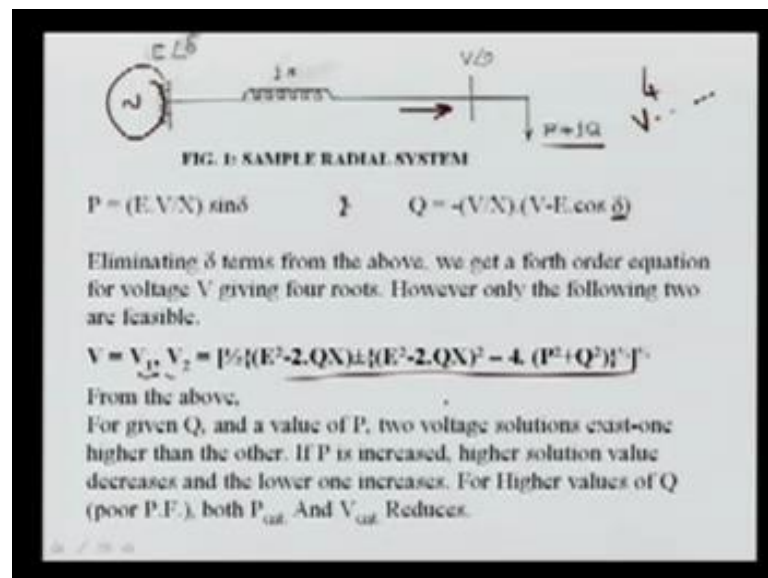


Power System Operations and Control
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Module – 2
Equipment and Stability Constraints in System Operation
Lecture – 12

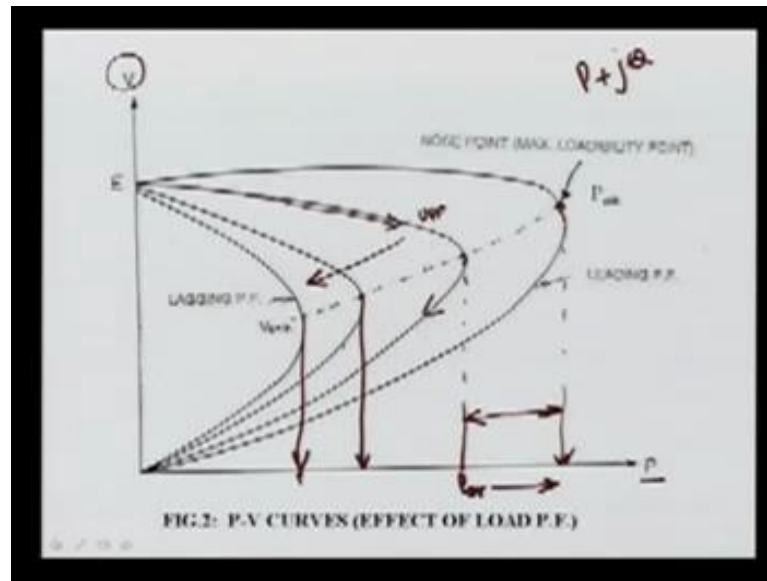
Welcome to lecture number 12 of module 2. In lecture number 11, we saw that is the voltage stability for the radial system and that we can analyze if your system is radial by the P V curve.

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Again to recap that here I used this system that is a system which is feeding power it maybe your generator, it maybe a service station. And there is a line that is a connected radial line is having jX let us suppose r is very very less. And it is feeding a load of P plus jQ for this is again I can write this power flow which is flowing here, this P value and the Q value we can write these 2 equations. If we are uniting here δ from these 2 equations we can get equations in terms of V and that will be order 4 and other terms we will be getting. So, means it will have the 4 roots, 2 roots are impossible, because they will be having the imaginary term an imaginary voltage is not possible. So, the remaining 2 we can get the V_1 and V_2 by this equation and you can get it very easily. So, for the different P and Q we can plot the curves.

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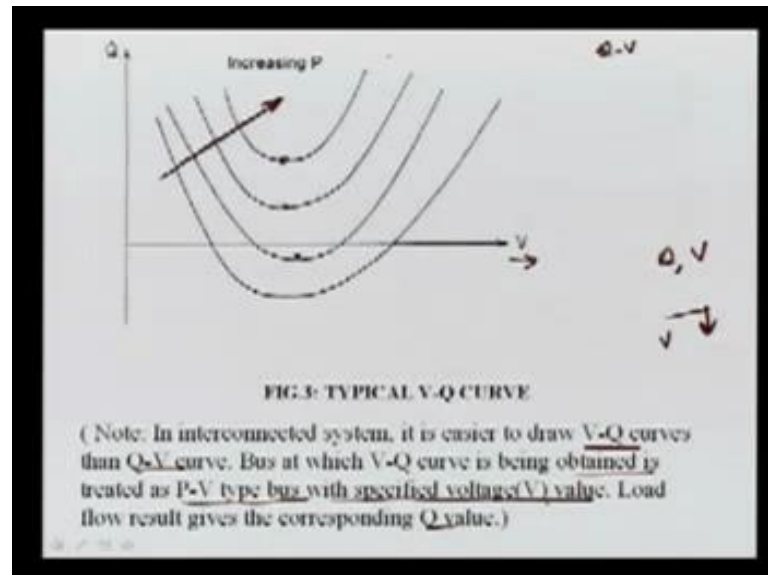


And then we plot the curve here that is called the P V curve means if your X axis is P and your y axis is your voltage magnitude at the load bus then we can get this here curve that is called P V curve. This point at this step here it is called the nose curve of this. If you are again now, for the different power factors here the curve will be different. If this is for your unity power factor UPF this curve which is here that is coming you can say po this curve then if your power factor of that load that is a Q you the Q if it is a plus or minus. If it is a minus means that is your leading power factor load and for that you can see our curve is here. Now, what we can see here? Means this P at this node this is the called the P critical and it is the voltage here we are getting that is a voltage critical. Now, if your power factor is leading you can see you can go as critical loading means maximum load that you can get here that is a more than this.

So, the source that, if you can improve the power factor of your load, you can load your power system without losing powers the instability due to the voltage. Because here now, you can say if your power factor increase now we have increase this much margin and then you can conjure more power. Similarly, the reverse is also true. Means if we are going for lagging power factor means it is your $P + jQ$ here Q is positive and P is positive. Then it is for the leading again this curve is keep on reducing and you can see you can only this load up to this critical value here for this power factor and again if you power factor is ((refer time; 03:19)) you can load up to this critical value. So, it is not

possible to go for beyond that, because it is not your system will be unstable and this is basically for this is true for your radial power system.

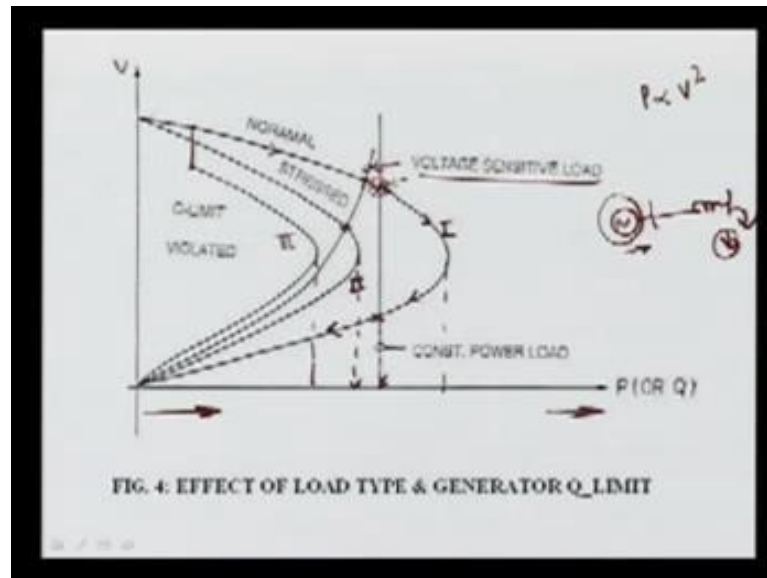
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You can also see the Q V curve this is your Q and this is your voltage curve this is basically very well known the Q V curve. In this QV curve you can see the curve here is approximately is just like a V curve means it is you can say this is V v and for the different. So, if we are keep on increasing your power if P is more you can say the Q requirement is going to be more and more. Means you can say more Q is going to require for the same voltage. If suppose your voltage here is a fixed value. So, then we can see your the power increase the Q increase will be more.

So, in an interconnected system it is easier to draw QV curves than this is vq curve than your QV curves. But at which this VQ curve is being obtained is treated as the PV type bus which a specified voltage value this is a voltage V. The load flow results give the corresponding Q value. Means you can keep on increasing the you can keep on increasing your load that P and then you can solve your power flow equation. So, then you can get the voltage for different P and Q. So, for that for any bus you can get the V and Q for various load increase you can draw accord and you can find this Q. So, this also gives the information about your system stability and these here these are the nose points.

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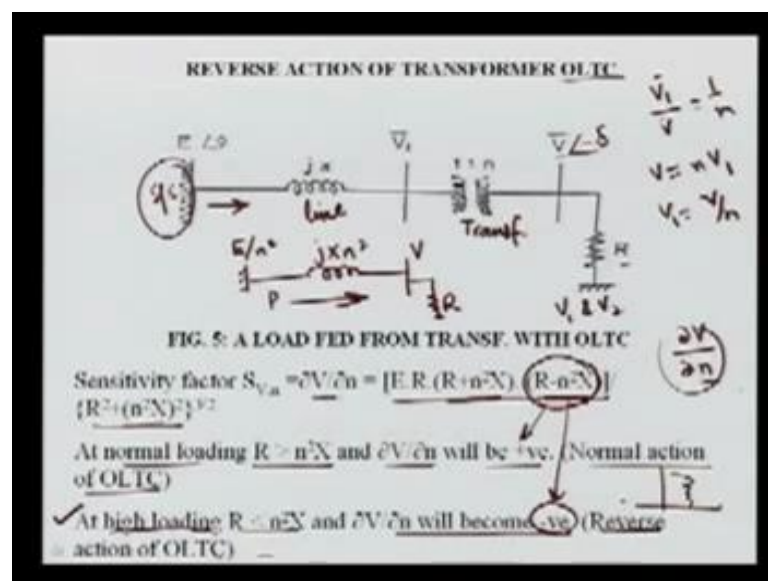
Now, let us see the effect of the load type and the generator Q limits. Here let us suppose your normal case here just I have represented as the first curve that curve which comes through here this is your curve this is your P and it is your V. So, you can say for any given load let us suppose your constant power load here this is a this curve is a constant power load you can see we are getting 2 intersection for here. It shows that we can have the 2 voltage solution 1 is very close to your operating point and other is unstable z1. So, this voltage is basically actual operating where your the voltage of the system will be this 1. Now, you can see if your system is stressed means again the highly stress system you can have another curve here that is 2 like this. Means you cannot load more in the stress system. So, you can see this line is not intersecting this it means that it is not possible to go for this much amount of power. Because the maximum power here will be only your critical power that is a P critical here you cannot get more power on that bus.

Now, again you can assume that if there is your load is a different type. Here the voltage sensitive load we have and it is a just take a P as changing your voltage is also changing. And this is just like a Quadratic curve you can say means your P is proportional to lets suppose V square. So, here you can say now the intersection here it is this and it is this. So, for the normal case you can go up to this loading and this will be operating point for the normal and for the stress you are going for this 1 and this will be actual operating voltage. Now, let us take the violation of Q limit you can see what happens? Your system is just following this at any certain point means if you are keep on increasing the

power this is the power increase that is here. Once you are taking more power there will be possibility that the Q limit of the generator is violating. Means it is here your generator this is your feeding la line and this is your load.

So, if you are keep on increasing this generator are service station will have some limited reactive power generation. So, if it is a hitting this limit what will happen? Suddenly this value will be dropping here and voltage here and then it will have another QV curve here. Now, in this case if it is violating now, you can see our critical loading that power that we can load here we can get the P now, again is much much less compared to your stress system as well as you can say normal system. So, this is your curve 3. So, the impact of this Q limit if we are having more reactive power in finite lets suppose reactive power then you can have this normal curve and then you can load as much as possible up to this point this is get converting. So, we can see if the violation of reactive power limits of the generators are reaching then our PV curve is reduced and then also we can load less reactive power.

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Now, we can again see the impact of the online load tap changing transformers already I explained what is OLTC in the previous lectures. So, this is again I have taken a system that is here your system that is a service station or you can say generator that is feeding a resistive load lets us take that is R. Here the voltage is V prime here bar means complex voltage here we are taking angle 0 means here we are getting angle 0 reference is your

this and the voltage here it is changing means this is your angle delta. So, the power will be flowing from this end and this load is consuming power. Here we have a line this is your transformer and this transformer having the term ratio that is 1 is to n. Means here you can see your V_1 over V will be your 1 upon n means this voltage divided by this is n 1 upon n 2. So, we are getting this ratio. So, now we can see your this V will be $n V_1$. Now, whole all the Quantity we can represent completely the complete system here now, we can if we are ignoring ideal transformer this side. So, we can write here this is your line reactance. And this is your V this is your r this is voltage V and this voltage will be transform and your j here x square and here this this we are getting this here it is upon n square.

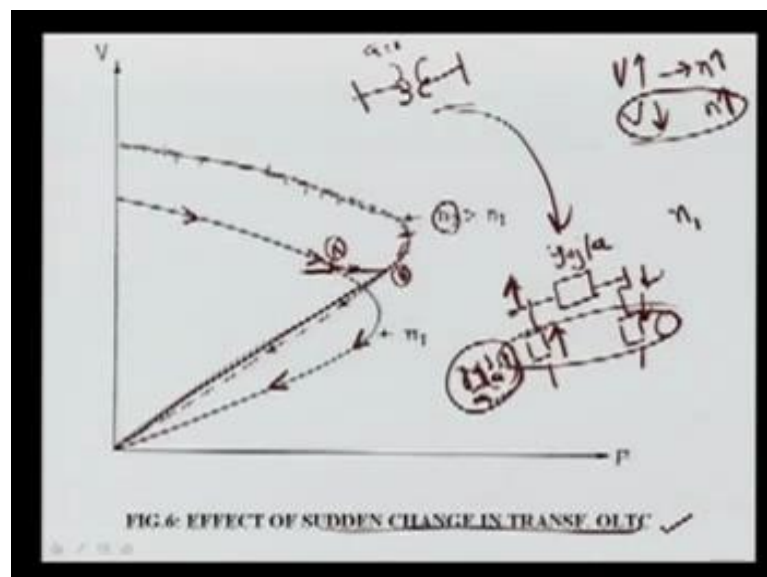
So, this voltage or you can say the V_1 I can write V_1 will be your V upon n . So, this voltage we are just putting this side. So, we can have this voltage sort of this sorry it is not n square. Now, again this is r and then now, we can write the power flow equation that is a P and we can write again the voltage equation. So, if we are writing the voltage equation in terms of P and Q and then this voltage again you will be getting your V_1 and V_2 as I in the previous case here only this; this X is changed you will get V_1 and V_2 . And from these V_1 and V_2 if we are going to differentiate this that equation you are going to get this much this is your expression and this is called the sensitivity. Means your change in the voltage with the change in tapping means if you are tapping is changed from V_1 to V_2 how much voltage is changed? And this shows that whether this value is positive or negative. From this expression we can say at the normal loading this r is greater than n square x . So, here this sensitivity factor will be the positive and this is called the normal action of your OLTC means this value is positive.

At the higher loading what happens? This r is less than n because higher loading means r is very very less compared to this x . So, this we can say here this may become the negative and this shows that it is a reverse action of OLTC. Means it shows that if we want to change the type your voltage is decreasing. If you are increasing the tap here from here you can see what the positive is and negative. It shows here it is your voltage are changing the tap voltage here V is increasing. If you are in this case in highly loaded case this is a reverse action means if you want to increase the voltage by changing tap as usual once we are going to change the tap for more taps more voltage in the normal conventional way in normal loading, but this reverses, because here you can see the

sensitivity becomes negative. Means if you are increasing the tap voltage is further reduced. And this further reduction in the voltage will cause further collapse and keep on means if your voltage here like this is here keep on reducing and finally, it is collapsing.

So, this is a reverse action of OLTC that is very very sever and that is a especially in high loading condition when from this equation you can say this $r + n^2 X$ here. This is always positive here $r - n^2 X$ that is only coming here. So, this value can be negative or positive, because that here denominator is always positive. So, this depends upon this factor only. And from here whether we can get negative that is positive or negative we can decide and it depends upon whether r is less than $n^2 X$ or these are more than that. So, this you can say the reverse action shows that when you want to increasing the tapping in for example, if you are increasing tap for increasing voltage. But the action is reverse means your voltage is decreasing and further your transformer will try again further and further and finally, voltage will be collapse. So, this is also very very adverse phenomena in the volt voltage stability case and it only happens at the higher loading.

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Again sudden change for example, if you want to change we know OLTC as I said it is online load tap changing transformers. They are the, and the loading it is the there will be some sensor circuit it will try to change the tapping to maintain the voltage. And here you know for increasing voltage of the V to want to increase here no doubt n must be

increasing. But in the reverse action of OLTC if we are increasing this this is decreasing. And that we do not want and, but you are the sensor everything it will try to increase only the tapping option is not there.

So, here it is a 1 case of the voltage collapse in other just i want to explain you the effect of sudden change in the transformer OLTC suddenly it is changed what is happening? Now, let us suppose you are operating at n 1 that is some fixed tapping your PV curve follows this again here i have drawn the PV curve. Now, at certain point what we are doing? You are just maintaining this n 1 constant are certain if reduction of the voltage goes behind certain value here suddenly we are changing we are increasing the tapping means we want to increase here what happens here?

The voltage will be trying to constant and it will come here and you can see at this n 2 tap our PV curve will be this much. What it shows that now, after changing from here to here this point a was stable, but here it is going to point b it is unstable in the sense of voltage stability. You cannot why it is less than the nose point and is below this. So, this portion from here to here it is unstable zone and this only this upper portion here it is your stable case. So, you can say suddenly change is also very dangerous and it may lead to your unstable zone and here once it has landed here.

Now, it is kept on reducing and finally, your voltage will be going 0 and your system will be collapsed. So, this is also 1 conceptions OLTC transformers sometimes also they try to although they try to improve the voltage, but some time they also get the voltage instability case. Now, let us see the, what are the, and all these cases basically why the OLTC this is happening? This voltage stability problem is related with the reactive power of the system. Because the tapping changing you know that tapping here is nothing transformer let us suppose here is there.

And if you are having some taps here what happens this transformer can be represented by here and a pi equivalent here the sun parameters here. If here it is a is to 1 I can write here the admittance of this is y_{ij} it is upon a and here it is y_{ij}^{-1} upon a 1 upon a minus 1 here it will be a 1 will be a capacitor 1 will be a inductor. So, this is nothing but is effectively just we are representing as a pi equivalent. What it does shows that 1 side it is a providing just like a capacitor 1 side is absorbing. So, again we are going to try to

maximize the reactive power means somewhere injection, but no doubt this total will be 0. So, this voltage here is increased this voltage is decreased and this is the action of that.

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Reactive Power Transmission in the Networks

- Reactive power is consumed not only by most of the network elements (lines, transformers etc.) but also by consumer loads. If it can not be transmitted, it ought to be generated wherever needed. Most of the loads operate at lagging p.f. and are voltage sensitive.
- Reactive power sources: ✓
 - Synchronous machines (Generators & Syn. Condensers)
 - Capacitors
- Factors affecting real power flow:
 - Mainly voltage phase angle difference and transfer reactance (X)

Consider a radial line having reactance x and sending end voltage as $V_s \angle \delta$ and receiving end voltage as $V_r \angle 0$. If sending end power is $P_s + jQ_s$ and the receiving end power as $(P_r + jQ_r)$, then $P_s = P_r = (V_s V_r / X) \sin \delta$

Handwritten notes and diagrams:
 - A diagram of a radial line with reactance x and current I .
 - A phasor diagram showing V_s and V_r with angle difference δ .
 - A diagram of a synchronous machine with field winding and rotor.
 - A diagram of a capacitor with $Q = I^2 x_c$.
 - A diagram of a bulb with $V_s = V_r \sin \delta$.

So, basically all these voltage instability phenomena this is the case of the reactive power and then that is a reactive power requirement in the system. What is the reactive power status that is reactive power loss as well as the reactive power generation? Now, reactive power is consumed not only by the most of the networks element like lines, transformers reactors, they absorb the reactive power consumed reactive power. But also it is consumed by the loads you know the most of the loads they are lagging they are operating lagging power factor why it is so, because what are the loads right now? Most of the loads it maybe your induction machine that is nothing but your induction motors. So, they operate you know it is lagging power factor, because they consume reactive power. We are having some bulbs here and these bulbs basically pure resistive. So, there is a unity power factor almost. We have the tube lights and these tube lights works again on the lagging power factor.

So, the most of the loads except synchronous motors our here the power factor comes in the lagging nature. So, the most of the loads operate at lagging power factor and are very sensitive to the load. So, if reactive power cannot be transmitted it ought to be generated wherever is needed. And also this reactive power or the voltage problem is the local phenomenon. It is not possible to transmit the reactive power from very far end to

the load centre; however, the real power it is true. For example, you can see in a practical power system a generator will be staying generating power very far from the load centre you can take the example of the up power corporation where most of the generating stations are thermal they are lying in the that is and also we are having some central. That is a NTPC power generators those are singrauli and rehand and they are great hub and they are generating more than 4000 megawatt power they are.

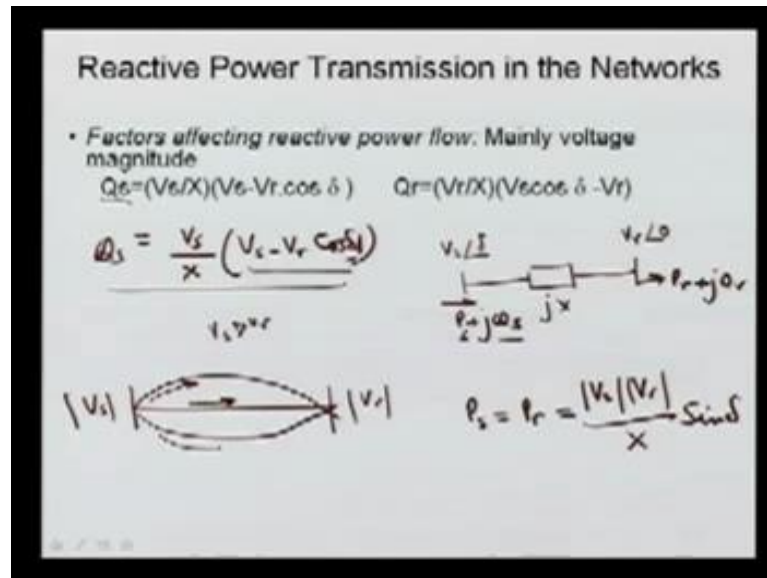
And then we are transporting those power to over the load central like Kanpur, Luck now and Delhi side your Noida side and Ghaziabad etcetera. So, what is happening? They are we are transporting the real power? At the same time reactive power it is not possible to transmit from those generator to again from Noida and Ghaziabad in Delhi side. So, it is if it is not possible to transmit you know again the possible due to the line constraints and other thing then it must be generated locally. So, we have to compensate those reactive power by generating locally in the local load centers. So, reactive power sources mainly are no doubt the synchronous machine those are generating the reactive power. First of course, the alternators or generators and another is your synchronous condenser. Synchronous condenser is the case it is nothing but a alternator it is an alternator which is generating the reactive power, but the P_e generation is 0. Means there is no electrical power generation it is simply running and it is improving the reactive power then it is called synchronous generator.

Normally it is not true, but what we do? We can have a synchronous motor where the mechanical load is 0 basically condensers are here. They are the motor nothing but they are the motors where there is no load on that motor, but it is rotating and we are just providing the reactive power to the system to improve the system profile or voltage profile of the system. Another reactive power source is your capacitor and again the capacitor putting in high voltage network is very difficult method ehv as well as uhv transmission system. So, we try to put these reactive power through the capacitors in the distribution network. So, the capacitors are normally installed near to the load centers. So, that it can be compensated the requirement of reactive power of the loads are made locally so that we should not flow the reactive power over the transmission line. Another source of reactive power is no doubt the line, but that line once it is a lightly loaded. It can generate the reactive power means here if again we can say this is a line.

Let us suppose it is having X here this will be your the loss here in the line that will be consumed reactive power is a $i^2 X$ if it is a X here. At the same time as we know that we are having the distributor parameters that is the charging capacitance of this line and that can be represented by here the π equivalent that is a capacitor. So, these are injecting the reactive power and that is nothing but V^2 upon your x_c . So, the total reactive power which will be supplied by this line is nothing but whether it is supplying or absorbing it depends upon your $i^2 X$ minus V^2 upon x_c . So, if this value is positive means this line is absorbing the reactive power if this value is negative means this is less than this so, we can say it is generating. And when it is that if i is less means loading is less then this component maybe less than this, because V is normally the rated voltage. So, the variation is very less, but this i can be from 0 to its critical loading. So, then this line can generate an especially this is the case when the system is lightly loaded. And for example, in the sometimes monsoon seasons or somewhere in the half peak se half peak hours even though in the midnight onwards the loading of the system is reduced. So, this value this line can generate reactive power.

So, the factor affecting the real power is mainly the voltage phase angle differences. This voltage phase angle difference and the transfer reactance. Because we know this P is nothing but if you remember it is your sending end it is your receiving end over $X \sin \delta$. So, this voltage we know it is very less affected because voltage change is not so, much, but if X is changed and the δ is changed. So, P is very much governed with the X and the δ . So, the mainly the voltage phase angle difference that is a δ and the transfer reactance X are the major factors those are affecting the real power flow in the transmission lines. Consider a radial line having reactance X and sending end voltage is V_s angle δ and the receiving end voltage as V_r angle 0 means we are taking as a reference. If the sending end power is p_s q_s and the receiving end power is p_r q_r then we can write this p_s here will be your nothing but that will be equal sending end power will be equal to receiving end. Again I am talking about the real power will be equal to your sending end voltage magnitude multiplied by your receiving end volt voltage magnitude divided by $X \sin \delta$.

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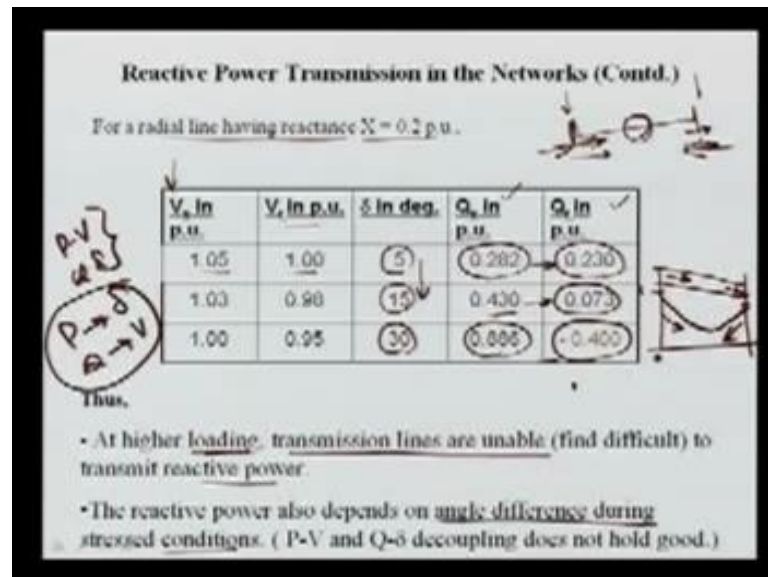


However, this reactive power this Q that is from the sending end basically what we did? Here it is your Vs angle delta and we have the transfer reactance of this whole system here this is your receiving end this is your Vr angle 0 and here I am taking it is a jx. So, this is your P plus jqx this is sending end and here receiving end we are getting what we are getting? We are getting here that is your pr plus jqr. So, this ps and pr will be equal, because there is a no loss here I have assumed this line is loss less. And then we can write this your Ps will be equal to your Pr. And it will be equal to your Vs magnitude although here I have taken magnitude here Vr magnitude divided by X sine delta. So, both will be equal. Now, but the reactive powers supplied source is Qs will be nothing but here you can see it is I can again slightly it is not clear your X and here is a Vs minus Vr cos delta. Now, again we can see that this Q is positive or negative that depends on this value. Since this value is very close to unity. So, it depends upon how much your Vr and Vs and Vr. So, if Vr is more means here it will be supplying the reactive power.

So, this variation, if we can ignore very less so, we can say if your Vs is more then Vr then it will be positive what it does mean? It shows that the reactive power flows from higher voltage to lower voltage. Especially it is true only in case of the ehv transmission line when this is very very small. So, it shows that this will be flowing here now; however, the Qr will be the here the different. Here it is a Qs delta cos delta minus Vr and here Vr upon X this side depends on this voltage. So, this profile of this here the reactive power generation i can see the voltage here lets suppose we are maintaining the

voltage somewhere at 1 per unit. So, voltage maybe here again like this or it maybe this if this V_s and this is your V_r . So, again it depends upon the loading of the system here this is called flat loading here it is called lightly loaded, because the voltage is more in the line and here it is heavily loaded and its exciting case is there.

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So, just I have taken 1 example for the same transmission radial system where this line is having the reactance that is X is equal to 0.02 per unit on 100 mva base. The values keeping the sending end voltage just is kept for 1 case 1.0 and the receiving end voltage is 1 per unit angle for the given load it is 5 degree we calculated this Q_s and Q_r and we found that Q_s is here 0.28 and your Q_r is 2.38. It shows that your system is lightly loaded and your here the V_s is more than your V_r and your this reactive power here the transmission line bus. Here that is flowing from here is more reactive power and here it is going less reactive power it shows that line is consuming some reactive power inside that. Again further we reduce the voltages again by 2 per unit here and then we what we did we just for the delta again for more loading and the again for more loading the voltage will be less.

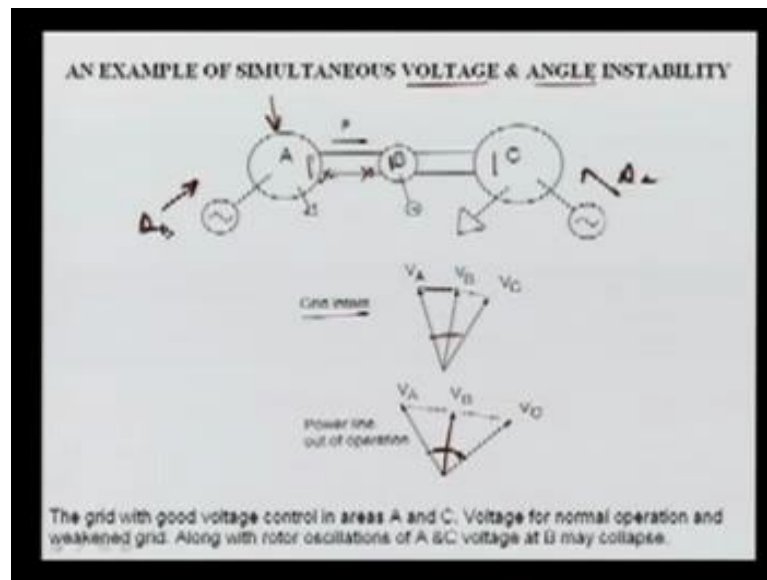
So, we saw the reactive power requirement will be very very high here and this is here going out that is very very less. Now, in this case the loading is more than this degree. You can say the loading 5 degree to 15 degree means it is almost you can say very high value that is increasing. So, it requires more reactive power from the source and here the

reactive power which is flowing is very very less that is almost 0. Now, you can further reduce the voltage and if we are here this angle is changed more. We can say the reactive power generation here is almost here 88 mvar. However, the other side that is here it is negative it shows that reactive power is again consumed by this line and the reactive power is flowing inside this line. So, here this is again you can see the profile of the this line the voltage profile. This is your I can say 1 axis here this is 0 here let us suppose 1 per unit here it is your V_r it is your 9.81 case 9.8. So, here 9.5 let us suppose is 9.5 so, what is happening?

You are the voltage profile it is a reactive power flowing from higher to lower voltage here also to your line profile will be like this, because we are maintaining the voltage. Because the reactive power, which is flowing from here to here and again from here to here in another case here what happens? One it was 1.3 and here point 9.8 the reactive power flow out like this means the reactive power is flowing here and here is less value and in other case also it was for 1 it was like this. So, if your loading is increasing this line is unable to transmit you can say what I want to say here the reactive power was transmitted. Here the reactive power was almost not transmitted just you can say this only completely this line is absorbing. In this case it is taking reactive power from both end. So, at higher loading it shows all this analysis suggest that the transmission line are unable are find difficult to transmit reactive power at this higher loading.

The reactive power also depends on the angle difference during the stress condition note down this angle is also very very important. And the PV and Q delta d coupling does not hold well. Normally you know in the load flow what we take this PV and the Q delta d coupling we take. Means we assume this P is coupled with the delta and the Q is coupled with V and based on that we normally go for decoupled load flow or fast decoupled load flow analysis we are going we are taking the solution. But in stress cases it is not possible that we cannot take these decoupling and then we have to go for the intact system and without considering the decoupling as well.

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To see an example of simultaneous voltage and angle stability here you can see this is area a. This grid with the good voltage control in area A and area C they are having very more reactive power and they are very strong in terms of voltage control. The voltage for the normal operation and the weak end grid here if the grid is there, there is another system that is b here and they are connected with the tie lines This system is very small this system is very large means they are having enough reactive power support Q and here also they are having enough Q_c here Q_b . So, power is transmitting from A to C via B.

If one of the line here trips open what happens? During the grid intact you can see the voltage here is very fine here almost equal V_A V_C and V_B and voltage here here and here is almost same magnitude and then you can say we have less angle. If this line is tripped what happens? Now, you can say the voltage of b is reduced angle is increased. So, both angle and the voltage stability coming together in this case, because here this impedance is reduce and this X is here more and we can transmit more power means delta is separated more. So, here you can say the delta is more and also the voltage at the B is reduced. So, here is a combination of of your voltage as well as angle instability.

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$$\Delta Q = (J_4 - J_3 J_1^{-1} J_2) (\Delta V/V)$$

Reduced Jacobian $J_R = (J_4 - J_3 J_1^{-1} J_2)$

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta V/V \end{bmatrix}$$

$$P = \sum V_i V_j \cos(\delta_i - \delta_j)$$

$$J_1 = \frac{\partial P}{\partial \delta}$$

$$J_2 = \frac{\partial P}{\partial (V/V)}$$

$$J_1 \Delta \delta + J_2 (\Delta V/V) = 0$$

$$\Delta Q = J_3 \Delta \delta + J_4 (\Delta V/V)$$

$$\Delta \delta = -J_1^{-1} J_2 (\Delta V/V)$$


Normally for whole analysis we analyze this reactive power up here means that is a load flow Jacobian this is called Newton Raphson power flow equations that we solve in polar coordinate Here again just I want to give brief about load flow now question again arise why we are using ΔV by V why not V ? You know our the mismatched vectors for power it is changing real power and change in reactive power. We go for the here Jacobian matrix here we go for the change in the state vector that is a delta and change in V . But here in the Newton Raphson load flow we normally divided by V here the question arises why? The main reason for this if we go for that differentiation for this j 1 and j here j 1 and j 3, because you can write this P equation here it is here j 1 is nothing but the element it is your ΔP upon $\Delta \delta$. Here you can see this P is a double summation here $v_i v_j$ again it is a function of cosine delta. So, this is a quadratic of voltage if you are differentiating with the differentiating this equation with the respect to delta we are getting still the quadratic here of the voltage.

But here if we are going for the j 2 j 2 it is nothing but your change in P by V means $1/V$ goes off. So, we try to multiply this to make it quadratic as you know the Newton Raphson load flow method is provides basically your quadratic conversions. So, we basically multiply here with V and here we go for V so, what does happen? This multiplication gives complete quadratic and then we get the faster convergence. So, this is your Jacobian if you are not changing the real power injection here. So, what happens? This will be 0 and we want to see the relation between your changes in reactive power

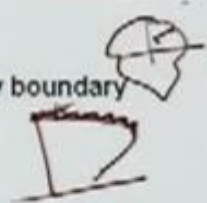
with the change in voltage. From these 2 equations now, I can write you can expand the first one. So, it is a $j_1 \Delta V + j_2 \Delta V$ upon V here is equal to 0. From here what we can write this change in delta is nothing but minus j_1 inverse here j_2 change in V upon V . If you write this second equation here we are getting change in reactive power it is a j_3 change in delta plus j_4 change in delta V by V . If you are substituting this delta here we are going to get change in reactive power Q will be here just you have to put this with the negative sign.

So, I can write here j_4 minus $j_3 j_1$ inverse here j_2 and here in total change in voltage by V . So, this called your J_r that is called reduced Jacobian and this gives information that the change in the reactive power with the change in the voltage at the any bus we can get that factor. So, this is the reduced Jacobian and this gives a lot of information lot of information about the system voltage instability. So, normally for the voltage stability analysis we use this Jacobian rather than only either J here we do not use this one. Because if we are going for coupling normally this equation we ignore it and we can say this is 0 and we can simply say ΔQ is your j_4 change in V upon V . If you are lightly loaded system is there so, this equation is valid. But for in all cases whether is lightly loaded or heavily loaded we can go for instead of j_4 here we can replace here it is a J_r and that is called reduced Jacobian. So, this gives very good information for analysis of the voltage stability.

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- **Three major methods of analysis are**
 - ✓ Repeated P.F. Methods, along with
 - Singular value decomposition
 - Eigenvalue analysis ✓
 - ✓ Continuation Power Flow. Uses predictor - Corrector method.
 - ✓ Direct Solution
 - Determines the voltage stability boundary using eigenvector approach or
 - Optimization technique.



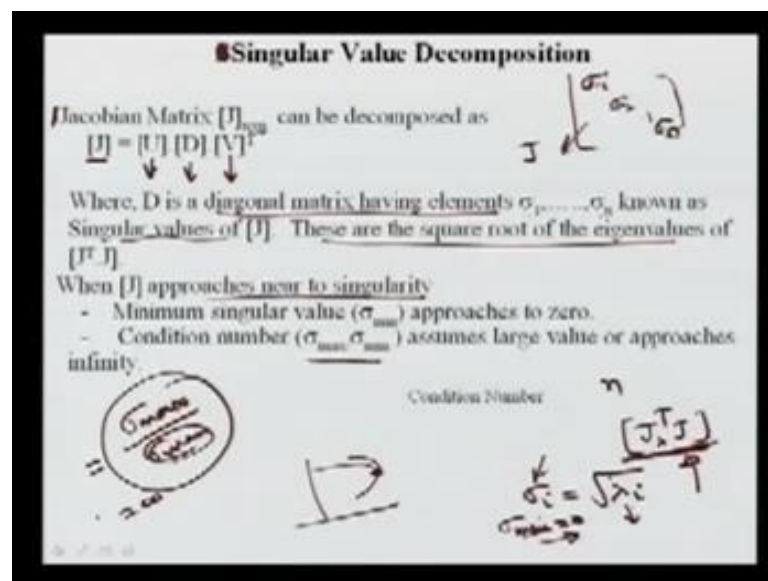
The measure methods for analysis of voltages by basically here I am talking about the voltage instability or a stability analysis. Our first one is the repeated power flow that is your repeated load flow power flow is similar to load flow methods along with the singular value decomposition or another method is your Eigen value analysis. Another method here the main problem here in the repeated power flow method is that this repeated power flow methods may not converge when your system is highly stressed. And especially at the nose point just we saw the PV curve at the nose point this Jacobian becomes singular means your this Jacobian here this will become singular and you cannot get the inverse and if you cannot inverse then you cannot solve. So, what happens? This repeated power flow sometimes very very difficult to get that value at the nose point. So, another method that is a continuation power flow methods are there and that uses your predictor and corrector steps and the 2 methods are coming together here we use the predictor and corrector. Advantage of this method is over this repeated power flow method.

Here in the repeated power flow means methods if you are using very good method for the load flow analysis here as you know this is PV curve only you can get the solution up to the upper portion here this portion this unstable curve you cannot get. But here in the continuation power flow. So, just I was explaining the advantage of continuation power flow over the repeated power flow method. Here in the continuation power flow again I will explain how that we can get the power flow solution using this continuation power flow method and that is I will discuss the steps that is a predictor as well as the corrector method. But here I would like to explain that in this PV curve here this is your PV curve we can get even though lower portion of this curve as well. So, we can get no doubt this and as well as here and we can go up to the nose point without any problem in this method what we do? First we go for the start from some initial guess then we go for the predicting this then we go for correction then we prediction correction prediction correction prediction correction and finally, we converge here and similarly we can go for the lower step.

So, here it is a corrector step this vertical axis and here horizontal is your predictor predictor steps and thus we can achieve this complete curve that is a upper as well as the lower side and this is very popular method for the voltage stability analysis. Another method that is third method is called your direct solution method. That in this method

means we can determine the voltage stability boundary using the eigenvector approach means here our system that is in number of n plane. We can get the boundary and in that in that boundary that we can use the Eigen vector approach. And then we can say whether your existing point is well within boundary or it is outside or it is on the boundary and then we can access the system voltage stability. Another is your optimization techniques that we can measure that how far we are and using that method we can determine our the voltage instability case.

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Now, let us see the first one is your singular value decomposition technique that is very very popular in very you can say that can be used. Here this the Jacobian matrix complete Jacobian matrix if you are having n bus system then you can form the Jacobian matrix this n and which this is order n cross n that can be decomposed into this form that is the Jacobian matrix here. We are having the 2 vectors orthogonal vectors u and V we can form in a such a way that this matrix d becomes or diagonal matrix. So, this d is nothing but your this having this element its only diagonal elements.

And these diagonal elements are known as the singular values of these Jacobian j and they are represented as sigma 1 sigma 2 to here sigma n will have n value and they are known as the singular values of j. These values are nothing but we can also instead of going all this we can determine a determine the sigma 1 and sigma 2 by getting another matrix here this j. This j transpose j you can say it is a square of the j. So, if we are

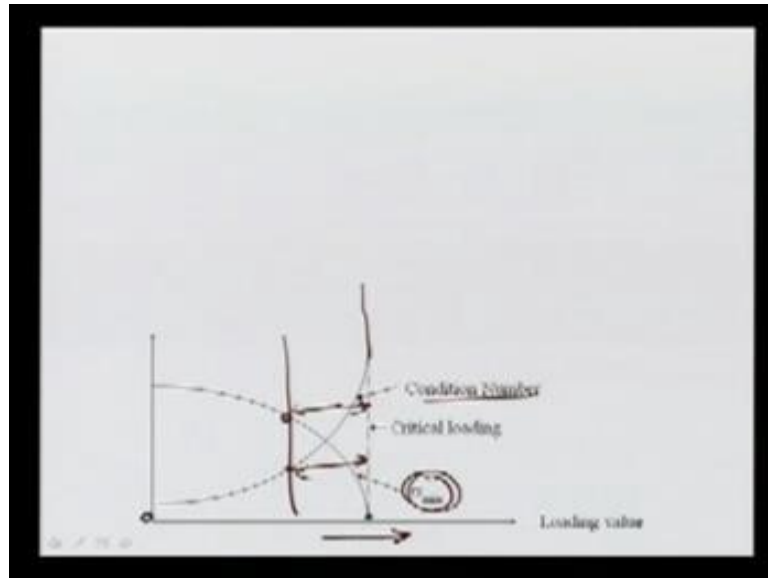
getting here the Eigen value here for this complete matrix not J . So, if we are getting that here λ_i . So, the square root of these Eigen value λ_i it is nothing but your σ_i .

So, you can what you can do? If you are having the J matrix completely you can multiply with its transpose pre multiply and then you can determine the Eigen values. So, you will get the some Eigen values let us suppose λ_i . This λ_i is not a Eigen value of J it is a Eigen value of the combined $J^T J$ matrix here. And then after that if you are taking the square root you will get here the singular value that is called σ_i when this J approaches near to the singularity? Means if it is approaching for the radial system lets suppose here the Jacobian become singular.

So, once it is approaching near to this singularity here this point singular point where the Jacobian this J determinant will become 0 means it is a rank deficient we also called. So, the minimum singular value here this σ_n what will happen? The Eigen value of this 1 Eigen value will have 0 and miss this J min will have 0 value. So, this will also approaching the 0 once this J is approaching to singularity. So, this singular value gives information that where you are how much that your system is operating how much away from your voltage instability means how much away from this nose point?

We use normally another number that is called condition number and that is defined as here σ_{\max} divided by your σ_{\min} . This assumes large value and approaches here means from here we are getting σ_{\min} also means we are having σ_1 σ_2 σ_n means we are having n values out of that 1 will be minimum 1 will be maximum. So, from that we can what we can do this σ_{\max} upon σ_{\min} gives your condition number and this normally is very very high or even though infinity at the when J approaches near to the singularity here at this nose point, because this becomes 0 means this is becomes infinite.

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Now, you can see the curve for this variation. Once your loading is keep on increasing here if loading factor is increasing this your condition number you can say here it is keep on increasing. Whereas, your this singular value you can say keep on decreasing and finally, here it is becoming 0. So, this shows that your condition number is increasing or you can say your singular value is minimum is decreasing. So, either you can use the condition number or you can use this and then you can see how far this is your critical loading where you are what is your this value of λ_{\min} then you can say you are at this margin. Similarly, from here condition number you can say how much you are away from your critical loading. So, this gives your margin and that is again it is the static voltage stability margin here this is nose point and then how away you are operating your system? So, this is this approach is known as singular value decomposition approach.

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Eigen Analysis Using System Jacobian

- System dynamics can be considered by augmenting the power flow equations to include dynamics. Many models possible.
- A common type of system model is mixed differential and algebraic equations (DAE)

$$\begin{aligned} \dot{x} &= f(x, y) \leftarrow \\ 0 &= g(x, y) \rightarrow \end{aligned}$$
- DAE equations can be analyzed by linearizing around an equilibrium point and calculating a reduced Jacobian. Linear set of equations use full system Jacobian

$$\begin{bmatrix} \Delta \dot{x} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{\partial f(x, y)}{\partial x} & \frac{\partial f(x, y)}{\partial y} \\ \frac{\partial g(x, y)}{\partial x} & \frac{\partial g(x, y)}{\partial y} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$
- Expanding the above equations and eliminating Δy a linearized relation in terms of reduced Jacobian is obtained.

Another approach that is again I was explaining that the eigen value analysis approach and that is again comes under the repeated power flow method. In this what we do? The system dynamics can be considered by augmenting the power flow equation to include the dynamics. That is a system dynamics include your generator dynamics your exciter dynamics your load dynamics your other whatever the parameters you want to include and then those device you can say dynamics equations you can include. So, the many models are possible again with how many these equations you are using and that can be included there.

So, we can write the common type of a system model that is a mix of differential as well as the algebraic equations. That is here the system dynamic model model of generators exciters and other load parameters load models this is your nothing but your power flow equations and that is your equality constants. So, this is combination of or you can say mixture of differential here is a differential equation this \dot{x} again is nothing but your $\frac{dx}{dt}$ and x is the state of the power system. So, here there is a 2 x and y are variable and what we can do here again this now, called the DAE here, because the differential algebraic equation we call it.

This DAE equations can be analyzed by linearizing around an equilibrium point and calculating or reduced Jacobian. Means again we can linearize this equation like here. You can say we can differentiate this here that is your change in \dot{x} will be here I can

say your change in F_X and here change in X plus your F here dy and again here change in y . Similarly we can write for here and we can write a Jacobian matrix here J . Expanding this equation and eliminating this your change in y here this is 0 if you can eliminate here from this equation.

So, we can get a equation in terms of this or linearize relation in terms of reduced Jacobian is obtained. So, and we can obtain like this ((refer Time: 49:55)) what happens from here? We can write from this equation you can see mean again let us here we can write in terms of Jacobian's term. Let us suppose j_1 and j_2 it is better $j_1 j_2$ here again here j_3 and j_4 . And this is your change in X change in y here corresponding to your change in X dot and here it is your 0. What we are going to do from here from the lower equation we can write again

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where the reduced Jacobian $J_R(x)$ is

$$\frac{\partial f(x,y)}{\partial x} - \frac{\partial f(x,y)}{\partial y} \left[\frac{\partial g(x,y)}{\partial y} \right]^{-1} \frac{\partial g(x,y)}{\partial x}$$

$$J_3 \Delta x + J_4 \Delta y = 0$$

$$\Delta y = -J_4^{-1} J_3 \Delta x$$

$$\dot{\Delta x} = J_1 \Delta x + J_2 \Delta y$$

$$\dot{\Delta x} = (J_1 - J_3 J_4^{-1} J_4) \Delta x$$

$$J_R(x)$$

This is your change in your X it is your j_3 plus your j_4 change in y it is your 0. However your this change in the differential equation here X it is nothing but your j_1 change in X plus your j_2 change in y . From here we want to trip this means I can write change in y actually X and y are the vectors. The various states are there various here y states are there. So, they are vector. So, it is not 1 Quantity. So, I can write here it is equal to minus j_4 inverse I can take this side and then i am just dividing here it is j_3 change in X and this value if we are trying to replace here in terms of y .

So, I can get this X dot will be your j_1 minus here $j_2 j_4$ inverse j_3 and here we are getting change in x . Now, this matrix is called the reduce J_r and it is the variable of X only. Now, this matrix will give the information about your again now, it is just like a steady state your power flow Jacobian and here we can analyze this matrix. And then we can get the information about the condition number another thing and we can say the system is stability conditions. So, here we can analyze using the Eigen value we can go for the Eigen value of this matrix and then based on that we can conclude several information.

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Eigen-analysis (contd.)

- If all the eigen values of system Jacobian have negative real part, system is stable. $T_R(s)$
- At max. loadability (static voltage stability limit or saddle node bifurcation), one of the eigenvalues become zero and others have negative real part.

Participation Factor Analysis REV, LEV

- ✓ **Bus Participation Factors:** Same as state participation factor but corresponding to load bus voltage state. Elements of participation matrix corresponding to critical eigenvalue are $(P_{ij}) = \frac{f_{ij}}{f_{ik}}$.
- ✓ **Branch Participation Factors:** When modal reactive power corresponding to only critical mode- i is perturbed by unity, these factors are defined as,

$$P_{jk} = (\Delta Q_{\text{max}} \text{ for branch-} j) / (\text{maximum } \Delta Q_{\text{max}} \text{ for all branches})$$
- ✓ **Generator Participation Factors:** When modal reactive power corresponding to only critical mode- i is perturbed by unity, these factors are defined as,

$$P_{mi} = (\Delta Q_{\text{max}} \text{ for generator-} m) / (\text{maximum } \Delta Q \text{ for all the generators})$$

The above factors are used for placement of shunt compensators, series compensators and generator controllers to improve static voltage stability

So, that is if all the Eigen value of that system Jacobian which i said here it is a J_{rx} it is not a power flow Jacobian it is a different than that. Because the power flow Jacobians are included there itself. Means here in that case we have used the dynamics of the system along with this one. So, if eigen value of the Jacobian means all the eigen values are having the negative real part then system is a stable is no doubt about that. At maximum loadability or you can say the static voltage limit or saddle node bifurcation 1 of the Eigen value becomes 0 and other have the negative real part what does it mean?

It shows that here in this case if you are reaching here as I said the Jacobian becomes singular means 1 of the Eigen value here that is a λ_i for this will be zero. So, this will be 0 and other will be only, one will be 0 and the remaining will have the negative part. So, at the higher loading condition here this and that condition is basically

saddle node bifurcation case means nose point or you can say static voltage limit case. So, we can obtain by knowing the eigen values. We can also determine the participation factors and that is called if you are using the participation factors then it is called the participation factor analysis. In that we can have the different participation factors it can be your bus participation factors, it can be your branch participation factor or it can be your generator participation factor.

The bus participation factor is the same as the state participation factor which we define i was talking about the small signal stability. It is nothing but it is the multiplication of your the vector which are obtaining that is a right eigen vectors. And it is left eigen vectors and then we can get the participation factor. But it is corresponding to a load bus state it is not a normal it is a if you are taking the bus state of the bus load bus then voltage participation factor can be this. The element of participation factor corresponding to the critical eigen value that you know just for each eigen value will have left eigen and right eigen vectors and then we can have the participation factor.

So, for the critical eigen value that we can go and critical eigen value is when we are getting the minimum critical value when that is a here in this j axis how close this the 0 point? The branch participation factors when model reactive power corresponding to only critical mode i is perturbed by unity. These factors are defined as p_{ji} is the change in reactive power loss for a for the branch i divided by the maximum reactive power loss for all the branches. Similarly, the generator participation factor when the model reactive power corresponding to only critical mode i is perturbed by unity these factors are defined as p_{mi} .

That will be equal to change in the reactive power generation of mx generator divided by the maximum reactive power change for all the generators. These factors are used for the placement of shunt compensators, series compensators and the generator controller to improve the static voltage stability. So, knowing the Eigen value we can go for these participation factors and then we can use these factors for improving the voltage state again here till now, I am talking about the static voltage stability and that will be used for that.

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Continuation Power Flow Method
Continuation and direct methods to solve load equations in AC/DC power system
available at: <http://www.power.electrical-engineering.com>

The load flow equations with a load factor (λ) can be written as

$$\vec{F}(\vec{S}, \vec{V}, \lambda) = 0 \quad 0 < \lambda < \lambda_{critical} \quad (a)$$

To solve the problem, continuation algorithm starts from a known solution and uses a predictor-corrector scheme to find out subsequent solutions at different load levels. It uses the concept of local parameterization with λ included in L.F. eqⁿ, no. of variables are now $n1 = (2N - Nq)$
 Where N = Total buses
 Nq = No. of P-V buses
 Writing (a) in differential form,

$$d\left[\vec{F}(\vec{S}, \vec{V}, \lambda)\right] = \vec{F}_S d\vec{S} + \vec{F}_V d\vec{V} + \vec{F}_\lambda d\lambda = 0$$

or

$$\begin{bmatrix} \vec{F}_S & \vec{F}_V & \vec{F}_\lambda \end{bmatrix} \begin{bmatrix} d\vec{S} \\ d\vec{V} \\ d\lambda \end{bmatrix} = 0 \quad (b)$$

Another is your there is a very important that is your continuation power flow method. This continuation power flow method as I said that can be used to get the upper and the lower bus voltage limits as well. So far we have seen that the 2 methods that is again the methods as I explained they are the 3 methods for analyzing the static voltage stability. That is your repeated power flow method that includes your the singular value decomposition techniques and your eigen value analysis and we saw that. Another that is very popular method is continuation power flow methods and most of people are using that. In continuation power flow there is advantage over this repeated power flow that at the nose point. At the saddle node bifurcation point that it is not possible to get the solution, because Jacobian becomes singular. So, we can approach very close to that if you are using very power flow powerful load flow solution.

But we cannot get solution at the nose point and also we cannot get the solution for the lower voltage it is sometimes we can get, but it is not always true because your initial guess must be different and there maybe always chance in that you will get this convert solution. Another as I said the optimization another I will discuss about this boundary criteria that we can measure that how far we are from that boundary and then we can say your system is stable. So, for the in this whole lecture this lecture number 12 I explained that the effect of OLTC there is once OLTC is operating normally the online load tapping transformer tap changes are used to improve the voltage. It is sensing the voltage and whenever the voltage is less it is trying to increase the voltage by changing the

tapping. So, sometimes if the voltage system is severely stressed in that case it may act in a reverse direction means if you are increasing the tapping voltage will be reduced. So, that will lead to your unstable power system in sense of voltage instability.

Similarly, also we saw there is a sudden change, no reverse active sudden change from 1 tapping to other tapping you may lead in the unstable zone and again your system will be unstable. So, the action of transformer is very very important especially when you are operating in the lower voltage region. So, these are the various again techniques again we are talking about the static voltage stability. We also see the dynamic voltage stability then we have to use some bifurcation techniques and based on that we will see the system performance. So, in the next lecture, we will see the continuation power flow methods, because we have to go for the predictor and the corrector method. Those methods are very very useful and that we will see how we are proceeding and how we are getting the solution both upper as well as lower curve of the PV curve. That is upper side of voltage as well as the lower voltage side. So, with this we will see in the next lecture on power this continuation power flow techniques.

Thank you.