrical is equal to your p electrical not plus change in p electrical. So, what happens if you want to calculate this change in Pnet if you put these values here and as here? So, you will get the change in the Pnet that is the real power will be change in the mechanical power from the not value, from the steady state value, from the during the steady state value just I am talking. So, this minus here you have to change in the p electrical value. So, also this Pnet since we have a change in the Tnet means the accelerating power has changed means it is not 0.

So, the speed will change and the speed will change from your omega naught. So, omega naught plus change in speed that will be multiplied by your Tnet that is a which is a difference between the T mechanical minus T electrical and thus here we can define it. So, the Pnet will be the multiplication of this factor multiplied by the tnet. And this Tnet is nothing but your Tnet not that is you are the in steady state Tnet and in a steady state it was 0. Because the mechanical was completely equal to your electrical if we ignore the losses. So, that plus change in the Tnet how much that is the changing and that Tnet change is nothing but your g Tnet change in the mechanical minus change in T electrical. So, if we will simplify this means Pnet can be also retain in the Pnet not plus change in Pnet that will be equal to you can multiply here.

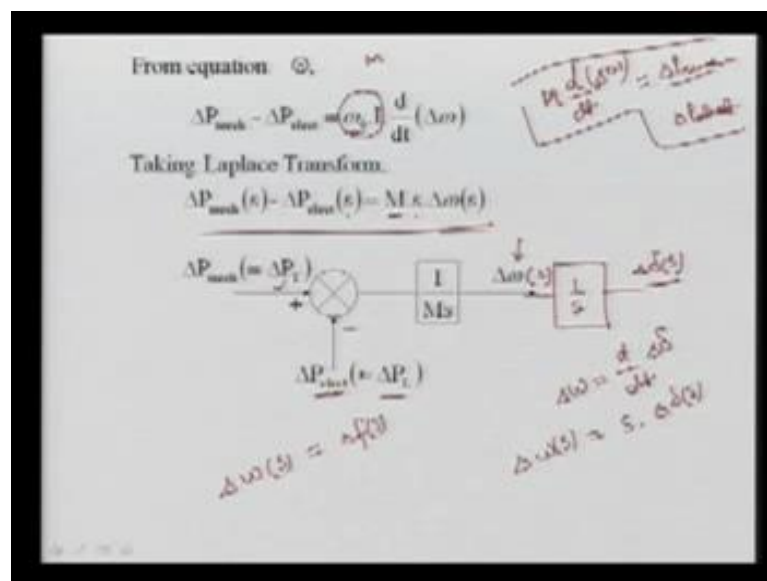
So, you will get the omega naught this factor is multiplied by this factor. So, this is this factor this factor multiplied by this factor will get this factor. This omega will be even change in omega is multiplied by the here this factor we are getting this another one. And omega naught if multiplied by the change in the Tnet we are getting this factor. So, will expand this will get the 4 terms now, what is Tnet naught? Tnet naught or Pnet not they are 0 why I have written here? Because in the steady state, what was this p mechanical was equal to your p electrical? So, this net is 0. So, here the change in the Pnet at the not in steady state it was 0. Similarly, this Tnet will be also 0, because if p is 0 the T will be 0, because the p is related with omega. So, if will put these values so, what will happen? This will be your cancelled out this will be your cancelled out and this will be your cancelled out.

So, what you are getting? You are getting change in Pnet that will be equal to your this term omega naught into the Tnet plus here you are getting omega into change in Tnet so, you are getting 2 terms. Now, change in the Pnet normally you can say this term is very small and this is also small. So, we are just changing the speed change multiplied by change in the net. So, this value is very small. So, normally we ignore this one also, because this is having very less impact, because this value is very fraction. So, we can ignore and we can put it 0 here already I have put 0 here 0 here it is 0 So, we are getting the relation the change in Pnet here that will be equal to the omega naught that is the best speed multiplied by the change in Tnet. So, from this equation here already I have use this one. You can see his Tnet change in the change in mechanical power minus change in electrical power here we have retain. That will be equal to your omega naught and this

is nothing but T_{net} is your change in mechanical torque minus change in electrical torque and we are having this expression.

Now, here what we are now, we are going to relate these term with our previous one here our intention is to relate in the T_{net} form here. So, to see that now, T_{net} what will be the T_{net} will be? T_{net} is the T mechanical minus T electrical torques that will be here, this mechanical plus change in the mechanical. And here minus torque electrical naught at this base case plus change in the T electrical what is happening? This and this will be cancelled out because they are in a steady state case both are equal. So, we are getting T mechanical here is it this change in T electrical. So, here this is what we are getting this is nothing but your t_{net} . So, this we can write the T_{net} now from above the previous equation here this T_{net} that is equal to I can write here this expression or we can we can say here this change in this or you can say T_{net} will be equal to I into d upon dt change in omega.

(Refer Slide Time: 46:03)



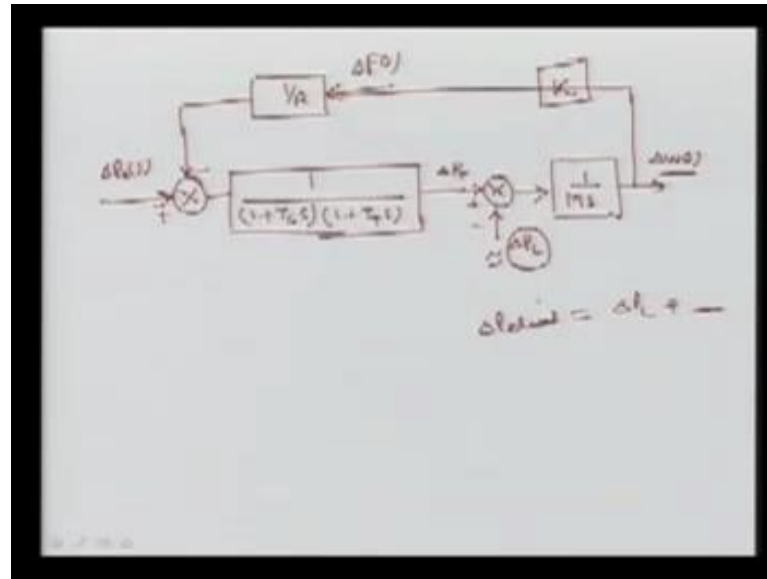
So, again we can simplify here from equation star I can say this change in the mechanical power now I am writing in terms of power, because all the quantity we represented in the power represent torque. In the power system it is a torque has no meaning, because it is very difficult to calculate the things in the torque. It is better to go in the mega watts and the frequency terms so that we can easily solve the problem. So, here the change in the mechanical and change in electrical powers that will be equal to be here I have

multiplied this and this is your nothing but the $\dot{\omega}$. So, here ω naught is multiplied and then ω naught into I is nothing but it is your M that is your angular momentum. So, now, I can see here we are having expression that $M \frac{d\omega}{dt}$ change in ω that will be equal to your change in p mechanical minus change in p electrical so, this p mechanical what is this? This p mechanical is nothing but your turbine output.

So, this is your turbine output that is coming and if you are taking here the laplace transform of this expression. We can have in the S domain. Change in the mechanical power in the S domain minus change in electrical power in S domain that will be equal to M that is angular momentum multiplied here that differentiation is there so, S term will be appearing in change in ω s . So, what we are getting? We are getting here the expression for this we can write in the transfer function. Here you can see the change in the mechanical and that change in mechanical power is nothing but the output of the turbine. That is a change in P_T which we used in the modeling of the turbine minus here the change in electrical. What is the change in electrical power? It is nothing but that is a load on the alternator. So, that is also is related with the change in p_l will see later on this p_l is not directly this electrical there is some damping also so, approximately we can say it is equal to the p_l . And that is coming to here is one over S if it is multiplied then we will get the change in here the speed.

We also get 1 relation the very carefully what we had? We had this change in ω is nothing but it is your $\frac{d\delta}{dt}$ change in δ . So, if we want to write here ω S that is nothing but your S into change in δ s . or here what we can do if you want to write in terms of δ . So, this is multiplied by a I I can say δ S here I can say it is 1 upon s . So, here we want to go for even through δ terms also we can get it, but our concern here is the frequency rather than δ . So, this is nothing but your change in ω is nothing but your changes in frequency both are same thing there is no difference. So, here I can write here this ω S change always here it will be your change in the frequency if we are using in the per unit system. If you are not using the per unit system then this ω is radian per second it is in hertz both are different will see some problem how we can going to relate it? Now, we have to now, combine all these models. Models means model of governor turbine and this generator let us see what we are going to have.

(Refer Slide Time: 49:49)



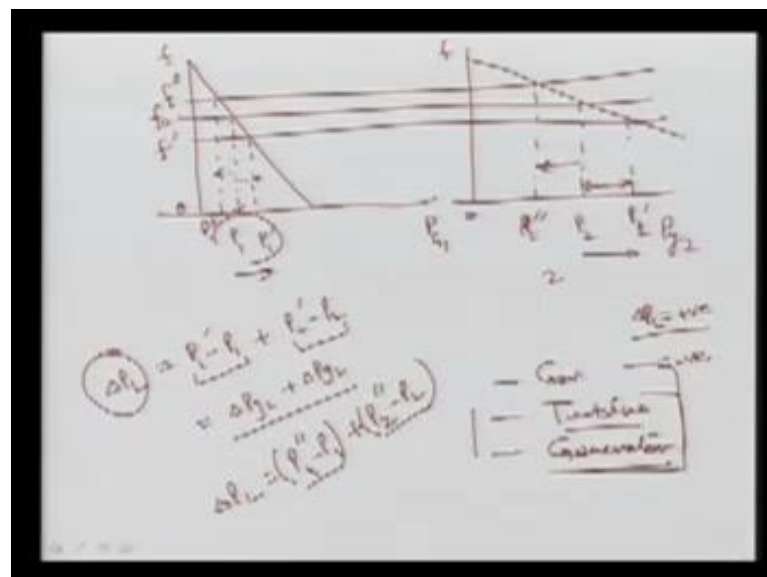
Now, we had here if remember here it was your the PC that is coming your change in the pcs here it was positive then we had some here the term with the negative. And we are had some drooping characteristic that is 1 upon R with the negative sign and here it was coming your change in fs. Now, this was coming and it was coming to the governor. So, now, the governor time constant and I have to now, write the governor and the turbine together, because I said that K_g multiplied by K_T is kept unity. So, both now, coming together so, I can write 1 here K_g into K_T . I can write 1 plus T_g that is a governor time constant S here 1 plus your T_t S here of your turbine. And here we are getting your change in PT and that change in PT is nothing but is the change in mechanical power and here again we are going to add this. And here we are getting change in this is your plus this is minus and this is nothing but your change in the load. Means change in electrical power and then here we are getting a turbo 1 over ms and here we are getting change in omega s.

So, this is the model that we are going to get this is also related with the change in the frequency if we are changing the unit. So, this frequency here is nothing but is coming here. So, the value here the change in the frequency if we are using in the per unit this change in here will be there. Otherwise you have to change with some constant here k and then it is fed to this means omega is related to your frequency and then it is a loop. Now, you can see this is a loop and we have to go for the solving and you have to see the system. And this is normally called this primary control and will see other secondary and

tertiary controls later on But here this load is not constant it is also affected by the change in frequency in the power session there are various loads they are also change if the frequency is changed how? For example, let us suppose your induction machine is there. And once the frequency is changed its torque is reduced or even the torque is changed and once the torque is changed means the output of that machine is changed.

So, we have to model the system we have some damping coefficient that d that is related with your frequency and then you are this load. So, we have to relate the frequency and this electrical load with that and then we will see how much we are going? So, that is why I said here it is nothing but it is approximity. So, we will see this we will try to model that this change in that is the change in p electrical here it is your change in the load plus some other factor and that basically the modeling of load will be the system here. Now, let us see in the power system the there is not only one generator there are so, many generators. So, and they are having the different R characteristic that is regulation constants R will be the different for the different one. How they will share the load if they are operating in the parallel? To see this as I said the characteristic that is we can draw here.

(Refer Slide Time: 53:22)



This is a load that is a p that is they are of generator 1 that is you can say $p_g 1$ and another generator here it is I can say $p_g 2$. Now, 1 generator is having this slope this is very deep and another is having very less slope so, what happens? Let us suppose they

are operating here. At the some this is the frequency operation frequency here and since they are connected parallely so, the operation of the frequency will be same. And let us take this is your f naught that is the nominal frequency means they are operating at the rated frequency. So, the loading of this generator will be nothing but this will be sharing here p_1 and this will be sharing your load p_2 . Now, you can also see here those are having here this less slope those are loaded much. Means they are taking more load here from you can say from 0. Now, if there is a suddenly release in the load or you can say let us we put more load on the system is very important. I just I change a change in the loading that is electrical power just we have increased this value just it is a positive what will happen? Means now, that load must be shared by these 2 generators.

So, and they must share in a such a fashion that steady state frequency is again the constant. And that is will be fixed and it will be of course, the lower than that, because the power is increased. So, what will happen? Here you can see now let us suppose here this frequency has said and that will be parallel here it is some by loop f prime. Now, you can see this will be changed from here p_1 to p_1 prime here this will be changed from p_2 to p_2 prime. So, the change in the loading means that is the difference here in the p_1 prime minus p_1 plus that is your p_2 prime minus p_2 that is nothing but your change in the load which has been taken place. So, this change in the here output of 1 change in the output it is nothing but here I can say change in $p_g 1$ plus change in $p_g 2$. So, you can see here this generator is sharing more compare to this one. So, those are having here less slope it will be loaded much compare to this. So, this is the parallel combination of this machine. So, you can see here this is similarly we can go for if load is released. Mean some loads are shed of then frequency will rise from this nominal. Here this is your nominal frequency thus I wrote here we are starting from 0 here.

So, the sharing also here the system frequency will be constant and then we will have that system frequency. Based on that we can calculate what will be the loading of the system and generators that and they will take care of that. So, you can see the change in here from p_1 to p_1 prime and here it is change from p_2 to p_2 prime. So, this is the load that is a if it is increased so, this value that they have to go for. If reduced then we can also similarly what will happen? If this is a negative means we have reduced the load what will happen? System frequency will go up and you can see here f double prime what will happen the now, net release you can say here now, we are getting here p_1

double prime and here we are having p_2 double prime. So, we have again you can say now we have shifted means we have reduced the loading here and from here to here and you can see that the reduction of load in this case is in 2 case it is much higher than your this case.

So, this is your frequency raise and again we can write your this change in now p_1 that is negative. It is nothing but how much we are going for this is here p_1 minus here p_1 . Again here this will be the negative I can say this double prime minus p_1 plus here p_2 double prime minus p_2 what will happen? Now, this quantity will become negative this will be negative and the finally, the change in load is negative means we are shedding of the load. So, this is the parallel combination of that. So, in this lecture now, I can in this lecture we saw the modeling of turbine and; however, we used for this analysis purpose or you can say simplicity. I have used only the turbine without the reheating, but if you are using if you are using some computers and other things then you can use the detailed modeling. Later on also I will use the turbine modeling without reheating. So, that we can see the performance of the system or you can say the various loops that are a load frequency control loops. I also model the generators along with the rotating mass of the turbine.

So, it is your generator plus turbine rotating mass and then we related that turbine output to the electrical output and that is electrical output related with the frequency of the system. And then I combine together all these 2 and then we saw that the complete block diagram of governing system, which I discuss here the governing system in the lecture 1 governing then turbine here the load and then we saw this performance. Another term is another model is left out that is your load model which I will discuss in the next lecture. And then your complete 1 loop that is a primary load frequency control loop is completed. And then will add the load model as well and then we will have a complete this ALFC primary loop. And we will see the performance that is steady state performance as well as the dynamic performance. And then we will analyze the loops and we will see what else are required for maintaining the frequency to its rated value. So, we will discuss in your next lecture.

Thank you.