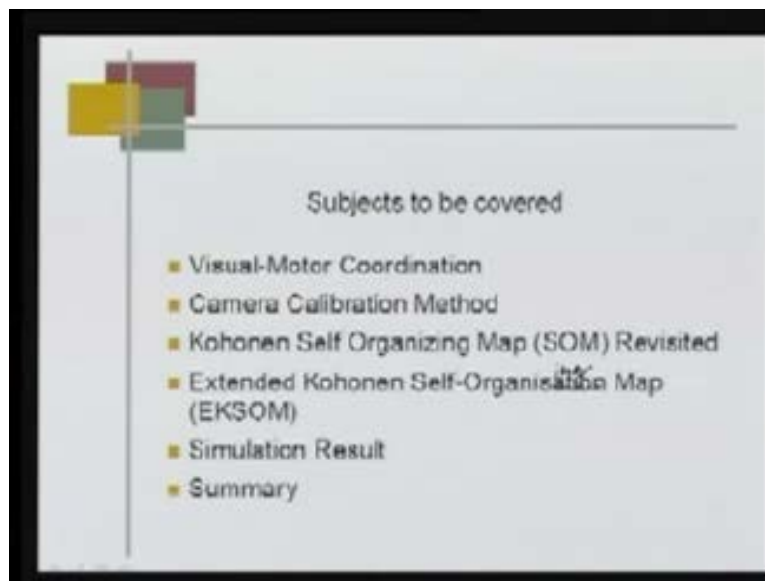


Intelligent Systems and Control
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Module - 3 Lecture - 7
Visual Motor Coordination using KSOM

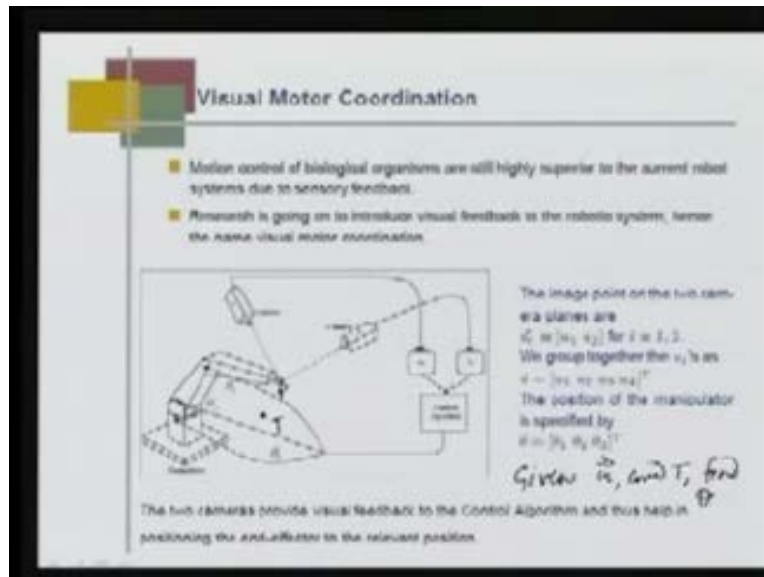
Our topic today is Visual motor coordination using Kohonen self organizing map; a little difference to what we have been talking on intelligent control, specifically neural control. We have been doing dynamic control but today, we will be learning how we can learn the inverse kinematics of a robot manipulator simply by learning. This is the seventh lecture on neural control.

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Subjects that we will be covering today: Visual motor coordination problem, Camera calibration method, Kohonen self organizing map, Extended Kohonen self organizing map, Simulation results and summary.

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Visual motor coordination motion control of biological organisms are still highly superior to the current robot systems not only due to sensitive feedback but, also very improved information processing capability of human brain. Many things also are not known about how information processing takes place in the brain. Research is going on to introduce visual feedback to the robotic system; hence the name visual motor coordination.

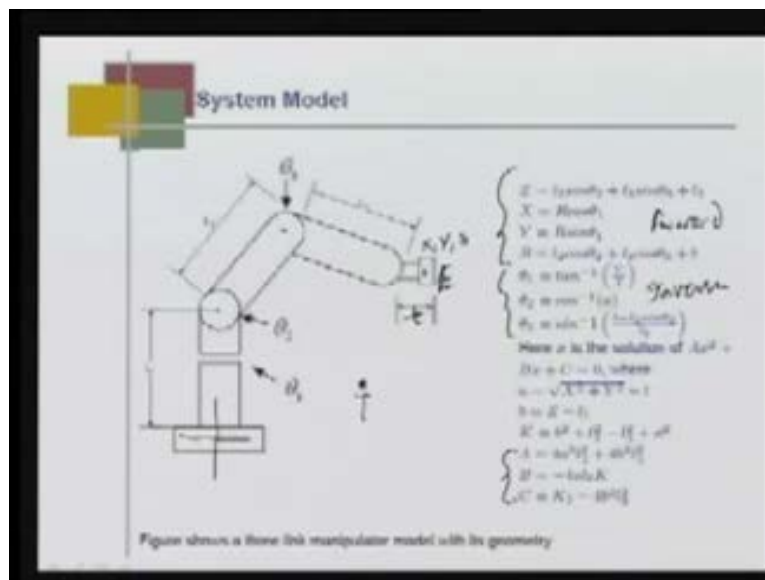
What you are seeing here is a robot manipulator. It is a three link robot manipulator. Two cameras are attached here camera 1, here camera 2. Camera 1 gives the input. Now camera looks at the work space and gives the information about the target point as well as the end-effector point u_1 and u_2 . This is the control algorithm and from control algorithm gives the command what should be θ_1 , θ_2 , θ_3 such that, the end-effector reaches any point in the work space. For example, this is my target point t and this is my end-effector point p . The p should match t by proper coordination θ_1 , θ_2 , θ_3 that has to be actuated such that, the end-effector finally comes to the t and this coordination takes place through visual feedback.

So, just like given an object, how I go and catch it? Now for example: this tip, I can touch the tip in this way I can touch the tip in this way. So, varieties of ways I can reach this point through visual feedback. This is called hand eye coordination. The image points on

the two camera planes are u_1 and u_2 we group together the u_i 's as one vector that is u vector $u_1 \ u_2 \ u_3 \ u_4$. You see although the object is in 3-D camera always gives the 2-D information.

The position of the manipulator is specified by $\theta_1 \ \theta_2 \ \theta_3$. The two cameras provide visual feedback to the control algorithm (04:11) to the relevant position. Obviously, what we are interested is given u vector and the target vector t ; find θ in steps such that, the end point reaches the target point. This is called visual motor coordination.

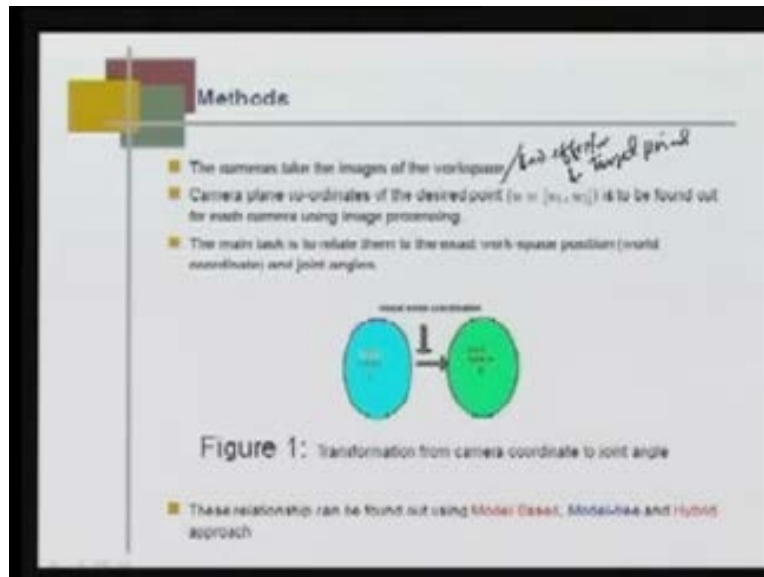
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The system model is more accurate than what we said this base which we will say first link, this is the second link and the third link. The base moment is circular that is θ_1 and θ_2 up and down and θ_3 also up and down in the vertical plane and this rotates in the horizontal plane. If you look at the relationship any point, say this point its coordinate is $z \ x \ y$ with respect to some main reference frame. This $z \ x \ y$ I mean, this reference is in this form from this point from the base $z \ x \ y$ and the $z \ x \ y$ is for example: given $\theta_1 \ \theta_2 \ \theta_3$, the end-effector has a specific position $z \ x \ y$, so this is my end-effector point.

A special position is x y and z this x y z means this end-effector has a position in the space in the special coordinates x y z that can be correlated with θ_1 θ_2 θ_3 in this particular manner. z is $l_2 \sin \theta_2$ l_2 is the length of the second link plus $l_3 \sin \theta_3$ here which is the length of the third link $\sin \theta_3$ plus l_1 , l_1 is the height. Similarly, the x position from here is fixed; this does not move; this simply rotates. Obviously, the x is $r \cos \theta_1$ where, r is $l_2 \cos \theta_2$ plus $l_3 \cos \theta_3$ plus t and t is this distance t . Similarly θ_1 is $\tan^{-1} y$ upon x . This is the forward kinematics that is given, θ_1 θ_2 θ_3 . How do I compute the end-effector position in the special coordinate in the space x y z ? Now, from this equation I derive, given x y z what should be θ_1 θ_2 and θ_3 ? This is called inverse kinematics. That is, θ_1 is $\tan^{-1} y$ upon x ; you can easily verify this θ_2 is \cos^{-1} small x , where x is the solution of $a x^2 + b x + c = 0$ where a b c are given by this formulas and θ_3 is $\sin^{-1} b - l_2 \sin \theta_2$ by l_3 . In this the three things that you need are a b and k you see that, you need here a b and k a , is given by $x^2 + y^2$ square root over this no time. This time is this from third link the end-effector as certain distance certain length. The end-effector link length is the t . This is forward kinematics and this is inverse kinematics. There is nothing to be solved. Mathematically, if I know this equation, if I know these parameters perfectly, inverse kinematics is solved. There is nothing to be done here. We are not interested in the mathematical way of computing inverse kinematics what we are interested is given a target point how do I learn step by step based on visual feedback such that, the error between this point and this point, this target point end point and target point, they match or this target point comes here. This is the problem that we are going to solve.

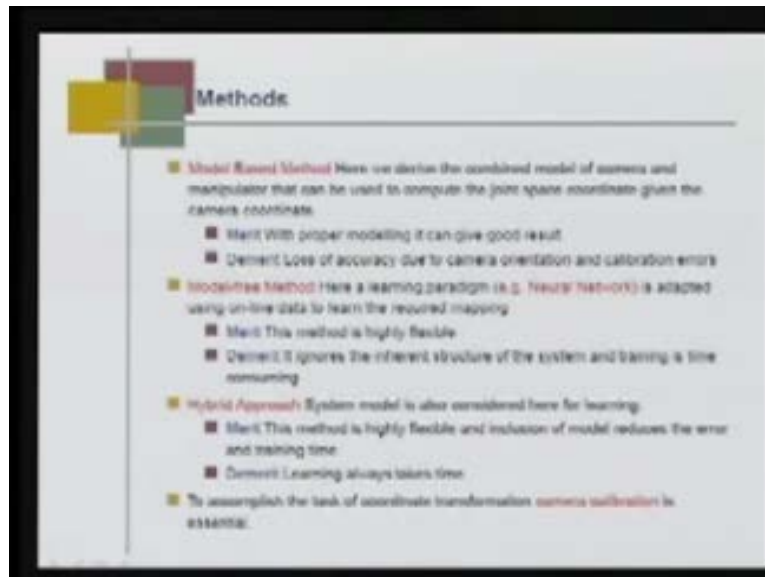
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Methods: The cameras take the measures of the work space. Work space means it takes the image of end-effector it locates, where is the end-effector and the target point. Cameras give us the information where is the end-effector at the moment and where the target point is or how far is target point from the end-effector. Camera plane coordinates of the desired point u is to be found out for each camera using image processing. We exactly find out what is the target position and the end-effector position from camera in terms of some vector quantity u .

The main task is, to relate them to exact workspace position world coordinate and joint angles. Given this camera input, means what camera gives you an image where, we get the object in terms of some pixels. Given that information, how do I correlate this information to the actual θ_1 θ_2 θ_3 such that, the robot manipulator end-effector finally reaches the end target point? So, these relationships can be found out using model based, model free and hybrid approach.

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Model based method here; we derive the combined model of camera and manipulator that can be used to compute the joint space coordinate given the camera coordinate. Merit is with proper modeling it can give good result; demerit is loss of accuracy due to camera orientation and calibration errors. Model based means, we involve the camera. The camera gives us the position. Earlier, the models that we showed you - this (Refer Slide Time: 12:11) model is inter-twined with the model of the camera and then we again go backward. Given the camera input what should be the θ_1 θ_2 θ_3 . This is the calculation; what is normally done is calibration of camera means. We take the known (Refer Slide Time: 12:38) x y z points and from there, we see the camera output. We correlate them in terms of certain parameters identify those parameters.

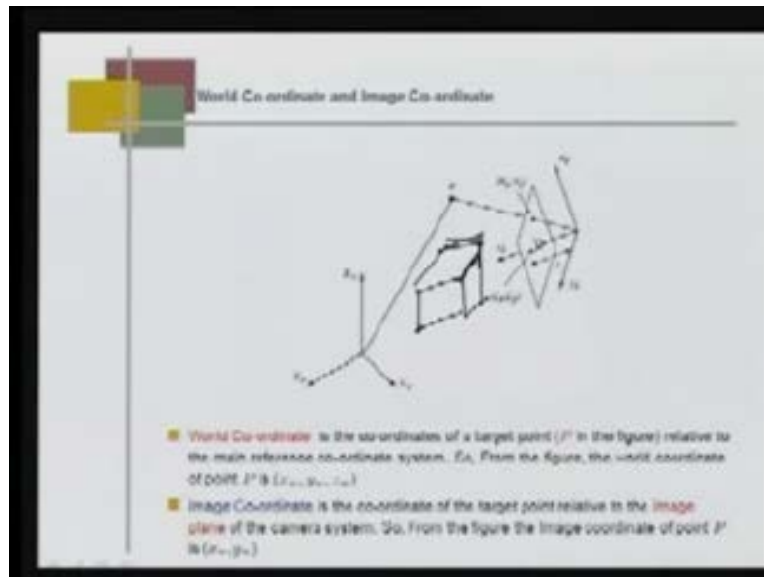
Then, inverse computing gives us from camera plane to x y z plane. This is called model based method with proper modeling; it can give good result but loss of accuracy due to camera orientation and calibration errors. So, calibration has to be done perfectly.

Model free method: here, we do not assume any model of the camera. Camera output is taken simply whatever is u_1 and u_2 . We really do not correlate that to what is x y z; there is no need. But based on that, we learn using the, Kohonen base self organizing map; we learn how we reach the target. It is just like hand eye coordination that is, if my object is

(13:53) somewhere how I catch it is based on visual; like I throw and then hold it - hold. By throwing the object up and holding it, I have the hand eye coordination that means, I am looking at the thing, I am observing the object moment and then I am trying to catch it. This is called visual motor coordination. When I am doing this job, I am not actually aware of any model on which I am working. Rather this whole thing I do while learning by practicing many times I learn how to catch. I really do not use the model of my arm; I do not really use the model of my eye but, it is simply a learning process this is called model free approach; this method is highly flexible. It ignores the inherent structure of the system training is time consuming- demerit. Hybrid approach system model is also considered here for learning. This method is highly flexible and inclusion of model reduces the error and training time; demerit learning always takes time to accomplish the task of coordinate transformation; camera calibration is essential.

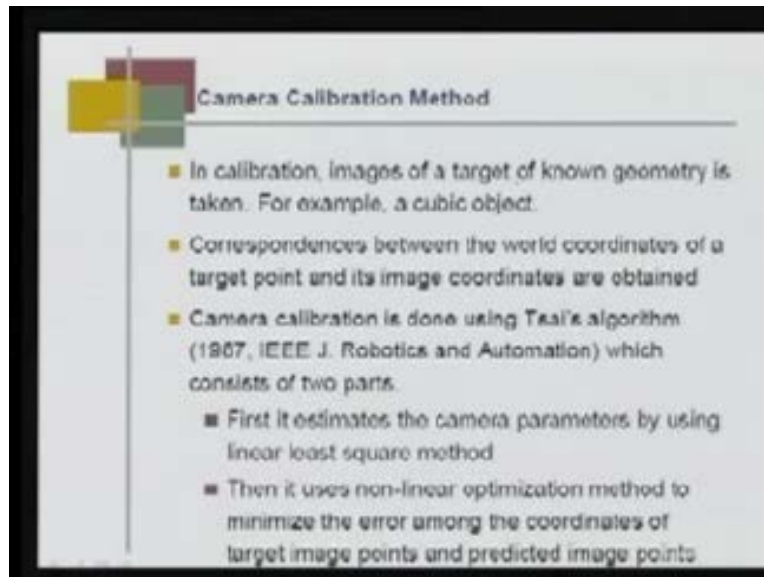
What is the overall thing that I told just now is, by putting a camera in the work space, robotic manipulation can be done using model based method where camera calibration is a necessity. Second is, I do not need camera calibration and still can do the learning simply by learning mechanism. The third approach is that, I can still introduce the camera calibration, do offline simulation for generating data and training. After I have done training on the offline model that includes the robot model as well as camera model, then I am almost done but, still calibration error may be there and the model also maybe there; there is some discrepancy. So, I start with these parameters; whatever the training parameters; I have already learnt and I do fine tuning in the field that is called hybrid approach. Combination of these two - this does not require any model, neither the model of the robot manipulator nor the camera model. This requires both the model of camera as well as robot manipulator. In hybrid approach, we take the positive of both and make a hybrid approach.

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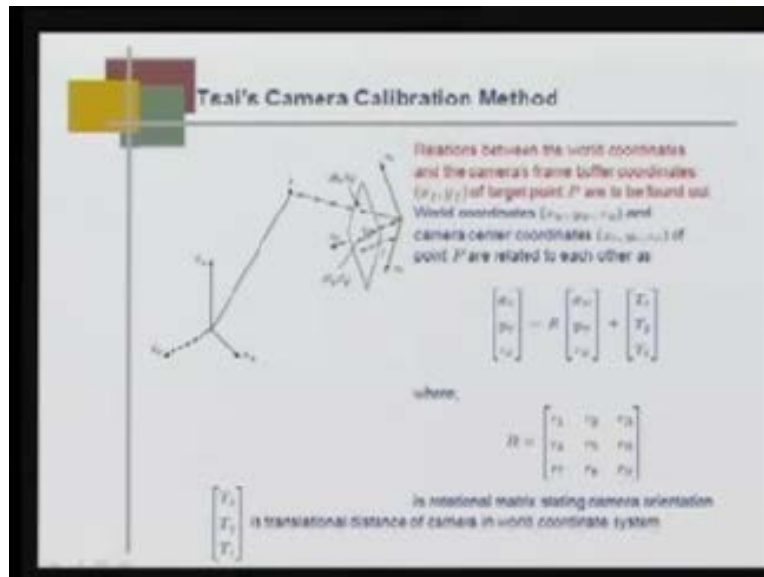
Now, before I go to calibration, I will explain world coordinate and image coordinate. World coordinate is you see that, this is my main reference plane and this is my p my target point or end-effector point. My end-effector is here or my target is here, where the end-effector is to go? The world coordinate is the coordinate spatial coordinate of this point with respect to this frame. Camera is also located in a specific frame which is x_c, y_c, z_c that is camera frame. If I put camera as the reference frame that is x_c, y_c, z_c the world coordinate is the coordinate of the target point relative to the main reference coordinate system. From the figure, the world coordinate of point is x_w, y_w, z_w . The image coordinate is this camera which is placed in this reference plane it has a focal plane. In this focal plane the object has a position and that position is x_u and y_u image coordinate is a coordinate of the target point relative to the image plane of the camera system. This is my image plane, this image plane center; central point is c_x and c_y , this is origin of image plane. From the figure the image coordinate of the point p which is here, this p is x_u, y_u .

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In camera calibration images of a target, known geometry is taken. For example: a cubic object that is, here (Refer Slide Time: 18:54) I place in this reference coordinate an object; this is a cubic object. With respect to this, I know what is given this point, this point, this point and this point. I know actually spatially known points. I generate in x y z coordinate that is, the calibration because meaning of calibration is, once I know given this point what is x_u y_u I should be able to predict. Given this point what should be x_u and y_u - this is called calibration. Correspondence between the world coordinate of a target point and its image coordinates are obtained. Camera calibration is done using Tsai's algorithm which you can find in IEEE journal of Robotics and Automation in 1987 which consist of two parts: First it estimates the camera parameters by using linear least square method. Then, it uses non-linear optimization method to minimize the error among the coordinates of target image points and predicted image points.

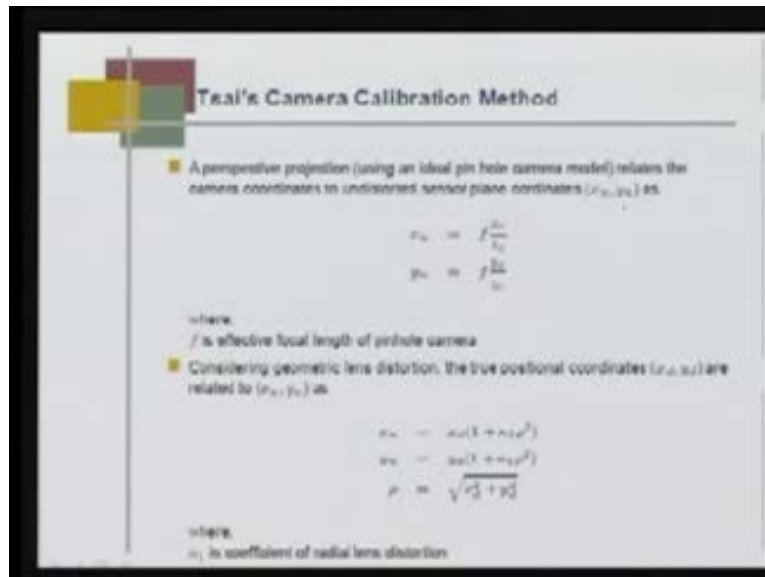
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I will just explain very briefly; you can actually go to this paper that I just referred to understand how the calibration is done. There are other calibration methods but, I am just focusing size calibration method. This is my world coordinate, this is my camera coordinate. First, I express the camera coordinate, this is camera coordinate which is x_c y_c z_c . The relation between world coordinate and camera frame buffer coordinates of target point p are to be found out. The world coordinate x_w y_w z_w ; this is the world coordinate of the end point, point p are related to each other. This p can be expressed either in this reference plane or in this reference plane. In this reference plane this is x_c y_c z_c and in this reference plane this is x_w y_w z_w . The point p here is either x_w y_w z_w in world coordinate or x_c y_c z_c in camera coordinate. If I assume that wherever the camera is spatially located that to be the origin, but in camera coordinate x_c y_c z_c can be written very simply.

That is, by rotational matrix r which is r_1 r_2 r_3 , r_4 r_5 r_6 , r_7 r_8 r_9 ; because this is a three link manipulator. Plus the translational term T_x T_y T_z where r is the rotational matrix taking camera orientation and T_x T_y T_z is translational distance of the camera in world coordinates. By using this, how many parameters will I need to compute x_c y_c z_c x_w y_w z_w ? Actually this is 9 plus 3, 12 parameters I need to convert from x_w y_w z_w to x_c y_c z_c , from world coordinate, the representation of p to the camera frame coordinate of p .

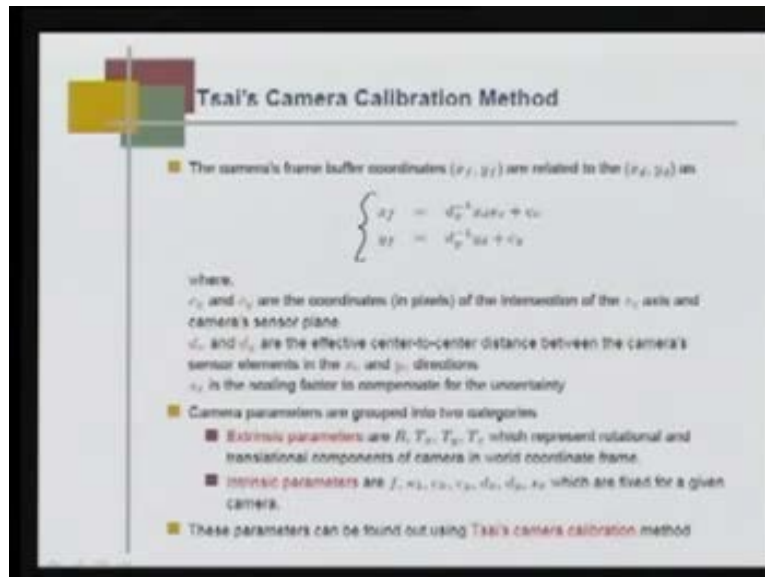
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Now once I have this p (Refer Slide Time: 23:12) represented in x_c y_c z_c then, there is a very easy relationship between how this p will be mapped to the focal plane of the camera. That is given by: x_u y_u is f is the effective focal length of the pinhole camera into x_c by z_c is the x_u and y_u is f upon f into y_c upon z_c . The perspective projection relates the camera coordinates to undistorted sensor plane coordinates as in this formula that is a simple formula. Normally, we do not observe x_u and y_u what we observe is x_d and y_d due to distortion. This distortion is due to geometric lens distortion.

Instead of x_u y_u , we get x_d and y_d that includes the distortion. Given x_d and y_d , I must know what is x_u and y_u and that x_u and y_u are given by this relationship where k_1 is the coefficient of radial lens distortion.

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Tsai's Camera Calibration Method

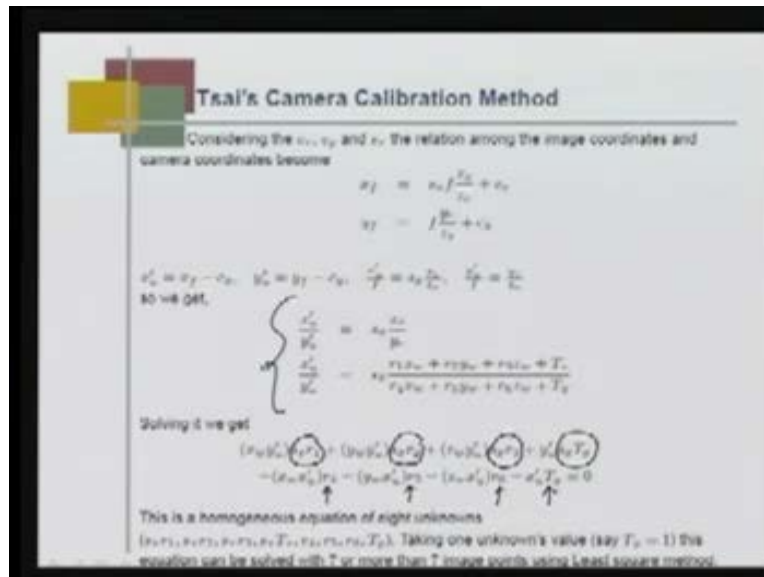
- The camera's frame buffer coordinates (x_f, y_f) are related to the (x_d, y_d) as

$$\begin{cases} x_f = d_x^{-1} x_d s_x + c_x \\ y_f = d_y^{-1} y_d + c_y \end{cases}$$
- where,
 - c_x and c_y are the coordinates (in pixels) of the intersection of the z_c axis and camera's sensor plane
 - d_x and d_y are the effective center-to-center distance between the camera's sensor elements in the x_c and y_c directions
 - s_x is the scaling factor to compensate for the uncertainty
- Camera parameters are grouped into two categories
 - Extrinsic parameters** are R, T_x, T_y, T_z which represent rotational and translational components of camera in world coordinate frame.
 - Intrinsic parameters** are $f, k_1, k_2, k_3, c_x, c_y, d_x, d_y, s_x$ which are fixed for a given camera.
- These parameters can be found out using Tsai's camera calibration method

Now, once x_d and y_d is given that I am observing the camera frame buffer coordinates which is, x_f and y_f . This I actually get from a computer image x_f and y_f . Given an object, what is the position of object is denoted by x_f and y_f which is, written by $d_x^{-1} x_d s_x + c_x$ and $d_y^{-1} y_d + c_y$ where c_x and c_y are the coordinates in pixels of the intersection of z axis and camera sensor plane d_x and d_y are the effective center to center distance between the camera sensor element in the x_c and y_c direction and s_x is the scaling factor to compensate for the uncertainties. By this method finally, we get x_f and y_f . The camera parameters are grouped into two categories. One is extrinsic parameter which is R, T_x, T_y and T_z that was we said (Refer Slide Time: 25:45) in this R, T_x, T_y, T_z and R is this matrix. Intrinsic parameters are $f, k_1, k_2, k_3, c_x, c_y, d_x, d_y, s_x$ which are fixed for a given camera. You saw here, we have the parameters associated with f . These are all computed already; this k_1 and ρ is you know $x_d^2 + y_d^2$ is the ρ ; so that is already known.

In this k_1 and f is there and in this is $d_x^2 + y_d^2$ s_x and also c_x and c_y which we said, c_x and c_y and you know that, this point is c_x and c_y . These parameters can be found out using Tsai's camera calibration method.

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This is a long list I will not give details. What I am trying to tell you is that, simplification we can do to write $x \cdot f$ is $s \cdot x \cdot f \cdot x_c \cdot z_c$ plus $c \cdot x$ and $y \cdot f$ is $f \cdot y_c \cdot z_c$ plus $u \cdot y$ ignoring the distortion, I can write this. Once I write this, we can now represent $x \cdot u$ dash is $x \cdot f$ minus $c \cdot x$, $y \cdot u$ dash as $y \cdot f$ minus $c \cdot x$. You know that, I know $y \cdot f$ similarly $x \cdot u$ dash upon f , f is $s \cdot x$ upon x_c upon z_c and y_u dash upon f is y_c upon z_c that is from this relationship. Finally, we get these two equations.

If I solve these two equations, I get this equation and you see that this is simply the objective is that, how I identify the camera model parameters that we started with world coordinate went to the camera coordinates. From camera coordinates, camera image plane; from camera image plane to the actual position of the target point in the camera image plane, which is x_f and y_f . Given x_f and y_f and given actual x y z in the world coordinate, how do I compute these parameters which is s x r_1 , s x r_2 , s x r_3 , s x t x ? Then r_4 r_5 r_6 m t y , these are the parameters. I already know x_w y_w z_w because, x_f is known c_x and c_y are given x_u this is known from t . So, these x_w y_w z_w are all given. Now, we can collect this data point 1,2,3,4,5,6,7 and 8. Eight data points, eight unknowns are here of course, here you see that, 2 unknowns are clubbed together. But, these 8 unknowns and these are known if I have many points I can always solve this using (29:39) technique. That is a homogeneous equation of 8 unknown s x r_1 , s x r_2 , s x r_3 , s x t

x , r_4 , r_5 and r_6 and t_y they can be easily estimated. Taking one unknown value say t_y equal to 1; this equation can be solved with 7 or more than 7 image points using (30:01) methods. Now, the other points given these combined values, how do I estimate s_x first? Once I estimate s_x then, I can easily estimate what is r_1 r_2 r_3 .

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Tsai's Camera Calibration Method

- We will get the estimated parameters r_1, r_2, r_3 and T_y directly
- For the rotation matrix R , $r_1^2 + r_2^2 + r_3^2 = 1$, $r_4^2 + r_5^2 + r_6^2 = 1$
- We have to find a scale factor c such that it satisfies the above equations as

$$c = \frac{1}{\sqrt{r_1^2 + r_2^2 + r_3^2}}, \quad r_i^* \text{ stands for estimated value of } r_i \text{ for } i = 1, 2, 3$$

$$\frac{c}{s_x} = \frac{1}{\sqrt{(s_x r_1^*)^2 + (s_x r_2^*)^2 + (s_x r_3^*)^2}} \quad r_4^2 + r_5^2 + r_6^2 = r_1^2 + r_2^2 + r_3^2 = c$$

Solving these two equation we will get the parameter s_x .

- With the value of s_x the parameters r_1, r_2, r_3 and T_y can be found out
- We can find the parameters r_7, r_8 and r_9 by cross product of first two rows of R

This is a simple method. Out of the eight unknowns we directly get r_4 r_5 r_6 and t_y . We have the information $r_1^2 + r_2^2 + r_3^2 = 1$ also $r_4^2 + r_5^2 + r_6^2 = 1$. When we estimate using least square we do not get exact values. So, assuming one upon $r_4^2 + r_5^2 + r_6^2$ to be c this root over and also c upon s_x is this quantity which you can easily see. Assuming that, this quantity root over $r_4^2 + r_5^2 + r_6^2$ I estimate is equal to $r_1^2 + r_2^2 + r_3^2$. Here the assumption is, $r_4^2 + r_5^2 + r_6^2 = r_1^2 + r_2^2 + r_3^2 = c$. By assuming that, we know the estimate of this so from there we can compute c . We also have estimate of this; we know this quantity, so we can easily find out s_x . With the value of s_x the parameter r_1 r_2 r_3 and t_x can be found out. Then, the next is r_7 r_8 r_9 they are found out because, their cross product of the first two rows of r which is r_1 r_2 r_3 and r_4 r_5 r_6 .

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Tsai's Camera Calibration Method

■ The parameters f and T_c can be estimated from the following equations

$$\frac{x'_c}{f} = \frac{r_1 x_w + r_2 y_w + r_3 z_w + T_x}{r_7 x_w + r_8 y_w + r_9 z_w + T_z}$$

$$\frac{y'_c}{f} = \frac{r_4 x_w + r_5 y_w + r_6 z_w + T_y}{r_7 x_w + r_8 y_w + r_9 z_w + T_z}$$

■ **Non-linear optimization**
The intrinsic, extrinsic parameters and the distortion are adjusted to minimize the image error

$$\sum_{i=1}^N (x_f - x_{pi})^2 + \sum_{i=1}^N (y_f - y_{pi})^2$$

where, x_{pi} and y_{pi} are the predicted image point corresponding to the world coordinate (x_w, y_w, z_w)

Finally, the non-optimization used to fine tune whatever the estimation has been done because, the estimation has certain approximation because, and distortion has been omitted during the estimation. For that a nonlinear optimization method is done.

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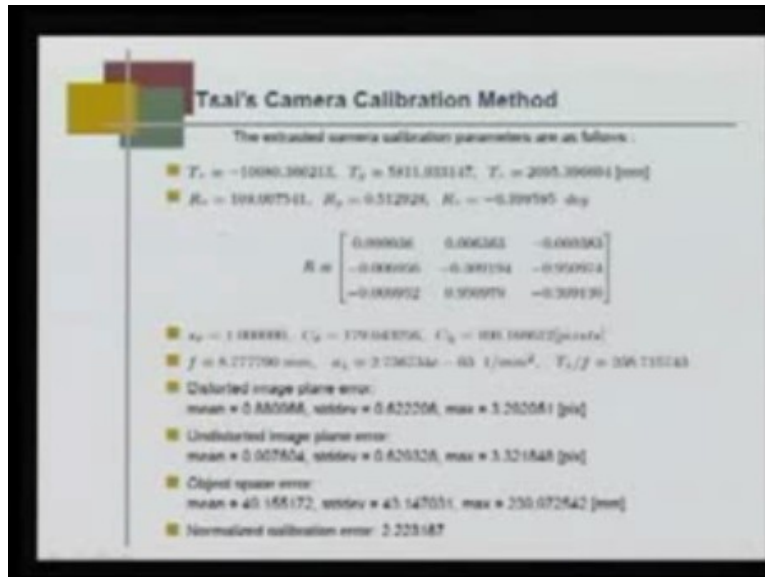
Tsai's Camera Calibration Method

Following list shows a set of points' world coordinates and their respective image coordinates.

x_w	y_w	z_w	x_f	y_f	x_w	y_w	z_w	x_f	y_f
0	32000	0	66	53	2450	30470	0	88	56
3450	28400	0	68	78	3450	22540	0	61	80
2450	19910	0	80	94	2450	10790	0	26	113
3450	15220	0	13	124	3200	27430	0	91	63
3300	28440	0	81	72	3300	22390	0	70	82
3200	19910	0	59	94	3200	17240	0	43	109
3300	14700	0	21	120	7130	30470	0	145	55
7130	27430	0	141	64	7130	24960	0	137	73
7130	22390	0	133	61	7130	17240	0	119	112
7130	14700	0	112	127	7130	12100	0	99	155
7130	9600	0	83	193	7130	7600	0	64	205
7130	7140	0	59	247	11130	30470	0	194	50

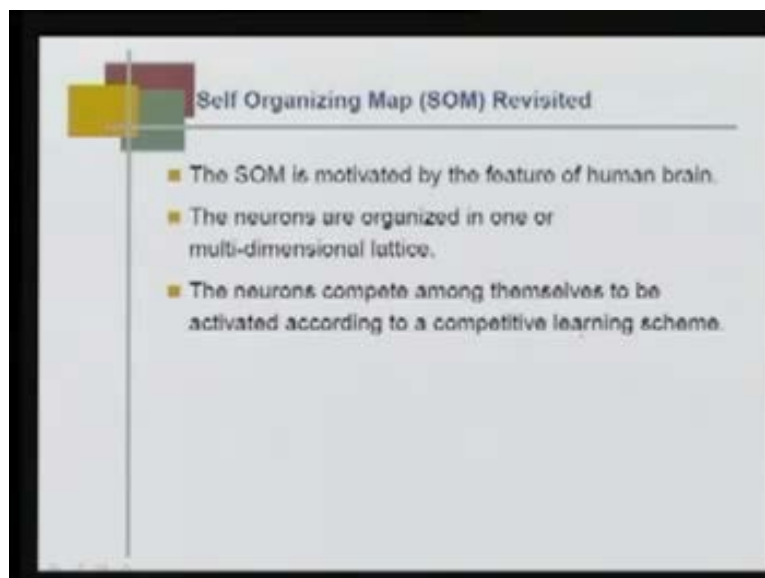
You see that, the object the approach is x_w y_w z_w . So, these are my actual points in the space coordinate and this is my actual image coordinate u x f and y f this I collect from the actual system.

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Then using the algorithm you can find out what is T_x T_y T_z , R_x R_y R_z and r is this rotational matrix s x c y f κ_1 t f by f t z by f .

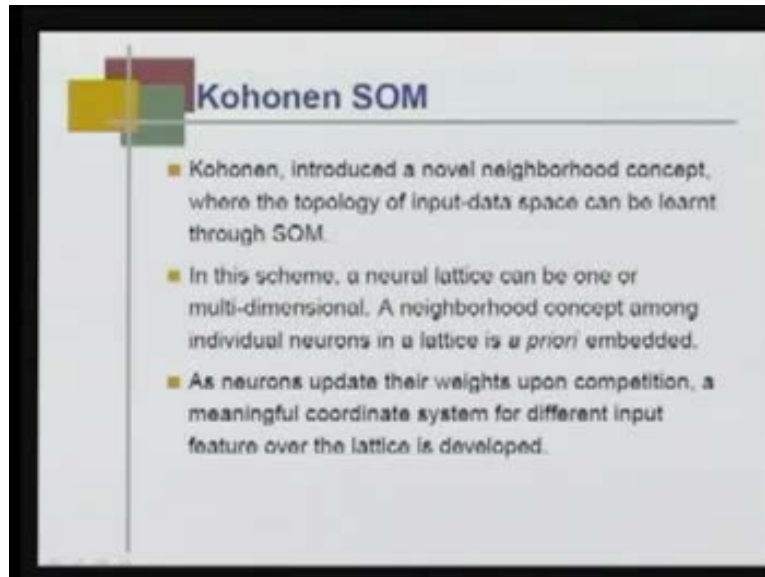
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All the parameters of the camera can be computed using this algorithm, what I just told you that, camera calibration helps us to find out the camera model. Given x y z in the actual world coordinate, camera model helps me to find out what should be x_f and y_f or which we say u_1 and u_2 . Once I know this camera calibration the advantage is that, I really do not have to do experiment to compute. Once I do the camera calibration, I can take this model incorporate in my robot manipulator or combine with the robot manipulator model I can generate data that is given x y z . I can randomly generate θ_1 θ_2 θ_3 , it will take two specific x y x z and from that, x y z , I can convert what should be my camera plane coordinates.

Now, we will talk about self organizing map that, we have already studied in the last class. The SOM is motivated by the feature of human brain. Neurons are organized in one or multi-dimensional lattice. The neurons compete among themselves to be activated according to a competitive learning scheme.

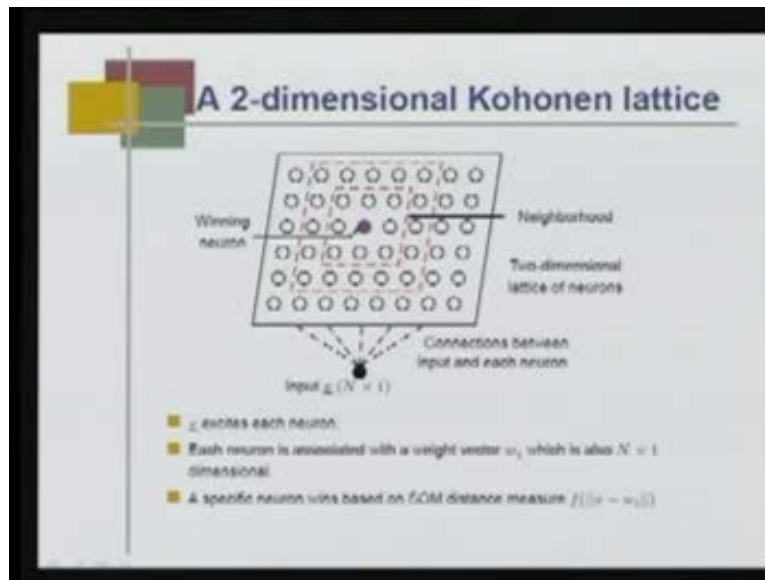
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In Kohonen, we introduced a novel neighborhood concept where, the topology of input data space can be learnt through self organizing map. In this scheme a neural lattice can be one or multi-dimensional a neighborhood concept among individual neurons in a

lattice is prior embedded as neurons update their weights upon competition. A meaningful coordinate system for different input features over the lattice is developed.

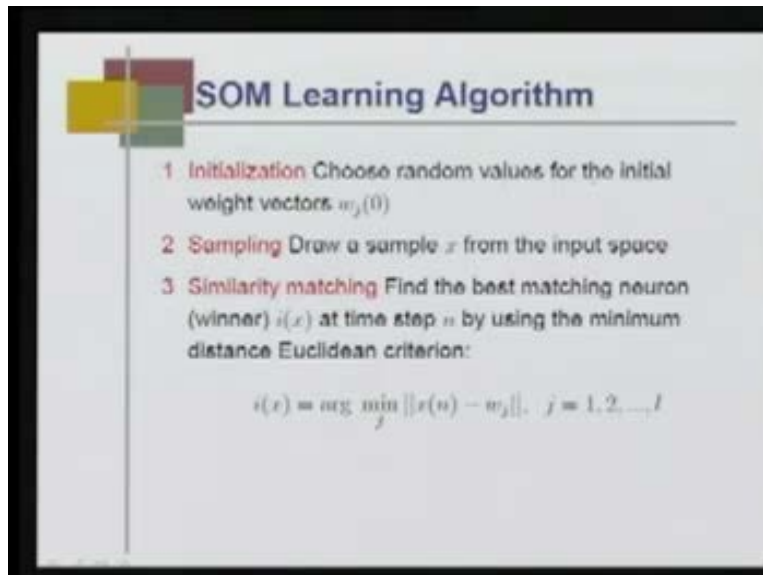
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Given a neural lattice like this is a 2 dimensional lattice and given the input space like for example: if my input data also is coming from a specific 2-D geometry then, the reference the coordinate points of this neurons would be such that, they would represent almost they will learn the actual topology of the input space. If the input space is 3-D they will also learn the 3-D topology.

If input space is 1-D they will also learn 1-D topology. The objective of a 2-D Kohonen lattice is my input data that excites all the neurons, one of the neurons becomes the winner. The winner is decided by the equilibrium distance between the neuron reference vector and the input vector. Each neuron is associated with weight vector \underline{w}_i ...a specific neuron wins best; on some distance measure this normally is an equilibrium distance measure.

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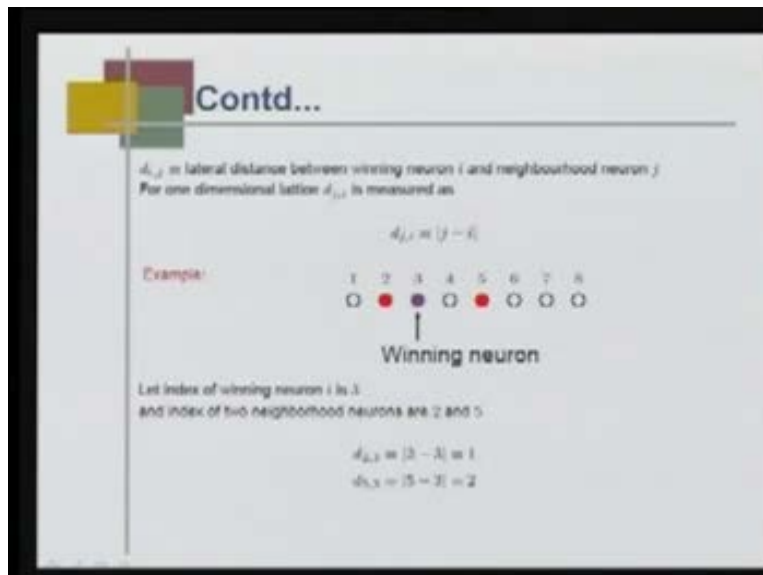
SOM Learning Algorithm

1. **Initialization** Choose random values for the initial weight vectors $w_j(0)$
2. **Sampling** Draw a sample x from the input space
3. **Similarity matching** Find the best matching neuron (winner) $i(x)$ at time step n by using the minimum distance Euclidean criterion:

$$i(x) = \arg \min_j \|x(n) - w_j\|, \quad j = 1, 2, \dots, I$$

The SOM learning algorithm is choose a random value for the initial weight vector w_j . Draw a sample x from the input space, find the best matching neuron at the time step by using the minimum Euclidean distance criterion.

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$d_{i,j}$ = lateral distance between winning neuron i and neighbourhood neuron j
For one dimensional lattice $d_{i,j}$ is measured as

$$d_{i,j} = |j - i|$$

Example:

1	2	3	4	5	6	7	8
○	●	●	○	●	○	○	○

↑
Winning neuron

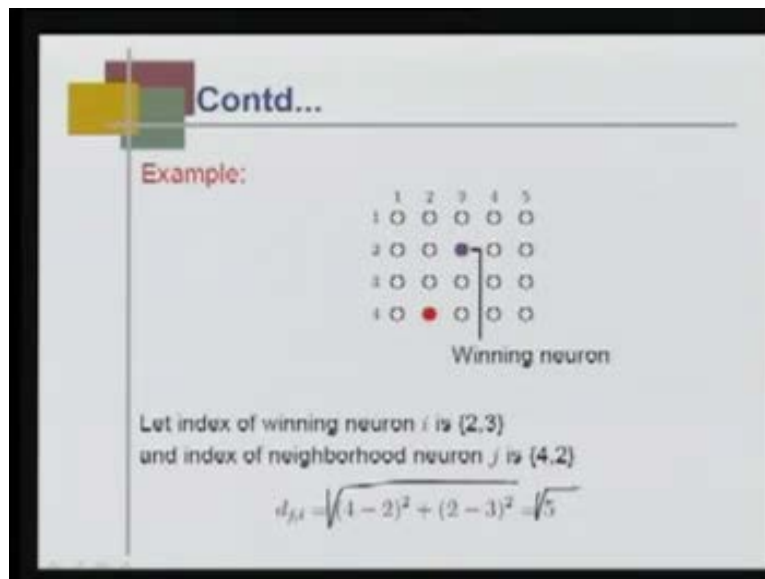
Let index of winning neuron i is 3
and index of two neighbourhood neurons are 2 and 5

$$d_{2,3} = |3 - 2| = 1$$
$$d_{5,3} = |5 - 3| = 2$$

Like for example: this is a 1-D lattice. This is my winning neuron and the neighborhood neurons like the number, this is my neighborhood neuron the distance is 1 and this neuron

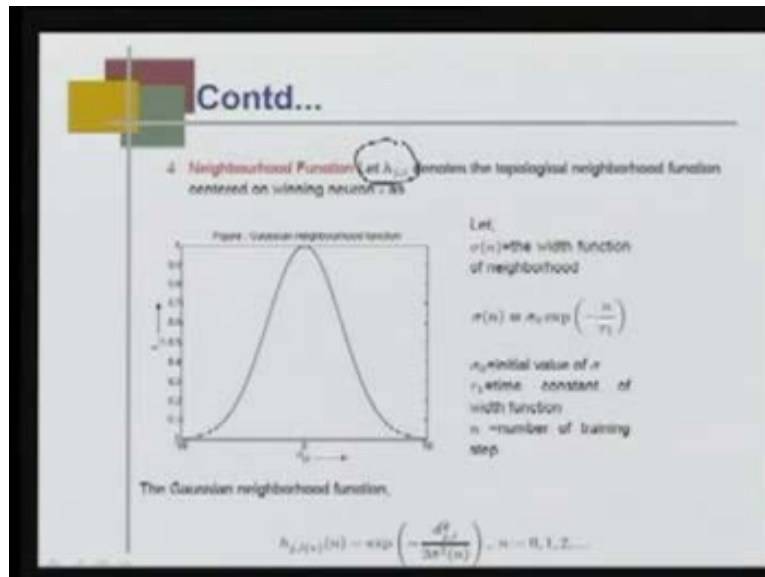
the distance from winning neuron is 2. While I find (Refer Slide Time: 37:27) the winning neuron I also find the distance of each neuron from the winning neuron. How far they are from these winning neurons? Given a neural lattice, we find first of all winning neuron based on the equilibrium measure distance. The neighborhood, the distance of the neighboring neuron from the winning neuron is computed. In this case, this neuron and this neuron has a distance then, we can say this is 1 and from here to here this is 2, from 8 to 3 of course, it is 5. If I think that 1 and 8 are connected then, you have to see that this distance the lesser distance is to be taken.

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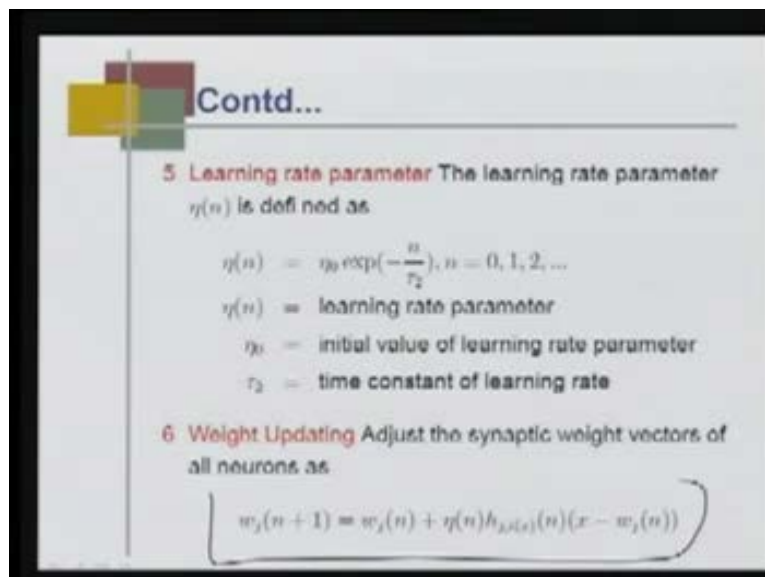
If I have 2-D lattice if this is my winning neuron then this neuron has a distance. You can easily see this neuron has a position 4 and 2. The winning neuron position is 2 and 3. The distance is 4 minus 2 whole square and 2 minus 3 whole square which is 5. Of course, you can also take the square root that is alright whatever the distance you say.

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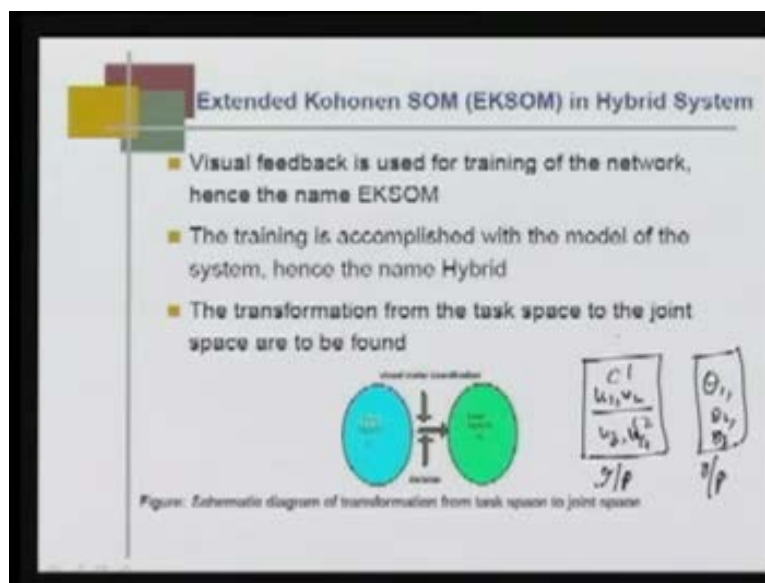
Given the distance, we define a neighborhood function h_{ji} ; h_{ji} is a neighborhood function. That means, it says, how close a specific neuron to a winning neuron that is, e to the power minus the distance square upon 2 sigma square. So, it is better that we have written distance like this. e to the power minus the distance square upon 2 sigma square.

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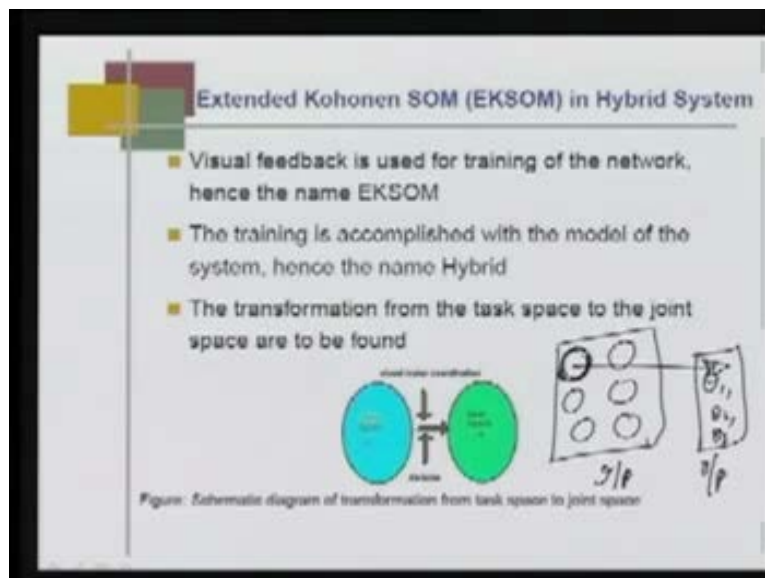
So, once I find out the winning neuron and their neighboring neurons with their neighborhood function then, the weights of each neuron is computed by this. This is my Kohonen algorithm that is, each neuronal index is updated based on its old reference vector; this is the learning rate; this is the neighborhood function. You know that neighbor function will be one for the winning neuron and will gradually decrease for the neurons that are farther away from winning neuron into this is my input vector that excites the neuron minus the reference vector of the neuron.

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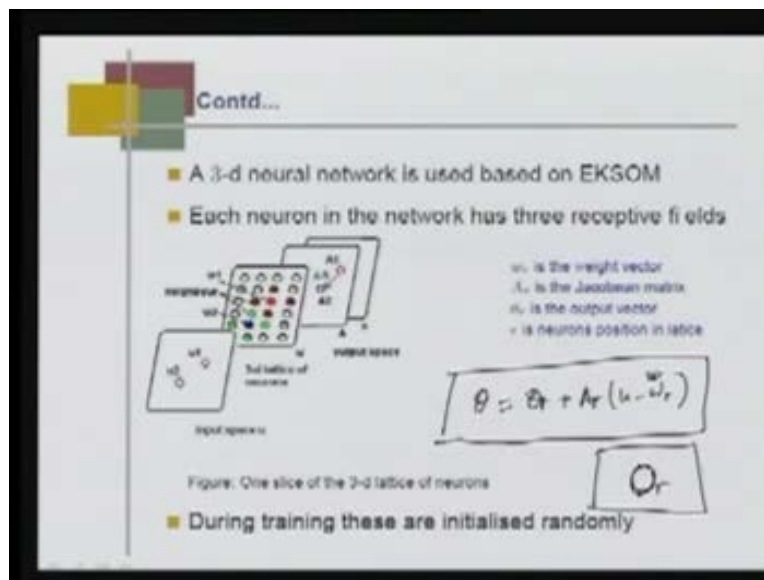
That is, Kohonen lattice it is all (40:45). Now we will be talking about extended Kohonen SOM: the visual feedback is used for training of the network. Here, the concept is little different. We try to correlate between input space and output space. In the visual motor coordinate my input space is u_1 u_2 camera 1 coordinate and u_3 u_4 is the camera 2 coordinate. So, this is camera 1 and this is camera 2. In that space objects are located. I get those point this is my input space and then my output space is θ_1 , θ_2 and θ_3 . So, one is that, we do the clustering of the input space. Meaning of clustering of the input space is that, I represent my input space using finite number of representative points which I said that is what the Kohonen lattice does.

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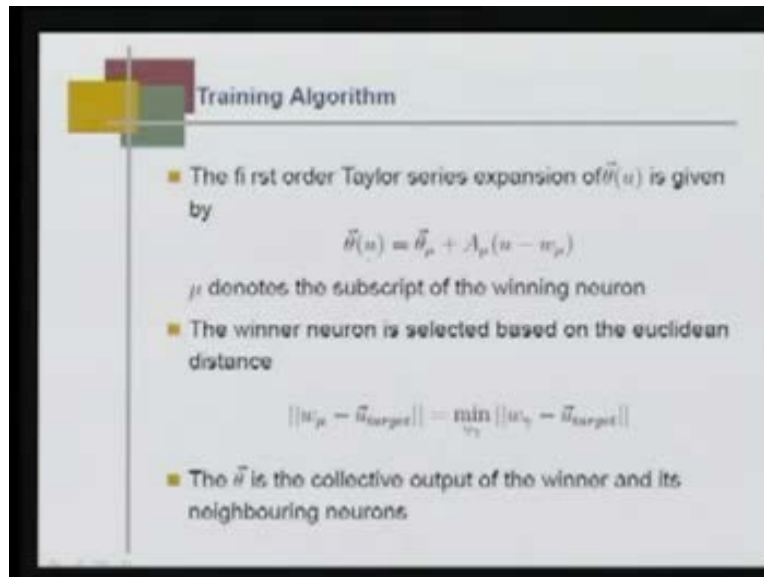
So, in a sense what I do is in input space, I represent some finite cells. These are finite cells which we say here, in this case Kohonen neuron. Within each neuron, we try to establish a relationship between the input space to the output space; that is the object. This is why, we see only Kohonen self organizing map is simply clustering. Extended Kohonen SOM is connecting this clustering to the output space. How do we do it is you know that, the relationship between the input space. The camera image plane coordinate to theta is actually nonlinear but, this we can assume to be linear in the discretized cell. That is the objective.

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This is what is done here; this is my 3-D lattice this is my u_1 camera 1 coordinate, camera 2 coordinate, each coordinate gives you x_f and y_f . You have this vector is 4-dimensional vector obviously; the w_y is of these neurons. This is the 3-D lattice and each neuron will have a reference vector which has also the dimension 4 by 1. Then, we transfer or we map the input space to output space through a Jacobian matrix A . By this relationship using Taylor series first order expansion that is, $\theta = \theta_r + A_r u - w_r$; w_r is associated reference vector A_r is the Jacobian. That means what I am saying is that, if this is my input space and this is my r th discrete cell, for each discrete cell associated, the discrete cell is actually a Kohonen lattice neuron. This neuron is associated with weight vector W_r and Jacobian vector A_r as well as a reference θ vector θ_r . θ_r given any other point u which is close to W_r then, the corresponding θ is computed by this linear equation. But unfortunately, we do not have the information what is θ_r , what is A_r and W_r , we have learnt using the Kohonen. We know only u ; we have to compute what is θ . We do not know what is θ_r and A_r , that has to be learnt. So, this is learnt using some feedback mechanism.

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A presentation slide titled "Training Algorithm" with a decorative header of overlapping colored squares (yellow, red, green). The slide contains three bullet points: 1. The first order Taylor series expansion of $\tilde{\theta}(u)$ is given by
$$\tilde{\theta}(u) = \tilde{\theta}_\mu + A_\mu(u - w_\mu)$$
 where μ denotes the subscript of the winning neuron. 2. The winner neuron is selected based on the euclidean distance
$$||w_\mu - \tilde{u}_{target}|| = \min_{\gamma} ||w_\gamma - \tilde{u}_{target}||$$
 3. The $\tilde{\theta}$ is the collective output of the winner and its neighbouring neurons.


Training Algorithm

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- The $\tilde{\theta}$ is the collective output of the winner and its neighbouring neurons

This I have already told you, how theta is related to parameter w_μ and θ_{μ} . The winner neuron is selected based on the Euclidean distance first. Theta is the collective output of the winner and its neighboring neurons. So, what is final theta (Refer Slide Time: 45:42)? Each neuron will predict given u_1 u_2 winning neuron will predict some theta, so also the neighboring neuron. The final theta is the overall decision making. But, the overall decision making is made by giving less importance to those neuronal decision which are further from the winning neuron.

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The collective output is

$$\hat{\theta}_i^{(n+1)} = s^{-1} \sum_r h_{r_1}(r) \left(\hat{\theta}_i + A_{ri} (\text{target} - u_{i-1}) \right)$$

$$s = \sum_r h_{r_1}(r)$$

↑ neighborhood function

This takes the end-effector to a position u_i , needing a correcting action determined by

$$\hat{\theta}_i^{(n+1)} = \hat{\theta}_i^{(n)} + s^{-1} \sum_r h_{r_1}(r) A_{ri} (\text{target} - u_{i-1})$$

$$i = 1, 2, \dots, n_2$$

n_2 is the number of visual feedback

This is my theta output the collective output. This is my individual output. This is my neighborhood function of that individual neuron and I sum that with over r , r is representing neuron. So, if my neural lattice has over hundred neurons, I compute all the responses and s inverse is simply the summation of the distance function. This is my distance function or neighborhood function and this neighborhood function is the highest value is maximum value is 1 and all of other values are less than 1. So, the objective is that, given this target in the beginning some kind of random θ_1 θ_2 θ_3 are generated because, we start from no knowledge. Then, slowly the objective of learning is such that, the end-effector should go to positions like from v_0 to v_1 until it reaches u target. That is the purpose of learning. I just learnt. This never happens systematically or randomly. I will show you a movie that would say how this actually happens.

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The learning scheme is as follows:

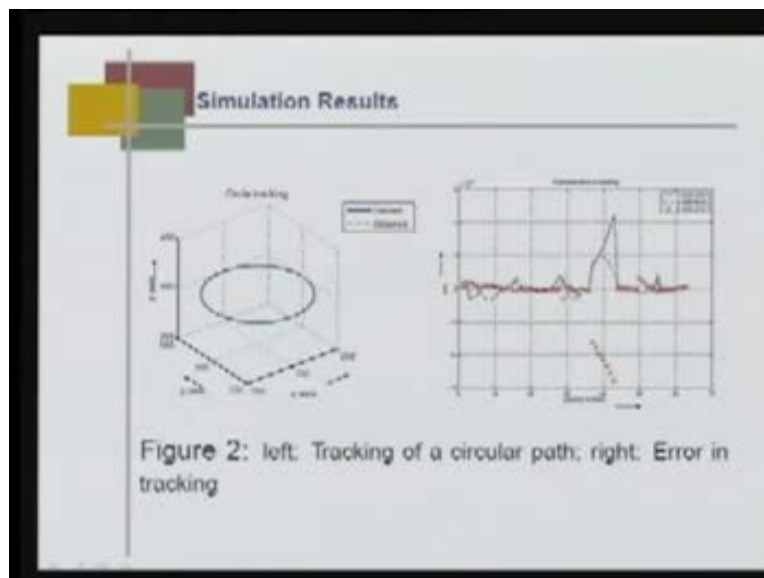
$$\left\{ \begin{array}{l} \Delta A_r = \|\Delta v\|^{-2} (\Delta \tilde{\theta}_{\text{out}}^{\text{net}} - A_r \Delta v) \Delta v^T \\ \Delta v = v_{n_1} - v_0 \\ \Delta \tilde{\theta}_{\text{out}}^{\text{net}} = \tilde{\theta}_{n_1}^{\text{net}} - \tilde{\theta}_0^{\text{net}} \\ \Delta \tilde{\theta}_r = \tilde{\theta}_0^{\text{net}} - \tilde{\theta}_r - A_r (v_0 - w_r) \end{array} \right. \quad \text{Stochastic Gradient Descent}$$

After completion of the movement stage, the neural units are adjusted by the following update rules:

$$\left\{ \begin{array}{l} w_r \leftarrow w_r + \epsilon_1 \eta_2 (u_{\text{target}} - w_r) \\ \tilde{\theta}_r \leftarrow \tilde{\theta}_r + \epsilon_1 \eta_3 \Delta \tilde{\theta}_r \\ A_r \leftarrow A_r + \epsilon_1 \eta_2 \Delta A_r \end{array} \right.$$

This is the learning scheme that has been using stochastic. This learning scheme has been derived by stochastic gradient descent that minimizes the error between the target point and the end point. So, this is how I learn the Jacobian matrix. How I learn my θ_r ? This is my final learning algorithm. My reference vector is learnt by this function that is, you can easily see this is my Kohonen learning algorithm θ_r by this expression where $\Delta \theta_r$ is given by this expression and A_r is given by this expression where, ΔA_r is given by this expression. Where, Δv is given v_{n_1} by v_0 , v_0 is give initial θ_1 θ_2 θ_3 to the camera point then v_0 . Like that, in n steps whatever is the output is v_{n_1} in n_1 states that is my Δv .

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Given this algorithm, now we try to learn a circle, we made the robot to track a circle. So you see that this is circle and the desired one is in blue and the obtained actual points that were covered by the robot manipulator is the green and you can easily see that almost very closely followed. Error point you see in this domain is obtained to the power minus 4. The error is very small tracking of a circular path and in this figure error in tracking.

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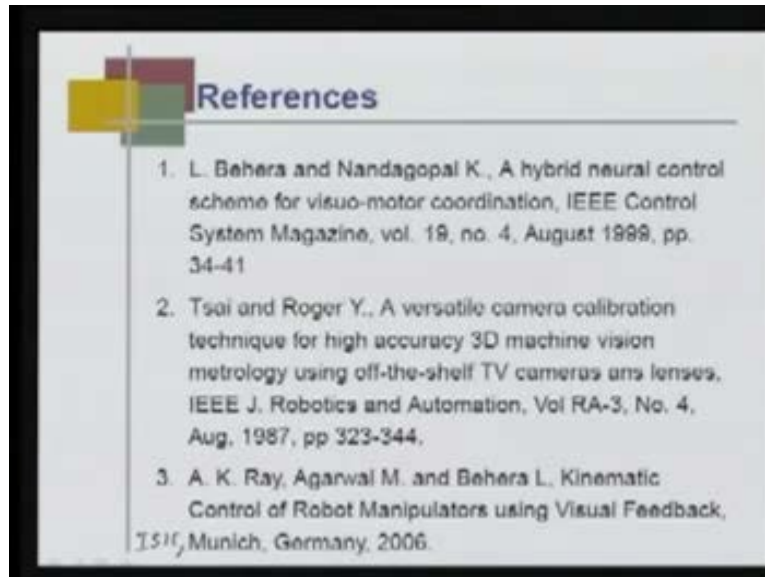
The slide is titled "Summary" and contains a list of topics discussed in the lecture. The text is as follows:

In this lecture, the following topics have been discussed

- Foundation of Visual Motor Coordination
- Various issues on the Camera Calibration Method
- The Extended Kohonen Self Organising Map (EKSOM) and its use in visual motor coordination

In this lecture the following topics have been discussed: Foundation of visual motor coordination, various issues on the camera calibration, Extended Kohonen self organizing map and its use in the visual motor coordination.

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References

1. L. Behera and Nandagopal K., A hybrid neural control scheme for visuo-motor coordination, IEEE Control System Magazine, vol. 19, no. 4, August 1999, pp. 34-41
2. Tsai and Roger Y., A versatile camera calibration technique for high accuracy 3D machine vision metrology using off-the-shelf TV cameras and lenses, IEEE J. Robotics and Automation, Vol RA-3, No. 4, Aug. 1987, pp 323-344.
3. A. K. Ray, Agarwal M. and Behera L., Kinematic Control of Robot Manipulators using Visual Feedback, ISIF, Munich, Germany, 2006.

The references are: one of our papers in IEEE Control System Magazine in 1999. For camera calibration you can follow Tsai's paper in 1987 IEEE journal Robotics and Automation. Also we have another paper regarding this work on international symposium on Intelligent Control, Munich, Germany 2006. Finally, I end this lecture showing you two movie files that will show you how the self organizing of the input space takes place and finally how robot really learns through steps reaching the target point.

Thank you.