Intelligent Systems and Control

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Module - 4 Lecture – 6
Fuzzy Model

This will be lecture six on fuzzy control. The topic that we will be discussing today is linear controllers using T-S fuzzy model. In the last class, we discussed how to design controllers for T-S fuzzy model when the input matrix is common for all subsystems. Now, we will consider the generic T-S fuzzy model and how do we design linear controllers for it.

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We will just revise our notion of T-S fuzzy model representation of nonlinear systems. , The approach of Controller design linear controller design. We will give two controllers for this, stabilizing controller design robust controller approach; we will propose controller I and as well as controller II - two different types of controller, simulation results: two-link manipulator, ball beam system and summary.

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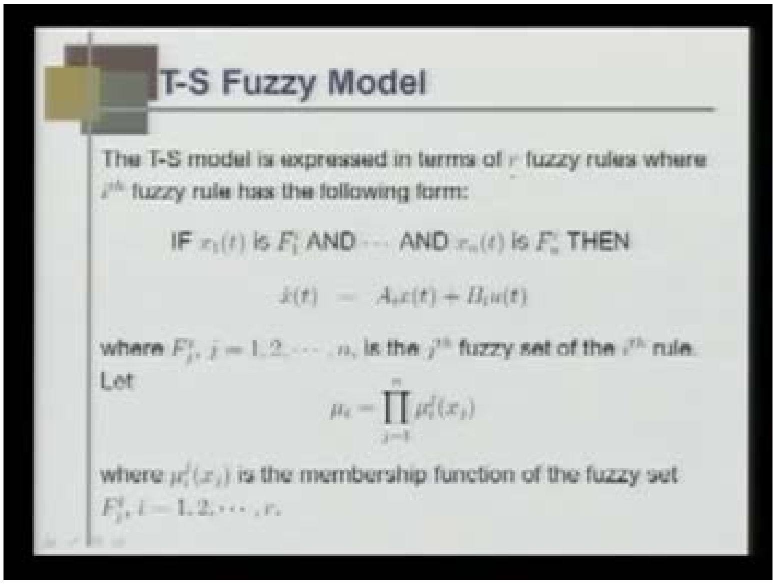

The T-S fuzzy model is expressed in terms of r fuzzy rules where ith fuzzy rules has the following form: if x_1t is F_1 i and x_2t is a F_2 i and so on until nx_nt is an F_nI then x dot t is a is A_ix t plus B_iu t. This is my ith fuzzy rule consisting of n states and each state is fuzzy variable and where the fuzzy variable F_jI is the jth fuzzy set of the ith rule. Then, the fuzzy index mu_I associated with the ith fuzzy rule is given by this (Refer Slide Time: 02:27) formula where mu_i j x_j is the membership function of the fuzzy set F_j i. i equal to 1 to r.

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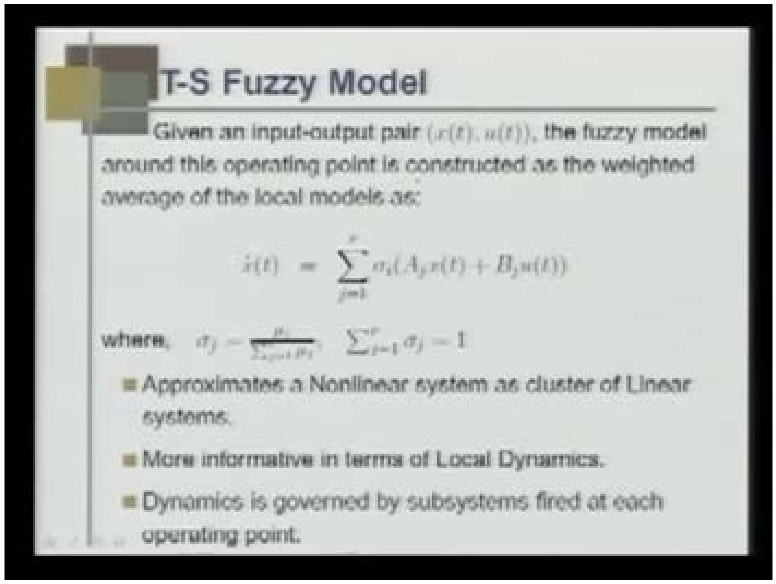

Given an input output pair x t u t the fuzzy variable around this operating point is constructed as the weighted average of the local model as x dot t equal to summation j equal to 1 to r sigma i A_jx t plus B_ju t. This is jth local model and with jth local model associated membership function is actually sigma$_j$, so sigma$_j$ is a normalized fuzzy index associated with jth local model. The summation of such local model multiplied with fuzzy index over j equal to 1 to r gives me the complete fuzzy dynamics in terms of T-S fuzzy model. A T-S fuzzy model approximates a nonlinear system as a cluster of a linear system. This is my linear system; this is my jth linear system, sigma$_j$ is the normalized fuzzy index associated with jth linear system. When the cluster of such a linear system if there are there, then if I fuzzy cluster these r number of linear systems then, I get the approximation of a non linear system. The advantage of the fuzzy system is more informative in terms of local dynamics because I can look at a nonlinear system in terms of linear system. Dynamics is governed by subsystems fired at each operating point. In this class we will talk about variable gain controller using single nominal plant.

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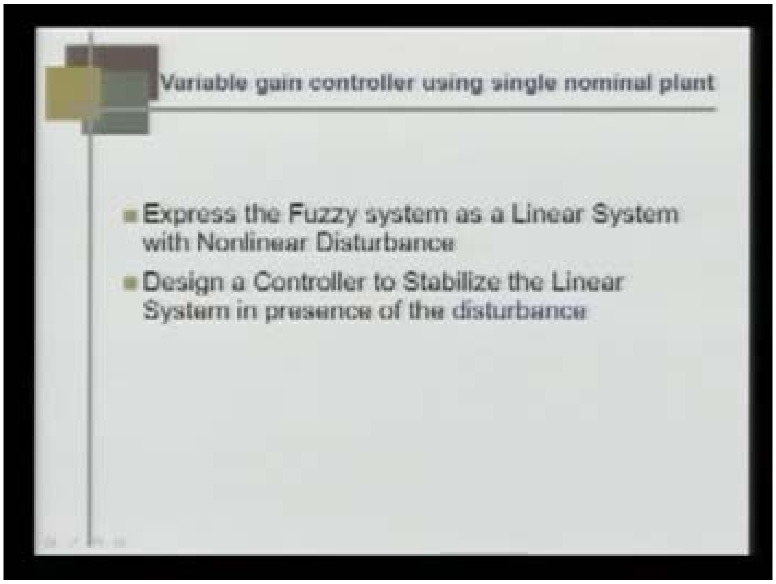

This is the first type of controller will be talking today and in this we expressed the fuzzy systems as a linear system with non linear disturbance. Our fuzzy system which is x dot sigma j equal to 1 to r sigma_j $A_j x$ plus $B_j u$, this is my T-S fuzzy model approximation of nonlinear system using T-S fuzzy model you can easily see that this has a very convenient form. It looks as very convenient form easy to handle and I can write this as Ax plus Bu plus disturbance term. This is my disturbance term (Refer Slide Time: 05:43), this is worst part I am saying here; express the fuzzy system is the linear system with nonlinear disturbance. Then design a controller to stabilize in the linear system in the presence of disturbance. The original plant was x dot is sigma_j $A_j x$ plus $B_j u$ j equal to 1 to r. So, this is my original plant.

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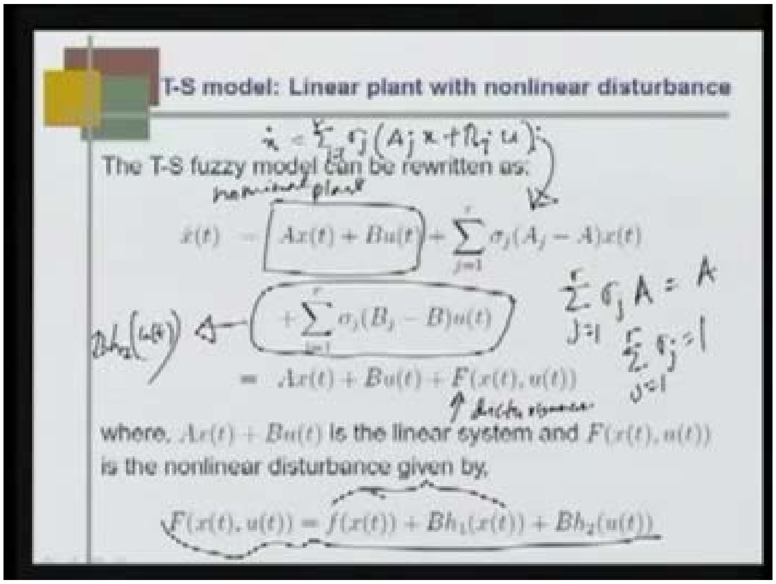

I can always write this plant you see as: A x t plus B u t this is the nominal plant. Once I separate from this as a nominal plant how do I write that this sigma$_j$ A_j x in that I have to subtract A x t which is here. So sigma$_j$ A_j minus A x t, I subtract that and j equal to 1 to I. Similarly, if I subtract b because, B_j is there I subtract b here. How can I do? You must know sigma j equal to 1 to r sigma$_j$ A is A; because, j equal to 1 to r sigma j equal to 1. You may worried that I should have only subtracted A outside no, this is one and this is equal to A so I can subtract this quantity here. Similarly, about B so x dot t is written in the particular form; this is my original dynamics; this can be written in this particular form. This can be finally written A x t plus B u t F x t u t and this is my disturbance term around this nominal plant where, A x t B u t is the linear system and F x t is a nonlinear disturbance given by. I represent this nonlinear system in terms of three different components. So, this is simply F x t B h_1 x t B h_2 u t. You can easily see that, B x t u t is obviously taken into account of this. This one is actually B h_2 u t you can easily see that this is a nonlinear term and that is given by B x_2 u t, we can express that. Similarly, these two terms combined represents this one. If I say this is the second term this is my second term. This term is represented by this. I wish that you understood what we are trying to do my original plant was given by this particular thing x dot is sigma A_jx plus B_ju this one I am representing this same dynamics as this. There is no difference between (Refer Slide Time: 09:30) this and this. I am representing this whole thing as A x t plus B u t is the first plant nominal plant and this F x t is disturbance and this has three component which the first component here I is this one. Sigma_j B_j minus B u t sigma_j equal to 1 to r and the second component is F x t plus B h_1 x t this component here. Always in controller design whenever we say any disturbance, we are not interested in exact represent of disturbance rather an upper bound. This is the principle of robust control; we want to know the upper bound of this disturbance. That means if I am designing controller for the worst case naturally the controller also will be a stable for the other cases. This is the robust control design principle is design the controller for worst case.

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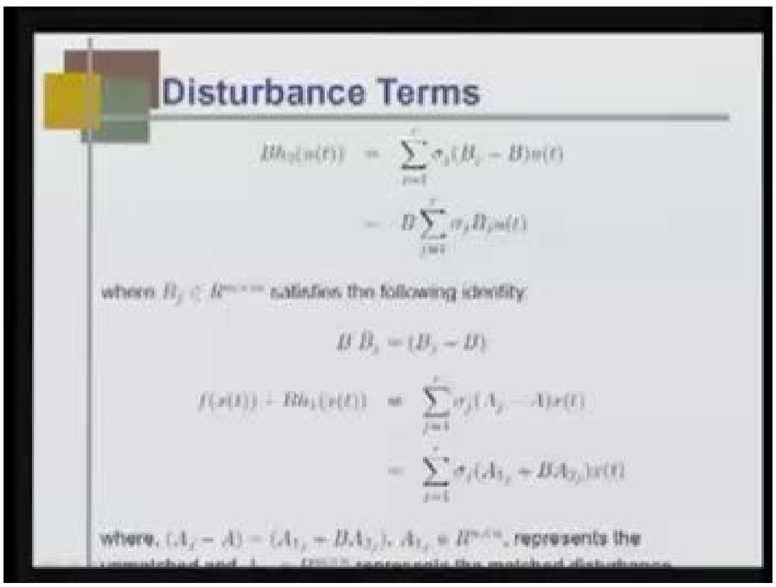

That is why we will represent first of all disturbance terms more elaborately, we will define the upper bound. Let us see the first disturbance B h_2 u t the component one as I said which is: sigma_j B_j minus u t j equal to 1 to r. We can write this equation as, I have to write in terms of B here multiplied sigma_j B_j bar t u t and this B_j bar is obviously such that the b B_j bar is v j minus v. This identity has to be satisfied, so B B_j bar is B_j minus B

Similarly, F x t plus B h_1 x t is this term, the second term, j equal to 1 to r sigma_j A_j minus x t and then this can be written in terms of two. Looking at this you can easily sigma_j A_1 j x t is F x t and sigma B_j plus x t is I can take of B and I can write this as B h_1 x t. So, here A_j minus A has to be written in terms of A_{1j} plus BA_{2j} where, the A_{1j}

represents the unmatched disturbance, unmatched means you see that, this disturbance is not magnified by u, means this is not with there with the control input. This is matched you see the b in to this and you see our normal plant is A x x dot is Ax plus Bu. So, anything with B means you can say that, this term is like an excitation because anything multiplied with B is kind of excitation to the system and this term is not with b. It is separate term unmatched disturbance and this is matched disturbance means this is disturbance excitation that is exciting system because, we say b is the control matrix. In that sense, A_1j is the unmatched disturbance and this is the matched disturbance.

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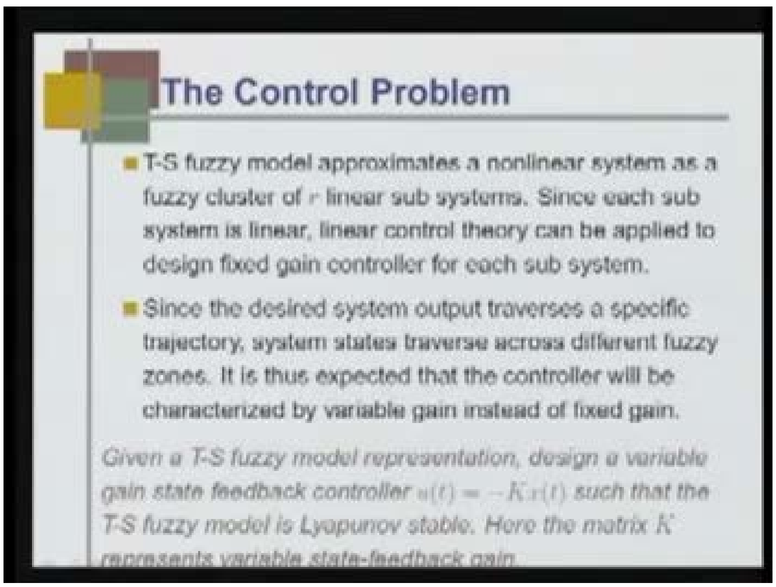

Now, we will talk about the control problem. The T-S fuzzy model approximates a nonlinear system as a fuzzy cluster of r linear subsystems. Since each sub system is linear, linear control theory can be applied to design fixed gain controller for each system design fixing controller for each subsystem. Since the desired system output traverses a specific trajectory system states traverse across different fuzzy zones. It is thus expected that, the controller will be characterized by variable gain instead of fixed gain which I have already discussed.

The control problem is given a T-S fuzzy model representation design a variable state feedback controller u t equal to minus kx t where, k is the variable such that, the T-S fuzzy model is Lyapunov stable. Here, the matrix k represents variable state feedback gain it is not a constant gain as in case of a linear system. Now, we will be talking about this disturbance measure. Since the nominal plant is linear while disturbance term is nonlinear one can possibly solve the control problem using the principles of robust control theory.

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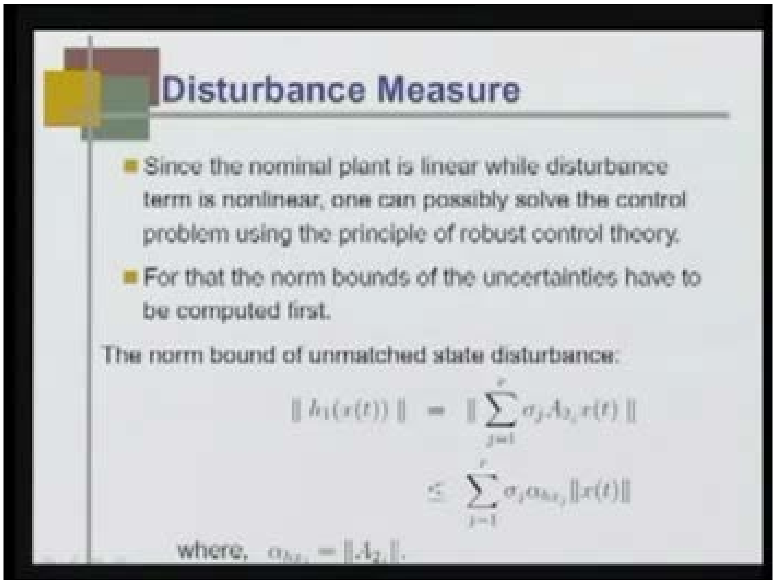

For that the norm bounds of uncertainties have to be computed first. The norm bound of unmatched state disturbance which is h_1x t. I will show you which is h_1 x t is 1 this one is matched state disturbance. Matched state disturbance h_1 x t is alpha j you see h_1 x t we have represented h_1 x t is this is h_1 x t. You can easily say sigma$_j$ A_2 x t sigma j equal to 1 to r. So that is what we have written here: h_1 x t is sigma$_j$ $A_{2\,j}$ x t known from j equal to 1 to r overall norm. I can represent this particular term using a triangular in a quality as less than equal to sigma$_j$ and this is the induced norm of A_{2j} is alpha h x j, alpha h x j represent the maximum singular value of A_{2j} and the norm of x_2 x t separately. This is a triangular in equality and what we are saying is that the kind of this disturbance is represented in terms of a major disturbance which is less than this quantity.

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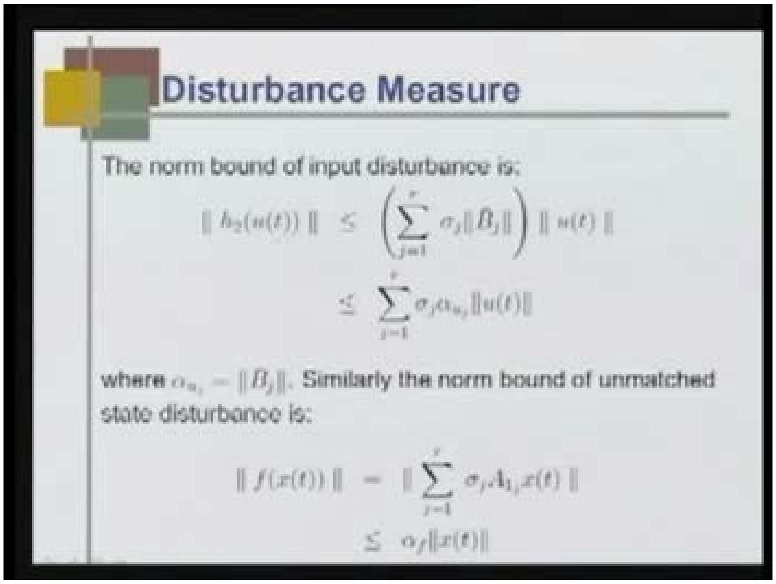

The norm bound of input disturbance is, which is h_2 u t you see h_2 u t given here is this one and this sigma_j bar B_j bar u t. This was actually sigma_j B_j bar u t j equal to 1 to r. This can be written in triangular inequality as j equal to 1 to r sigma_j and norm of B bar j into u t norm and we can write now B bar j norm the maximum norm. I can put here the maximum norm of induce norm of B bar j is alpha u j alpha u j is the norm of B bar j that means this is maximum singular value of B bar j. This inequality gives a disturbance measure for this quantity where we already know alpha u j, we know sigma_j. Similarly, the norm bound of unmatched state disturbance which is F x t the previous one is unmatched. This is unmatched one F x t and F x t is sigma_j A_{1j} x t, so this whole norm can be written as again less than equal to this A_{1j} alpha_f is norm of A_{1j} induced second norm A_{1j} this is alpha_f.

I wish that you have understood now what we have been talking about the disturbance measure so once the disturbance measure I define, we will be now telling a theorem which says that: if I design a state feedback controller, if I have a state feedback controller for the system, what is the system now, my system is: x dot is Ax plus Bu plus f x plus B h_1 x plus B h_2 u. This is my system which my original system is simply sigma_j A_j x plus B_j u sigma j equal to 1 to r. This is same as this quantity we said we approximated and then we found out the measure upper bound of effects h_1 x and h_2 u and then, we are saying that, the system will be stable if this u control input u is given by minus sigma j equal to 1 to r sigma$_j$ gamma$_j$ B transpose B x t this is my control law and gamma$_j$ satisfies this particular condition where alpha h x j and alpha u and alpha m alpha f they are all disturbance measure as we define just recently. (Refer Slide Time: 20:19)

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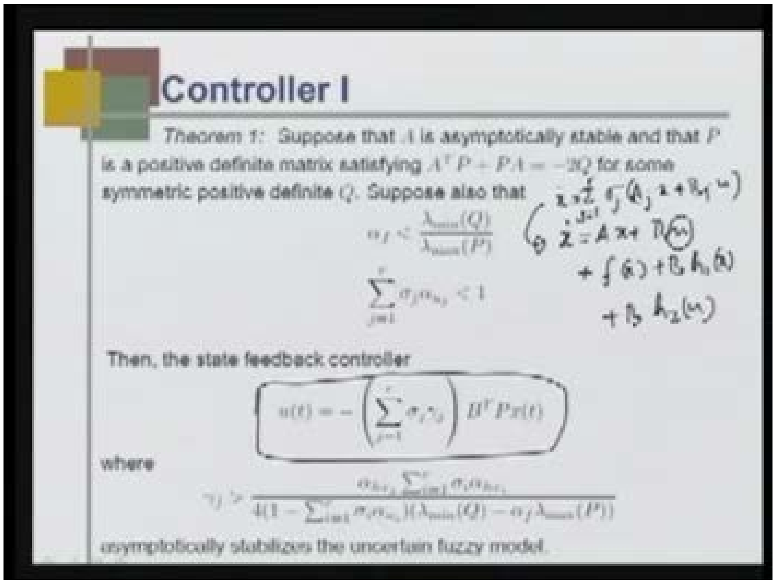

Now, this can be only valid if the two conditions are satisfied and that is alpha f is less than equal to lambda$_{min}$ the minimum eigen value of Q and lambda$_{max}$ is the maximum eigen value of P where, A transpose P PA is equal to minus 2Q. You know that is my A is nominal plant model and for a nominal plant model A, I can always find out AQ for such that, I have also P which satisfies A transpose P plus PA equal to minus 2Q and so given this P and Q the alpha$_f$ which is here, this alpha$_f$ is the induce norm.

You see alpha$_f$ is the induce norm of A_{1j} and this A_{1j} is coming from our term A_j minus A this can be written as A_{1j} plus A_{2j} this is what we have shown earlier A_j minus A is A_{1j} plus BA_2j. So A_j minus A A_{1j} plus so this A_1j is the induce norm is alpha$_f$ means the maximum singular value A_{1j}. This is the theorem and we will just prove this theorem again repeat what is this theorem implies that means, if I propose a control law u t where the gamma$_j$ satisfies this condition then, the system will be stable provided alpha$_f$ is less than this identity as well this identity is true. Now, consider the Lyapunov candidate v equal to x transpose Px this is the theorem one proving, we trying to prove the theorem one. Now, the time derivative of v is given by v dot is two x transpose is P x dot (Refer Slide Time: 23:13).

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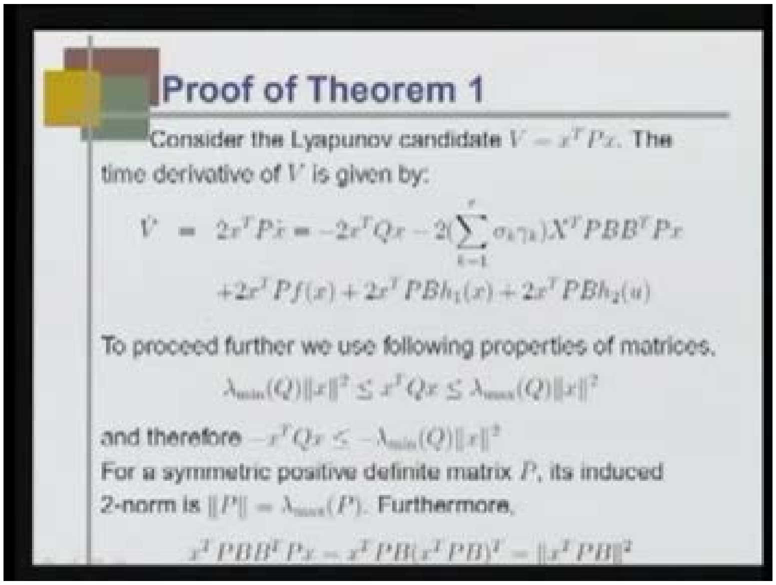

My x dot, as I have already told you is Ax Bu plus f x plus B h_1 x plus B h_2 u and u is we have already given u to be so u is we just define u is here this is our u and if I write down that u is sigma minus j equal to 1 to r sigma_j gamma_j B transpose P x. This is my u so before I introduce u inside what I will do is that, this is my V dot is 2 x transpose P x dot and I can write this expression by introducing this x dot inside here. If I do that what I will get 2x transpose P x dot. So, there is A x so what I get is that 2 x transpose P A x so this two and we know that A transpose P plus P A is minus 2Q this combined with the knowledge that 2x transpose P A x this can be written as x transpose A transpose P plus P A x. You see that A transpose P plus PA is equal to minus T Q then, I can write 2x transpose P A x transpose A transpose P plus P A into x and this quantity is now minus 2Q. We have already said that given a we achieve this A transpose P plus B transpose P A equal to minus 2Q if you can write. This writing that this first term 2x transpose P A x can be written as minus 2x transpose Q X, so, this is minus 2x transpose Q X, the first one. So, from x dot I took care of it AX. Now, let me take care of Bu so Bu is this quantity, so how do I write it is B u if I put it here, so 2x transpose P Bu, so 2x transpose P B and u has B transpose P x this one and the other quantity is that sigma j equal to 1 to r sigma_j gamma_j. So because, we have already j inside so j we have k equal to 1 to r sigma_k gamma_k, so this quantity is given by 2X transpose P B u and again we have multiplied here f x here x dot has also another component f s, so that is 2 x transpose P into f x. Similarly, B h_1 x 2 transpose P B h_1 x similarly 2 x transpose P B h_2 u. So, all that here what we have done instead of x dot, we have replace this and we can write the equation like this. Here, further what we can do we can rewrite this term as kind of a using the properties of matrices that is x transpose Q x. It can be bounded by two quantities the lower bound is lambda minimum that means the minimum eigen values of Q x norm square and the upper bound is lambda_{max} singular value of lambda eigen value of Q norm square because, Q and P there are symmetric matrices. Hence, the singular value maximum singular value is same as maximum eigen value and therefore minus x transpose Q x, if I write this minus I can write minus x Q x transpose Q x is less than equal to minus lambda_{min} Q x norm square.

For the symmetric positive definite matrix P that induces 2-norms is P norm is we can say induce norm is lambda_{max} P. So, this is my maximum value of P and further more if I look at this expression That this is not capital x this is small x bold x, x is a vector small x. So, x transpose P B x transpose P is 2 x transpose 2 x transpose P. This is not capital x. So, x transpose P B transpose B x is this quantity, this can be written as x transpose P B again x transpose P B transpose which is B transpose x. In this I can write this is a norm noun x transpose P B norm square, this is matrix theory. Taking all the relation to this account and using norm bounds on certain elements. We get, norm earlier V dot is this so using the norm bound using norm bound means this is less than this quantity. Similarly, we can find this is less than this quantity, this less than its norm bound; this is less than its own norm bound; this is less than own norm bound.

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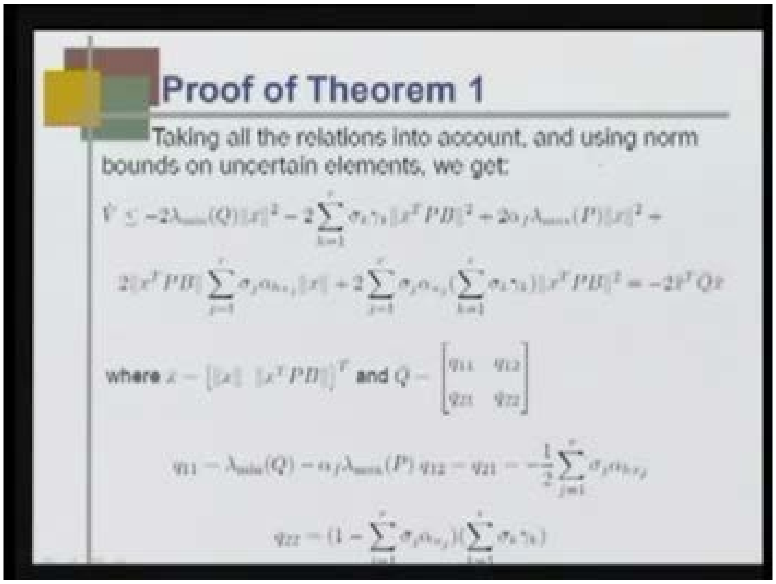

V dot can be written as a less than some quantity and that quantity is here V dot is less than minus 2 lambda_{min} Q x norm square. This is the norm bound of the second term; this is the norm bound of the third term; this is norm bound for the fourth term and this is norm bound of the fifth term, we have five terms. By putting that, this particular expression, so what we did is that, we first derived what is the direct derivative of Lyapunov function and then we expressed that in terms of norm bound using the sine less than equal to and then this particular quantity can be written as: minus 2 x bar transpose Q bar x bar where, x bar is this two quantity x norm and x transpose P B. This you can say one element and another element this x bar is a vector norm, this is first element and this is a second element and Q bar has four element q_{11} q_{12} q_{21} q_{22} where q_{11} is given by this quantity that is minimum eigen value of Q minus alpha f lambda_{max} of P the maximum eigen value of P. Similarly, q_{12} q_{21} is half of this quantity and q_{22} is this quantity and you may be wondering how we got this but, I will just explain to you in a very simple understand you can easily see that, I can now add all x norms square together so that, if r that you can see that, this x norm square if I take common, I get here two lambda_{min} 2 and 2 alpha x lambda x P. So, if I combine them I can get lambda_{min} Q minus alpha x lambda x and lambda alpha f lambda_{max}. This is my q_{11} so you see that, if I write this expression as this and I have taken 2 out, 2 is common here. So, if I take 2 common out so I have also taken negative outside, so this becomes lambda_{min} Q minus alpha f lambda_{max} P which is this quantity q_{11} into so the point is that, you can easily see this quantity and with definition x bar and this and this is we can write out q_{11} x norm square plus q_{22} norm x transpose P B square and you can easily see q_{12} plus q_{21} x norm and x transpose P V.

What you saw that, V dot is less than this five terms and I am trying to represent five terms in terms of minus 2 x bar transpose Q bar x bar. If I define x bar is this two terms vector of two terms and Q bar is vector matrix of 2 by 2 then, this x bar transpose Q bar x bar is this quantity q_{11} x bar x norm square q_{22} x transpose P V norm square plus q_{12} q_{21} x norm into x transpose P V norm. So, you can easily see here that, obviously I have to find out this is x bar transpose Q bar x y is this quantity. So, minus 2 if I take common here, what I get is that easily by comparing the coefficient of q_c q_{11} will be the total coefficient of x_1 bar. That is lambda_{min} x minus alpha f lambda_{max} p which is here q_{11}.

q_{22} is the coefficient of x transpose P V norm square, so this is one term x transpose P V square and another term is x transpose P V square. If I take minus 2 common here I get here minus 2 common k equal to 1 to r k equal to 1 to r sigma_k gamma_k sigma_k gamma k. This is the first term and second term is here where, this is sigma_j alpha u j j is equal to 1 and sigma_k gamma_k k equal to 1 to r. So, this is q_{22} and then q_{12} plus q_{21} is the coefficients half x bar and x transpose P V you see that, this all one term. Since we have q_{12} and q_{21} the coefficient is simply j equal to 1 sigma_j alpha H x j, so this is my coefficient and I have two term, so I can easily do that by dividing them equally and making q_{12} equal to q_{21} so q_{12} to equal to q_{21} which is minus half because, here if I take minus 2 common I am getting minus half here minus j. So, minus half sigma_j alpha H x j equal to 1 to r, so I wish that you understood how we finally wrote V dot in terms of a quadratic function x transpose Q bar x bar and instead V dot what is the advantage of this is we can write V dot is x bar transpose Q bar then, if this is Q bar is positive definite if Q bar is positive definite since there is negative sign here, V dot is negative definite hence the system is stable. We can find the positive definite of Q bar using Sylvester criterion all principal minor should be positive. So, lambda_{min} Q minus alpha lambda x mean P is greater than zero and determinant of Q bar which is this quantity determinant of Q bar is greater than 0. So, this quantity the first minor for first alignment for this q_{11} actually, so this q_{11} has to be greater than 0, so this quantity gives you if you go back to theorem one this identity and by equating this identity determinant of Q bar has to be greater than zero.

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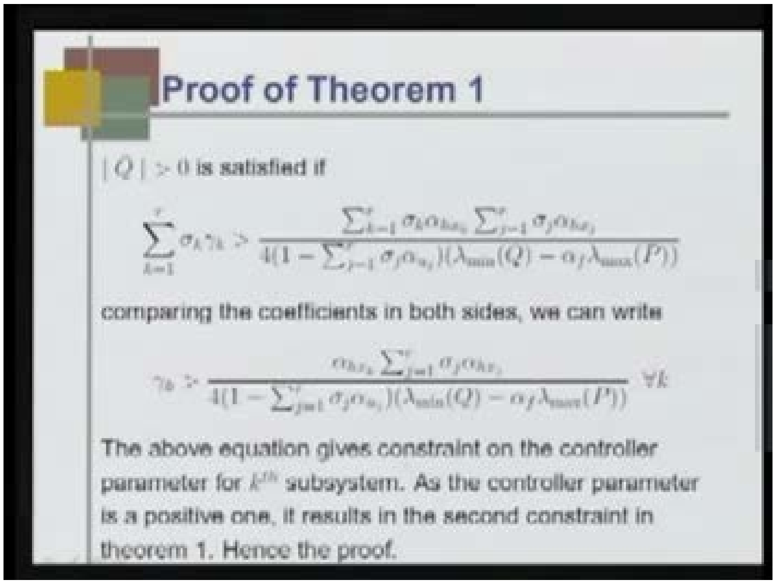

Then by doing some manipulation Q bar; this is the Q bar quantity and then if satisfied, you get this quantities greater than this, comparing the coefficient both sides, you can write: gamma_k is greater than this for all k. The above equation gives constraint on the controller parameter for kth subsystem as the controller parameter is the positive one it results in the second constraint in theorem one which is gamma_k.

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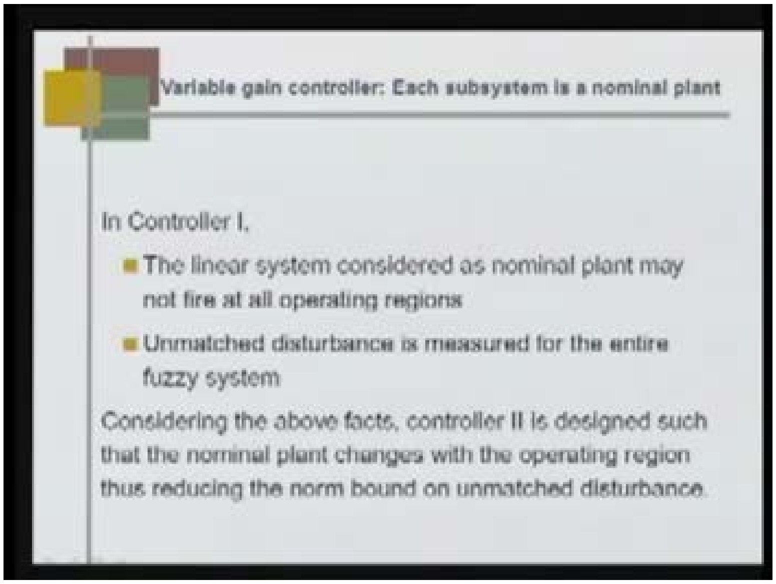

We actually prove that, the theorem is correct and in the controller I the salient points are: the linear system considered as a nominal plant, may not fire at all operating results as system is traversing from one point to another point x_1 to x_f x to x_f. Then all the nominal plant, we have selected one of the system matrices a and b of specific system to be nominal plant. It may not fire all the time as it moves. Unmatched disturbance is measure for the entire fuzzy system considering the above fact controller II is designed such that, the nominal plant changes with the operating region thus reducing the norm bound on unmatched disturbance. What we are trying to say here in the controller this one this desired criteria before that we can implement the controller, the alpha_f is less than lambda_{min} Q by lambda_{max} P because, this nominal plant is the norm bound on the nominal plant.

Because, distance between the nominal plant and the actual plant where, the system rule is fire a specific rule is fired corresponding to that in a plant and this alpha_f kind of measure distance measure between the nominal plant and actual fuzzy the plant associated with fuzzy rule that has been fired. Hence this condition becomes little too harsh. To make that relaxed what we are doing is we are now talking about second controller that, the nominal plant changes with the operating result. As operating zone changes so which ever rule is fired from that rule we take the plant if I have two rule fire so I consider each of them as a nominal plant and so what I am trying to do in the second controller that, let us think that two rules are fired: rule i and rule j. So, rule j is … rule i is x dot is A_i x plus B_i u and rule j is x dot is A_j x plus B_j u so this is i. In this second controller what we are aiming is that, we consider all of them to be nominal plant. This is my nominal plant and also this is my nominal plant, so I design a controller u around this plant and then the fuzzy blending of the controller for both the plants is the overall controller gain. That is the idea for second controller which we did not do for the first controller considering kth subsystem to the nominal plant the T-S fuzzy system can be written as this particular one where x dot t is represented around kth plant associated with the kth rule. Similarly, where we did the disturbance term, so F_k can be written has in terms of three disturbance term as we saw for the controller I. All the approaches are same only thing little bit difference will be there.

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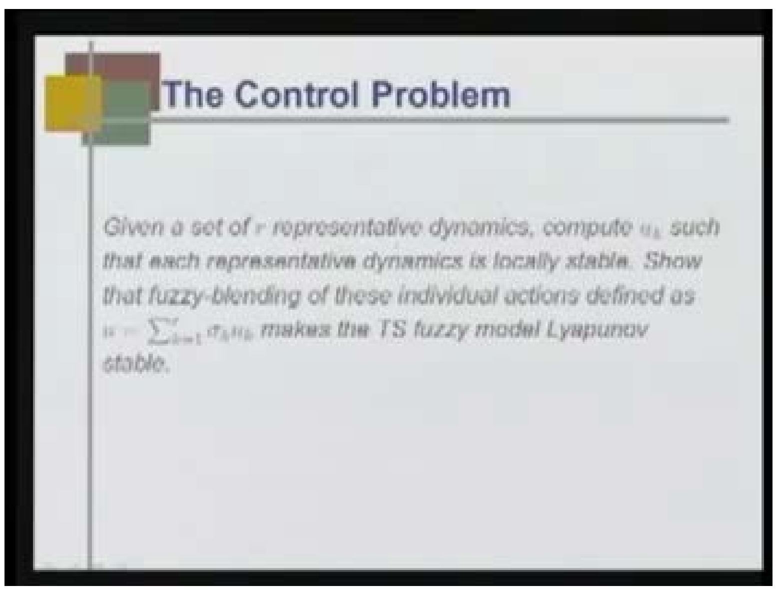

A controller problem given a set of r representative dynamics compute u_k such that, each represented dynamics is locally stable so that, fuzzy blending of these individual actions defined as u makes T-S fuzzy model Lyapunov stable. That is what I said here, r represented dynamics mix two represented dynamics mix if two rules are fired or all rules can be fired actually in principle. That is why, r represented dynamics compute v_k for each subsystem and then fuzzy blending of u k equal to 1 to r sigma_k u_k makes the T-S fuzzy model Lyapunov stable. This is the idea which comes from our the first class the last class we discussed and again the disturbance measure the way it has to be computed for h_{1k} x t u t and f_k x t.

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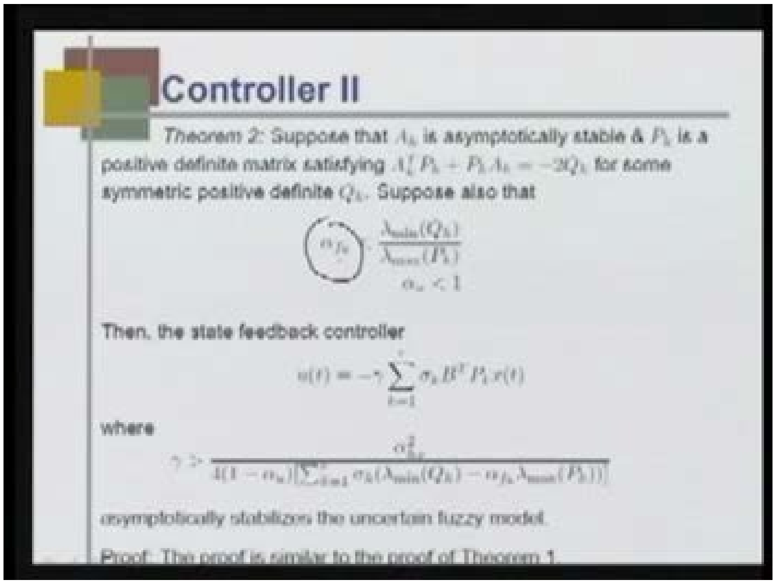

Then theorem two the controller II is suppose that, A_k is the asymptotically stable and P_k is the positive definite matrix that is fine A_k transpose P_k plus $P_k A_k$ is minus $2Q_k$ for some symmetric positive representation Q_k. Suppose also that alpha_{fk} is less than this quantity and alpha_u is less than one then the state feedback controller u t is equal to minus gamma k equal to 1 to r sigma_k B transpose P_k x t where, gamma is greater than this quantity asymptotically stabilizes the uncertain fuzzy model. So, here you see that, this is called k equal to 1 to r sigma_k B transpose P_k x t and this is called fuzzy blending of the controller where, we find the gamma has to be greater than for this system to be stable. We also relaxed the minimum condition that is required for implementing this controller where alpha_{fk} is the distance between kth plant and the corresponding jth plant which is also fired and normally the distance would be less. The proof is similar to the theorem one so I will not explain that in this class you can this is an exercise for you that, how this theorem can be proved an exercise. It is similar just like we moved to theorem one theorem two can be also proved.

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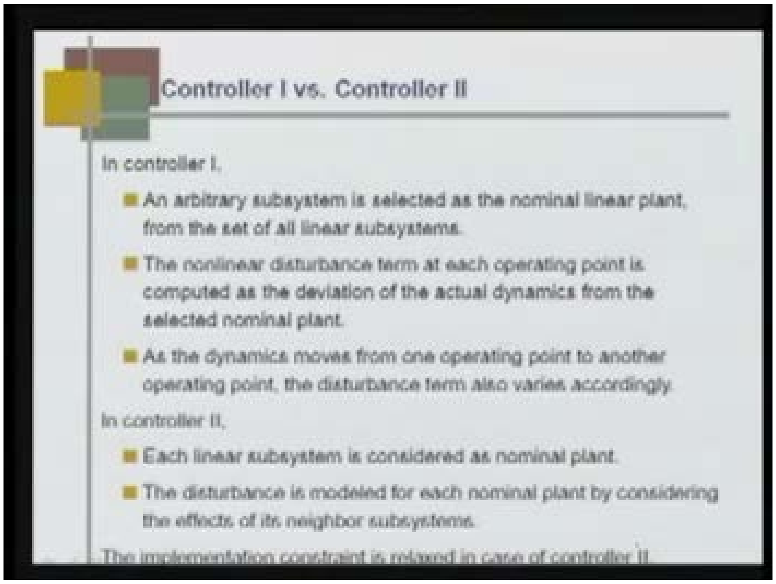

This is our theorem two we define two controllers and the salient point difference between controller I and controller II: An arbitrary subsystem is selected as a nominal linear plant from the set of all linear subsystems. The nonlinear disturbance system at each operating point is computed derivation of actual dynamics from the selected nominal point. As the dynamics moves from one operating point to another operating point the disturbance also varies accordingly; whereas, in controller II, each linear subsystem is considered as a nominal plant. The disturbance is modeled for each nominal plant by considering the effects of its neighbor subsystems. The implementation constraint is relaxed in case of controller II.

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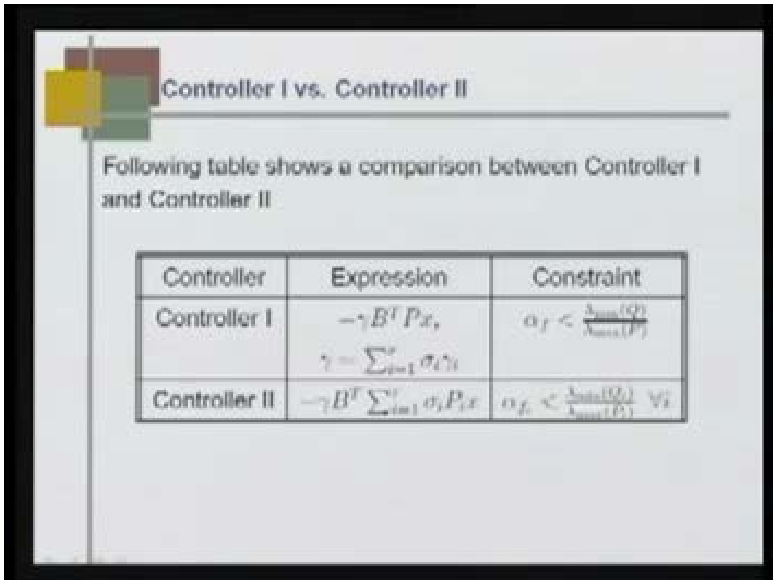

This is our controller I where the structure is minus gamma B transpose P x gamma is given by this and this gamma makes this controller time varying the gain is time varying, x is my state so minus gamma B transpose P is the time variable quantity it is not a constant quantity because gamma is varying. In controller II if you look at here gamma is a constant quantity but here sigma_i P_i makes this again variable quantity but, the design principle between Q_1 controller I and controller II are different; whereas, the same principle of robust control has been used to design controller I and controller II.

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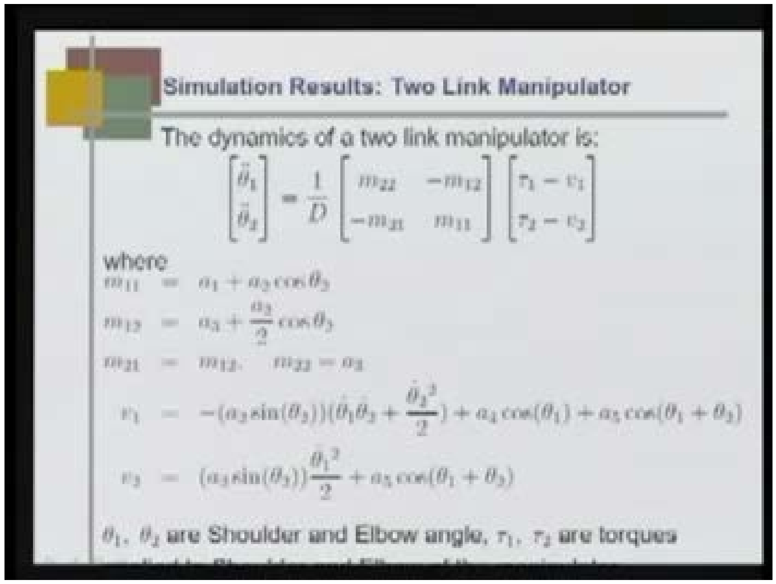

This is our two link manipulator and this is our dynamics we have already discussed a lot about this. I will not discuss a lot about how we find a two link manipulator dynamics. Theta$_1$ double dot and theta$_2$ double dot these are the two link angular accelerations and tau$_1$ tau$_2$ minus v_1 v_2 where v_1 is given by the quantity and v_2 is given by this quantity and theta$_1$ and theta$_2$ are shoulder and elbow angle, tau$_1$ and tau$_2$ box applied to shoulder and elbow manipulator and these are the m_{11} m_{12} m_{21} and m_{22}.

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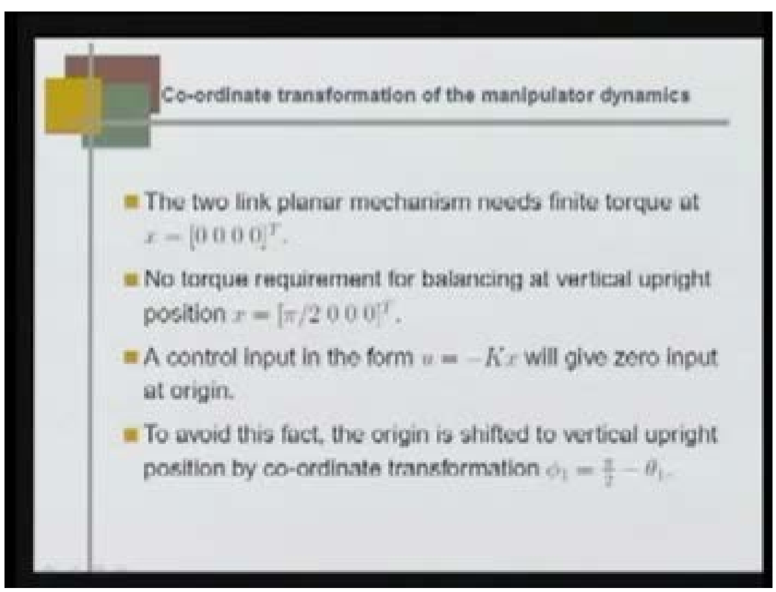

Here we do a little changes little transformation that is two link planar mechanism needs finite torque at x equal to 0 0 0. That means if I have a robot manipulator like this and if I want to keep this robot manipulator; this is my 0 0 0 position or due to gravity it will fall down so hangs at every joint; this joint and this joint (Refer Slide Time: 48:19) I have to keep some finite torque I have to apply so that the manipulator remains stable. But, you see that, other case the vertical position if I keep the manipulator here the torque required is zero. That is when the no torque requirement the torque is not require for balancing at vertical operating point. If I keep two link one above the other this is called vertical operating position which is state wise pie by 2 0 0 0 then, you see the system does not require at equilibrium in any control action. The control input if I assume my u is minus K_x so you see that, we will give zero input at origin and because it will give zero input around origin. If I define this as origin I require finite torque it is not possible but here, I require zero torque hence I can define this to be origin and I can implement u equal to minus K_x. To do that the origin is shifted to vertical upright position by co ordinate transformation phi_1 is equal to pie by 2 minus $theta_1$. So, $theta_1$ has been transferred to theta phi_1 by pie by 2 and we make this as the origin, this reversed.

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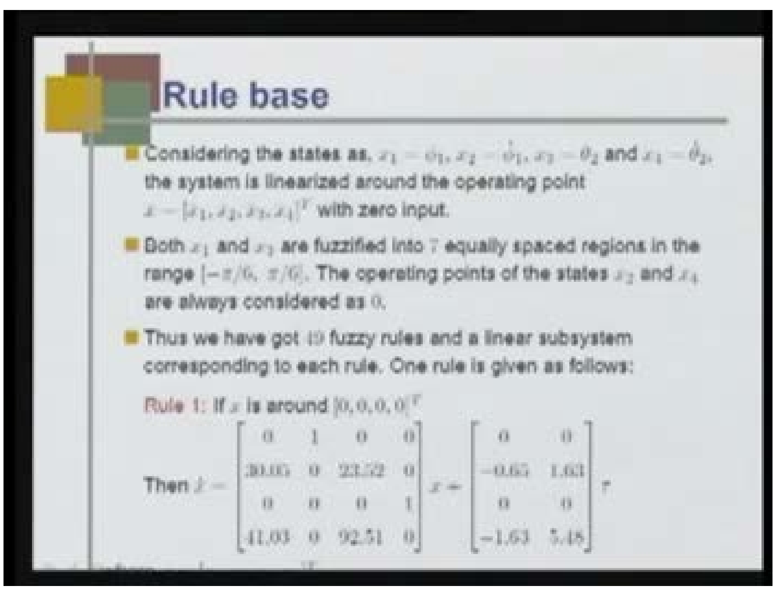

Doing that transformation what we do so is we have this transformation and then the rule base: Considering the states as: x_1 equal to phi_1 x_2 equal to phi_1 dot this is transformed

$theta_1$ and transformed $theta_1$ dot x_3 is $theta_3$ and x_4 is $theta_2$ dot, two did not change that. The system is linearized around the operating point with zero input both x_1 x_3 are fuzzified into seven equally specified reasons in range minus phi by six the operating point of the state x_2 and x_4 are always considered as 0. Thus we have 49 fuzzy rules and a linear subsystem corresponding to each rule. So, one rule is given as follows: this is just taking an example so we have 49 fuzzy rules you understand because what I am trying to do is that we are keeping here because we have 4 states.

But what we are trying to do is that linearizing x_2 and x_4 we are always making a 0. So, hence x_1 and x_3 they are varying and x_1 is fuzzy partition into seven as well as x_3 is fuzzy partition into seven equally specified reasons. By doing that we have 49 rules, so if x is around 0 0 0 by linearizing using taylor series expansion you had x dot equal to this quantity A x plus v tau. Similarly, I can linearize using this dynamic I am giving this dynamics. I can use this dynamics to linearize and then I get this, so you can just do it given the plant model. Once done that, this is my linear subsystem around the equilibrium point around the vertical upright position.

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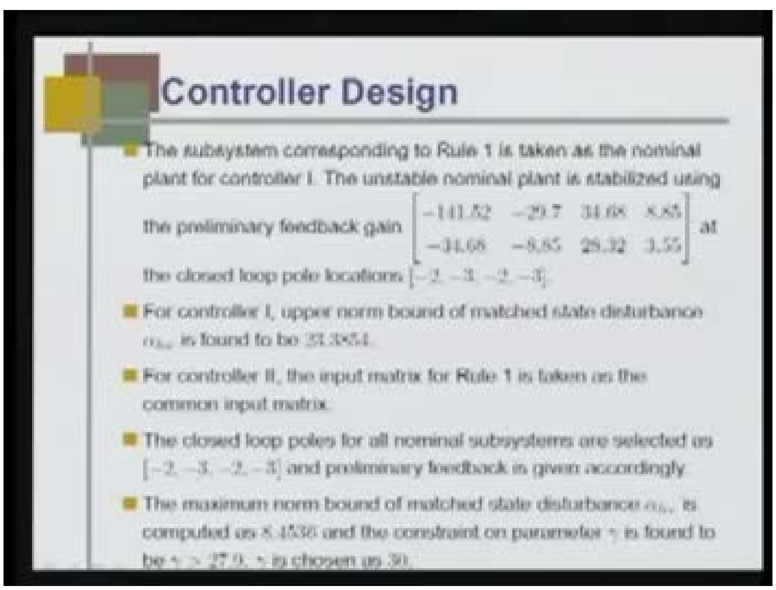

Done that, you see that in the beginning I have to stabilize the nominal plant. To stabilize the nominal plant, we place the poles at minus 2 minus 3 minus 2 minus 3 and we got these are the state feedback. You can use any mat lab program or (52:14) formula or pole plus technique then you get this gain. So, for controller I upper norm bound is the disturbance $alpha_{hx}$ is bound to be 23.3854 and for controller II the input matrix for rule one taken as common input matrix. The closed loop poles for all nominal subsystem are selected as minus 2 minus 3 minus 2 minus 3 minus 2 minus 3 and the preliminary feedback is given accordingly. Then maximum norm bound of matched state disturbance $alpha_{hx}$ computed as 8.44536 this is for controller II and the constraint on parameter gamma is found to be greater than 27.9 and gamma is selected as 30.

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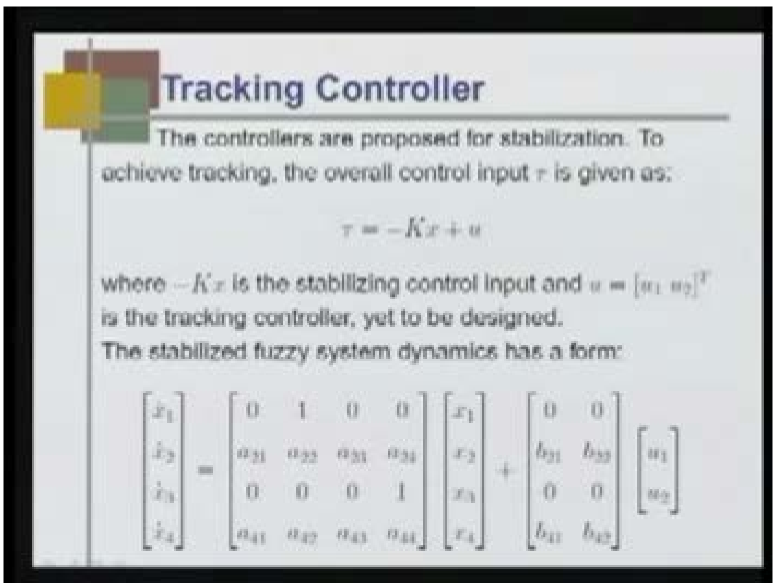

Controllers are proposed for stabilization. To achieve to tracking those the overall control input tau is given as: tau is minus K x plus u where, minus K is the stabilizing control input and u equal to u_1 and u_2 is the tracking controller yet to be designed. We can easily design the stabilized fuzzy system dynamics has a form this particular form and we have to give now the tracking controller u_1 u_2 so that, it tracts any desired trajectory. The output equation y is my $theta_1$ position and $theta_2$ position and this using this equation the y double dot which is x_2 dot and x_4 dot if x_1 dot and x_4 dot by y then x_2 dot and x_4 dot you know that x_2 is x_1 dot and x_4 is x_3 dot. Using that principle I can write y double dot is a_1y plus a_2y dot plus bu where, a_1 is given by the matrices given by a_{21} a_{22} and a_{41} a_{43} a_{21} a_{22} a_{41} a_{43} a_{21} a_{23} a_{41} a_{43}. Similarly, a_2 is a_{24} a_{42} a_{44} and b is similarly here b_{21} b_{22}

b_{41} b_{42}. So, let the decided output vector be y_d and the error vector is defined as e equal to y minus y_d.

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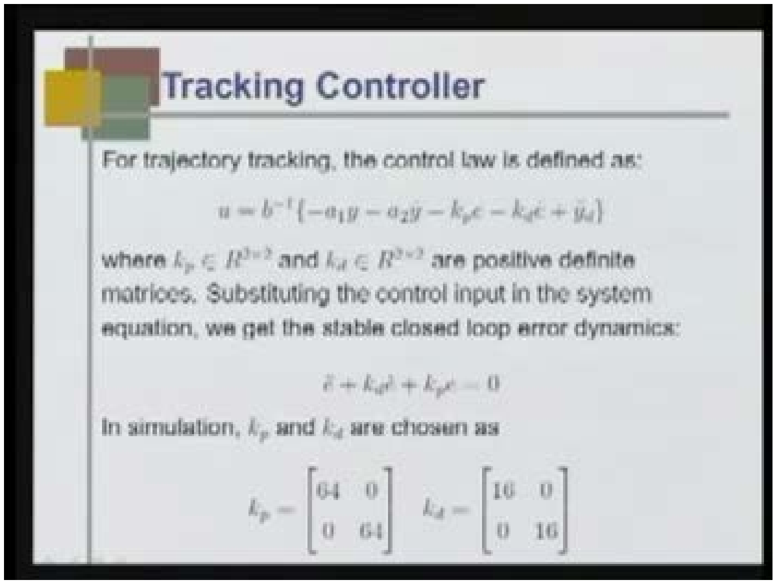

Then I can say u is this is my tracking control and if I design this tracking controller I get the closed loop error dynamics as this. If I take k_p and k_d and this is a stable dynamics and tracking is possible.

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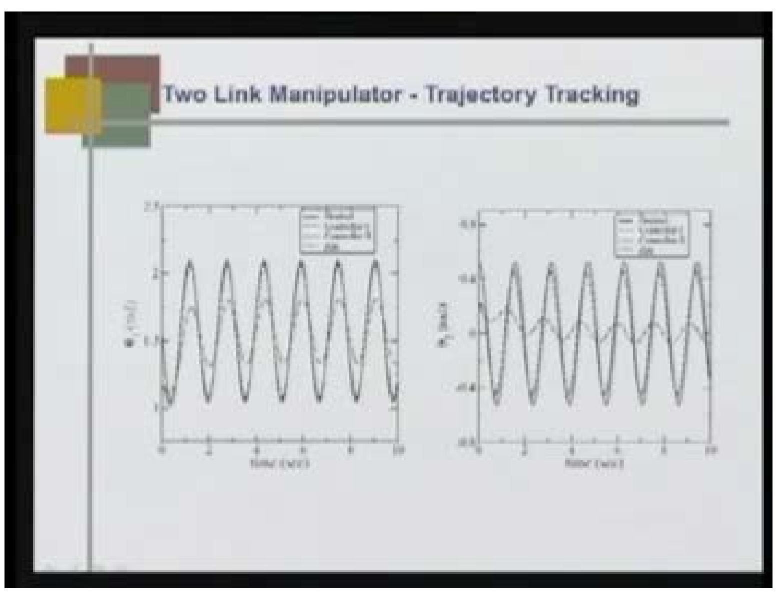

Two link manipulator trajectories tracking you can easily see that, desired trajectory and controller I and controller II they are very much following; whereas, this other one which is not following is actually proposed by Jack which is a fixed gain controller for T-S fuzzy model and it is performing very badly. Similarly here also at a tracking at this joint 1 and this is joint 2. This joint 1 position tracking for both control one and control two is very good and here also for both controller I and controller II is very good. But, Jack which is we compare with another algorithm given by Jack as I said is not able to do properly.

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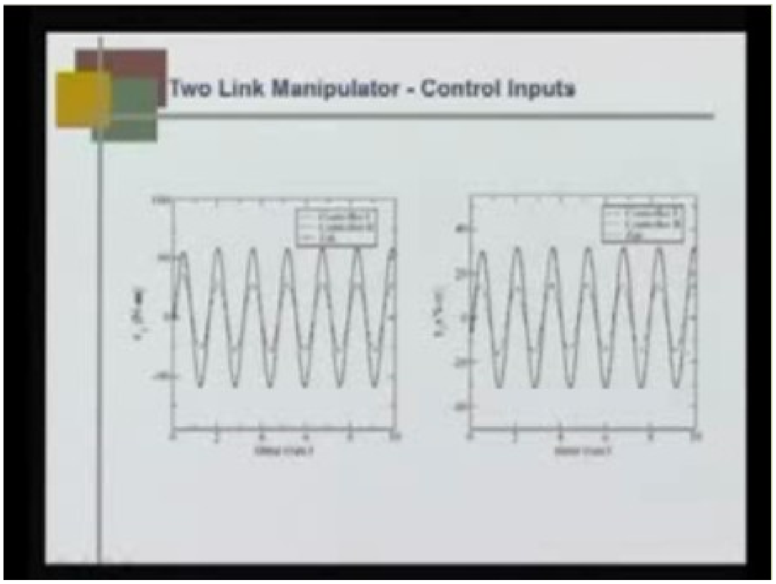

Control input this is the control input tau_1 and tau_2 which is very smooth.

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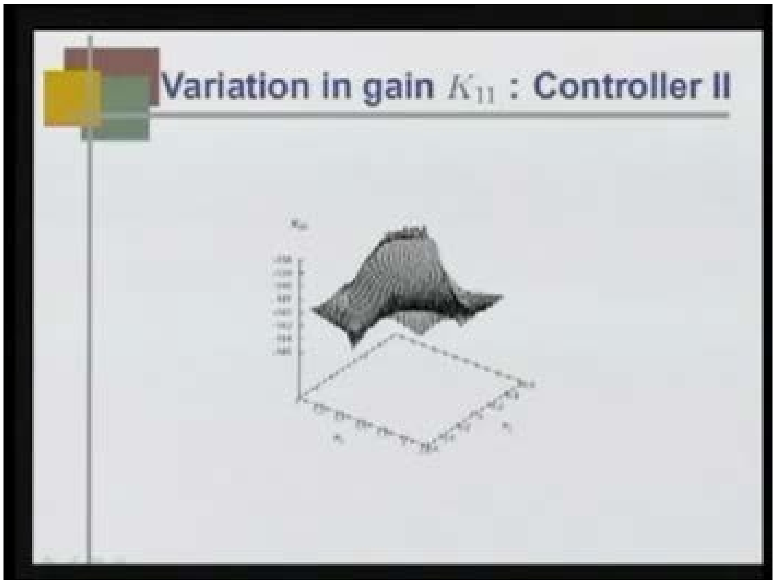

Controller parameter you see that, how the controller is varying at different operating zones theta$_1$ theta$_2$. If you see that is not a flat surface it is constant it is varying so variation in gain K_{11} controller II you see that, how it is varying over a range.

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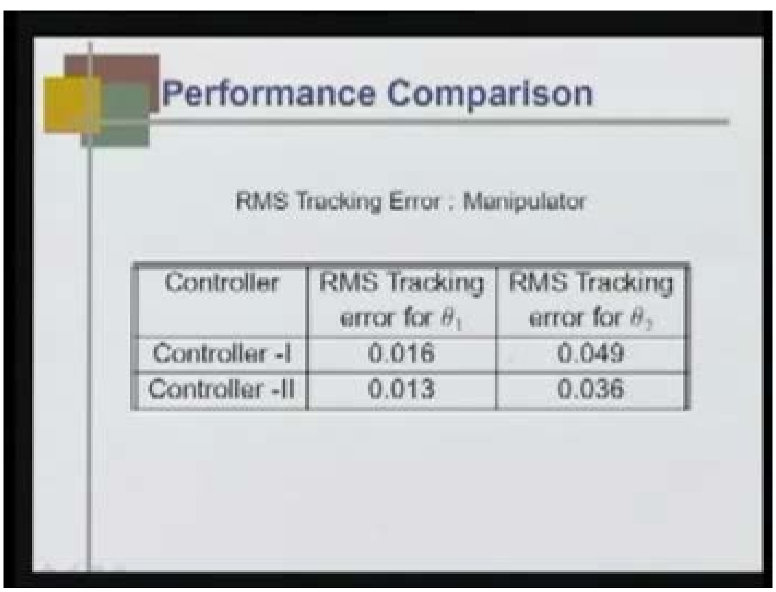

Performance comparison controller I RMS error is 0.016 and 0.049 and controller II 0.013 and 0.036 the performance has been improved.

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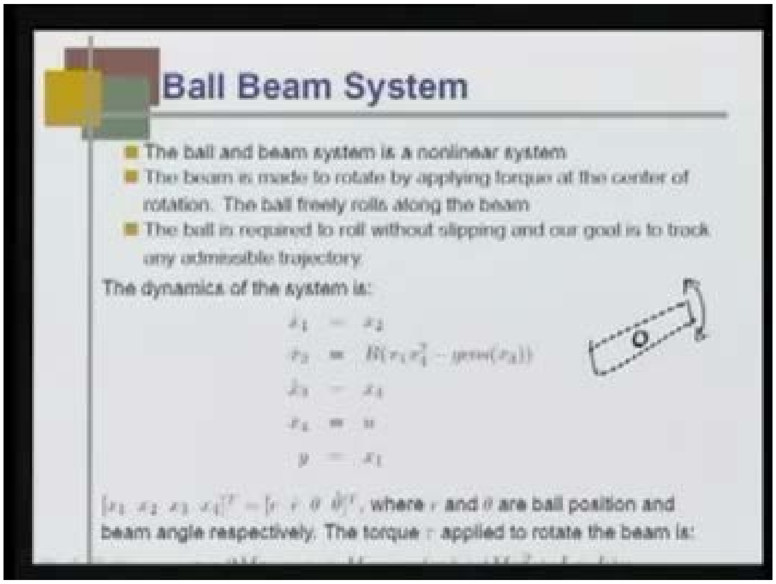

Similarly we can simulate the system for ball beam system. What is the ball beam system is, I have a beam and on this I place a ball and the ball like it is like this beam and I place a ball here and the ball will roll over the beam and the controller is that I do up and down this beam. That means this beam is made up or down such that, the ball always remains in the center point and this is the dynamics and for that also we have designed the controller. I will not go in details of this. You see that, this controller for ball beam system we could not implement controller I as well as Jack controller. We could implement only controller II because of the relaxation that this controller provided.

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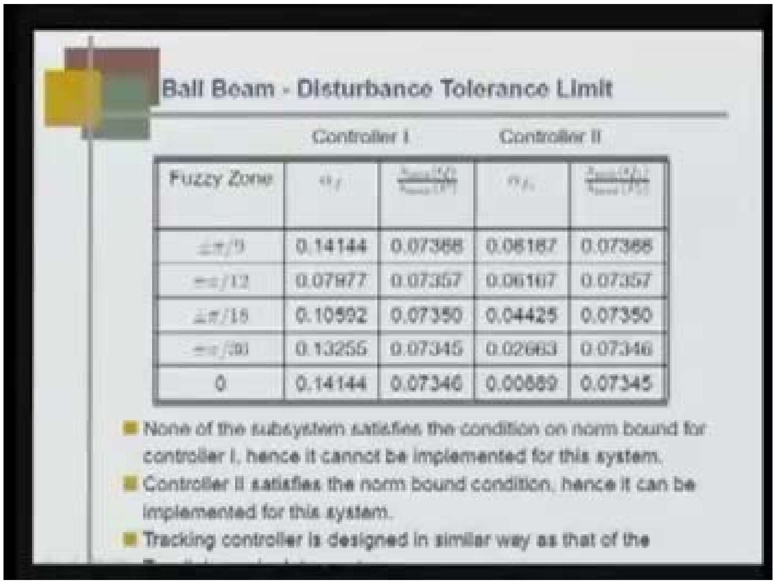

This is called the relaxation and this was not satisfied for ball beam system whereas it is satisfied for controller II for ball beam system. None of the system satisfies the non beam system non bound for controller I. Hence, it cannot be implemented for the subsystem. What I am trying to show is I am trying to show you another system for which the controller I cannot be implemented but controller II can be implemented; control two satisfied norm bound condition. Hence, it can be implemented for the system.

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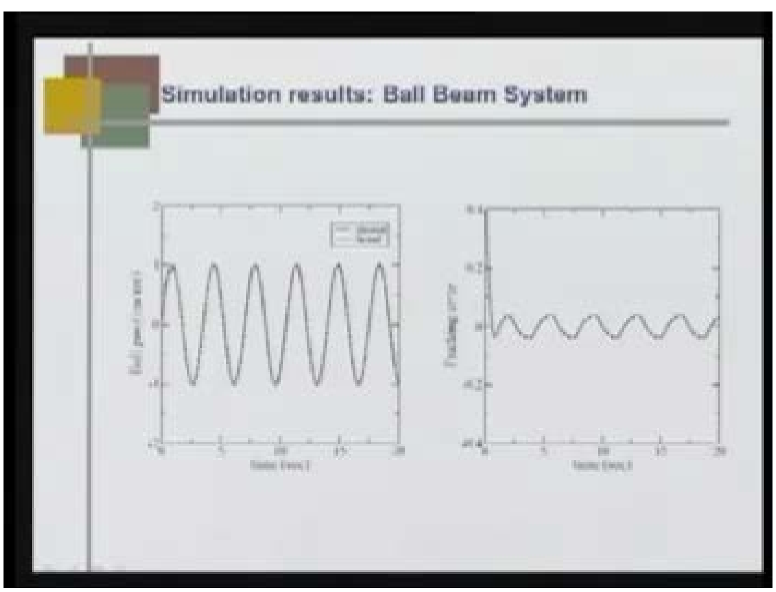

You see that for ball position the desired and actual very perfect tracking and this is the tracking error. Similarly beam angle you see that and controller input.

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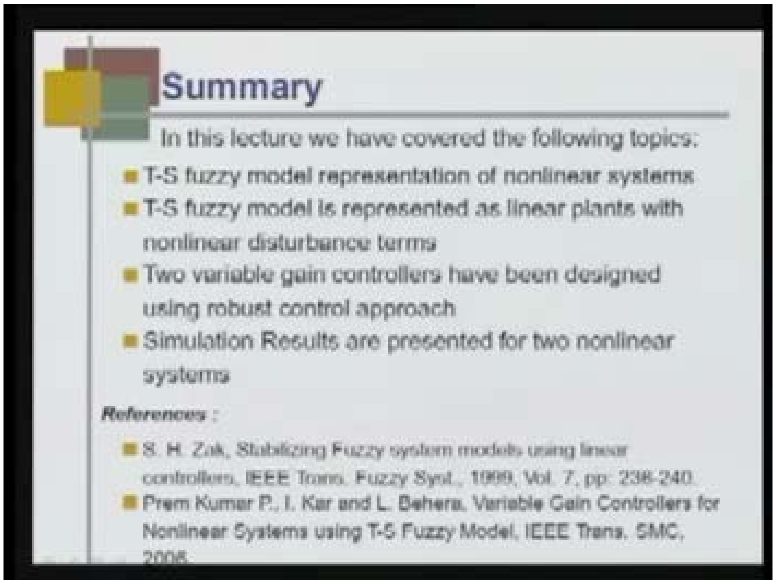

Summary in this lecture, we have covered the following topics: T-S fuzzy model representations of nonlinear systems. T-S fuzzy model is represented as linear plants with nonlinear disturbance terms. Two variable gain controllers have been designed using robust control approach. Simulation results are represented for two nonlinear systems showing the comparison. For the references you can see that, we have two papers here, the first paper is Zak paper that is there in IEEE Transactions Fuzzy Systems in 1999. The second is our paper which is in IEEE Transaction Systems Man Cyber net in 2006 which is called variable gain controller nonlinear system using T-S fuzzy model. Thank you very much.