Intelligent Systems and Control

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Module – 1 Lecture – 5

Nonlinear System Analysis: Part 2

This is lecture 5 of module 1. We will continue the discussion that we did in the last class on nonlinear system analysis.

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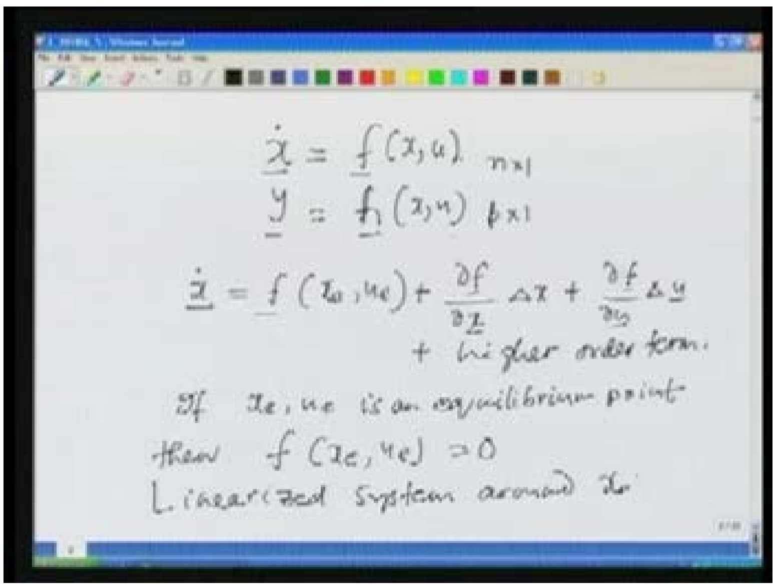

What we did in the last class was we represented a general nonlinear system in a state-space format. These are your states (Refer Slide Time: 00:58) and this is your output. f is a nonlinear function, it is a vector; h is also a vector. These are all n-dimensional vectors and y is a p-dimensional vector. What we said (Refer Slide Time: 01:20) we said how a nonlinear function can be expanded using Taylor series expansion around some point. This is my point and around this point, I want to expand this function. Then I can write del f by del x into delta x plus del f by del u into delta u. This is the first-order expansion and there is some higher order term. If x_e is an equilibrium point, then f xe_1 u_e is 0.

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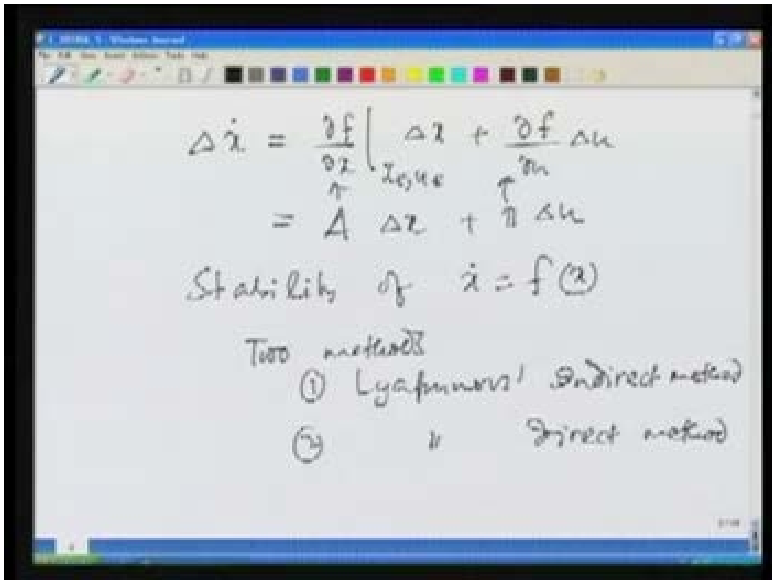

The linearized system around x_e and u_e is given by delta x dot is equal to del f by del at x_e, u_e into delta x plus del f by del u into delta u. In our Taylor series expansion, we ignore the higher order approximation with certain condition. We said that this is A matrix and this is your B matrix. So this is your A matrix and this is your B matrix (Refer Slide Time: 03:33). For stability of x dot equal to f x, there are two methods we discussed: Lyapunov's indirect method and Lyapunov's direct method; this is what we discussed. In this indirect method, we linearize.

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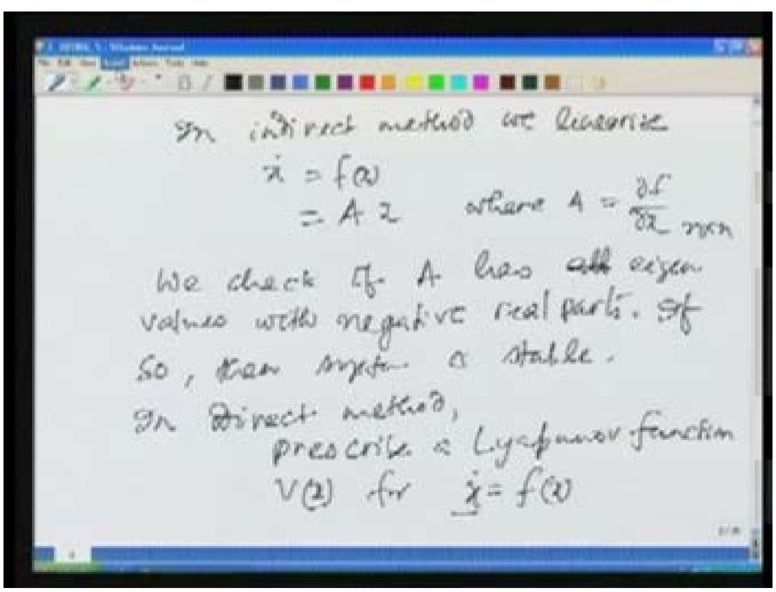

In indirect method, we linearize x dot equal to f of x as A x, where A is del f by del x. This is an n-by-n matrix. We check if A has Eigen values with negative real parts. If so, the system is stable – that is what we discussed in the last class. Then, we discussed Lyapunov's direct method; prescribe a Lyapunov function v x for x dot equal to f x. and then after finding Lyapunov function, we have certain properties for Lyapunov function v_x (Refer Slide Time: 06:20) and we had discussed that.

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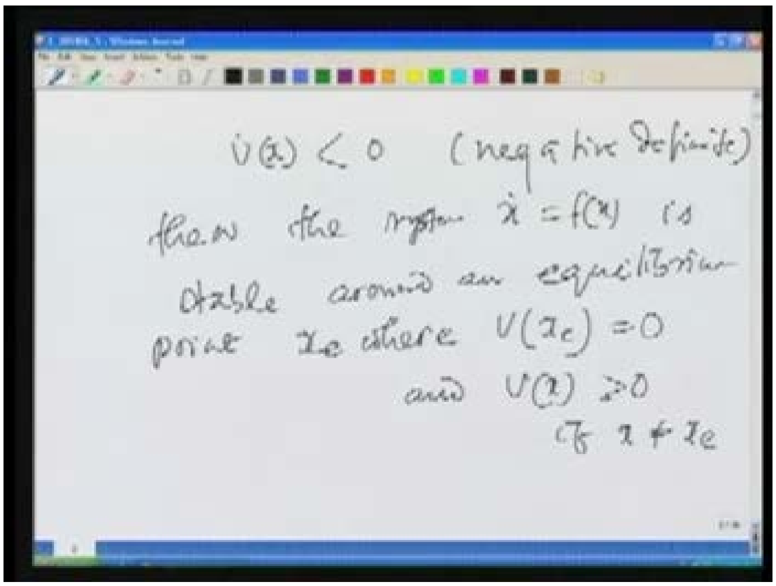

Then, we have to make sure that v dot x, if negative definite, then the system x dot equal to f x is stable around an equilibrium point x_e, where v x_e is 0 and v_x is greater than 0 if.… (Refer Slide Time: 07:25). This is the summary of the last class. Now, we go to the next further discussion.

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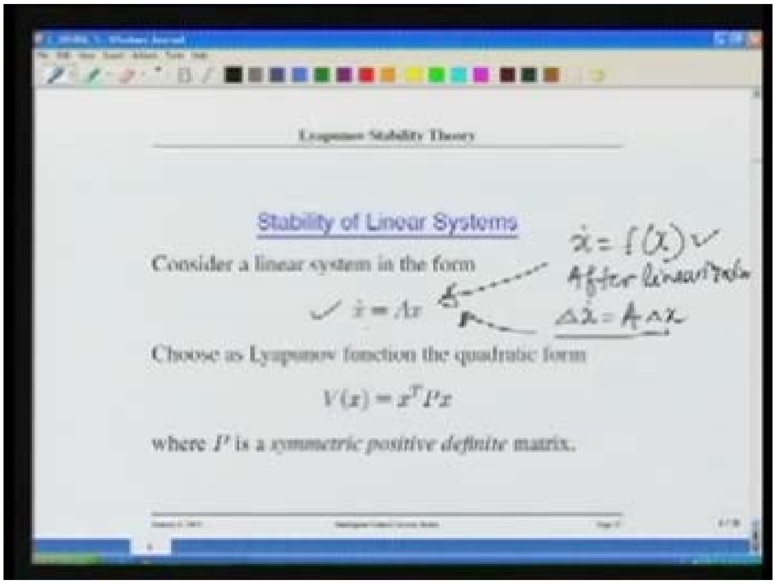

If we have linearized the system, now our actual system is x dot equal to f x. If you linearize this system, after linearization we write x dot equal to x and you say that (Refer Slide Time: 08:05), you see that linearization makes so you just make our symbols very simple. The linearized system is actually delta x dot equal to A delta x but for convenience, we represent this equation as x dot equal to A x of the same system x dot equal to f x. This is our original nonlinear system (Refer Slide Time: 08:30), this is the linearized system and then we concluded how to determine the stability of the linearized system.

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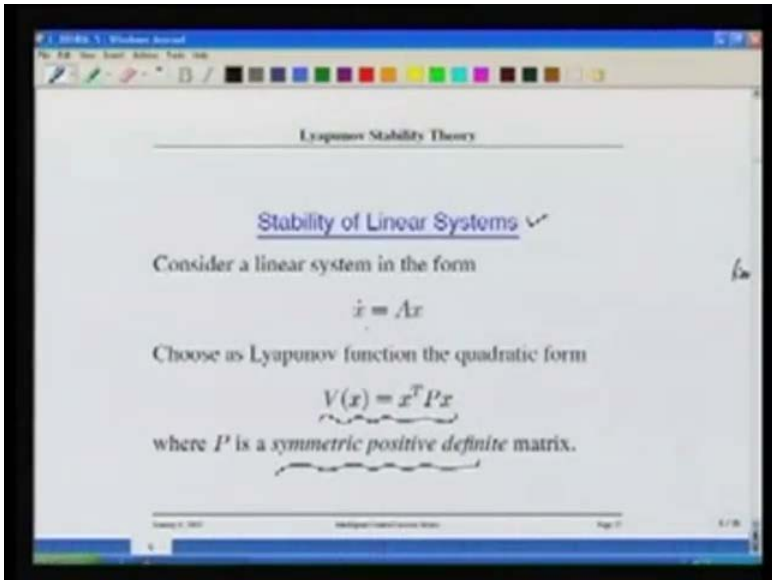

We discussed in the last class the direct method and indirect method. Now, can we use this Lyapunov stability theory for stability of linear system, which is x dot equal to A x? The answer is yes. It is very interesting to see that for x dot equal to A x, there is a very scientific method to say what exactly a Lyapunov function is. It turns out that the Lyapunov function is V x is equal to x transpose P x. It is a quadratic form Lyapunov function and this is a valid Lyapunov function. One really need not break his head to find out what a Lyapunov function is for a linear system. It is very well defined: x transpose P x and where P is symmetric positive definite. That is very important.

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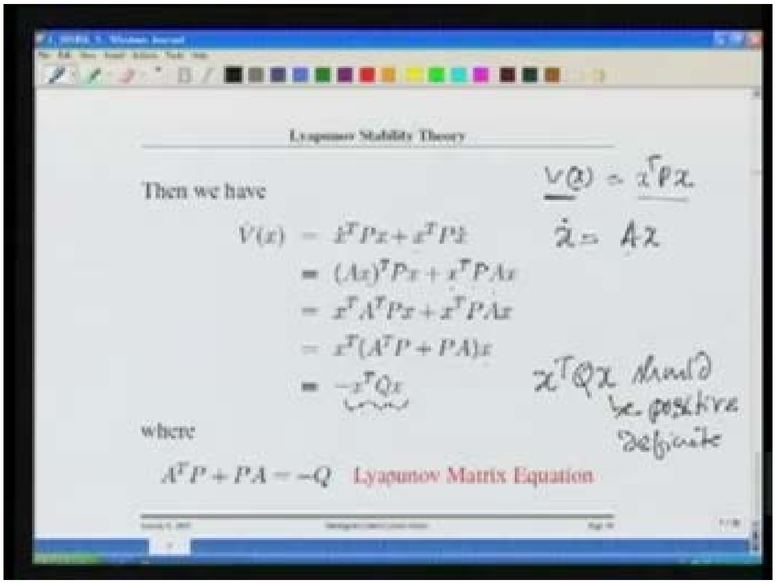

Once we have defined V x is x transpose P x, this is our Lyapunov function (Refer Slide Time: 09:58), then we differentiate V x, which is V dot x. It is obviously x dot transpose P x plus x transpose P x dot and our x dot is A x. You replace x dot by A x. So, (A x) transpose P x, then x transpose P and x dot is A x. By simplification, we find x transpose into A transpose P plus P A into x, which is minus x transpose Q x. For stability of a linear system, according to the stability theory, this quantity x transpose Q x should be positive definite because if x transpose Q x is positive definite, then V dot x, which is negative of positive definite, becomes negative definite. What is the condition? If it is stable, then this should be positive definite. If x transpose Q_x is positive definite, the definition of a quadratic function to be positive definite is that Q should be positive definite.

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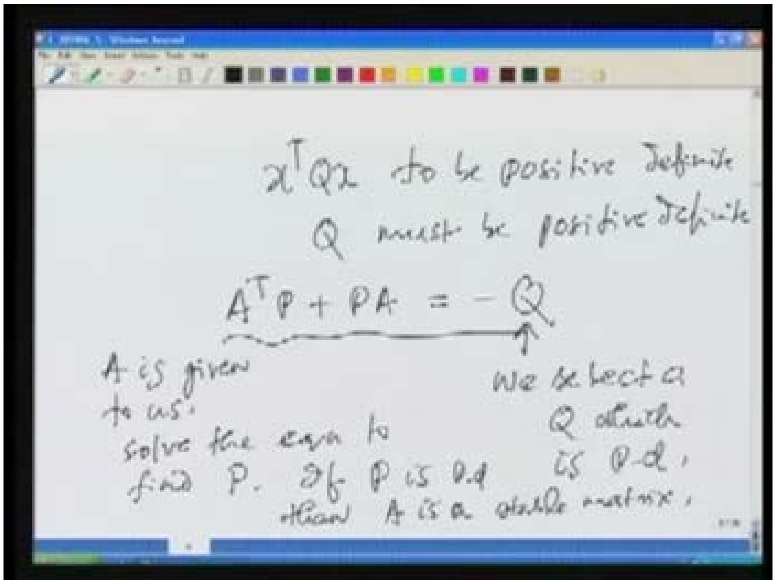

Let me write here that x transpose Q x to be positive definite, Q must be positive definite. Our identity is A transpose P plus P A equal to minus Q. This is our identity from here, V dot x, this is known as Lyapunov matrix equation (Refer Slide Time: 12:05). Popularly, this is also known as Lyapunov equation. If this Lyapunov equation A transpose P plus P A equal to minus Q…. For stability, Q should be positive definite. We can always arbitrarily select Q to be positive definite, we already know what is A, and determine if P comes out to be positive definite.

If that is the case, then the system is stable. We will go by circularity. So A transpose P plus P A is minus Q. We select a Q, which is positive definite. A is given to us. Now, we solve this equation with a specific positive definite Q and then solve the equation to find P. If P is positive definite, then A is a stable matrix, that is, the linear system is stable.

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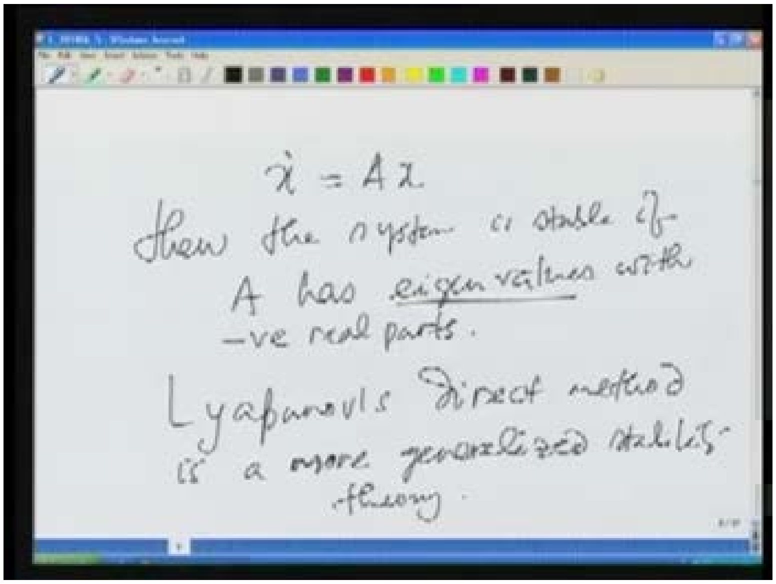

Already, we know that if x dot is equal to x, then the system is stable if A has Eigen values with negative real parts – all Eigen values (Refer Slide Time: 14:12) negative real part. That is a well-known theory, we already know about it, but we now say that even the Lyapunov stability theory can be used. We can use the direct method of Lyapunov stability for a linear system. In that sense, Lyapunov's direct method is a more generalized stability theory because using this, we can determine the stability of both a linear system as well as nonlinear system – this is important. Lyapunov's direct method is a more generalized stability theory and can be applied to both a linear and a nonlinear system.

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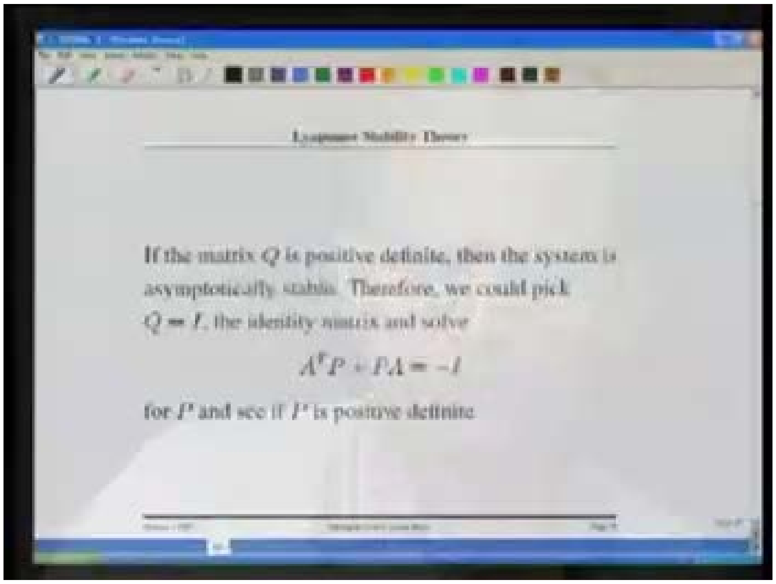

We already discussed that if matrix Q is positive definite, then the system is asymptotically stable. Therefore, we should pick Q equal to I, the identity matrix, which is a positive definite matrix and find if P is positive definite. If P is positive definite, then it is stable.

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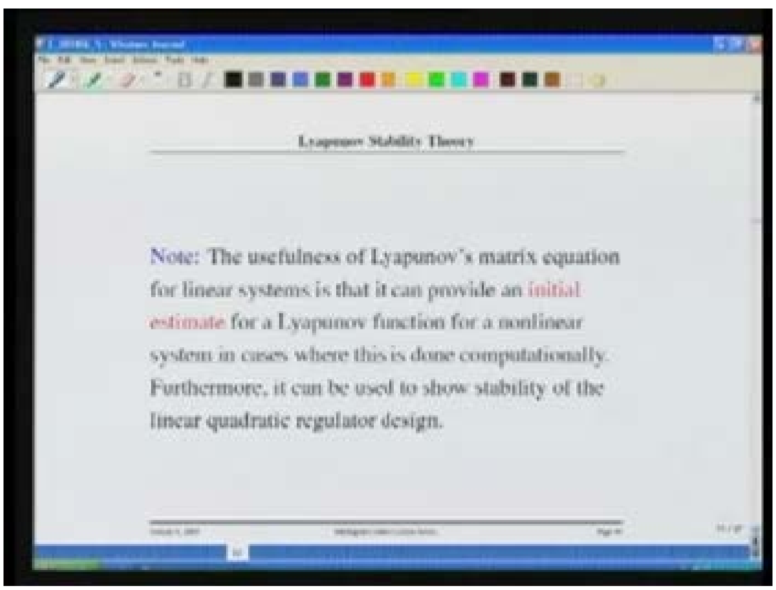

The usefulness of Lyapunov's matrix equation for linear systems is that it can provide an initial estimate for a Lyapunov function for a nonlinear system in cases where this is done computationally. Furthermore, it can be used to show stability of the linear quadratic regulator design. Sometimes, we are not very clear how to find out the Lyapunov function for some linear and nonlinear systems and there, the Lyapunov matrix equation becomes handy.

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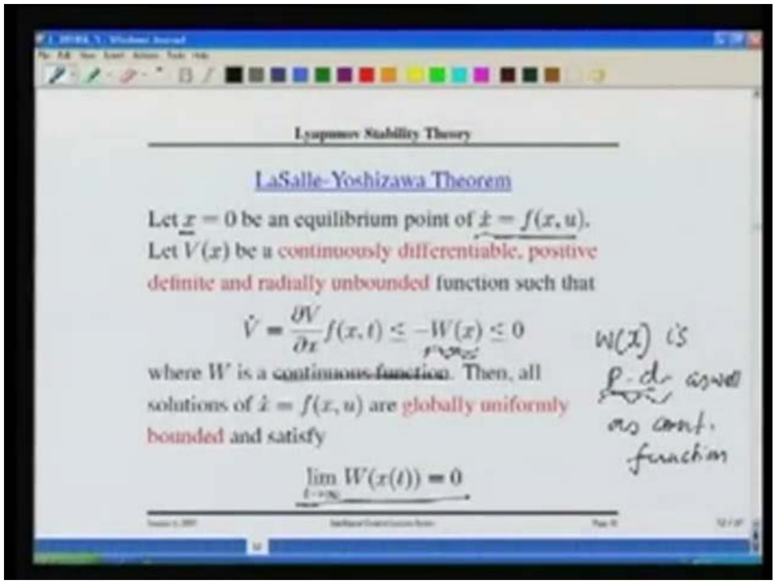

This is an important theorem, the LaSalle–Yoshizawa theorem. It is an addendum to the already existing Lyapunov stability theory using direct method. If x = 0, origin, this is an origin and it is the equilibrium point of this nonlinear system, then assume that V x is continuously differentiable, positive definite, radially unbounded. That is the property of a Lyapunov function. Then if the time derivative of the Lyapunov function V dot is less than or equal to minus some function of x (Refer Slide Time: 17:15), where W x is a positive definite function, then…. Not only W x here, W x is positive definite. W x is a positive definite function.

Then, all solutions of x dot are globally uniformly bounded and satisfy W x t tends to 0. We already said that if b dot is negative definite, then the system is stable. Now, we are only simply saying this W x is a positive definite as well as a continuous function. Earlier, we simply said it is a positive definite. Now, we are saying that it is a positive definite as well as a continuous function. When these two are together true, then what happens? This is also true. W x t … t tends to infinity, will become 0. This is LaSalle–Yoshizawa theorem and this theorem is very important to determine the convergence of trajectories, as we will see later in this course.

If W x is positive and definite, then the equilibrium point is globally uniformly asymptotically stable.

Now, we will discuss more examples using these concepts. What we discussed now were the Lyapunov direct method, the indirect method and how to apply Lyapunov's direct method to a linear system plus the LaSalle–Yoshizawa theorem, which gives the convergence property of the function that W x, which goes to 0 as t tends to infinity. Now, we will talk about now Lyapunov's examples for Lyapunov's direct method.

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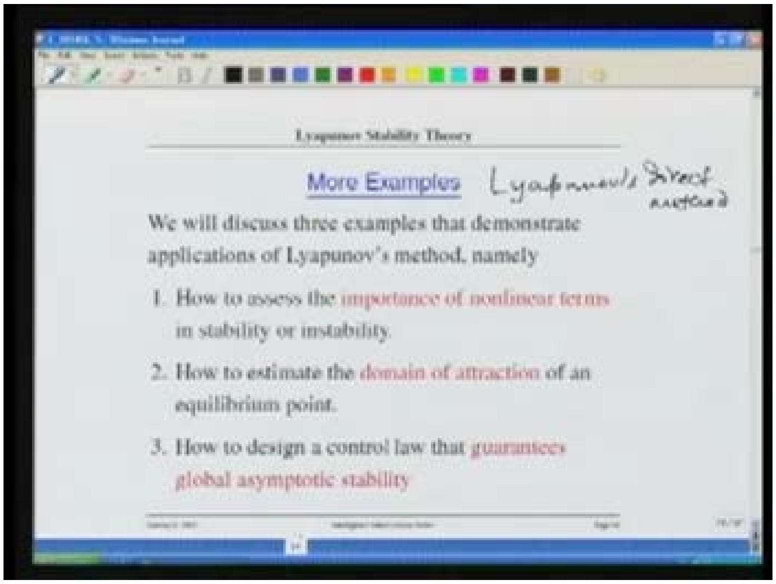

We will try to answer these three questions: how to assess the importance of nonlinear terms in stability or instability, how to estimate the domain of attraction of an equilibrium point and how to design a control law that guarantees global asymptotic stability?

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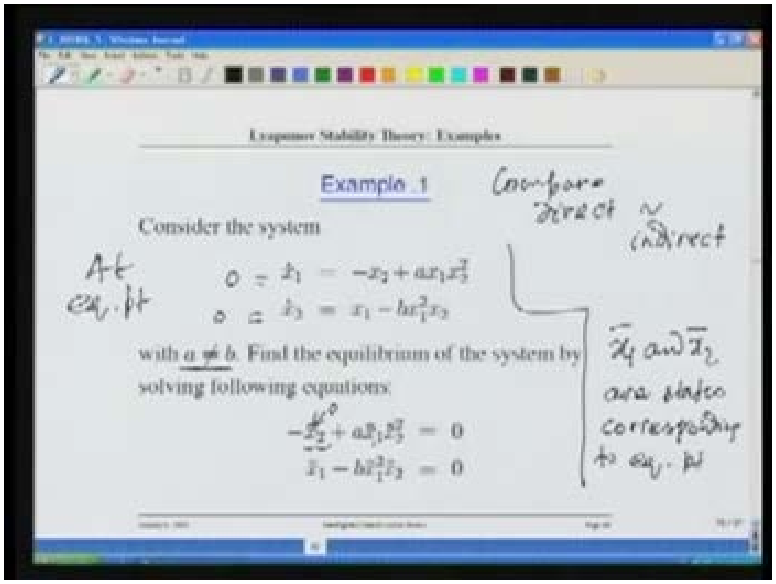

We will take one example here. In this example, will show how using Lyapunov's direct method, we can find out the domain of attraction. x_1 dot is minus x_2 a x_1 x_2 square, x_2 dot is x_1 minus b x_1 square x_2, where a is not equal to b. This is an important condition. First of all, through this example, we will compare direct versus indirect, that is, we linearize the system, look at the stability, linearize (Refer Slide Time: 21:33) and then as a whole, we apply the direct method and look at the stability and we will see what the difference is. x_1 bar and x_2 bar are the states corresponding to the equilibrium point.

If you put x_1 bar x_2 bar here, then at the equilibrium point, the dynamics becomes stand still, there is no dynamics meaning this becomes 0 and this becomes 0 at equilibrium point (Refer Slide Time: 22:31). This is very important – at equilibrium point. What is equilibrium point? It is a point where there is no more dynamics. So x_1 dot x_2 dot all become 0. If we put this, then we find what the equilibrium points are.

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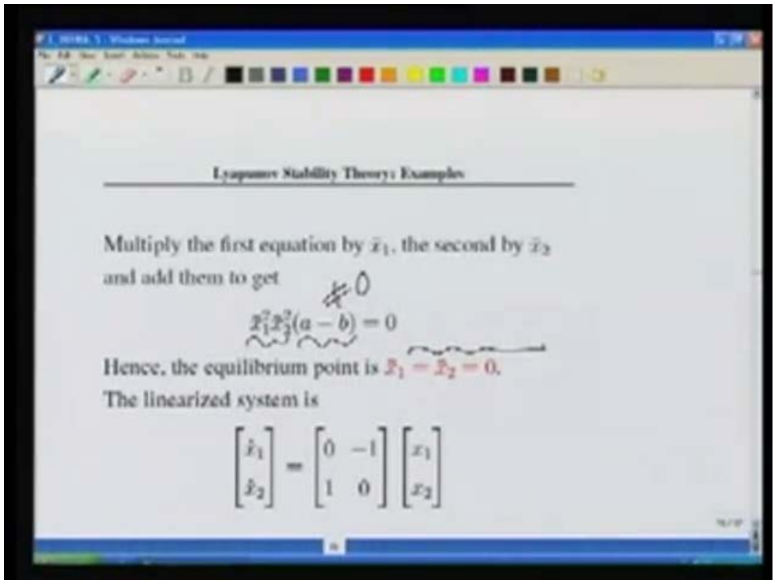

Obviously, you will see that at equilibrium points, the solution because a minus b is not 0. Since this quantity is not equal to 0 (Refer Slide Time: 23:02), obviously x_1 square and x_2 square has to be 0.

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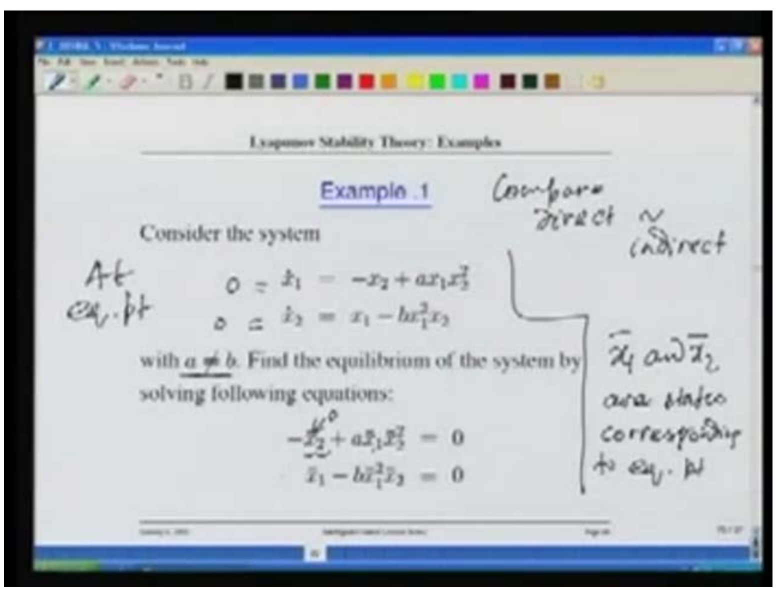

If you go back to the previous one, if you consider in this x_2 equal to 0, that is, if this is 0, then x_1 has to be 0. Similarly, if x_1 is 0, then this quantity becomes 0 and so x_2 has to be 0.

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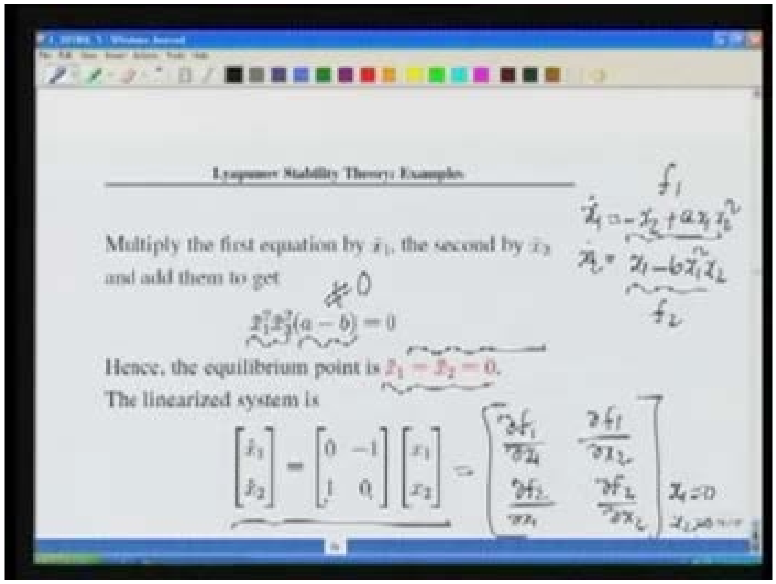

Finally, the net result is the origin is the equilibrium point. x_1 equal to x_2 equal to 0 is the equilibrium point. We linearize using the linearize concept. This is our result x_1 dot, x_2 dot (Refer Slide Time: 23:46). How did you find this 0, -1? Please verify. This is actually (del f_1 by del x_1, del f_1 by del x_2, del f_2 by del x_1, del f_2 by del x_2) at $x_1 = 0$, $x_2 = 0$. What is f_1 in this case? x_1 dot is minus x_2 plus a x_1 x_2 square and x_2 dot is x_1 minus b x_1 square x_2. This is our system. This is my f_1 (Refer Slide Time: 24:50) and this is my f_2. You differentiate this f_1 with respect to x_1, which is 2 a x_2 square a and if you put $x_2 = 0$, this becomes 0. Similarly, del f_1 by del x_2 is actually -1. This is -1. At any value, this is -1.

Similarly, if you differentiate f_2 with respect to x_1, the first element is 1, second element is 2 x_1 x_2 into $-b$ and by putting $x_1 = 0$, $x_2 = 0$, this second term becomes 0, so 1 is retained. You can also verify that del f_2 by del x_2, which is minus b x_1 square, at $x_1 = 0$ becomes 0. I hope it is very clear to you how to linearize a nonlinear system around an equilibrium point. This should be very clear to you.

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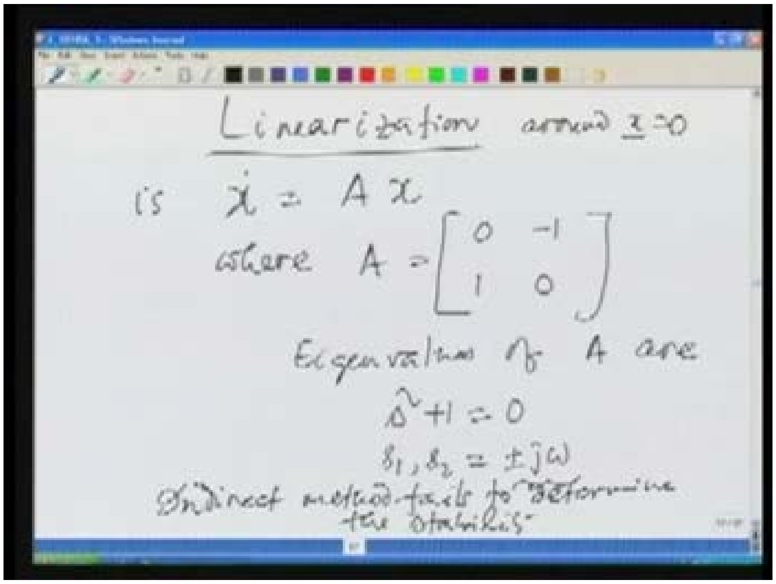

We found out that x dot is equal to A x. The linearization around x = 0 is x dot is equal to A x where A is (0, —1, +1, 0). This is the matrix we obtain. You can easily check that the Eigen values of A are s square plus 1 equal to 0. So s_1, s_2 are plus or minus j omega. The Eigen values do not have negative real part. By using Lyapunov indirect method, we cannot determine the stability. The indirect method fails to determine the stability. For saying stability or instability, all real parts have to be either negative or all real parts have to be positive. If it is negative, then it is stable; if all are positive, then it is unstable. But what happens when the Eigen values are 0 or like in this case we found out plus or minus j omega – 0 real part? In this case, the indirect method fails, it is silent, it does not say anything about the stability, but let us go to the….

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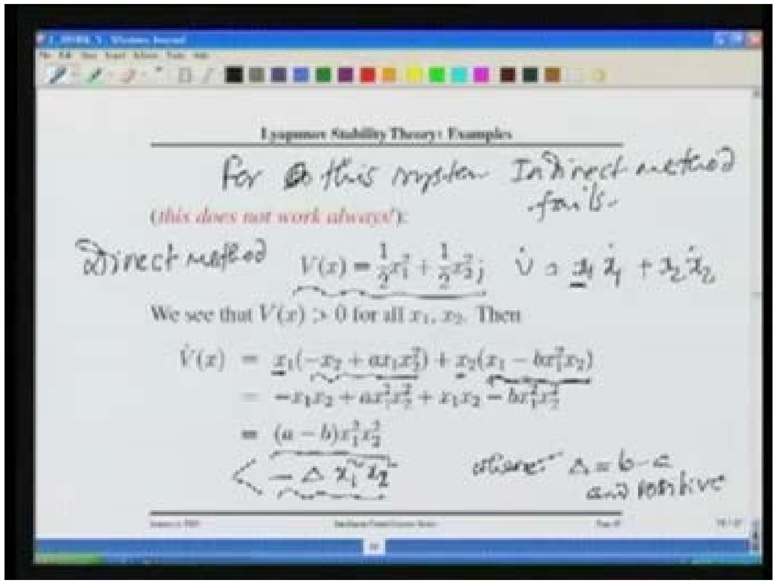

For this system, the indirect method fails. Now, go to the direct method. We apply the direct method. In the direct method, we assume the Lyapunov function, which is a quadratic Lyapunov function. We have two states x_1 and x_2. It is simple, it is half x_1 square plus half x_2 square and we differentiate V dot x. This, as you know, is our f_1 (Refer Slide Time: 29:05) and this is our f_2. You replace because if I see here, V dot is x_1 x_1 dot plus x_2 x_2 dot. That is what I am doing. x_1 is here and this is my x_1 dot, this is x_2 and this is my x_2 dot.

If I finally simplify it, I get a very nice expression here. This expression is a minus b into x_1 square x_2 square. You can easily see that this is negative if minus delta x_1 square x_2 square, where delta is b minus a and is positive, that is, if a is less than b, then this becomes a negative definite quantity (Refer Slide Time: 30:30) and hence, the system is stable. This answer is very simple. If a is greater than b, the system is unstable; if b is greater than a, the system is stable using the direct method. That is the interesting application.

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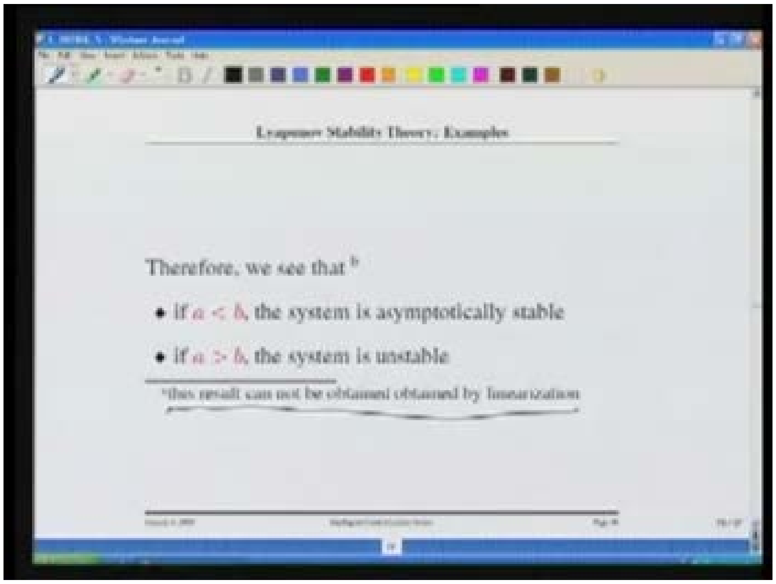

We see that if a is less than b, the system is asymptotically stable, but if a is greater than b, the system is unstable. This result cannot be obtained by linearization using indirect method.

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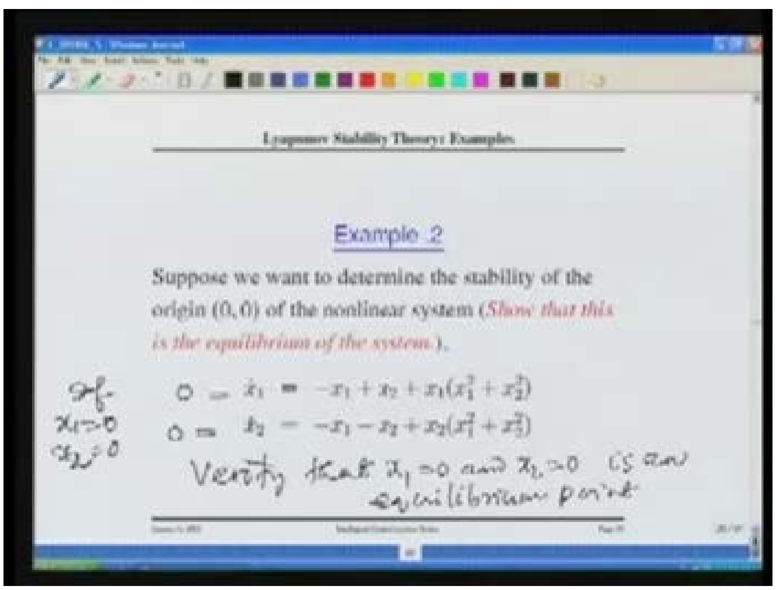

Now, we go to another example. In the first example, we found what is the advantage of the direct method over the indirect method. In some cases, the indirect method is silent about the concept of stability, but the direct method gives that. We take another example where x_1 dot is this and x_2 dot is this. You can verify. This is an exercise for you: verify that $x_1 = 0$ and $x_2 = 0$ is an equilibrium point. You can easily see that because if this is 0, all 0s, then this also becomes 0. So obviously, this is an equilibrium point. Now, the question is what is the stability about this equilibrium point? How stable is this equilibrium point?

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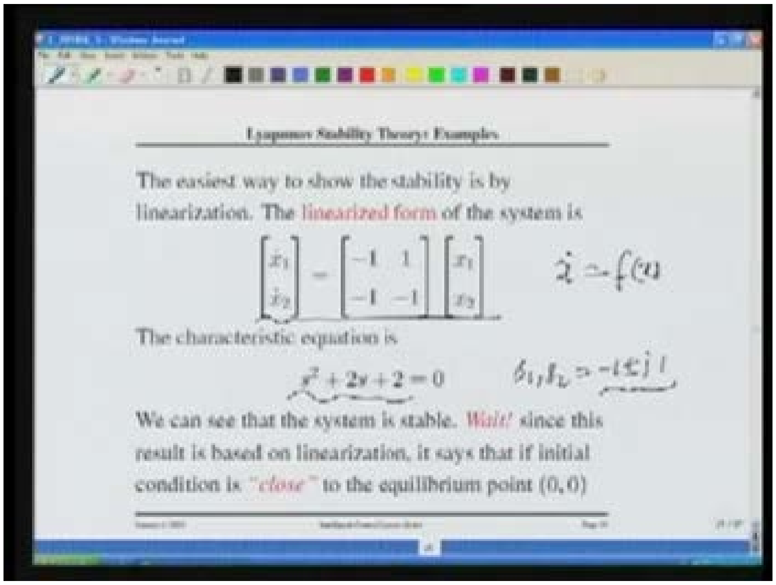

This is the question. We linearize and again, we go back to the indirect method, linearize the system, verify that the linearized system of the original x dot equal to f x is (—1, 1, —1, —1) into (x_1, x_2). In this, you can see the characteristic polynomial concerning this A matrix is x square plus 2 s plus 2 equal to 0. You can easily see that this particular system has all Eigen values that are negative, because s_1 and s_2 are —1 plus or minus j_1. Obviously, this system is stable.

The indirect method also gives us the solution, that is, the system is stable around the equilibrium point, but it cannot say how stable this equilibrium point is – there is no quantification about it that if I disturb the system from the equilibrium point, that is, the origin to a very… how much I can disturb from this equilibrium point and still say the system is stable. We will not get that answer using the indirect method. If the initial condition is close to the equilibrium point (0, 0), then we can say the system is stable using the concept of Lyapunov's indirect method.

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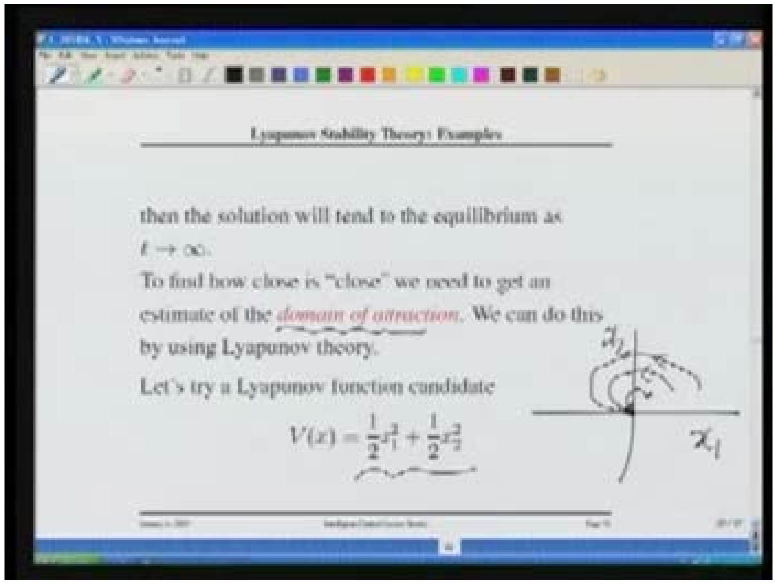

But now, let us go to the direct method. The indirect method cannot say what is the domain of attraction, that is, this is my x_1, this is my x_2 and this is my origin. The indirect method said that if I disturb this origin to this point (Refer Slide Time: 34:28), then I can come back here, but how far can I disturb this particular origin? Can I disturb it to this point? Further? Where will I fail? Let us go to the direct method to answer this question. This is called domain of attraction or region of attraction. V x is half x_1 square plus half x_2 square.

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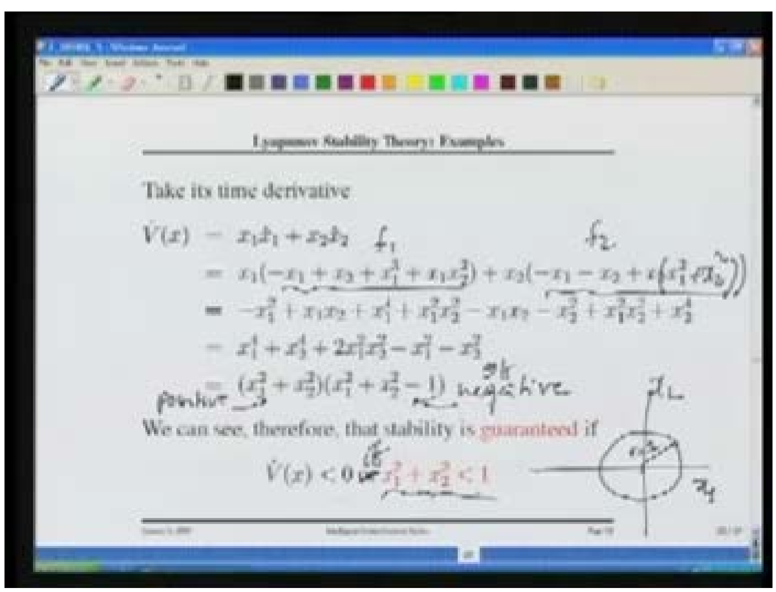

Apply the derivative, x_1 x_1 dot plus x_2 x_2 dot. I think there is some mistake here. (Refer Slide Time: 35:20) This is our f_1 (Refer Slide Time: 35;26) and this is our f_2. If we simplify this V dot x, then we get an expression like this: x_1 square plus x_2 square into x_1 square plus x_2 square minus 1. This will become negative definite or this V dot x is negative definite if x_1 square plus x_2 square is less than 1; that means these quantity is negative. You can always say that this quantity is positive.

This will become negative definite. This is always positive, so this will become negative definite if this becomes negative. This becomes negative if x_1 square plus x_2 square is less than 1 and that is the condition for which the system is stable. Obviously, this gives me a region of attraction, because this is my x_1, this is my x_2 and this is my origin. Obviously, this is a circle of radius 1; this is r equal to 1. If my initial system states are anywhere inside this region, this is always positive and if this can be made negative, then the system V dot x is negative definite. Using Lyapunov direct method, we conclude this system is globally stable. But then, the region of stability is not globally stable but is asymptotically stable and that stability is guaranteed as long as your system initial states are disturbed and the disturbed initial states lie inside this circle. The circle has a radius 1. x_1 square plus x_2 square is equal to 1.

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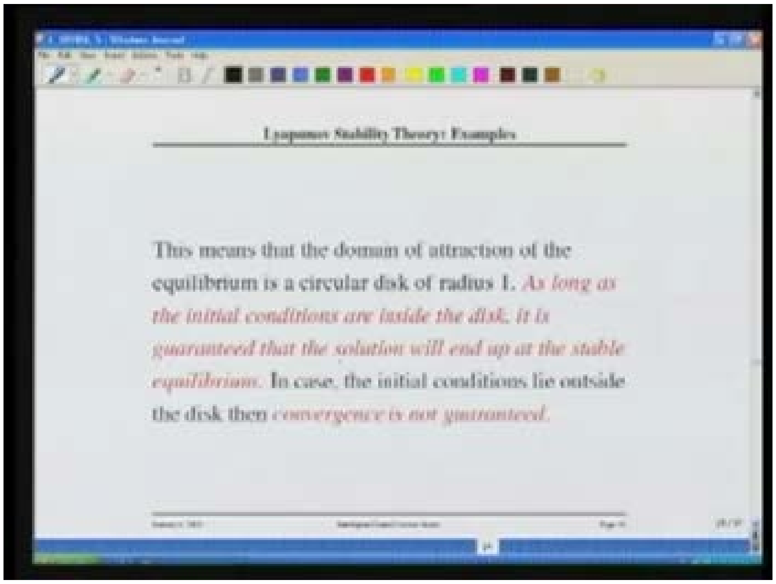

This means that the domain of attraction of the equilibrium point is a circular disk of radius 1. As long as the initial conditions are inside the disk, it is guaranteed that the solution will end up at the stable equilibrium, that is, the origin (0, 0). In case the initial condition lies outside the disk, then convergence is not guaranteed.

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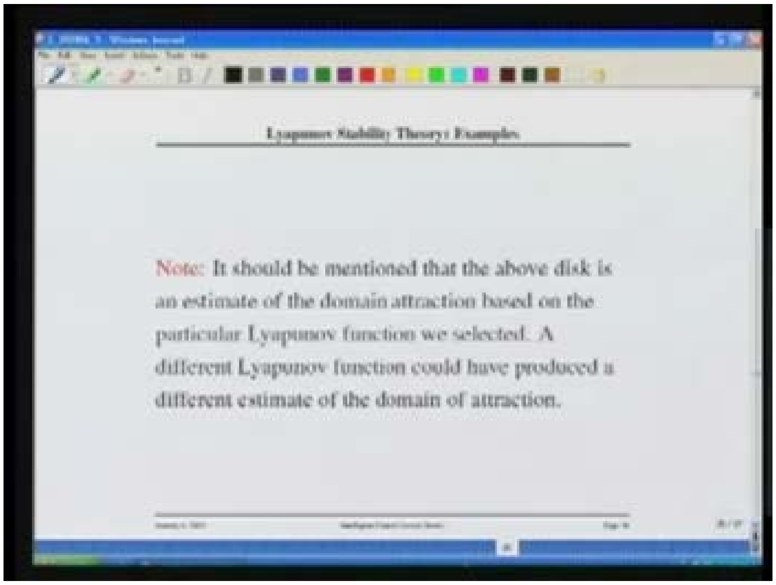

It should be mentioned that the above disk is an estimate of the domain of attraction based on the particular Lyapunov function we selected. A different Lyapunov function could have produced a different estimate of the domain of attraction. Another Lyapunov function may again determine a domain of attraction where the domain of attraction that we found will be a subset, or it may find a domain of attraction that will be a subset of the domain of attraction that we just found out.

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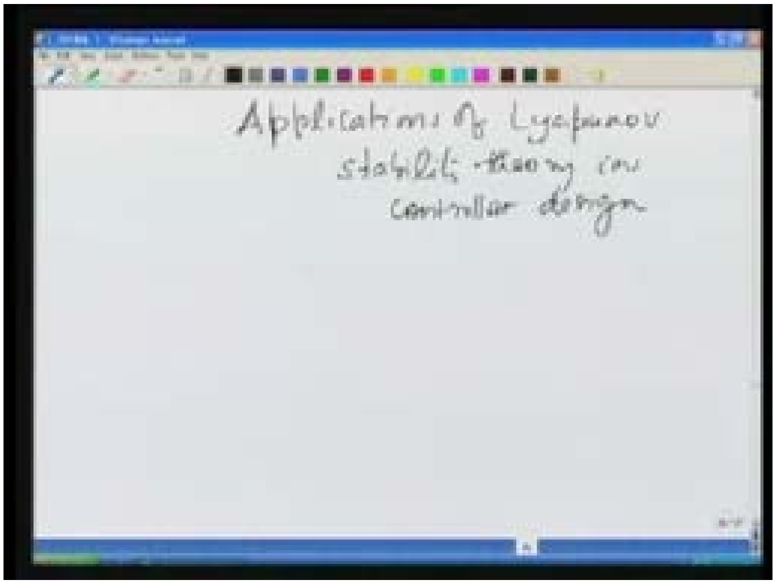

We just gave certain ideas about how to determine stability using Lyapunov stability theory with both the indirect and direct method. Now, we will go a little further in terms of application to control system design. We will just take some examples in this class. This is a trajectory-tracking example.

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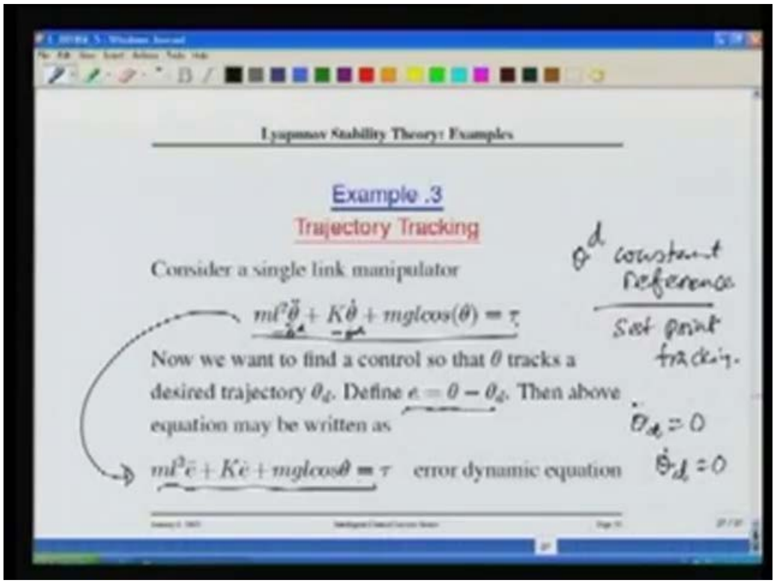

What you are seeing is that this is the dynamics of a single link robot manipulator. This is one link, a rigid link. This rigid link is mounted on a motor. This rigid link is mounted on a motor and this moves in a vertical plane and there is a motor. This is my motor here, this is my rigid link and this rigid link moves in the vertical plane. The motor rotates this rigid link in the vertical plane. There is some friction here, there is some friction the motor shaft on which the link is mounted. The equation is m l square theta double dot plus K theta dot plus m g l cos theta equal to tow. theta is this angle (Refer Slide Time: 40:20).

The objective is to find a control law so that theta tracks a desired trajectory $theta_d$. I am moving my hand like this, this is the way and this should follow any trajectory that I want it to follow, whether it is a sinusoid trajectory or a set point. This is a sinusoid trajectory with constant amplitude, maximum amplitude; it goes from one point to another point and it can also go set point (Refer Slide Time: 41:01) – any trajectory. Now, we define trajectory tracking.

This is my $theta_d$. $theta_d$ can be a set point; $theta_d$ also can be a sinusoid trajectory; it can be also exponential trajectory. This is $theta_d$. This is initial theta and final theta is here and follows there. The objective is what should be my control law, how much torque this motor should develop and apply to this link such that the link follows this desired theta angle, desired angle. The position of the link is given in terms of… the angular position is always following given $theta_d$, the desired angular position. For that, what we do is we define e equal to theta minus $theta_d$. The error gives us an idea of how close my actual theta is – the angle of my link from the actual trajectory. By putting e equal to this, we can show that the error dynamic equation is m l square e double dot plus K e dot plus m g l cos theta is equal to tow.

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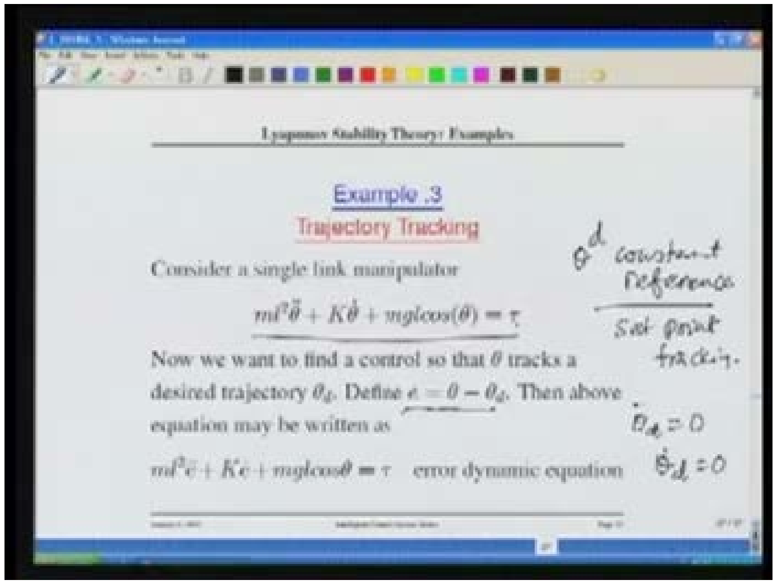

This is an example that we will show an application to control of design using Lyapunov stability theory. In this, we will take a single link manipulator. A single link manipulator means you have one rigid link here and this rigid link is mounted on a motor shaft in such a way that this rigid link moves in a vertical plane (vertical plane and not horizontal plane – it is in a vertical plane) and we are assuming there is some friction at the shaft where the link is moving.

We can write the dynamic equation as m l square theta double dot plus K theta dot plus m g l cos theta is equal to tow. If you have a little difficultly with this dynamical equation, kindly refer any book on robot manipulators and you will understand the dynamics of the robot manipulator from the book. Now, we are not interested in how we got the dynamics. We are interested if given a dynamics, how to control the robot manipulator, a single link manipulator. This is my dynamics, this is my applied tow, torque. tow is applied by the motor here at this point.

This is my single link, the motor is applying a torque here and this torque is making this link move vertically at some angular position theta. The objective is theta should follow a particular trajectory $theta_d$, but here, $theta_d$ is a constant reference. So although we say trajectory tracking, this is actually set point tracking. This is my dynamics and I want that whatever my initial angular position of this link, it has to go to a desired $theta_d$, whatever my actual angular position. How do I actuate a torque here by the motor such that this link goes to any point, any angle?

We define an error theta minus theta$_d$, how far it is from the reference point. You can easily see that theta$_d$ double dot is 0 and theta$_d$ dot is 0. This is because theta$_d$ is a constant reference and so, theta$_d$ double dot is 0 and theta$_d$ dot is 0. Using these two ideas, we can always add theta double dot minus theta$_d$ double dot. Similarly, here, theta dot minus theta$_d$ double dot. I subtract here theta$_d$ double dot (Refer Slide Time: 46:13) and I subtract here theta$_d$ dot because this is 0 and this is 0. My dynamics remains unchanged, but this gives me an error dynamics, where m l square e double dot plus K e dot plus m g l cos theta is equal to tow.

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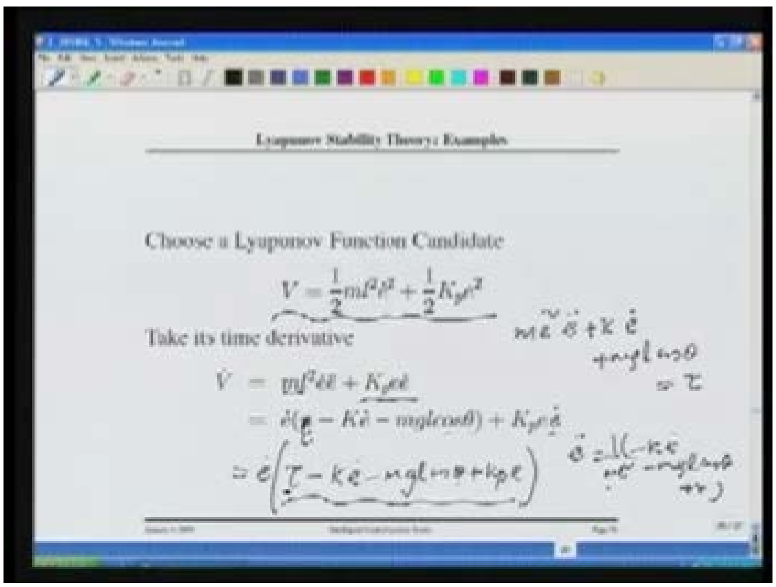

Now, for this error dynamics, I define a Lyapunov function, which is half m l square e dot square plus half K_p e square. As I said in the last class, we have proposed these Lyapunov functions based on some experience, it is not exactly a science, it is just an art based on our experience. We have to make sure that we have proposed a Lyapunov function that is positive definite. There are so many ways but for this, we design this and the time derivative of V dot is m l square e dot e double dot plus K_p e e dot. This is not u, this is actually tow (Refer Slide Time: 47:28). You see that we found out just now m e square e double dot plus K e dot plus m g l cos theta is equal to tow.

From here, we can write e dot to be minus K e dot minus m g l cos theta plus tow and 1 by m l square. This m l square is multiplied with this m l square (Refer Slide Time: 28:09) and that becomes 1. We have here tow minus K e dot minus m g l cos theta and then, here also, this is K_p e e dot (Refer Slide Time: 48:25). Now, I can take e dot as common, then K_p e will come inside; so if I take e dot as common, then this is tow minus K e dot, this is the torque developed by the motor, minus m g l cos theta plus K_p e. This is V dot.

Obviously, to make it negative definite, I should adjust this quantity in such a way that this becomes a negative definite. Can you think how this can be made negative definite? You see that this can be made negative definite if tow is m g l cos theta minus K_p e. We can see that, because in that case, this term will be minus K e dot and you have so many ways to propose what should be my controller law. Do you how see how we are using Lyapunov function or Lyapunov stability theory to design a controller? We do not know what is tow. We have to design tow. we are given a dynamics, we proposed a Lyapunov function, we found out the time derivative of the Lyapunov function and we reached here. Here is an option for you – how do you select tow in such a way that V dot is negative definite such that the system is stable for that controller you proposed.

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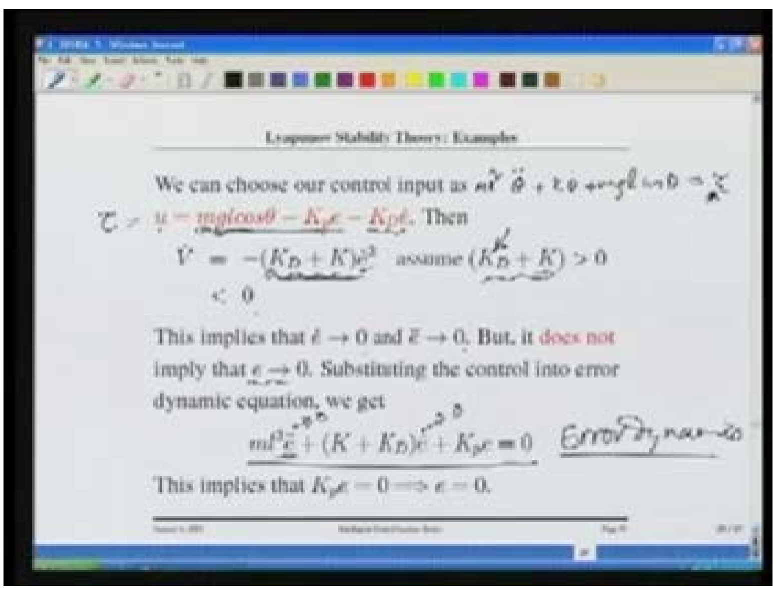

Obviously, if I simply make u equal to m g l cos theta minus K_p e, then in the previous slide, we saw that simply it can be equal to e dot minus K e dot square, but that does not guarantee me asymptotic stability – you will see that. That is the reason why we added another term called minus K_D e dot. By doing that, V dot becomes minus (K_D plus K) into e dot squared. Hope you are very clear about this. Why? It is because the initial K may be positive negative. Now, K_D plus K, because K_D is my extra parameter I provided and even K may be a very small quantity. The K_D e dot was actually introduced to make this quantity reasonably large. Now, this is a positive quantity, e dot square is a positive quantity, so V dot becomes negative definite. This implies that….

You see that the LaSalle–Yoshizawa theorem says when V dot becomes a function like this…. This is a continuous function because e dot is a continuous function and this particular quantity is a positive definite (Refer Slide Time: 51:54) and these will converge to 0; since e dot square converges to 0, e dot also converges to 0 and because e dot converges to 0, e double dot converges to 0, but does not guarantee, this does not imply that e goes to 0. If e dot goes to 0, e double dot goes to 0, but does not imply that e goes to 0. We take this control input and replace this control input in our….

This is u or tow (Refer Slide Time: 52:33). Normally, the control input is denoted by u, but actually this is tow and our actual system is m l square theta double dot plus K theta dot plus m g l cos theta is equal to tow. This tow is here and I bring this quantity and put it here. If we put it there, you get this equation (Refer Slide Time: 53:03). This is complete error dynamics. This is the actual error dynamics given the control law. Once this error dynamics is known, from here you can see… because e double dot is 0, converges, e dot goes to 0 and obviously, e has to go to 0. This implies e is 0, e dot is 0 and e double dot is 0. That means whatever might be the initial position of my link, it will exactly go to the desired position. I hope you appreciate this particular symbol as an example.

What you saw is that an example of how we can actually use Lyapunov's stability theory to design even a controller. We use Lyapunov stability theory and LaSalle–Yoshizawa theorem to show how finally the single link manipulator will follow a given set point exactly with no steady state error. This is very important because in this course, most of the time, we will use this Lyapunov stability theory to show that our system is stable and trajectory convergence is there.

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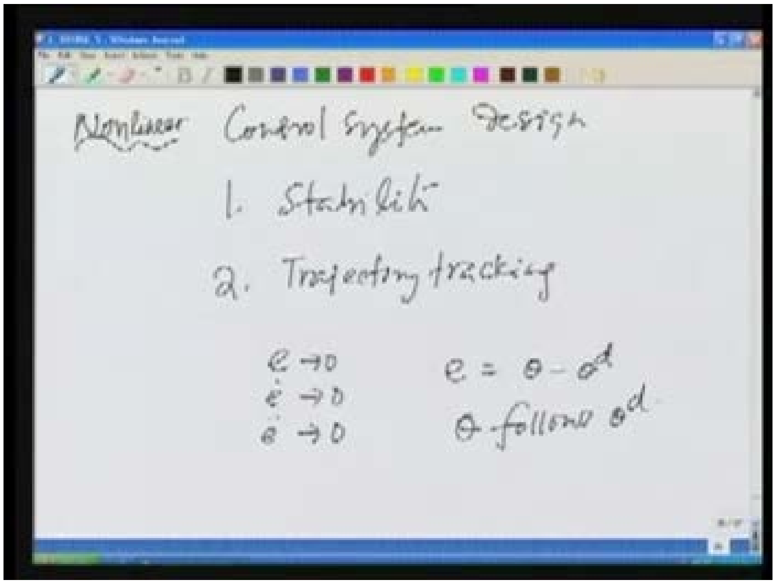

In any control, the most important is that in control system design, of course nonlinear, when we say intelligent control, it will always apply to a nonlinear system. The first is stability and then trajectory tracking. In this case, we showed using Lyapunov's stability theory that e goes to 0, e dot goes to 0, e double dot goes to 0, where e is theta minus theta_d. That means theta follows theta_d and that is the objective of intelligent control, how accurately…. But we have not done. This is a classical control system and we will also talk about this. These are the basics we must have.

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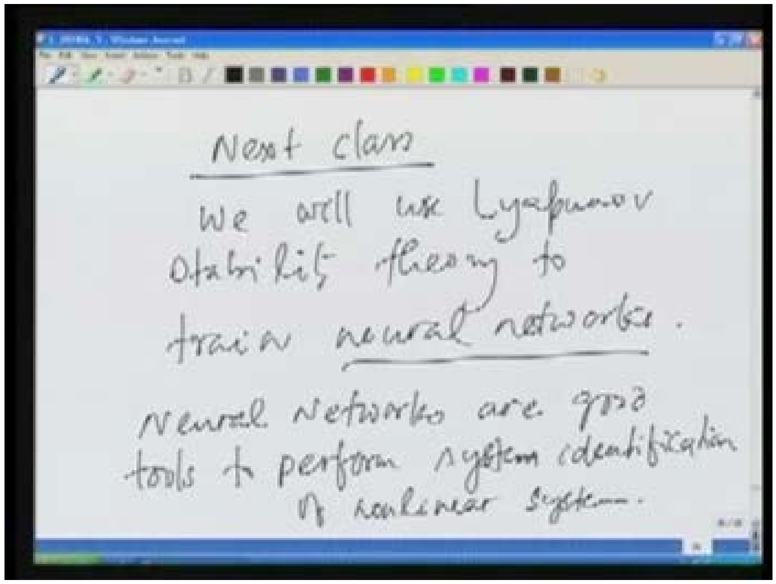

In the next class, we will discuss how we can use this Lyapunov stability theory. We will use the Lyapunov stability theory to train neural networks. Why are we learning this? It is because these neural networks are good tools to perform system identification of a nonlinear system. That is the reason I introduced now what is Lyapunov stability theory. We will use this concept to train neural networks in the next class.

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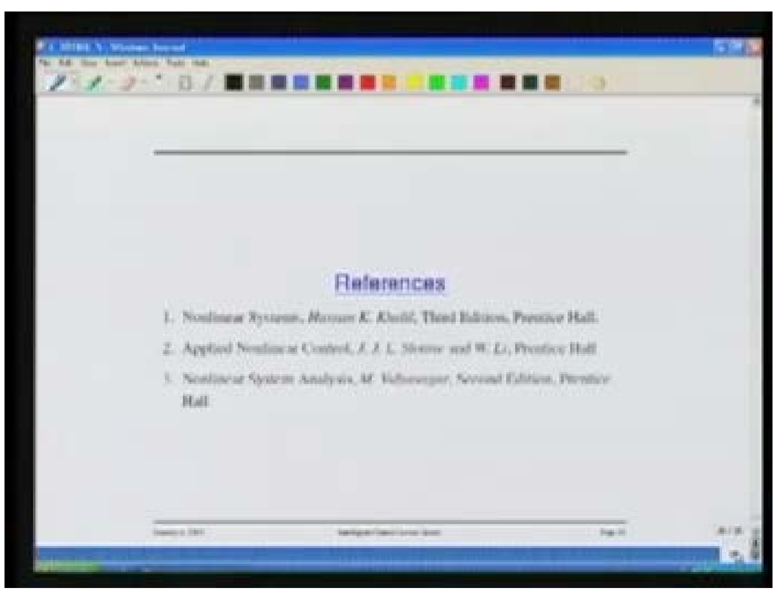

For your further reference, here are the same books that I mentioned in the last lecture. These are the books that you can follow for grasping the concepts or the ideas that we discussed now. Thank you.