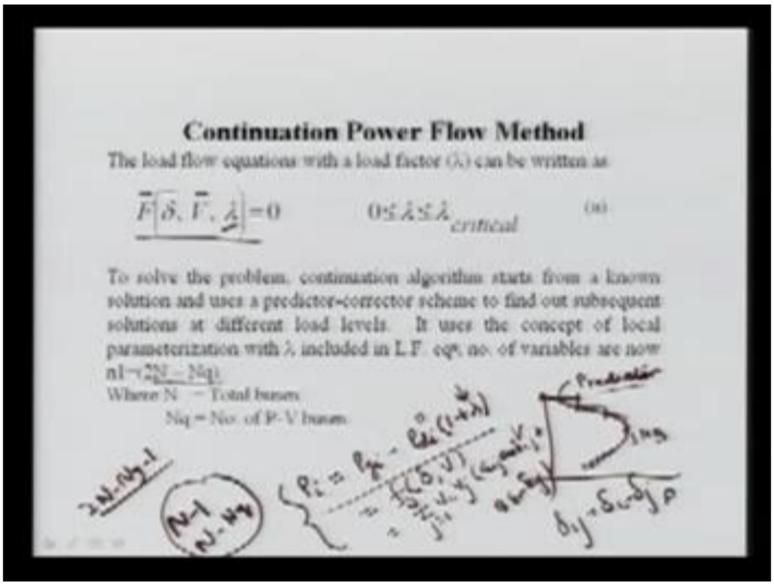
Power System Operations and Control
Prof. Dr. S.N. Singh
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Module - 2
Frequency and Voltage Control
Lecture – 13

Common the lectures number 13 of module 2. In the last module, we saw that load flow problems and load flow solution algorithm cannot give the solution at the notch point. So, in this lecture, we will see the continuation power flow method which can give you the solution at near to the notch curve as well.

(Refer Slide Time: 00:39)



But if the notch curve, if you remember here it this is an if you draw the PV curve this is your real power p and this is your voltage V then this here this is your PV curve. And the point here that is also know, n as a saddle node bifurcation point here the load flow solution will not get converts ,because the determinant of the load flow jacobian becomes singular here. So, we cannot get the inverse of that even though more over that this is if you are approaching this point this point is called saddle node bifurcation point. Before this here even though load flow solution sometimes may not converge due to the ((refer time: 01:20)) conditioning of jacobian, because the jacobian becomes air condition the inverse will be not proper and the finally, we cannot get the change in the voltage and angle using the Newton raption method.

So, this continuation power flow method is the method by which we can calculate even though this whole portion of the solution till the notch point and even all the lower point as well. Now, in this continuation power flow methods, which utilizes that conventional power flow method again, but we go for the several steps. And the normally we utilize the 2 steps 1 is called the corrector step and another is predictor step. Means with the predictor step first we increase the load and then we draw a tangent point we linearly calculate this predictor. Let us suppose we start from here at this point then we can have a tangent here vector this up to certain load change here the load is changing in the tangent direction and then this step is called your predictor step. And since we have moved here then it is a point is not the actual solution our actual solution point is this.

So, we go for corrector step and the similarly again the tangent point here then here and the finally, we can reach to the notch point. So, this is the sequence of predictor and corrector steps applied or used in the continuation power flow method. Here what we do we keep on increasing the load. Load means load at the load buses. So, it is a if there is a one factor that is called lambda that is a load factor here normally used if you are increasing from 0 means if 0 means your increase is almost there is no it I ting at the from the base case. For example let us suppose we can write the load injection at any bus that is I can say here Pi that we can write the power generation at that bus minus p demand at that bus. This p demand I can say not means it is a base case or the load flow or even a initial loading of the bus ith. Now, if you want to increase the load at this bus I can say I can use here 1 plus a term lambda lambda can vary from 0 to certain lambda critical value.

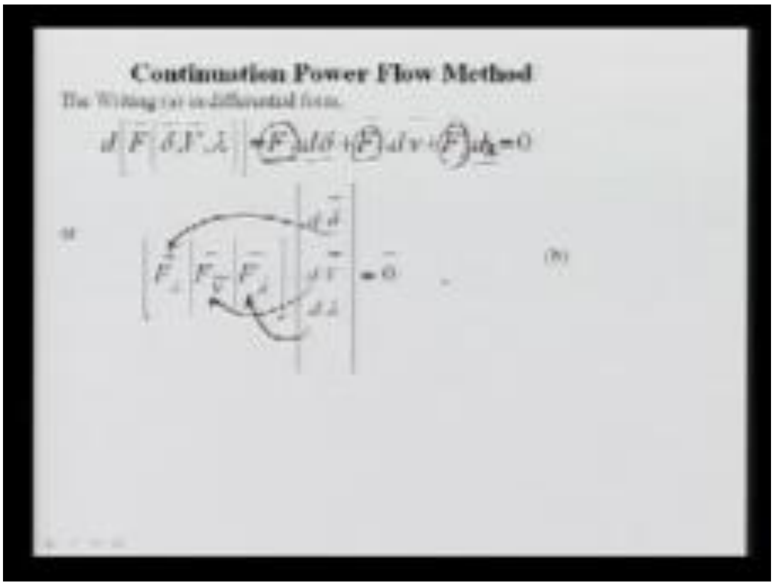
So, if your lambda is increasing means lambda is 0 means it is a base case solution base case injection and then that is a point I can say here at some point here starting some point. Now, once your lambda is increasing then your demand at that bus is the total demand is increasing by that factor 1 plus here that is lambda. So, here your p injection will be changed and then we have to solve what will be the voltages and the angle at the various buses in the system. In this continuation power flow program we can calculate the voltage and angle as well and the method here we apply the predictor method first and then corrector methods I will explain these methods later on. Now, what this equation? We know that it is p injection at any bus that we can write in terms of voltages and angle functions.

Here I can say it is a function of your delta that is a voltage angle and here is the V the voltage magnitude vectors. If you remember in the power flow equations normally this vector at the ith I can write here the summation of here vi vj here j is equal to 1 to nv or you can say n total number of buses here i can write gij cos delta ij plus b sine delta ij. So, this is we can say this is a function of voltages vectors and as well as the angle vectors. Here delta ij is equal to nothing but your delta i minus delta j. So, this whole function we can write is a function of function of angle and voltage. So, in the total if include this 1 as well. So, I can say this whole representation can be written in this form that is a vectors here this F vectors and it is for all the buses except your slag bus and if it is a real and reactive since it is a reactive power real power.

So, it will be n minus 1 equation and since it be load flow equation is also for reactive power. So, it will be your N minus Nq, Nq is number of generators they are reactive power including reactive power sources. So, the total number of equations here the power flow as you know, it is your 2 N minus Nq minus 1. Now, you can see here we have number of variables now, or here the number of variables where equal to the number of equations and we were able to calculate the value that is unknown values, because we had the number of equation equal to the number of unknowns. In this case we have another unknown that is lambda that is we had we do not know, what will be the lambda at this point f. So, right now, the total number of variable is 2 N minus Nq, because another unknown value is used. So, in this case we have the number of equations are lesser than number of unknowns.

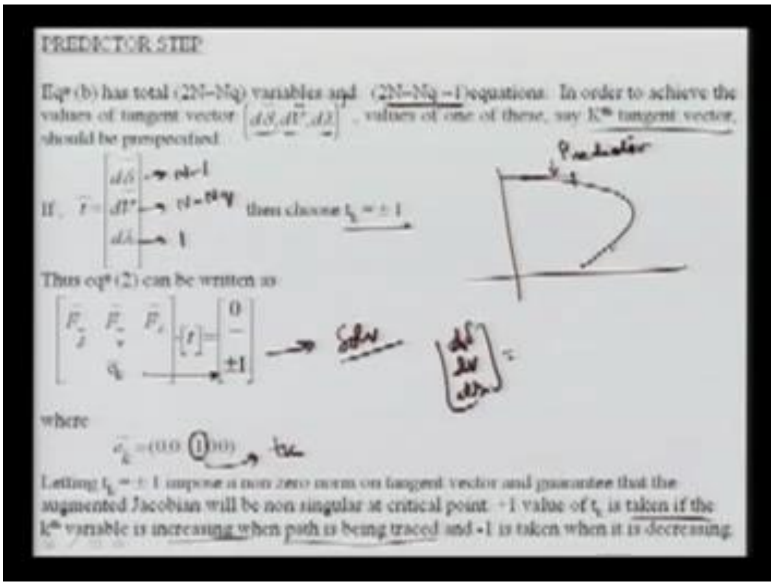
So, we have to use some mechanic we have to fix 1 value and then we can go for the predictor and the corrector methods. To solve this problem that is F of that is it is nothing but your load flow equation here including a load parameters load factor that is changing. So, to solve the problem continuation algorithm starts from a known solution known solution means you have to be start from the base case where lambda is 0. And uses a predictor and corrector scheme means 1 predictor and 1 corrector to find out the subsequent solutions at the different load levels. It uses the concept of local parameterization with lambda included in the load flow. Already we have included here lambda you can say you can see here and here the number of variables are now, increased by 1 and total number of variables that to be known here 2 N minus Nq.

(Refer Slide Time: 08:04)



Now, here let us see how we can derive? So, the equation which I wrote here that is a equation here if you can differentiate we can write in the differential form with the reference to your delta voltage. And the lambda we can get this differential equation or this vector means here 1 jacobian here F is a vector and that is we have the differentiation after change in the delta. And here this is a factor that is a differentiation of the voltage means we are differentiating this function F with respect to voltage then it is change in the voltage and here with the change in the delta here we are using. So, in this case you can say the change in the lambda here is 1 factor here, 1 factor here and 1 factor here and these factors are calculated from the base case value. In this another form we can write here and that will be 0 here you can say this is multiplied by here; this is multiplied by here and this is multiplied by here. So, this is your we can say it is retained in the differential form.

(Refer Slide Time: 09:11)



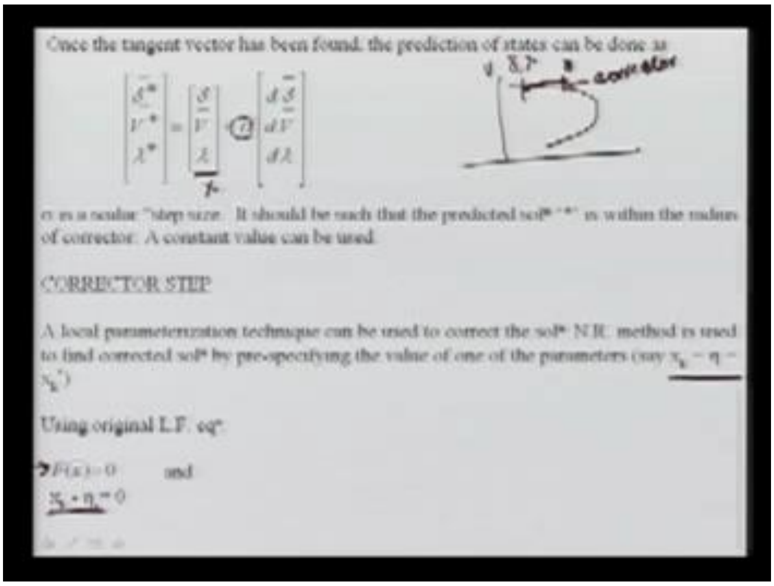
Now, what is a predictor step? First the number of the equation b has the total 2 N minus Nq variables. However, we have only the equations again i am repeating it is 2 N minus Nq minus 1 equation. In order to achieve the value of the tangent that is a vector here the value of 1 of these say kth tangent vector should be specified what happens here again if we will see the PV curve. In the predictor step always what we do? Let us about this is your actual pv curve just we want to calculate if we start from here. Then in the predictor step we draw a tangent to this point here and we go we need increase the loading and then we go for the correction step.

So, this step is your called the predictor step and since we have predicted this basically we go for this calculating this value with the increase in the load. And then keeping the load constant we calculate the actual solution using any conventional load flow algorithmic algorithms and normally we use Newton raption method So, in this as I said number of equations are less than number of unknowns that is 1. So, we have to specify 1 tangent vector and here that is your here the change in the delta V and lambda are nothing but the tangent to this surface. So, let us just t is the here vector that is a here we are of vectors this value here this value and this value this lambda is 1, because we are assuming that we are increasing with the constant load factor in this case. So, the lambda delta just we require it is nothing but your N minus 1 and the V we require N minus Nq.

So, the total you can see here it is a 2 N minus Nq unknowns are there so, what we do? We choose 1 kth out of this whole 1 we fix either plus 1 or minus 1 we will see which 1 may be plus 1 or minus 1. So, the equation 2 can be retained again if we will see the equation 2 here this equation 2 here we can write another 1 and that we can add 1 more equation here to increase the number of equations so, that we can solve it tk easily. So, tk here there's 1 value we are putting the plus minus 1. So, here we are put it here and then this is the t is the vector of change in your delta change in voltage and change in lambda and this is equation. Were this ek will be nothing but the 0 0 1 and 0 0 this will have only 1 value this is the vector here all the 0's and 1 value which will be multiplied by the t that is 1 is specified here. Means the 1 here out of these n 2 N minus Nq 1 will be having 1 and that value will which when will be fixed 1 it will be carrying only 1 1 other will be it is 0.

So, all the elements will be 0 except value 1 and that is corresponding to any kth value out of this. Letting tk is equal to plus minus 1 impose a nonzero norms on the tangent vector. And guarantee that the augmented jacobian will be non singular at the critical point plus 1 value of tk is taken if the kth variable is increasing when the path is being traced. If the value this tangent all these are the tangent value if these values are increasing here taken positive 1 if it is increasing while the tracing means while going here if it is increasing then it will be taken 1. And it will be take a negative when it is decreasing normally this side it will be taken negative value. So, once the tangent vector has been formed means now, we can solve this 1 easily we can solve this equation and now, what we can get? We can get the vector t means we can get here change in delta change in voltage and change in your lambda we can get here from this 1.

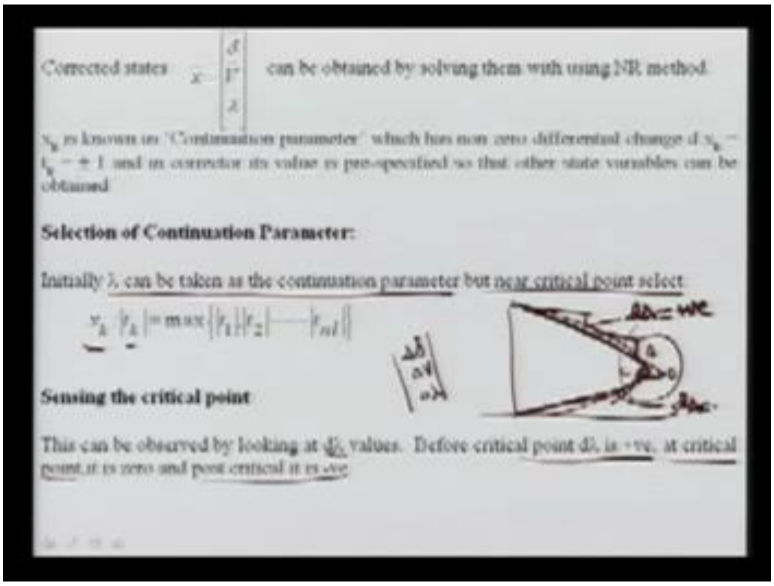
(Refer Slide Time: 13:36)



And then once you have obtained this then we can add with with your static point means base case and then you can add with the sum here we have use sigma or scalar step scalar value and that is known as the step size. It should be such that the predictor solution star is within the radius of corrector or constant value is normally used. So, we can get the optimal value and then this is your predictor step means from here from any point in this curve you have be started from a you have just made a tangent here. And then you have reached at this point that is your star point which we have written. So, starting with here this lambda's lambda here and your just we have obtained this. So, we have reached at this point. Now, this point is not the actual solution, because it is not on the surface of this curve means that is an approximate solution. So, what we have to do? We have to again go for that correction that is a error which we have encountered which we had been taken.

So, that step is called the corrector step and that corrector step just let us see how it is used this is a correction. In this corrector step a local parameter parameterization technique can be used to correct the solution. The Newton raption method is used to find the corrected solution by pre specifying the value of 1 of the parameter means 1 parameter we have to fix. And then we have to calculate the remaining 1 and that value which we are fixing here that is a Xk. Xk means any value out of these all these this is your nothing but it is X vector in that you are fixing 1. And then you are going normally this is a nothing but a load that load you have increased that is fixed and the remaining we calculate this. So, using the original load flow equation here that is your Fx bar that is a x vector here and then we are having fixing 1 kth valuable variable, because we are having more number of unknowns than equation. So, we are adding 1 equation here and then we can solve it and this is called a corrector step.

(Refer Slide Time: 15:50)



The corrected state that is a x bar here is a this this can be obtained by solving them using the conventional Newton raption method. Xk is known as the continuation parameter the kth is known as the continuation parameter which has nonzero differential change that is a dx should be tk and that is should plus minus 1. In the corrector it should its value is pre specified so, that other state variable can be obtained. Now, you can see in this method what we are doing? If you can see if you are going for the corrector and the predictor step let us see here again this is your that is we have on this PV curve here. Here it starting from this point and then you have a taken at tangent here and then you are correcting here again you have taken a tangent you are correcting and here again you have taken a tangent and then you are correcting.

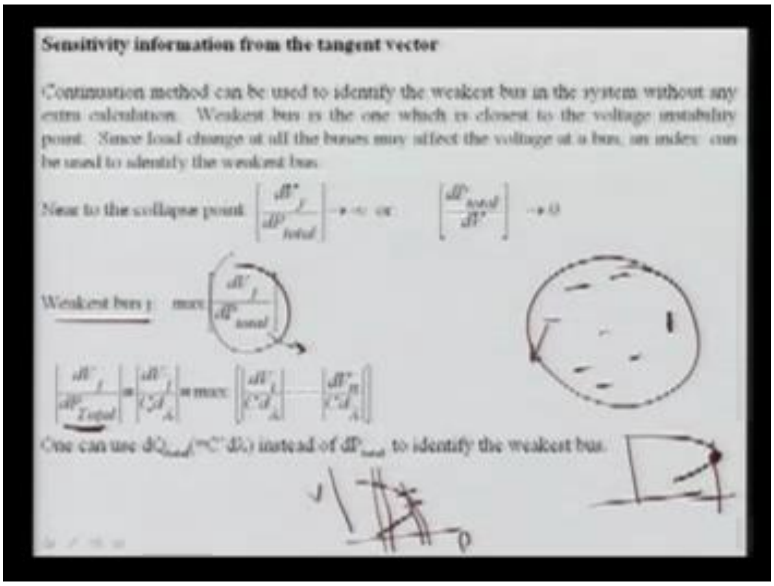
Now, you can see the point here at the d and this is the point e at the point d for the corrector step this will not converge. Because this is an outside the, it is not solvable point at all so, you cannot solve this one. So, what you have to do instead of the, you can estimate the load here what we did? We estimate the load we have just moved here tangent vector we estimated the load here then at that load we got the solution and then point e. Here you can say point whatever you can say b from b we took here the 1 tangent we move at the t, but this d point here is out of the solutions unsolvable again the non convergent and unsolvable both are the different set. There is a possibility that the non convergent solution means the method is not accurate and it is not giving the converge solution. But there is a possibility that the solution exist, but in this case you can say it is out of thus it is not cutting here for any demand this is a it is a unsolvable point.

So, what in that this case we normally fix the voltage means we never estimate the load there in this case fix the voltage and then we correct this and then we can reach this e point. So, in this corrector step normally the step ((refer time 18:15)) in such a way that we can get we should not go some where beyond and then we can get the solution. So, this method is no doubt is very slow and it is very time consuming compare to the conventional power flow method. So, the slow as you know, here every time we are moving with a small distance tangent 1 and then we are using the predictor by solving the 1 load flow then we are correcting 1 load flow.

So, keep on doing this till we are moving to the notch point. But this method will give the converge solution at no doubt in any case it will give the converge solution. So, the selection of continuation parameter is very very important. So, initially lambda is can be taken as the continuation parameter, but near to the critical point here at this point what should be your this continuation parameter that parameter this Xk or you can say tk is the maximum of all this tangent. So, what is a difference this t what is the t 1 t 2 all t n there are nothing but your change in your change in your delta change in your V and change in your lambda.

So, here which we have the highest value that is taken as a continuation parameter near to the critical point or you can say notch point since in the critical point this can be observed by looking at the change in the lambda values. Before the critical point normally here in this change in this will be positive at the critical point it will be 0 and the post critical it is negative. Means here during this zone here change in the lambda change in the lambda is always positive. Here at this point it is 0 and lower point here it will be your change in lambda it will be negative.

(Refer Slide Time: 20:12)



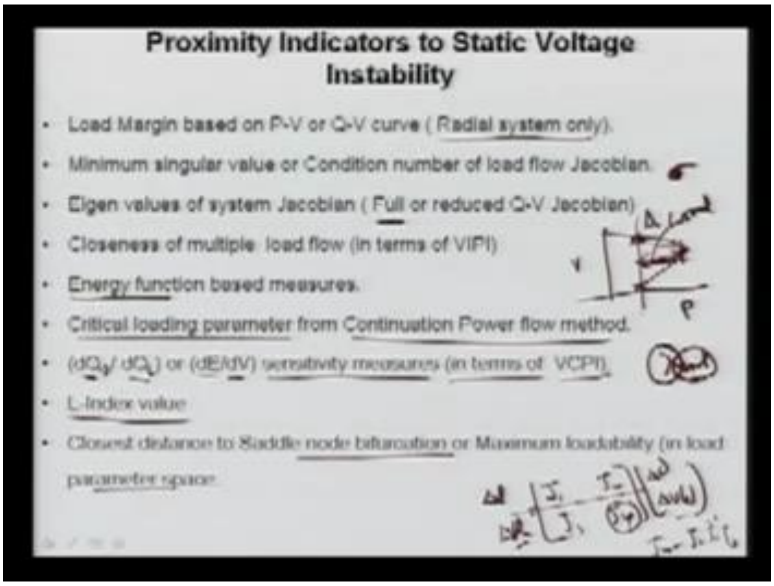
Now, the, we can get other information as well from this continuation load flow method. And that is basically we can get the sensitivity information that sensitive information that we can get from the tangent vector, because tangent vector is change in the angle voltage and your loading. So, continuation method can be use to identify the weakest bus in the system without any extra calculation. There is no need to go for extra calculation and based on this continuation method we can get a weak bus system in this. So, that we can know, which one is the weak weakest bus and very close to the voltage instability point. Means weakest bus is the 1 which is closest to the voltage stability point since load change at all the buses may affect the voltage at a bus and in thus can be used to identify the weak bus in the system. You know this here whole big system let us suppose the voltage here at this bus and you are keeping on increasing there various buses in the system.

If you are changing here even the load here the voltage at this bus will affected it may be less affected, but it will be no doubt. So, here it can be defined as the near to the collapse point or notch point collapse point is nothing but the saddle node point here. Again I have to say it means here the collapse point is this is your collapse point that is notch point sometimes called saddle points. And here also called the singular here the jacobian becomes non singular. So, the near to the collapse point the change in the voltage at any bus here with the change in the p total how much change that is a p total means changing from one point to another point that is tends to infinity. Means it is very very high value means if you are changing here your load the change in the voltage will be very very high.

How you can see here? Again from this you can see at this bus means let us draw only the notch point here very quickly here. This is your p this is your V you can see here if you are changing the value this p and this p the voltage change is very small, but you can see here and here this voltage was here and now, the voltage is here. So, at the notch point this value is changing to infinite. So, what we normally do? We can get the maximum out of all this at which bus it is the voltage change is the maximum and that will be the weakest bus in the system. So, if want to go for the several weak buses then you can rank them and then you can pick up which are having the highest value are that is change in the voltage of that bus with the change in the total power and here we are talking about the real power. The reverse of this if you can go this will be change in the total real power with the change in voltage it is tending to 0 the reverse is also true.

So, what we can do? It is the change in the total here it is nothing but some factor c and change in the lambda. The change in lambda here this is also see the some constant, because some initial loading and then we are changing the lambda. So, this is also we can say change in voltage with the change in here you can say lambda that will have the maximum value. We can also use for the reactive power means here instead of p total we can use some q total as well and then we can also identify the weakest bus using here through the total reactive power change and then again we can have the different buses for the different one So, this sensitivity information is very use to identify the weakest bus in the system and that weakest bus system is very close to the voltage instability point.

(Refer Slide Time: 24:20)



Now, the proximity indicators to the various proximity indicators to static voltage instability again the voltage in stability can be classified in to the 2 types. One is the static another may be your dynamic voltage instability will come to the later about the dynamic instability dynamic voltage instability. To see what are the various proximity indicators for this static voltage instability? First one is the load margin based on pv or qv curves and it is true only for the radial power system means here this is your as I said this p and this V especially it is violate this one how much margin you are having from here?

Let us suppose you are loaded at this point. So, this this distance is called the load margin means point here is a notch point this is your actual operating point and this is your card the load margin. So, we can say if you are having more load margin in the radial system we can say your system is more stable. If you are operating very close to notch point then you are very close to the voltage instability condition and that is true only for the radial systems only. Similarly, we can also go for the cubic curves and again we can see and we can analyze that how close we are with respect to the voltage instability.

Another indicator is already I have discussed that that is the minimum singular value or condition number of the load flow jacobian. Means at the, this here we can go for the load flow jacobian here we can calculate. And then based on that we can calculate the sigma's if you remember here sigma that is a condition number. Or you can say minimum singular value and means the, we can calculate the jacobian the Eigen value which will be having the minimum value critical. And based on that we can calculate corresponding to that we can calculate the minimum singular value already I have discussed that.

So, this is one criteria that we can determine another using the Eigen value of the jacobian. And especially we can use either full jacobian means com using both real and the reactive power flow equations. Means that is a full we can use your J 1 and J 2 as well means here J 1 J 2 J 3 J 4. Here this corresponding to your change in p here change in q and that will be equal to your change in delta here change in V by v. So, this is called full jacobian, but we can also use the reduce jacobian and that is called the cubic jacobian here we can use the J 4 means what we do?

We assume if there is a no change in the p we can go for the q V relation and then that will get the reduced jacobian corresponding to J 4. And this is normally J 4 minus here J 2 J 1 inverse here J 3 and you can get it. Another is the closeness of multiple load flow solution in terms of voltage instability proximity indicators. So, VIPI normally as power flow equations are non-linear and we can have the various solutions. So, multiple solutions are there again if you are keep on moving to the critical point the solution becomes 1 and after that we cannot have the solution at all.

So, the solutions for this radial system you can see for a given p we are having 2 voltages. So, it has 2 solutions and then you can see if keep on load is increased here only 1 solutions. Means both solutions are converging and both solution will have the same value. So, in the power flow we can get the multiple solution and then we can find the closeness to the multiple solutions and then we can go for the different index we can form and that is called the voltage instability proximity indicators we can use. Another is energy function based measures means we can use the energy functions.

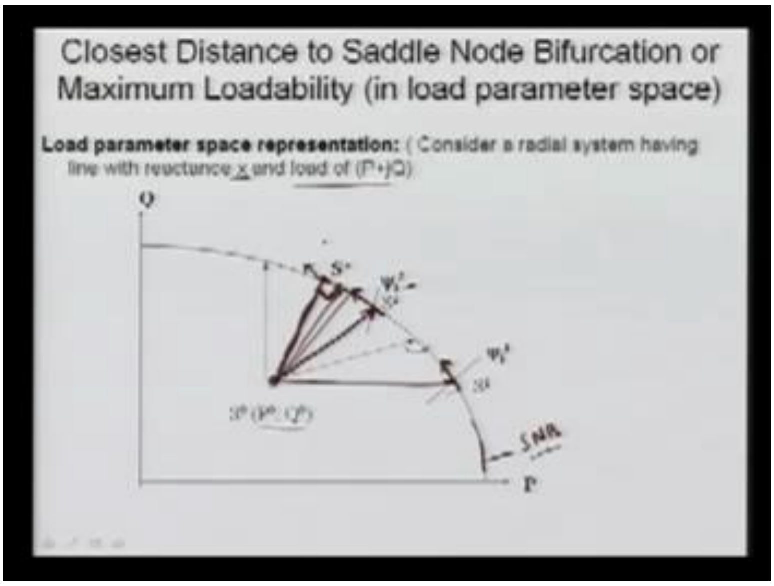
And we can see how much energy needs from boundary how much energy means you are having and you can we can use we can formulate the problem in terms of energy functions. The critical loading parameters from the continuation power flow methods here means here the critical loading parameters as you know, if you are operating lets suppose at this point a with the help of continuation parameters you can find that how much you can load the param your system to reach this notch point. That is a critical lambda critical I can say means you can only load your system up to that lambda critical.

So, knowing this criticality we can say how much you are close to the voltage instability? And this can be obtained using your continuation power flow method it is just I described other that we can use the change in the real here the reactive power generation to the change in load or change in e divided by change in rho V that is your sensitivity measures. And this gives your voltage control proximity indicators means here if you are changing the voltage at 1 bus how much voltage change in other system are that is a excitation voltage.

So, that can also give your some indicators for the voltage instability and again it is static in nature. We can use the l index separate to pull having suggested that l index is also very useful and the powerful methods to calculate this. To know, that how far you are from the voltage instability point and this index can be also used. Another is your closest distance to the saddle node bifurcation or the maximum loadability in load parameters space. As I said here we can say how much thus we can how much distance just we have closest distance and that point is called your saddle node bifurcation point.

So, these are the various methods those are suggested in the literatures and and here most of the solutions here some one is very fast some one is very slow some gives only some relative information some gives some more information. So, we can analyze the static voltage and stability using these various methods. Now, let us always we try to calculate here the last point which I said here the closest distance to the saddle node bifurcation point. Here in the 2 bus system it is very easy to calculate that this distance you can measure here. But what happens in this multi bus system or the several bus system then it is very difficult to calculate this.

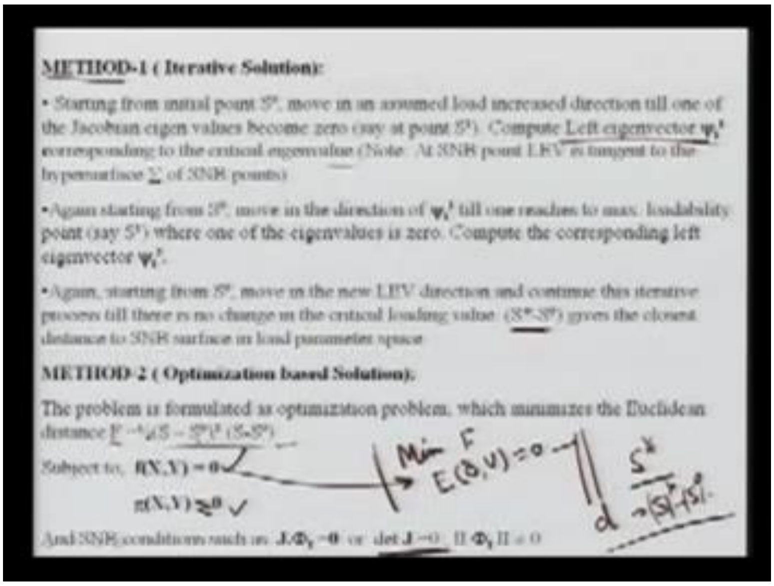
(Refer Slide Time: 31:26)



So, what normally we do? We go for some method to calculate for example, this is your pq space means this is your the space at all the buses means here pi and qi and that you can say it is a 2 n space in this in this axis is there So, you are some where at point x here and that is called your s not means your complex power is a p plus jq naught here you are having this. Now, I want to know, this boundary is your saddle node boundary. So, it is your hyper surface it is not a simple surface. It is a hyper surface of the n dimension hyper surface and we want to see how much you are close that. So, we want to calculate this distance from this boundary.

So, this minimum distance and that minimum distance is always once it is here it is that you can say it is a perpendicular to that point. So, if it is forming 90 degree this is not your minimum loading this is not your minimum loading, because this angle is more this angle is more. So, we have to determine this s star and that distance s star minus s not here the magnitude. That will give you the closest distance to the saddle node bifurcation point or maximum loadability. In the load parameter space load parameters again that is the p plus jq. So, consider a radial system having line with the reactance x and load p plus jq.

(Refer Slide Time: 33:00)



Now, how we can determine that the difference between your s naught to s star 1 method that is a method number 1 that is you can use the iterative solution method and that starts from the initial point S S naught and moves in an assumed load direction. You can assume any load and increase in the direction any assumed direction till 1 of the jacobian Eigen value of jacobian Eigen values becomes 0 say it is at point s 1 you can see here means here you are moving you are increasing the load. Now, here simply just we have increased the real power keeping your reactive power constant, but you can go in any direction. So, what happens you are keep on increasing increasing increasing and you have reached at this point, because at this is a surface where it is a saddle node bifurcation means one of your jacobian Eigen value become 0. Means this jacobian becomes singular at this surface this surface is you can say it is called singular node bifurcation point. So, you are moving here at the point s 1 you have reached your jacobian becomes singular.

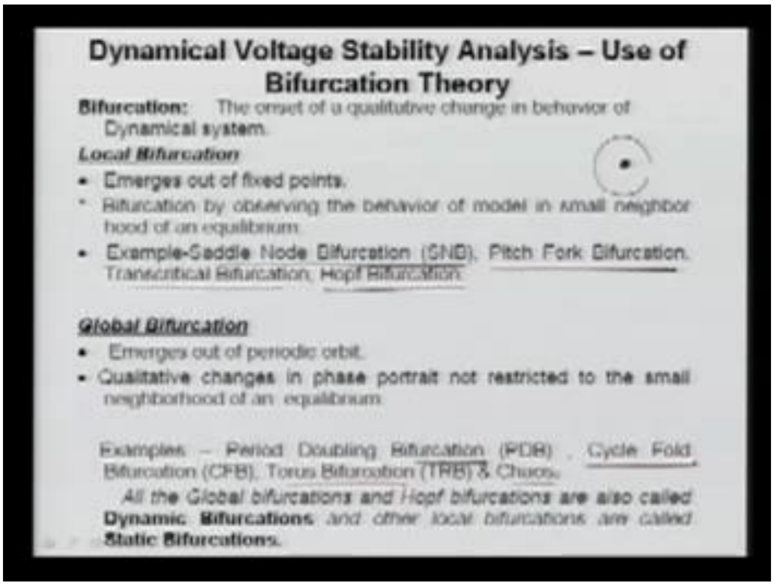
So, corresponding to the Eigen value which is 0 you can calculate your left Eigen vector that is a psi 1 corresponding to the critical Eigen value that is 0 in your case. This you have denoted at the saddle node bifurcation point this is tangent to this surface. This is your surface means this is your surface and this value will be the tangent to this. So, what you can do? Just you have calculated here and then you got the direction tangent. So, now, after getting this direction here now, you can again come here and now, you can move your load in that direction here. So, in that case you may come to another point lets suppose here in that direction here and you have achieved your S 2 again this S 2 is not your optimal point means this is not the distance here, because the minimum distance is here we want to calculate this minimum distance. So, this point also S 2 you have to calculate here as you know, this boundary your jacobian determinant become 0 means you will have 1 Eigen value 0. If they are having that 1 Eigen values then corresponding you can calculate the left Eigen vector again. And let us suppose you got the psi I 2. This will also give you a direction here and that will be a tangent.

So, again with this direction calculation again you come here and then you can move to another point here and the finally, you can again moving the tangent and you have arrived s star. At the s star now, this will be here this point here there will be no further change. Means here no change in the critical loading values and this star minus s naught gives the closest distance to the saddle node bifurcation surface in the load parameter space. So, let us see a next method that is method number 2 and which uses the optimization method in this what we do? We formulate the whole problem into 1 objective function and that objective function is nothing but it is the Euclidean distance that is f. And which minimizes the distance between the minimum distance it finds out and that here you con this distance we have to find out. So, what we from starting from here we have to move at this boundary. This boundary is again saddle node bifurcation point and then when it gets the minimum value then it will stop and again subjected to the various constraints. So, here your objective function is a scalar objective function f which is a difference between your s that is a we are going to find out what will be the value of f minus s naught where this your initial operating point was there.

So, here it is a transpose 1 so, here s is the vector means vector at all the buses there is a SI means s 1 s 2 s 3 and so on so forth. So, here just we are difference then we are multiplying with the transpose means we are getting here a scalar value. The half is used is since 1 constant optimization will not change at all if you are multiplying by any constant only we have since here if you are differentiating it here this 2 will be vanishes. So, this is your objective function means we have to minimize this function f that is minimum of f subject to various constants constraints here are nothing but your power balance equations that is here that is your I can say f that is a delta and here V is equal to 0. In this case what we are going to achieve? Here this value f delta here this is corresponding to both the real and reactive equations. So, you have the equality constraints then you have your inequality constraints means that is a limit on your various things limits on your power generation, limits on your the voltage limits on the ((refer time: 38:35)) so, we can have the equality constraints here.

And we have the inequality constraints and this is your conventional optimization problem and that can be solved by any non-linear programming techniques here this problem is highly non-linear. So, the selections of this method all although you must know, there are so, many optimization methods are there even though in non-linear. But they are some are having inherent problem and then you have to choose very and also there is a initial guess is very very important. So, initial guess we start here from s naught and then we can find some initial guess and then we can get the optimal solution. So, at that point that there is a weak condition here the jacobian the determinant here becomes 0. So, we can optimize and we can get it and then here the solution here you will get it here star point. And then you can calculate the distance that is your d distance will be your star minus s snot.

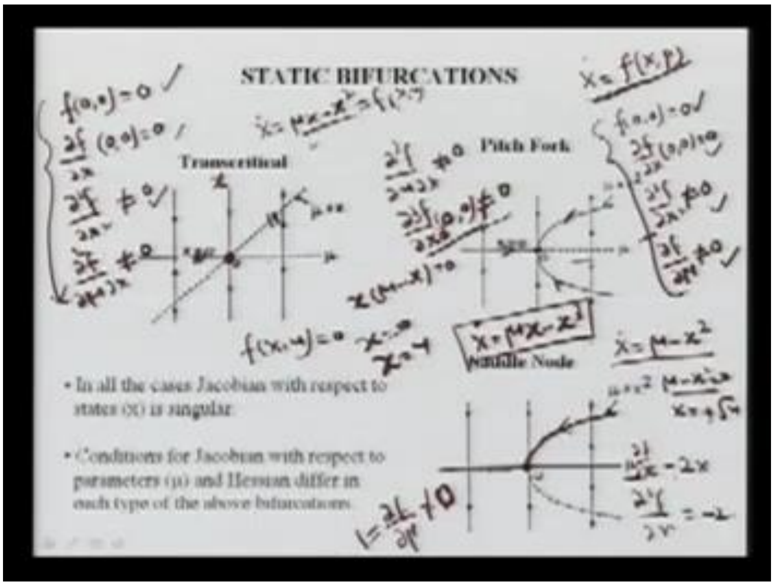
(Refer Slide Time: 39:46)



And here then you can say how much loadability that you have in your system? And that gets your information about the voltage instability point. Now, come to the dynamic voltage stability analysis and normally for that we use the bifurcation theory. The bifurcation is nothing but a qualitative change in the behavior of a any dynamical system. Whenever there is a qualitative change in the behavior of the dynamical system then that point is known as the bifurcation point. There the 2 type of we can classify the bifurcation into the 2 categories 1 is your local bifurcation and another is your global bifurcation. Local bifurcation basically emerges out of fixed points fixed point are nothing but your initial equilibrium points. Bifurcation by observing the behavior of model in small neighborhood of an equilibrium. Means equilibrium point here it will equilibrium point is this then we normally go for the small neighborhood of this one we go for and we analyze this. Example of this local bifurcation is nothing but your saddle node bifurcation that is SNB the pitch form bifurcation transcritical bifurcation and another which your hopf bifurcation.

This local bifurcation is nothing but it is your static bifurcation which also known except your hopf bifurcation. Global bifurcation is nothing but it is your dynamic bifurcation that emerges out of periodic orbit. The qualitative change in the phase portrait not restricted to the small neighborhood of an equilibrium point. This has a several examples and that examples are periodic doubling bifurcation PDB, cyclic fold bifurcation and the torus bifurcation and chaos of course, chaos is one of them. So, all the global bifurcations and hopf bifurcation are also called the dynamic bifurcations. Means here in local if include this hopf bifurcation then it becomes a static and if include the half here in the global it becomes the dynamic bifurcation. So, we can categorize in the dynamic bifurcation and in static bifurcation.

(Refer Slide Time: 42:06)



Now, let us see the various bifurcations here. First we can see this saddle node bifurcation that is a very very as already in static we discussed the saddle node point, saddle node bifurcation point that was nothing but here you can say it is a notch point here and we were calculating. So, the saddle node bifurcation you can say in all the cases jacobian with respect to the states is singular. Here you can say here your jacobian becomes singular, here it becomes singular, and here it becomes singular. So, this is the point were your jacobian becomes singular. So, in all the cases jacobian becomes singular the condition for the jacobian with respect to the parameter. Means parameter that is a load or whatever you can say here mu parameter and the Hessian differ in the each type of above bifurcation.

So, this parameter value is keep on changing, but all are having the jacobian point here these are the bifurcation points at this point. So, in the saddle node bifurcation point that we can categorize here you can see here this is a stable part and it is here you can say suddenly lower portion is unstable. So, this is your stable and here this is your unstable. So, it is a qualitative change at 1 point you can say the system becomes stable and then suddenly it becomes unstable. Then this point is called the saddle node bifurcation means the point between the 2 stable and unstable points in generally you can say. So, this is your saddle node bifurcation. To have the condition for the saddle node bifurcation it is the defined as that if you are having means we can any dynamical system I can say it is x dot is equal to your any function f that I can say here x into some p. P is your disturbance vector that is a changing something change.

For example, let us suppose is a load or mu that is I have considered and x is the state vector and it is the describing state. Now, if it is your dynamic system that is a x dot it is nothing but dx upon dt that is changing with the time. Now, for the condition of this here for the saddle node bifurcation first conditions here this f 0 0 here is equal to 0. Means here if you are putting x is equal to 0 and the p is equal to 0 it is 0 you can see here at the mu 0 everything here 0. Means here mu value is starting from 0 here you can say x is also 0 and need 0 at this point. This is the first criteria another here your del f with change x here and again add the 0 0 it will be 0. Here your double differentiation of x x here and then will be not equal to 0 and also del f here del mu in this case p here it should be not 0. To have the example for this so, this is these are the condition for your saddle node bifurcation.

To have an example, let us see and the example here that is your x dot is equal to mu minus x square. Now, this is your dynamical system and then you can see here x dot is equal to 0 here we are getting the solution means your equilibrium point here that is the mu minus x square ((refer time: 45:47)) 0 means I can say x is equal to plus minus mu. So, this is your solution point now, when x is 0 and mu is 0 we are getting this value. Now, if you are differentiating this this function this is your fx. So, if you are differentiating with the x you are getting minus 2 x is equal to your del f upon del x. So, I am putting 0 then again you are getting 0 so, you are satisfying this one. Now, if you are double differentiating this means your del f del x square is minus 2 and it is not 0 so, you are getting this one as well. If you are differentiating with respect to here mu and you are getting your del f upon del mu you are getting unity which is not 0 means it is not 0, because it is equal to 1.

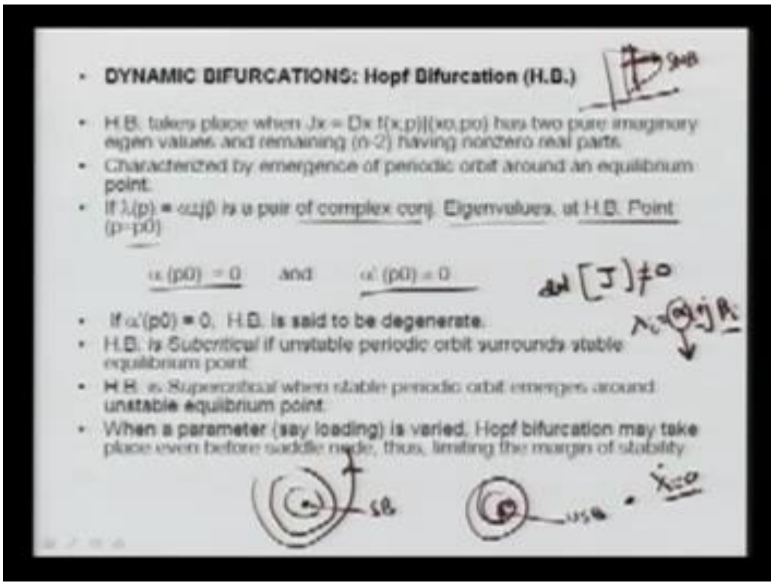
So, we are meeting this one and then it is called the saddle node bifurcation. Here if you are you can see here the point which is coming here and here it is suddenly it is on a stable and it is called a saddle node. And this is nothing but an equation of parabola that is a mu is equal to x square here that will be static. Now, let us come to your other bifurcation point let us see a transcritical. In the transcritical the condition for this 1 is the similar means we can have a condition that is a f 0 0 is equal to 0. Here a change in your f with your x 0 0 is 0 here your del square f upon del double differentiation is not equal to 0 and here we want to have del f over del u del x double differentiation here should not be 0. So, these are the 4 conditions for your transcritical and the example for this if i can put here x dot is equal to your mu x minus x square. You can see if you are putting mu is equal to 0 and x is equal to 0 here and this is nothing but your f x and mu here if you are putting this x 0 mu 0 you are getting this if you are differentiating this with respect to x.

So, you are getting and putting 0 you are going to get this. Here if you are double differentiating this you are getting minus 2 which is not 0. And if you are differentiating with respect to your mu then you are differentiating with respect to x we will not get 0 and then you are satisfying this. So, this is your transcritical bifurcation here you can say the point of the bifurcation again here the point just we are starting that is here f f x u is equal to 0 means if you are solving this is what is showing. This is showing your x u minus x is equal to 0 means its ((refer time: 48:54)) s is equal to 0 and here your x is equal to x is equal to mu means you are having the 2 if you are drawing here mu and x you can see this is your x this is your mu. So, one will be here starting point that is 1 equation and your mu is equal to x is a linear equation and this is your linear equation. So, at this point here you can say this is the stable point the stable curve this is your stable curve, but this 2 dotted lines are unstable. So, at the point were stable and unstable your characteristics are meeting it is called transcritical here you can see it is a transcritical.

So, the conditions for the transcritical bifurcation are this and the example for this as I explained here. Another one is your pitch fork bifurcation this is slightly different than your saddle node bifurcation. But you can say this here this diagram is almost similar here only difference here this is a quadratic here it is not there. It is only this curve which coming here and this point it is stable and unstable, but in the pitch forth it look likes a fork. And you consider this is a handle and this is your fork and then it is called pitch fork The condition for the pitch fork bifurcation here we go for all these 3 points here and we add several more. So, these 3 are true here and then we are adding here. So, that is a mu f about mu u or mu x here that is not equal to 0 means you can say transcritical plus 1 extra and that extra here is your triple differentiation here with respect to x 3. Here it should be not 0 at here at point 0 0 here it should not be 0.

So, here we can see at this point an extra point is added and then at this point we can say this is your stable, this is your stable this is the stable and here this 1 is not stable. So, this is the 4 curves means 1 2 3 and 4 are combining here out of that 1 is unstable and 3 are stable. In this case 2 are stable 2 are unstable here 1 is a stable another is unstable. So, 1 2 and 3 and you can say now, it is a pitch forth bifurcation The example for this here we can have a example that is a x dot is equal to mu x minus here if we put x 3 then this is the example of a pitch forth bifurcation (refer time: 51:43) So, this bifurcations are used where for the dynamic analysis and that is very very important. So, another dynamic bifurcation here this is all 3 are the dynamic bifurcations another one is what your that is called hopf bifurcation and that comes under the local bifurcation category.

(Refer Slide Time: 51:58)



So, the dynamic bifurcation that another is hopf bifurcation and that is very very useful for the getting your dynamic stability in the terms of voltage here I am talking. So, hopf bifurcation takes place whenever the jacobian have the 2 pure imaginary Eigen values and the remaining having the nonzero real part. Again you can see here in the saddle node point bifurcation here I said the jacobian here complete here it will have the determine is equal to 0 means 1 of the Eigen value means it is a rank ((refer time: 52:44)) so, 1 of the Eigen value will be 0. But mean the hopf bifurcation here the determinant is not 0, but the Eigen values here the pair of pure Eigen value means normally I can say the lambda i it is nothing but your alpha i plus j i can say beta I so, here this will be 0. So, we can say we will have the pure and again it will be the conjugate. Always if you are having the imaginary Eigen values it will be in the complex conjugate pair means 1 will be the plus another will be minus.

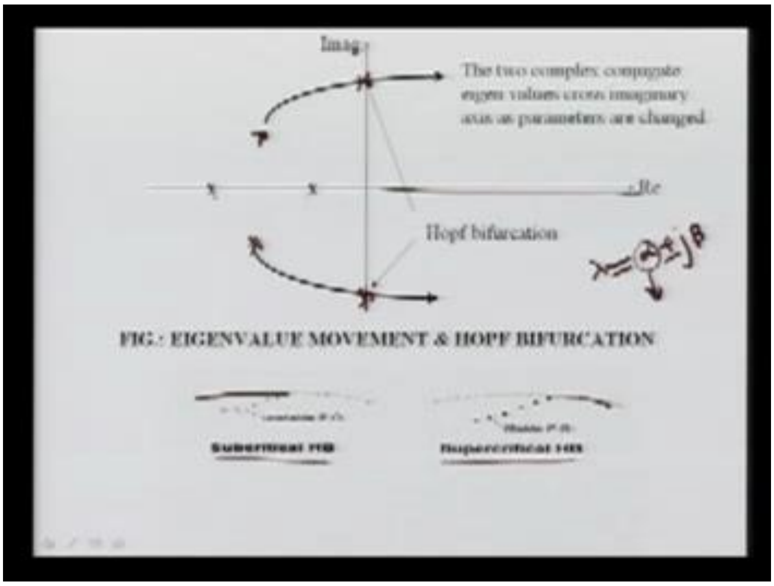
So, here this real part of this Eigen value is 0. So, this is not the determinant 0, but here the real parts of the 2 Eigen values are 0 and then here we can achieve and that is known as hopf bifurcation point. It is characterized by emergence of periodic orbit around an equilibrium point means it is a periodic point around 1 equilibrium point this is a point that is nothing but your here I say x dot is equal to 0 this will give you the equilibrium point. So, if the lambda that is a here that is a parameter of loading parameter p or disturbance parameter then this is Eigen value here this is a lambda is Eigen value wherever in the continuation power flow lambda I was using the loading loading factor.

So, this lambda is the different than the continuation power here is the Eigen value. So, Eigen value corresponding to any loading parameter p that I can say alpha plus minus j beta is is a pair of complex conjugate. The Eigen values at the hb point here it will be having 0 and for others it will be remaining it should not be 0.

So, if here alpha remaining some of the alpha if it is 0 then we can say this hopf bifurcation is degenerate. Also this hopf bifurcation is classified in to the 2 different categories 1 is called sub critical and another is called super critical. Means in sub critical if unstable equilibrium orbit surrounds the stable equilibrium points. Means here there is a possibility that this x not whether the solution here is stable or not. So, we can have the equilibrium point here putting x will be 0 whether that is a stable or not. So, if it is a stable and it is surrounded by unstable periodic orbit means here this is your unstable points here unstable points and it is going out. And this is your stable equilibrium point then it is called sub critical means here it is stable outside it is surrounded by unstable orbits. How that in the super critical? Here your equilibrium point here is unstable 1 is stable equilibrium point.

And then it is a surrounded by here you can say a stable orbits and that it is called super critical when a parameter says the loading is varied hopf bifurcation may take a place even before saddle node and thus limiting the margin of the stability. So, for in the static what we were considering this is you are this point and we were going up to this saddle node bifurcation point. And then we are saying if your loading is here then this di this distance is your margin that you are having to the collapse point. But this saddle node point may be even though earlier to that saddle node point and then you cans not load beyond that, because your system becomes unstable. And that is called hopf bifurcation point and then we have to measure the distance from here to this one to the dynamics of the nature of the system.

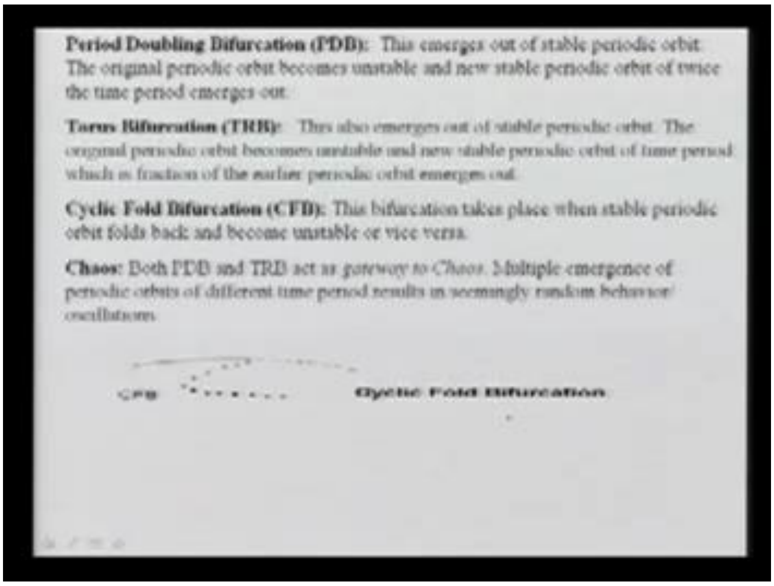
(Refer Slide Time: 56:50)



So, you can see in any system here the your Eigen values may have a real only may be having the complex conjugate and may be having the positive means here you can say your lambda can be alpha sorry here lambda can be here alpha plus minus j beta. There is a possibility if beta is 0 means you are having and this value can have the negative 0 or positive. If this alpha is positive this zone then it is called unstable at all. If it is negative then it is in here your left hopf plane and then it is a stable and if beta is 0 then it will lie on the pl axis. There is a possibility that here alpha and beta are not 0 then we will have the pair of root cl the pair of Eigen values and if you are keep on increasing your load then what will happen?

They may move and they may worry and here they may cross at this imaginary point so, at this point, when they are crossing the imaginary point that is called your hopf bifurcation point. So, the 2 complex conjugate Eigen values cross in imaginary axis as parameter are changed and here we are talking about the load parameters. So, this is your at this point you can see this becomes 0 and only we are having the pair of this Eigen value that is imaginary values. Now, the sub critical here and the super critical you can say this is your solid line is your stable line and this point is called unstable periodic orbits and it is surrounded by stable. So, it is sub critical. Here your this is a stable periodic orbit and here is your stable zone here it is unstable. So, it is surrounded by your unstable.

(Refer Slide Time: 58:59)



So, this is your hopf bifurcation. So, we have seen that the bifurcation for the static we have the different criteria's and then we can go for the analysis of the using the different proximity indicators. And then we can analyze the static voltage instability whereas, if you are going for the dynamic voltage instability you have to use the bifurcation techniques. And this hopf bifurcation is one of the varying part in which normally occurs before the saddle node bifurcation point. So, you have to go to determine the hopf bifurcation point and then you can say how much you have your dynamic voltage stability margin. And then you can decide your parameter have a system operation and also in some times case of the ((refer time 59:44)) 1. So, in the next, we will see the various other bifurcation techniques. And then we will get some examples for how this other devices like a facts that can improve your dynamic stability as well.

Thank you.