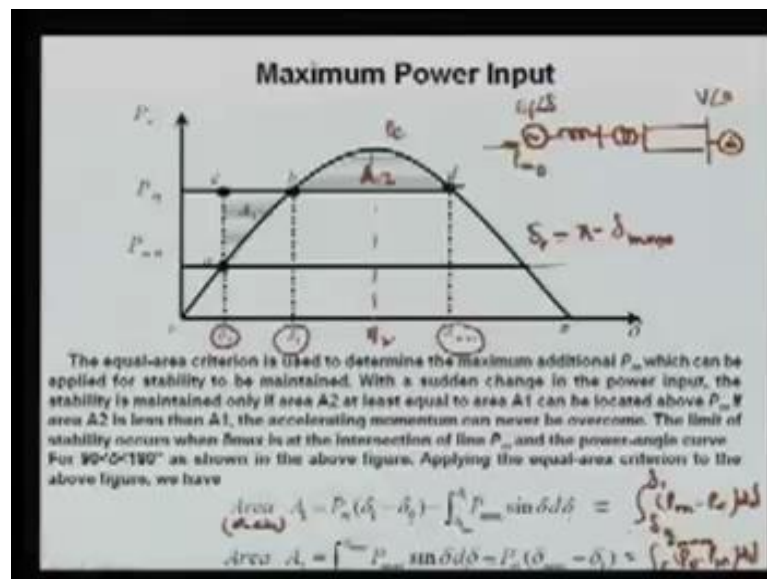


Power System Operations and Control
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Module -2
Equipment and Stability Constraints in System Operation
Lecture 7

Welcome to lecture number seven of module two. In previous lecture, just we have seen that equal area criteria can be used to analyze the stability of the system, and that is only applicable for this single machine Infinite bus system or two machine system. And the last example we saw, that if we are increasing, the sudden increase in the power, how your system will be reacting, and what will be the stability of the system. Now, let us determine what will be the maximum power that can be increased, on any machine over on over what is loading, present loading, and that is basically governed by the stability of the system.

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Let us take your machine is loaded again this machine, now your ultra. This is your synchronous machine. It is a here, it is a reactance, and this is the transformer, and then we have two lines, and then it is connected with the Infinite bus here, that is your Infinite here. So, this is your voltage angle zero; that is V infinite. Here it is Ef angle delta. So, in this case, here your Pm that is your mechanical power; that is entering, and now I can say this is Pm not. For this case, your Pm naught here where your machine is operating at

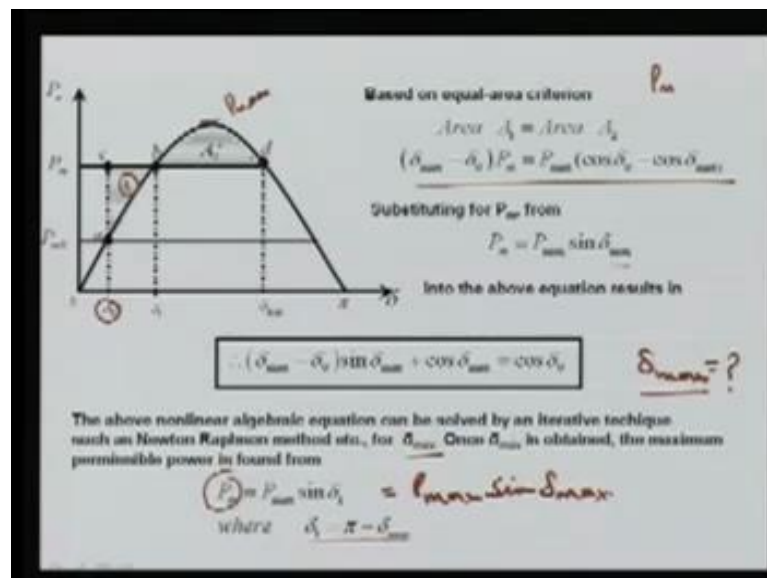
angle δ naught, and this is your P_m and this A point; means A point will be the operation when your feeding, this mechanical power is P_m not. As again I explained that the operation point will be the Intersection of this mechanical power characteristic; that is P_m line here, and then that is your P_e line, this is your P_e curve, which is a function of $\sin \delta$. So, the Intersection here the point A now the question I want to know, how much power that we can increase that is your P_m you want to calculate, so that our machine should be stable without losing synchronous.

To know this we have seen that is equal area criteria, is used to determine the maximum additional P_m , which can be applied for to stability to be maintained, with the sudden change in power Input. The stability is maintained only If the area A 2. This area, this is your A 2, at least equal to area A 1. Means this area here, it will at least equal this, means this area may be larger, but it should not be less, but If this area is less than this A 1, then your system is not stable. So, we want that maximum power that we can Inject, suddenly we can change on the loaded power system, or on this generator, that can be determined; that means, area A 1 will be equal to area A 2, and then we can what will be the determine the P_m value. To determine we have to calculate this δ_1 , or we can have calculate this δ_{max} , because this δ_{max} is nothing, but here δ_{max} will be, or you can say is a π minus δ_{max} will be your δ_1 . So, to determine δ_1 , we want to know δ_1 , then we can get the point P_e , because the P_e characteristic we know. If we put this δ value in that one here; δ_1 in that P_e characteristic, we will get the point P_e , and thus we can determine the P_m . We will see in the following lines. So, here now this area one, here is escalating power. Means this area as I said, so to maintain the stability area A 2 at least equal to area A one, can be located A above the P_m , If area A 2 is less than area two. If this is less than area two the accelerating momentum can never be overcome.

The limit of stability occurs, when δ_{max} is at the Intersection of the line P_m here, means this P_m we want to determine, and the Intersection of P_e and this P_m will give your δ_{max} , and this will be always here, will be δ will be more than ninety degree this side, means here in this zone, here it is your $\pi/2$ that is 90 degree. Applying equal area criteria to above figure, we can have this area A 1. This area that is acb here you can say area acb, or you can say area one will be nothing, but integration from δ naught to δ_1 P_m minus this P_e curve. So, means this is nothing, but here

it is Integration of delta naught to delta 1. Here this Pm minus Pe d delta. So, this Pm is constant. So, Integration of this we can take the Pm outside. So, it is a delta 1 minus delta 2 first term, and another term this Pe with the minus sign we are integrating from delta naught to delta 1 that is P max sine delta d delta. Similarly, this area 2, it will be the Integration here from delta 1 to delta max. And now it is Pe minus Pm, because this curve is over, and then this minus this Integration from here to here, will be your area 2. And then we can know this Pe is a function of sine delta. So, it is integration from delta 1 to delta max P max sine delta d delta minus this Pm is constant, so delta minus delta e delta 1.

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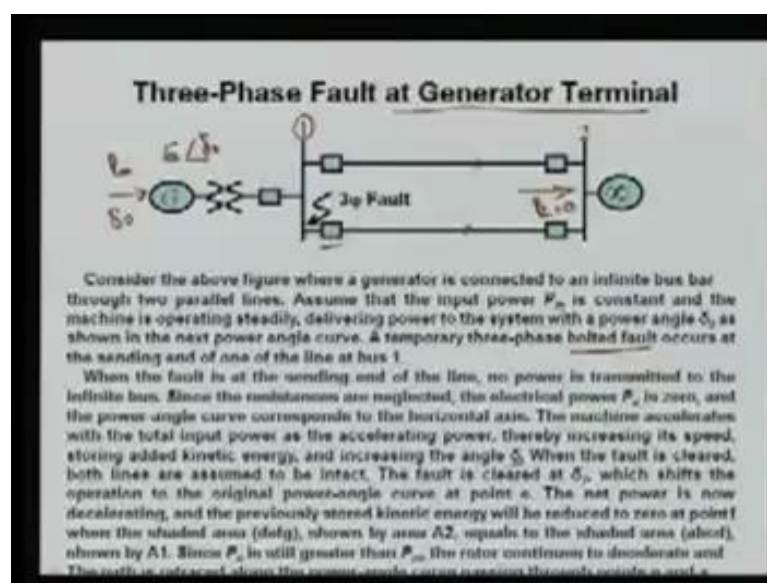


So, based on the equal area criteria to the system is stable, at least the area A 2 should be equal to area A 1. So, this area one for maximum Pm, the area A 1 should be equal to the area A two, and then from the previous equation, If we will equate we will get this expression. Now, here the Pm we are getting here, and that Pm is nothing, but again we can get the Pm is equal to Pmax sine delta max. Means this value the Pm, will be again that will be your Pmax sine delta max we can get this point, and then you can substitute this. So, we will get a expression of us a delta max value, and this is a non-linear equation; means we have the sine delta max, we have another delta max here, we have cosine of delta max, and cos delta naught is known to you. Delta naught is original, where your machine was operating, that can be calculated, because Pm is known to you, Pm naught here. So, this equation is a non-linear algebraic equation, and can be solved

by any Iterative techniques, including your Newton Raphson method, Gauss Seidal methods, Gauss Iteration method so on and so forth. So, that we can determine δ_{max} . Once this δ_{max} , means here our Intension is to find δ_{max} . Once you can determine the δ_{max} , then the maximum power P_{max} will be $P_{max} \sin \delta$ here; this P_m , or you can get this one directly.

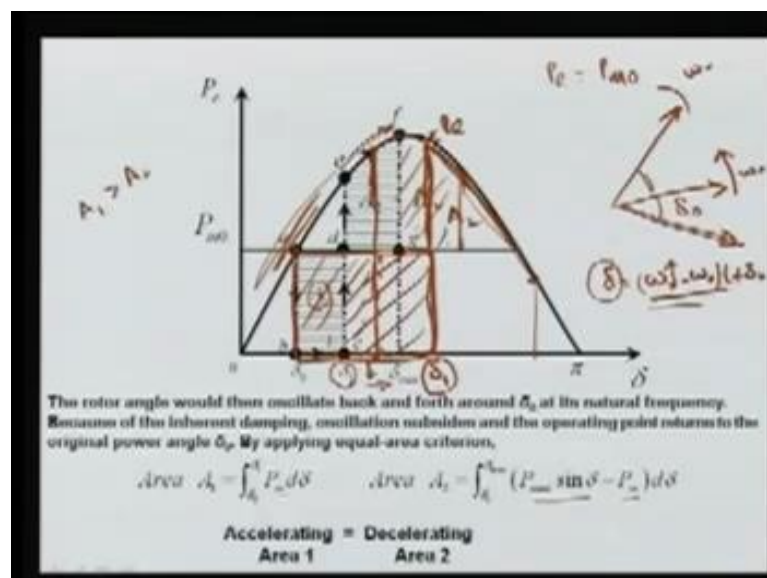
So, you even though where δ_1 is your $\pi - \delta_{max}$, or here you can write this simply $P_{max} \sin \delta_{max}$. So, this will be the maximum power that you can load above this P_m not, without losing their stability. So, this is one way to determine that how much you can load suddenly. Again question here, if you are continuously increasing, then your maximum loading here is up to P_{max} . But the sudden increase here for the sudden, means it will be accelerating and decelerating, because here is an acceleration power, here machine is a decelerating, here this zero, and here is again escalating so on and so forth. So, since why this area A 1 is called accelerating, because the P_m one, P_m is more than P_e , so what will happen. Your machine will accelerate, and it will store the kinetic energy; however this area A 2 is decelerating area, because your P_e is more means the power electrical power that you are taking from the system is more, means here, then your power that you are feeding to the turbine, so, this is a decelerating. So, accelerating power must be equal to you decelerating power, then the P_m can be determined.

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Now, let us see another consideration; means how this equal area criteria, can be useful for determining the other stability criteria. Let us suppose there is some fault. So, if we consider the three phase fault; the three phase fault can be a bus fault, it can be in the line fault, and it can be anywhere in the line either in sending or in receiving end or in between line. So, let us first consider the three phase fault at the generator terminal, here that is this terminal we are boarding. Basically this is a generator, this is a GT generating transformer and this is a terminal, where we can say this is bus one. So, again this system is similar to our previous system; that here a generator is connected to infinite bus bar through two parallel lines; line one, and line two. Assume that the input power again here is P_m is your constant, and the machine is operating steadily delivering power to the system with an angle δ . Here means your δ is E_f means, if you are here I can say E_f during their steady state, and that will be shown in the next term. Our temporary three phase fault that is bolted fault. Bolted fault means the fault without any up fault resistors, means directly three phase to ground fault, occurs at the sending end of one of the transmission line at the bus one here; three phase fault let us suppose has occur here in line two. When the fault is at the sending end of the line, no power is transmitted to the infinite bus. If the bus fault is here, what will happen. There will be power which will be flowing to Infinite bus. So, your P_e at that case, it will be your zero. Since the resistance is neglected of the line, the electrical power P_e is zero, and the power angle curve corresponding to the horizontal axis. Means you can see what will happen.

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Your this P_m here A was your operating point, because your P_e here is equal to your P_m not; that is P_m naught bus you are getting. So, suddenly there is a fault, what is happening your P_e becomes zero. So, this is suddenly coming to this point, and then till the fault is persisting here, it is going and at the delta. Once fault is cleared, and then again it is going back to the P_e . Again the P_e is going to be delivering to the Infinite bus. So, in this case the machine accelerates with the total input power, as the accelerating. Means the total P_m , because we are not taking P_e from that generator. So, this machine will get escalated, means your input power is more than output power, so, whatever power you are feeding as you know the energy conservation law, then this energy will be stored in terms of kinetic energy. And if the kinetic energy of any system Increasing, means speed of that machine rotating mass is increasing. So, there by it increasing its speed, storing added kinetic energy, and increasing the delta. If machine is accelerating what is happening, now delta will be changing, the delta is angle if you see what is now question is delta.

Delta is, let us suppose this your reference angle, any or rotary reference angle. Here this is your delta not, and your machine here this is a rotating two fluxes here. This is your synchronously rotating here, and it is actual rotor speed is ω_r ; this is ω not. So, this difference that angle; that means, your delta is ω_r minus ω not; that is δ with delta not. So, if though machine is accelerating this is increasing, this is your synchronous speed. So, what will happen this term is increasing with the time. So, your delta is increasing. Here delta naught is any represent, so this delta naught will be zero. So, once your machine is accelerating, means your delta is accelerating, increasing, and once it is decelerating means your delta; that is angle delta I have defined with this one, it is a decreasing. So, that is retaining here the Increasing angle delta. When the fault is clear, and it was assumed that the fault was temporary, and the fault it cleared without tripping the of the line, because it was the bus fault, and the bus fault is cleared. So, both line assumed to be Intact. Means your Impedance of the system is not going to be changed. So, again it will follow the P_e characteristic. The fault is clear at delta one, which the operation to the original power angle curve at point e; here this is your point d. Means your fault is cleared here, means it will suddenly follow this delta curve, because this is P_e here P_e is zero.

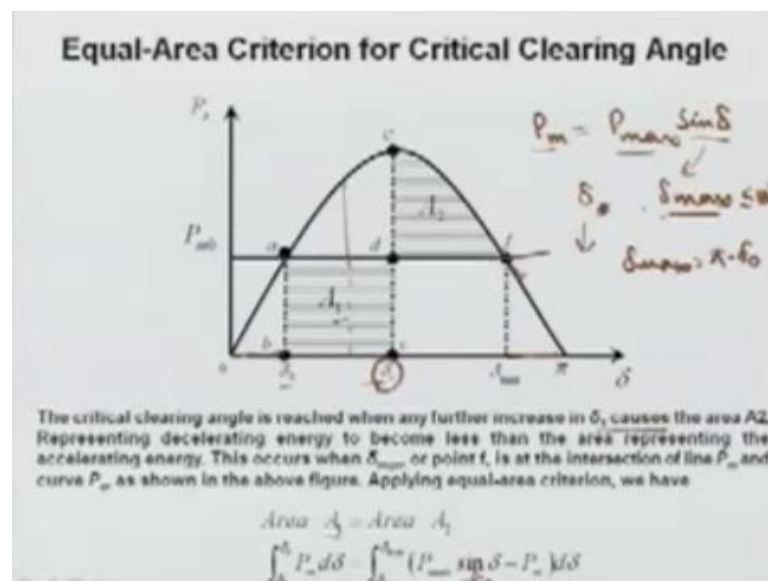
So, this P_e it was fault running at this point, suddenly due to the fault this P_e becomes zero; this is your zero P_e this is a P_e axis. And once fault is cleared, then it will again just it will follow the P_e curve here; that is here it is your P_e curve it is a function of sine delta. So, it is coming to your e point. The net power is now decelerating, and the previously stored kinetic energy will be reduced to zero at point f, when the shaded area is shown by the area two, equal to the area shaded in abcd shown by this. Now, you can see this area A one, as I said this one is your accelerating power. Means rare in this area A 1 your machine is accelerating, and you are storing kinetic energy. Now, once this kinetic energy is stored in the machine, what is happening at the e point, you can see this is now electrical power is more than your mechanical power. Means you are taking more power. So, whatever the energy is stored in the machine that will be taken out, and this area here it is called your decelerating. Means here it was accelerating, now from here it will decelerate till f, when again this your acceleration will become zero.

So, again you can see the delta is keep on increasing here and it is delta max. Here again your decelerating power is zero, then again it will find that this is more than this it will try to retard, and again it will coming back. So, it is again there will be oscillation around point A, and finally, due to the damping of the system, it will be damp out, and your final steady state operate of the system it will be stable operation. Now, the question again here arise, If this area is less than this area. If this is area is more A 1. Means your area A 1 is greater than two, and till what point, till certain range here, then machine will be unstable. So, we can the rotor angle would then oscillate back, and force around here this not, means here it will be oscillating at its natural frequency, because of Inherent damping in the system. The oscillation subside, and the operating points return to the original power angle delta naught here; that is at point A.

So, by applying the equal area criteria, this area A 1 that is integration from delta naught to delta naught to delta one, here that is Integration of here this completely P_m minus this zero. So, this P_m delta, and area A 2 will be delta 1 integration from delta 1 to delta max. Here this P_e minus P_m d delta, means accelerating area one will be equal to accelerating area two. Now you can see, how much you can move ahead this is f. Let us suppose you have increased further, means it is not clear at this one. Let us suppose your machine, fault is clear at this point what will happen. Now this area here it is up to this area, and once fault is clear now it will follow here, and then it will go up to here area now area

two will this much completely, and it will be your area two is this much. So, still your system is stable. Let us suppose further you have increased. Let us suppose your delta 1 your fault is cleared after longer time. So, this area now is registered here, and now you have come down here, and again you can see the area this is bounded here A 2, is now less than your area A 1 complete this area, this area here is larger than this. So, in this case your area is more than A 2 and the system is unstable. So, what we normally do. We try to determine what should be value of delta one. Here what I am going to explain, that how much this delta 1 that can be maximum value means the fault can be cleared, so that our system will be stable. Why we are very concern about this this delta one; that is maximum delta 1 at which we can clear the fault at which our system will be stable, this is known as the critical clearing time. Critical clearing angle delta If you are calculating this delta If you are calculating in the time then it is called the critical calculating time CCT, or here delta 1 is your critical clearing angle.

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In the next slide we will see; that is delta C is critical clearing time, that is what will be this value you want to determine this, so that you fault is cleared before that. So, in that case suppose your fault is cleared. So, this condition will be always cleared here this area A 2 will be always you can say more than your area A 1, means your system will be stable. So, from here this complete area is always larger, and then A one. So, we can say our accelerating power will be always less than your decelerating power. So, the system will be stable. To determine this critical clearing time, that is very important. The critical

clearing angle is reached when any further Increase in delta one; that is delta 1 here, causes the area A 2, representing decelerating energy to become less than the area representing the accelerating energy; that is your A one. This occurs when the delta max here or the point f is at the Intersection of the line point Pm, here this is line Pm, and this is curve Pe then it is Intersection point f as shown in this figure.

Now for this critical value, here this point cannot come below, because this is decelerating up to this point Pe is more than Pm. So, this is decelerating. You cannot go beyond this because if you are coming here what will happen your Pm will be more than Pe and again it will accelerate. So, your system will be unstable if it is moving beyond this point. So, we have to this area A 2 should be less than this delta max, and then we have to determine this delta c, so that we can know if your fault is cleared less than that angle, then we can say our system is stable. To be very critical for getting the maximum value of delta at which it is cleared for this stable operation. This area A 1 should be equal to the area A 2 here. So, area A 1 that is your ABCD will be equal to your DEF be, this is area two. So, this area for this case it is you Pm Integration from delta naught to delta c, is equal to your Integration from delta c to delta max. Here Pe minus Pm, so it is Pm max here this is nothing, but pe minus Pm delta d delta.

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Integrating both sides, we have

$$P_m (\delta_c - \delta_0) = P_{max} (\cos \delta_0 - \cos \delta_{max}) - P_m (\delta_{max} - \delta_c)$$

Solving for δ_c , we get

$$\text{CCA } \delta_c = \cos^{-1} \left(\frac{P_m}{P_{max}} (\delta_{max} - \delta_0) + \cos \delta_{max} \right)$$

In addition, it is possible to find the critical clearing time for the machine to remain stable. To find the critical clearing time, we still need to solve the nonlinear swing equation.

Swing equation $\frac{H}{\pi f_b} \frac{d^2 \delta}{dt^2} = P_m - P_e$ $P_e = 0$

$$\frac{H}{\pi f_b} \frac{d^2 \delta}{dt^2} = P_m \Rightarrow \delta = \left(\frac{\pi f_b}{2H} P_m t^2 \right) + \delta_0$$

If $\delta = \delta_c$, $t_c = \sqrt{\frac{2H (\delta_c - \delta_0)}{\pi f_b P_m}}$

Critical Clearing Time (CCT)

So, integrating both side, you can say left hand side here, your Pm delta c minus delta naught will be there, your another side here it is a Pm cos delta c minus cos delta max

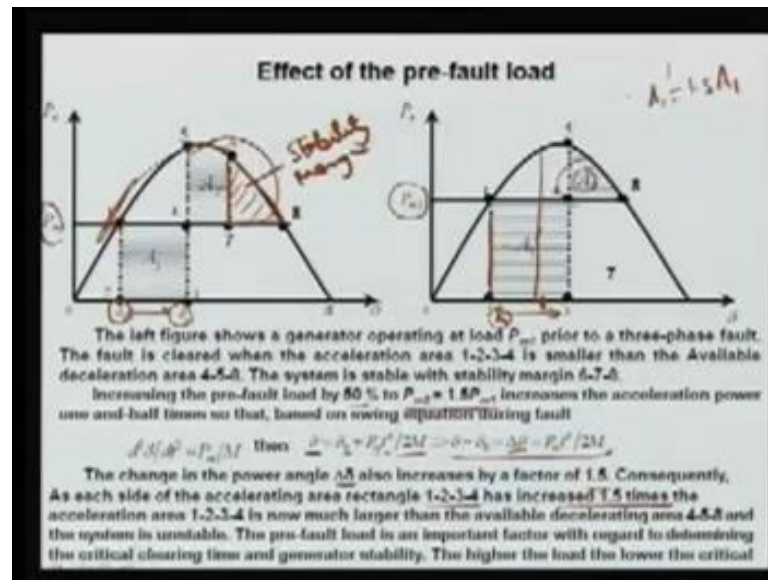
minus $P_m \delta_{\max}$ minus δ_c . So, here this you can say this δ_c this δ_c will be cancelled, and we are getting a 1 equation of δ_c very easily, and that we can say $\cos \delta$ will be this function or we can say δ_c will be $\cos^{-1} \frac{P_m}{P_{\max}}$, δ_{\max} minus δ_0 plus $\cos \delta_{\max}$, and δ_{\max} can be determined again here we know this is P_m , is your P_{\max} , here $\sin \delta_{\max}$. So, if you have solve this knowing P_m P_m as well as P_{\max} , you will get the two values of δ_{\max} . Means here in the previous case, If you will write this equation this P_m will be equal to this is P_m at this δ point, it is your P_{\max} here $\sin \delta$ If you will solve here, knowing this P_m , knowing this you will get the two values of δ 1 you will get the δ_0 , another δ_{\max} and these value less than equal to 180° .

So, since we are talking up to the π degree, so you will get the two values. So, first one the small value will be your δ_0 here in this case; that is here, and this value δ_{\max} you can get the another value that will be again the π minus δ_0 will be the δ_{\max} , means here δ_{\max} will be nothing, but your π minus δ_0 . You can see from here it this angle will be equal to this angle. So, now we can get this by, because you are knowing this δ_{\max} . We know this δ_0 and then P_m P_{\max} we know, and then we can determine the critical clearing angle and this is known CCA, and that is one of the major system stability. Means if your fault is cleared before this angle your system will be stable, and if it is cleared after this your system will be unstable. In addition it is possible to find the critical clearing time, because angle is very difficult to say what is angle and α measure, and the time is very important means at what time your fault is cleared, and that is called your critical clearing time and that can be calculated again by using the swing equation.

Your swing equation is nothing, but your H upon π is $\frac{d^2 \delta}{dt^2}$, here the double differentiation of δ with respect to time that will be equal to your P_m minus P_e , and this P_e is nothing, but your zero, because here in this curve you can see this is your P_e is zero. So, you want to Integrate from here to here, where the P_e is zero, or we can say this is P_e we can write the δ we can differentiate this equation, we can solve this equation. Here we will get the π f not upon twice H P_m t^2 plus δ_0 , because it is starting from δ_0 . So, δ_0 is added, and then this is with respect to time thus δ is increasing. So, δ is equal to now δ_c If you will put this value. So, at that time your t will become t_c , means at δ is equal to your δ_c , your time

will become your t_c . So, the t_c will be nothing, but here, it is δ_c minus δ_{naught} here, twice H is multiplied, divided by twice $f P_m$ under root it will be your t_c , and this is called your cct ; that is a critical clearing time. So, this time can also be determined.

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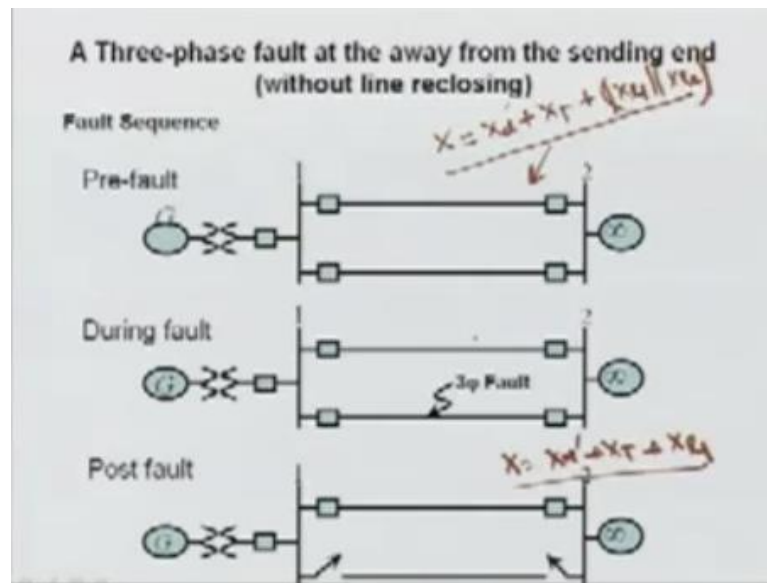
Now, let us see the effect of pre fault load, means earlier your system was loaded at the P_m one. Now in this what we are going to see let this loading. In this case again your fault same system; there is a three fault at the bus say, the curve is similar to the previous one. So, it was operating at the 0.1, that is angle at δ_{naught} the fault was cleared after the δ_c and this regard the A_1 it will equal to A_2 and system is stable. Now you want to your machine was loaded more than that your $P_m 1$ value. Let us suppose your value is $P_m 2$ and that your value is $P_m 2$ 1.5 times more. Means this is more than what will happen. In this case also critical clearing time let us suppose the same fault is clear for the same duration. So, what happens you can say, here again the 0.2 it was δ_{naught} , and again this δ_{naught} is different from this δ_{naught} . But this duration from two to three is same from here as well as here. So, this $P_m 2$ is 1.5 times more this I have already written $P_m 2$ is equal to 1.5 times of $P_m 1$ here.

Now, you can see this now area A_1 ; that this time is same, this area is increased by 1.5 times means this area was A_1 . Now this area one A_1 prime mean A_1 prime, will be 1.5 times of A_1 , because this height was Increased by “one point five” times where this axis same. So, this area is increased. Now you can see, and at the same time your area 2 is in

decrease. Means here what is happening, this area is now shorter, smaller. So, we can see this again, that the Increase in the pre fault load by 50 percent, the P_m 2 Increases the acceleration power, one and half times here. It is increased by one and half times. So, based on the swing curve equation during the fault, here Integration of $d\delta$ upon dt square, P_m upon m , then we can write this δ this δ naught $P_m t^2$ upon $2m$. Again it is written in terms of m your momentum. And therefore, we can see this is your angle. The change in the power angle δ here, this δ also increases by a factor 1.5 times consequently, at each side of accelerating area rectangle one to three has increased by 1.5 times this is Increased by 1.5 times the accelerating of one two three of here means this area is Increased than this one, is now much larger than allowable decelerating area 4 5 8, means this area is much less compared to this area.

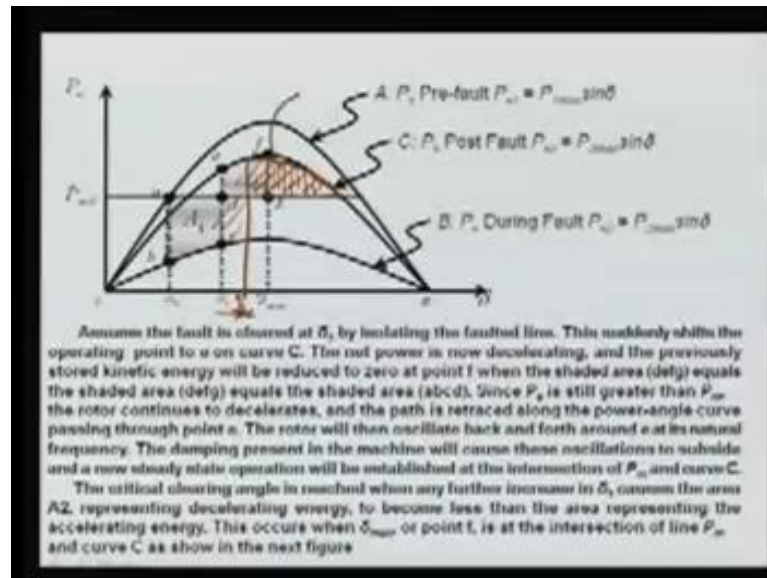
So, what happens if your pre fault load is more, your system is again critical clearing time is lesser. Means you must clear your fault well before here so that we can have area A 1 better more than your area A 2, means we should have some margin we will see. Now question what is this area, because you A 1 is equal to A 2, your machine will here decelerating is equal to your accelerating power, and finally, your machine will oscillate around the point one, and finally, it will be stable, it will be stable. So, area 6 7 8 is called the margin, and that is known as stability margin, stability margin of the system. So, if this margin is larger, than we say our system is more stable. So, this gives your relative stability. Means this margin is an Indication of stability of the system. in this case you can say the stability margin, If area is equal to this stability margin is zero, means in the case at the certain critical clearing time, as I said here in this critical clearing time, this area is equal to area two there is no margin here left out. So, if your system is cleared after this angle your system will become unstable, but If it is cleared here, then we will have some margin here, and we can say our system is stable.

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Let us say the three phase fault in the different scenarios. So, far just we have discussed the three phase fault; that is occurring at the bus one, but let us it is a fault is occurring in the line, and followed by the tripping of the line. So, during the pre fault again your system as the previous, just we have consider all the slides, same here your generator, generating transformer two lines and connected to the Infinite bus system. So, during fault what is happening, now fault earlier it was at bus one, now I am taking fault as line two, and that is your three phase fault. And this fault is cleared only after tripping of this line. So, this is your post fault scenario, means your line here is tripped fault is cleared. Now what is happening, the system impedance here, is a different your due to the fault Impedance is different, and now after tripping this Impedance is a different. Now in this case what will be your X , and using this X here it your X_d prime; that is of your generator, your X_T of your transformer, and plus here X_{L1} parallel to X_{L2} , If you remember this value with I was using in lecture number six. So, this is your X in this case. Now in this case what will be happening, your X will be your X_d prime plus your X_t plus your X_{L1} , only one line is there. So, now you can say this Impedance is lesser than this X one, because the parallel Impedance of the two systems is always less than Impedance of small. So, here this value is now more compared to this. Now during the fault again the Impedance of the system is reduced, whether the reducity is less than this because this is a faulty, and less power is flowing into the system.

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To understand this now you can see these curves, we have now three curves; one is your pre fault, another is your post fault, and then another is during the fault. So, let your curve is A, outer curve from 0 to π , and that is denoted by your a and it is called the pre fault curve. Means we are all the Impedances are intact, where X is your X_d prime plus X_d plus the parallel Impedance of the two lines. Since Impedance is less in that case, so what is happening this, this curve will be higher why. Again you remember this is your VE upon X_d prime, means X prime simply sine δ , If this value is small then P_e will be larger value. So, it is outer where it is follow. Now after the fault one line is tripped. So, what is happening your Impedance is Increased, that let us suppose your point your c curve that is curve here, that is following this, means you can see this curve.

During the fault what is happening, again there is a bolted fault, Impedance is again increased tremendously and the power which is showing Inside the Infinite bus is reduced, and let us suppose this one curve. There is a curve, and where it is denoted at the during fault and it is denoted at the $P_e 2$, that is during post and one is pre fault. Now, you can see the loading of the system was P_m not, before fault, your operating point was A, because it is Intersection of pre fault curve A and your P_m naught curve, and this is your operating, that is pre fault operating point, and the angle of the excitation voltage your δ_0 naught is this one. Now, let us fault we have to apply. Once fault has come what will happen, during the fault that which is occurring here, this is suddenly it is shifting to another curve, because it will always follow the P_e curve here. Now the P_e

curve it is now during the fault we have this curve, that is curve B, and it will be following this curve.

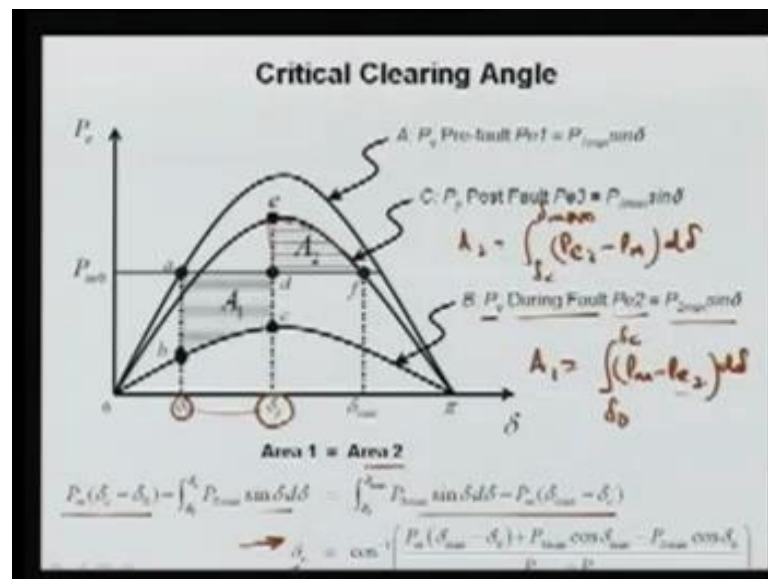
Now, what will happen, now in this case your P_e which is fed to the Infinite bus is reduced, because here it is P_m is more and P_e is less, so machine will accelerate. Accelerate means here your δ will Increase. Let us suppose at δ_1 , your fault is strictly cleared and this fault clearing is associated with this tripping of the line, means suddenly we have to reach the post fault condition; that is point e of this curve that is the post fault curve P_e . And then what will happen, this is your accelerating energy, this curve, then here it is going to decelerate till f, till the point when the A_1 will be equal to A_2 , and then again it will go back, and then it will be again oscillating, and your final point of operation will be now here. It will be another, it is your k point will be here the final point, because it will be oscillating around this point, and your system is stable, if damping is there and the system will subside.

So, this is the condition when it is the fault, means during fault, curve means this during fault, this is post fault, this is a pre fault. So, this is retaining in this with the fault location away from the sending end, the equivalent transfer reactance between bus bare is Increased, lowering the power transfer capability. And the power angle curve is represented by the curve b during the fault, because less power is fed. Finally, the curve c represent the post fault power angle curve, assuming the faulted line is removed. When the three fault phase occurs, the operating point shift Immediately to point B on curve B. here that is coming during the fault, and the axis of mechanical Input power of this machine accelerate the rotor, there by storing kinetic energy, and the angle this is Increased here up to this point. So, this is your point of movement, here it is coming back, and then finally, attained it would be attaining now, later that it will come follow this one. It will come here and then go back. So, in this curve we will see, that this area again is here again, I can say in this vertical hashed line.

This is basically your margin that the system had. Means your system is stable no doubt, and this area if this area is larger, we can say our system is more stable. If this area is more stable if this area is very small, then we can say our system is less stable. So, what happens there is a possibility, that to fault even though if it is not cleared then here δ_1 less than that r at δ_1 . So, this value will be kept on Increasing. So, there is a possibility, let us suppose our δ is increased here. So, this machine will go up to

accelerating here, and this complete area, will be equal to If area A 2. Then this angle again theta delta c is called as a critical clearing. Means the critical clearing angle is the angle that beyond that fault cleared that system will be unstable, and if it is cleared before that angle, or you can say that time then your system will be stable, and there will no margin means that is a critical clearing time.

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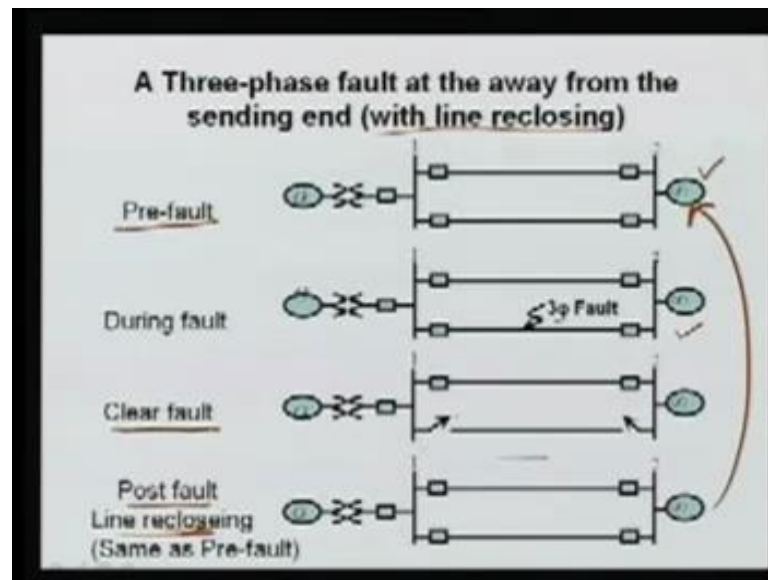


So, to calculate this, the critical clearing angle, means I want to calculate this delta c at which this area up to this point; means that is your Pm line. This area A 2 will be equal to your area A one, then we can say our system is margin less stable, means angles beyond this If fault is cleared system is unstable. To know this critical clearing time, because we have to plan our system; means your protective device, Including your relays, as well as circuit breakers that must clear the fault, if any fault occurs before this angle, and we can also calculate the time. So, this delta c is the dead line, less than that fault must be removed otherwise your system, this generator will be out of the unstable. So, this area A 1 will be equal to your area A 2 and then we can determine again your delta c as the previous example. Again this area is nothing, but now Pm minus Pe again the Pe here of the during fault that is Pe 2; means I can say this area A 1 will be your Integration, from here delta naught to delta c, and this one is your Pm minus Pe 2 d delta. So, this is your area A 1.

Similarly your area A 2 will be your Integration from δ_c to δ_{max} , and this is your P_e now curve, it is your c curve that is P_e three minus P_m d δ . So, if we equating this I am integrating for this case you can say integrating this P_m is constant. So, we will get this $P_m \delta_c$ minus δ not; this term. Another term here is with the negative sign, integrating from δ_{naught} to δ_c of this P_e 2 and P_e 2 is your P 2 max, P_e 2 max we are integrating here. Similarly, for another area two we can write in this fashion and again you can see this $P_m \delta_c$ here $P_m \delta_c$ it is this side it is plus plus it will cancelled out. So, we can determine this δ_c that is a critical clearing angle, it is a cosine Inverse this $P_m \delta_{max}$ minus δ_{naught} plus P_m 3 plus max $\cos \delta_{max}$ minus P 2 max $\cos \delta_{naught}$ divided by P three max minus P 2 max. So, we can determine the critical clearing angle for the different decision.

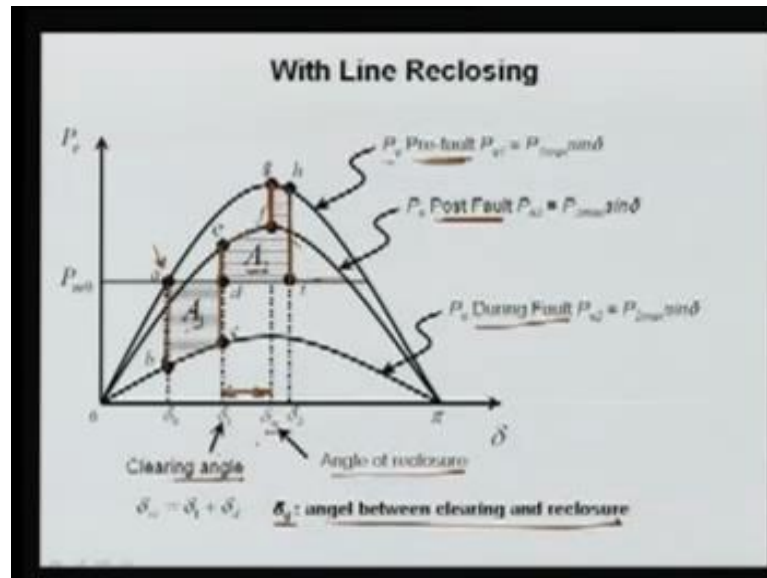
Similarly, we can go for the different one let us see in that case just fault is cleared, by the tripping of the line, but there may be the possibility that the circuit breaker those are in the line, they may have some re closing facility. Why we go for the re closing, because whenever there is temporary fault occurs. There may be possibility, that this circuit breaker is opening and fault is automatically cleared, and after the closing, it was found that there is no fault. For example, let us suppose your there is a flux over on the Insulator, due to the fogging, due to the motion on Insulator, there may be possibility that is line, due to the higher line voltage, and there is a flash over on the line Insulator, and there will be the fire. So, your circuit breaker will be clear, simply that it will be opening, and then it will try after few cycles. Again depends upon the circuit breaker consideration type and make etcetera. So, it will try to re close the fault to see, whether fault is cleared or not for example, let us suppose some word, has come to the wire solitary wire. So, there is a depth three phase wire, three phase circuit. So, once this any live animal or bird there is on that phase here. Once that will die automatically that is again all these phrases are operating in Isolation. So, the fault is automatically cleared after one saw. So, the circuit breaker normally for ESB transmission line, they will have re closing facility.

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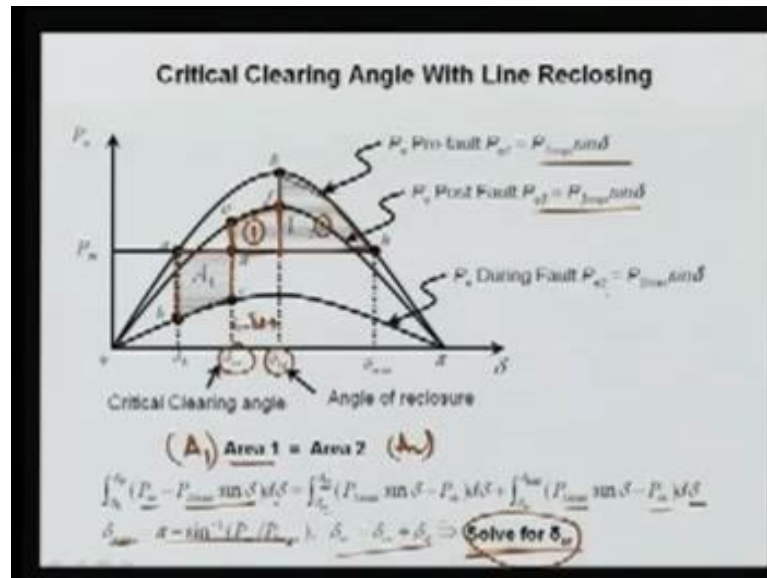
So, with that we can see the line re closing as well. Means here our pre fault system as usual in the previous case, during fault we have the three phase fault. Now we hear the fault is cleared by that tripping, and again it is the line is re closed with the help of re closing facility of the circuit breaker after this cycle. And again is your system configuration here is similar to the pre fault condition. So, now, what will be the scenario to see this. Again we have now three you can see three categories; here this is equal to this, means your pre fault, and your post fault with the line re closing is your same Impedance. So, same curve, and that curve is here, it is written that P_e of Pre fault. Now, during the fault, again it is similar to this. And the post fault that the fault which has occurred here sorry. So, this fault here, you can see this Impedance is same. Here this line is cleared. So, this is fault clear. So, this Impedance is just like in the previous case post fault.

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So, just we have here curve this curve here, is a basically the fault clear at that time fault is clear, but it is not a post fault. Post fault is again here If this condition where is again it follows the same P_e . So, in this case here your operation is at point A. Again it is a Intersection of P_m not, and you are the P_e angle; that is pre fault, and this, after this there is some fault your system is coming here, that is your during the fault condition, it will be try to follow here. And then fault is clear, there is a clearing angle here, it will try to go at this point, because the P_e during the once fault is cleared it is following this, now till it is accelerating this line was opened it follows this and at the same time circuit breaker try to close It. Means it is suddenly here it is Increase here before that here it is a decelerating completely, it is your after few cycle it Is. What is happening now, If this is coming and now again following this, and then it is coming back, because this area is now more. So, it is more accelerating, because we are taking more power here. So, now, area again for stable, here this area one will be equal to area two. And in this case you can see area of margin here it is very high, because line is not true. So, you can feed more power. So, your system is more stable. So, the outage of line, again create some criticality of the line. If you saw in the previous case it was this area only, now we have this area. So, your system is more stable. So, outage of lines sometimes creates some Instability and the system is very near to the stability margin. So, we want this R_c is called angle of re closure. We want to know, and this your δ_d angle between clearing and the re closure this angle. We can determine, and we can know that it is cleared at this point, and then we can see what should be the critical clearing time in this case.

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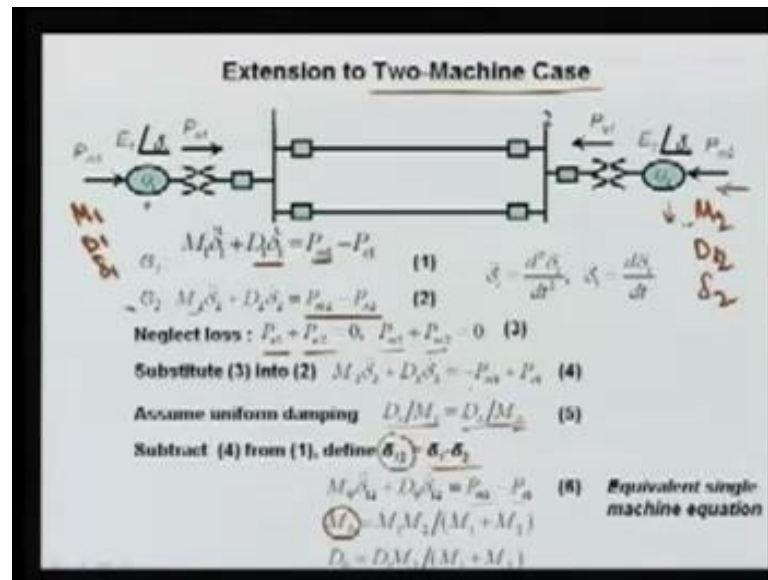


We can again similarly determine. Now again you want to get the what should be the critical clearing time here delta cr, having the re closing facility this, this angle is given to you that is delta dt it is to note, because it depends upon the circuit breaker configuration automatically, it may re close may be one cycle re closing may be one and half or two cycle re closing. Means one cycle it will wait and then again it will re-close. And again the fault is persisting what will happen, this will open, circuit breaker will open and it will not re close again and again, it is only once. So, in this case we want to determine what will be the critical clearing angle this one. So, that knowing this specialty in your circuit breaker. You want to determine that your circuit breaker fault must be clear before this angle, otherwise your system become unstable. So, in this case your area again using equal area criteria this area, should be equal to area this now. Your area defghd. So, this is the complete area now you want to know.

So, area one or you can say A 1 should be equal to area A two. Now this area A 2 will be nothing, but the Pm minus this Pe 2 of integration, that will be equal to from here to here. Now we have the different curves and different area. Now we can divide this area in two parts; one is here one, here is another is two. So, one is your nothing, but your either Pe 3, because it is a Pe 3 here, Pe 3 max minus Pe, and it is from delta cr to delta rc re closing angle plus this area this area is from rc to delta max Pm 1 here minus Pm delta. And now you have to solve, because this Angle you know you can determine this knowing the Pm and Pm 1 max, you can calculate delta cr knowing all this value. So,

here we can get the value, we can substitute δ_c here and we can get this value δ_{cr} ; that is your critical clearing angle, and then we have to design our system based on that one.

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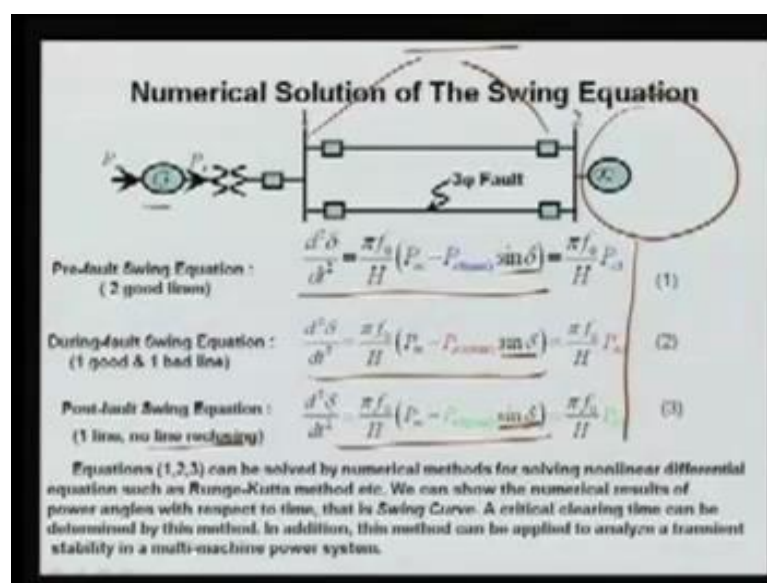
Now, same theory can be extended, can be applied to the two machine phase, in which here this two machine having the different Inertia constant. Here it has a different Inertia constant, and then we can add, and we can apply together. So, in this case we want to apply the equal area criteria for the two machine case, is a special case, and this two machine case can be represented by equivalent a single machine connected to Infinite bus. Here we have the generator one, and another is generator two. This generator two is having all the parameter; that is denoted by the two here suffix. So, here it is having M one, it is having D 1 damping constant, it is having delta on side it two, so it is two. This case it is your M 1 it is your D 1 and delta 1. So, we can write the dynamic equation; that is a swing equation, even then including damping. So, here the damping term is Included. Normally till now we have ignored the damping, because it is related with the speed of the system.

So, this is M 1 delta 1 double dot that is double differentiation of delta 1, delta 1 of here it is delta 1 written; that is feeding power Pm one. Here the mechanical power that is coming to this generator is Pm 1 minus Pe. Similarly for generator two we can write this M 2 delta 2 double dot plus d 2 delta 2 dot plus that will be equal to your Pm 2 minus Pe

two; that is coming from this side If losses are neglected what will happen. This P_{e1} P_{e2} in this system the total P_e here that is inducting that must balance that must conserve, energy cannot be going anywhere. So, losses are zero means the energy which is feeding that will be the balance crowd, means whatever you are feeding it must be taken by some machine. So, this P_{e1} plus P_{e2} equal to zero. Similarly the energy, mechanical energy that is coming into the system the P_{m1} plus P_{m2} is also equal to zero. Now, substituting equation three into two here, even two substitute the equation three here.

So, substitute the equation three into two, we will get here your $M_2 \ddot{\delta}_2$ plus D_2 here this side, and this we are just replacing, complete this 2 equations; that is P_{m2} and P_{e1} in terms of P_1 we can write this minus P_{m1} this, and it will be plus P_{e1} . Assuming the uniform damping, means damping here is a D_1 upon D_1 that will be equal to D_2 upon M_2 , or you can Ignore the damping. If you are also Ignoring that is no problem, then it is very simple, but if your damping is uniform. So, D_1 upon M_1 and D_2 upon M_2 is equal. Also we define the δ_1 minus δ_2 that is a δ_{12} . So, we can write a equation $M_{naught} \ddot{\delta}_{12}$ plus δ_{12} delta naught delta 1 single dot is equal to P_{m1} minus P_{e1} , and this $P_{mnaught}$ will be the basically the parallel combination of this angular momentum that is M_1 and M_2 . So, M_{naught} will be equal to your M_1 into M_2 divided by M_1 plus M_2 . Similarly D_{naught} will be D_1 into M_2 divided by M_1 plus M_2 .

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So, this is your equal length system, and we can again apply our equal area criteria. Now, for the single machine, it is very easy, no doubt to know, whether system is stable or not. We can analyze and we can calculate your critical clearing time, we can see the stability of the system, and we can only know the relatively stable system. We want to get what is the delta variation, it is not a time domain simulation at all. It is only giving Information about the stability of the system, means your system is stable or it is not a stable. But that it is not only our concept. We want to know what is the system behavior during the period, means what is the speed deviation, how speed is changing, what is your delta is changing that is also require, and for that we have to go for solving differential equation for different Intervals. So, this is the numerical solutions to the swing equations. We have written the swing equation with this during the pre fault condition.

During the faulted condition this equation, and post fault here If know line re closing we are not considering means fault is cleared, by the outage of the line means this line is tripped, after certain time only post fault is this only one line is existing along with this generator transformer, and this is your base equation. So, the equation one two three here, can be solved by the numerical methods, for solving non-linear differential equation, why this delta here this is also delta this non-linear function here, this non-linear non-linear. So, this differential equation is a non-linear differential equation of second order, because of D^2 . And that can be solved by the various methods. Methods may be your Eulers method, Trapezoidal methods that is called kutta methods. We can show numerical results of the power angles with respect to time; that is a swing curve. Our critical clearing time can be determined by this method. In addition this method can be applied to the analyzer transient stability in a multi machine system.

For the single machine system, they as said the equal area can be easily determined, and we can say our system is stable or not stable. We can also expect what is the margin available with you, if fault is cleared at particular time so we can know the relative stability, but we want to know the performance that is a time domain simulation is required. So, we have to solve these three differential equations. More over whole this swing equation here, we have modeled a simple classical approach, and that is not valid. Means we have to go for the detailed modeling of the machine. Here we have to Ignore we have made seven six seven assumptions, that can be Incorporated and then there will be several hundreds of machines in a big power system. So, all these will be solved for

the system, then we can see the performance that the delta etcetera variation of each Individual generators, and then we can say, whether the system is stable or not.

So, in this lecture we have seen the equal area criteria for the application for the various, configuration of various fault with re closing, without re closing, fault at bus, fault at line, and then we can saw this equal area criteria can give you the relative stability. It will give Information about without, whether it means whether your system is stable or not at the same time. It can also with the margin that is stability margin available with the system, and it is very useful and very fast no doubt, but only the limitation of this is that, we have made several assumptions; those may not be valid for the power system. And also it is not possible to formulate to form the system into single machine Infinite bus, equivalent and then we have to go for the complete and the real time petrol system for the simulation. And for that we have to solve these differential equations for all the machines, not only one machine, this is the case of one machine. So, we have to go in this Infinite bus system there will be several generators, and we have to include.

Again this generator is not a classical generator. Classical generator means is a generator which we have model in the classical form, means we have taken only this generator with the some Inertia constant, then we can go for several order of modeling of synchronous machine. It will equip with your excitation as well, it will be equipped with the governing system. So, all dynamics will be, dynamical equation will be clubbed together along with algebraic equation, because here the power flow of the different transmission lines, they are not connected with the transmission lines generating stations, the different transmission lines that they will be there. So, we have to form this algebraic equation, and then we can solve. Some times what we do, we this all the buses, those are not connected we try to eliminate. We will see in the later sections. Here this they will be eliminated only the generator terminator buses are kept Intact, where there is a fault is occurring, and then we can analyze the system behavior, means we can see the relative motion of the Individual generator with reference to any particular generation, and then we can say whether your system is stable or not stable. So, with this, now this chapter seven, lecture seven is closed.

Thank you.