Intelligent Systems and Control

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Module – 2 Lecture – 3

Fuzzy Rule base and Approximate Reasoning

Today, we will be discussing fuzzy rule base and approximate reasoning. This will be lecture 3 in module 2 of Intelligent Control, which is Fuzzy Logic in module 2. Last class, we discussed the relation. We learned what a fuzzy relation is. Today, we will be dealing with the inference mechanism from a fuzzy rule base. What is a rule base?

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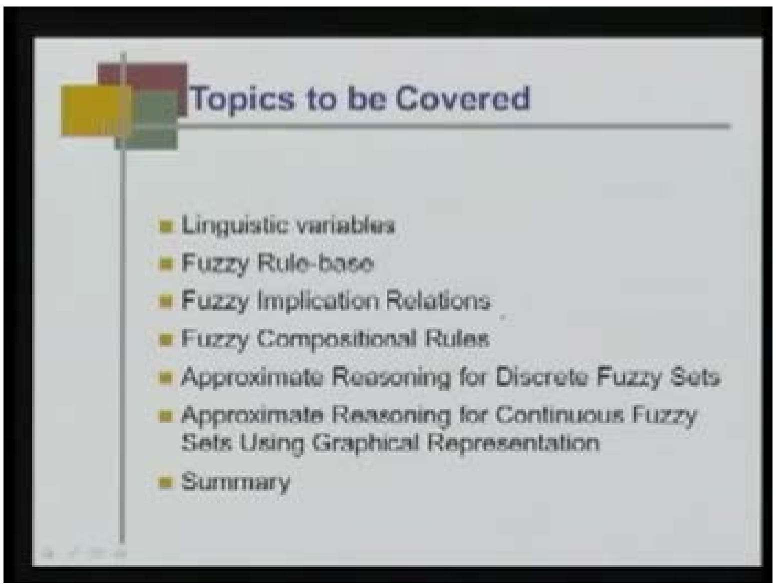

Topics to be covered today are linguistic variables, fuzzy rule base, fuzzy implication relations, fuzzy compositional rules, approximate reasoning for discrete fuzzy sets, approximate reasoning for continuous fuzzy sets, and summary.

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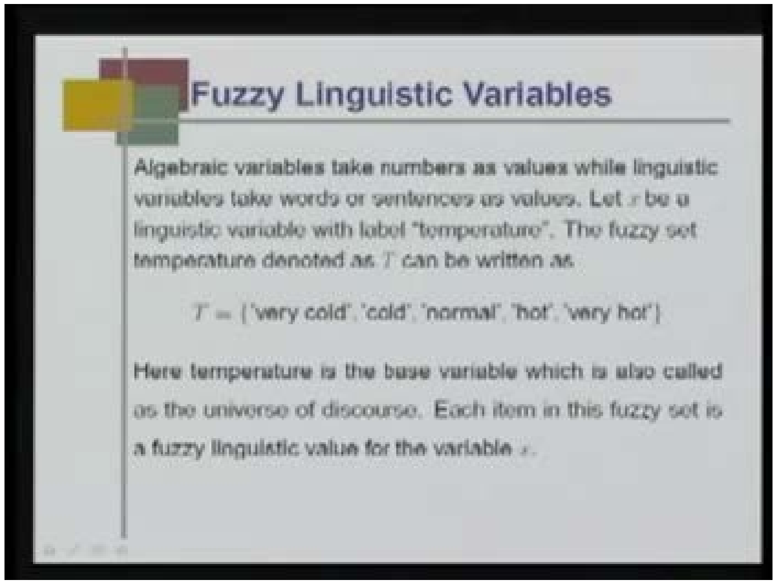

What is fuzzy linguistic variable? Algebraic variables take numbers as values, while linguistic variables take words or sentences as values. You already know that if I have a variable x, then if it is an algebraic variable, it may take natural numbers or real numbers depending on what kind of variable it is. Similarly, a fuzzy variable x will take values that are linguistic values. For example, let x be a linguistic variable with a label ‘temperature’. The universe of discourse is temperature. In that universe, I am looking at a fuzzy variable x when I describe the temperature. The fuzzy set temperature denoted as T can be written as T = very cold, cold, normal, hot or very hot.

From place to place, people may use different connotations for describing the temperature but we have taken for example's sake this particular set to describe temperature in natural language. Looking at the temperature, one may say it is very cold or cold, normal, hot or very hot. Here, temperature is the base variable, which is also called as the universe of discourse. Each item in this fuzzy set is a fuzzy linguistic value for the variable x. If temperature is a fuzzy variable, then the linguistic values for this fuzzy variable are very cold, cold, normal, hot, and very hot.

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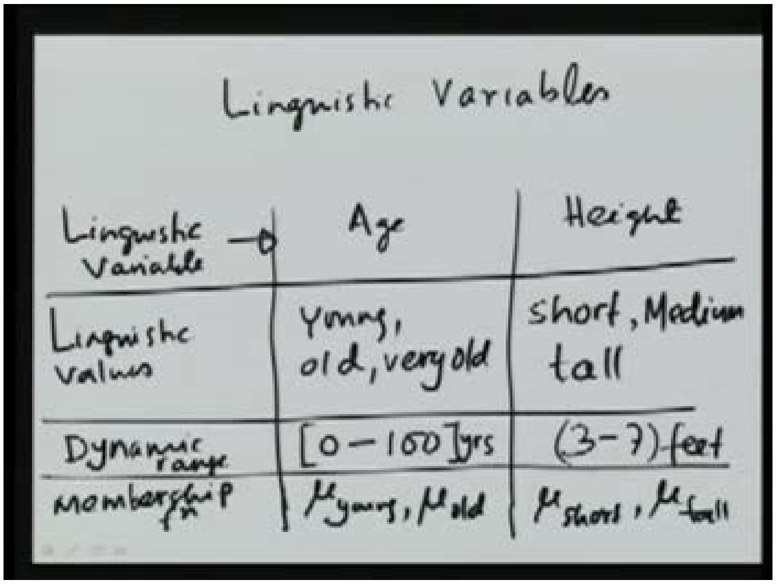

We will take some other examples of linguistic variables. For example, we can talk about age, which is one linguistic variable; height is another linguistic variable. Let me say that this is the linguistic variable. One is age and another is height. Then, I can say that the linguistic values for age would be young, old, and very old. This would be the linguistic value for age, whereas for height, we can say short, medium, or tall.

These are the linguistic values for height and for each category, we can say there is a defined dynamic range. For age, we can define of course 0 to 100; very rarely, people live for more than 100 years. We can say 0 to 100. If I say young, probably the prime should be… anybody who is less than 25 years would be young. I can define old to be around 50 years and very old as those who are around 70 years. Like that, we can define the dynamic range for age whose linguistic values are young, old, and very old. This is years (Refer Slide Time: 06:26).

Similarly, we can easily say 3 to 7 feet for height – short, medium, and tall. We rarely see a person who is less than 3 feet and also persons who are above 7 feet are very rare. Between 3 and 7 feet, we can define the dynamic range for the height and the linguistic values can be ascribed as short, medium, and tall, and then after dynamic range, we define membership function. How do we define?

Given an age, we have to find out how much that membership… – young or…. For example, if somebody's age is 40, how young is he? The membership value may be for a 40 year old, we can say how young he is 0.5 and for how old he is – maybe, he is in the middle between young and old. So, we can say 0.5 membership mu_{old}. A 40-year-old person may be ascribed as a membership function under the category young to be 0.5 and membership function onto the category old to be 0.5. Similarly, for height also, we can define mu_{short} membership function and mu_{tall} and so on. Given a crisp value for age or for height, as many linguistic values are there, we get that many membership functions. For each linguistic value, we get a specific membership function.

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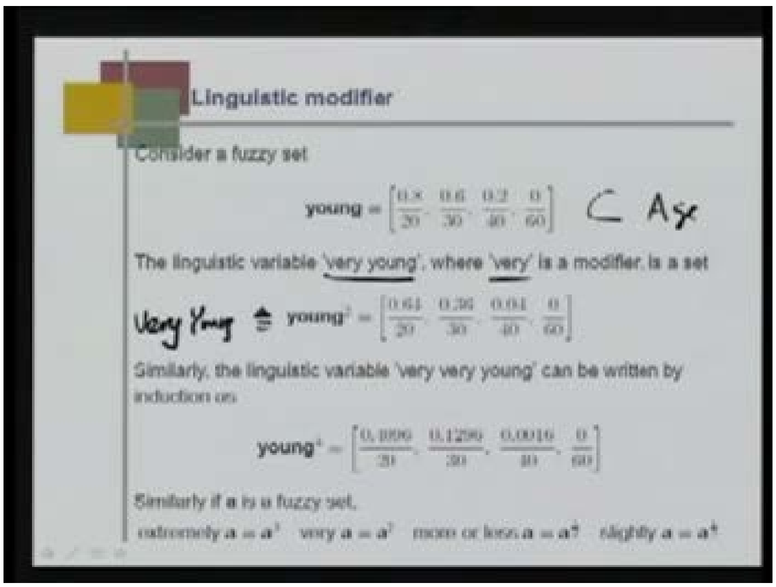

Once we talked about linguistic values, now we will introduce another term called linguistic modifier. Consider a fuzzy set young. Now, if you look at the set young, those who are 20 years old the membership function young is 0.8, for 30 years old it is 0.6, for 40 it is 0.2, and 60 obviously is no more young, so 0. If a fuzzy set young is given, which is a subset of a fuzzy set called age…. I can say this is a subset of age. Age is the superset or the universal set for defining young. Now, the linguistic variable very young where very is the modifier…. Once I define what young is, in very young, very is the modifier. So, we can probably define that their membership function as very young to be young square, that is…. (Refer Slide Time: 10:07) What does young square mean? This membership function gets squared, that is 0.8, becomes 0.64, 0.6 becomes 0.36, 0.2 becomes 0.4, and 0 is 0.

Given a specific set, if I ascribe to the set and the modifier, probably we can define some kind of rule by which the very young can be defined. It is not that always very young will be young square. This is just one way to evaluate a linguistic value very young given young. Similarly, the linguistic variable very very young can be written by induction as again young to the power of 4. If I define very young to be young square, then very very young has to be young to the power 4. Obviously, in that case 20 is 0.4096, which is 0.64, the membership function square. You define a 0.64 square is 0.4096, 0.36 is 0.1296, 0.04 is 0.0016 and similarly 0 is 0. This is the notion of linguistic modifier.

Another example. Similarly if a is a fuzzy set, I can define extremely a to be a cube, very a is a square, more or less is a to the power half and slightly a is a to the power 1 upon 3, so cube root. So, cube root of a, square root of a (Refer Slide time: 12:06), a square, and a cube, depending on whether the modifier is extremely or very or more or less or slightly. This is where we can deal with linguistic modifiers.

We gave some idea about the linguistic values and linguistic variables. These are necessary because in the traditional sense, when we express worldly knowledge, we express them in natural language. So here it is. From computational perspective, such worldly knowledge can be expressed in terms of rule base systems.

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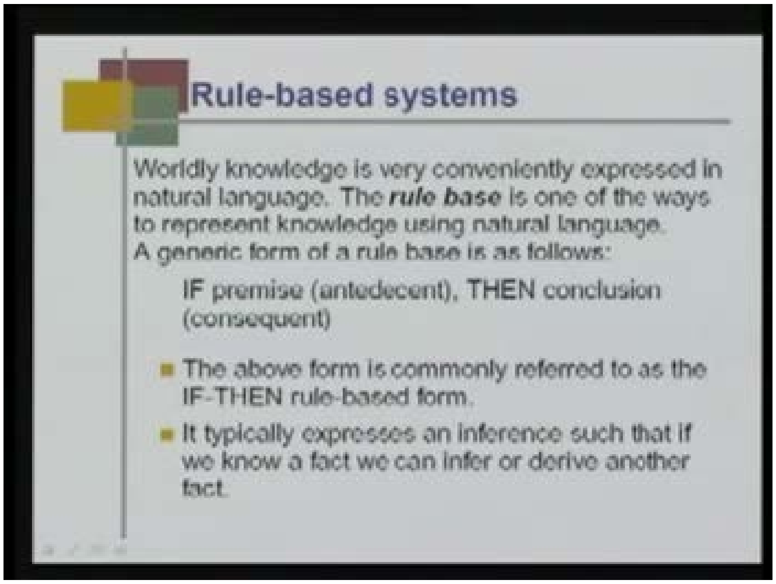

Worldly knowledge is very conveniently expressed in natural language. When we describe the worldly knowledge, natural language is the best way to describe them. The rule base is one of the ways to represent knowledge using natural language for computational purpose of course. A generic form of a rule base is as follows. What we said is that the worldly knowledge can be represented in terms of rule base and a rule base is described as if premise or antecedent, then conclusion is consequent.

The above form is commonly referred to as the IF-THEN rule-based form. It typically expresses an inference such that if we know a fact, we can infer or derive another fact. Given a rule, I can derive another rule or given a rule, if I know a rule and the associated relation, then given another rule, I can predict what should be the consequence.

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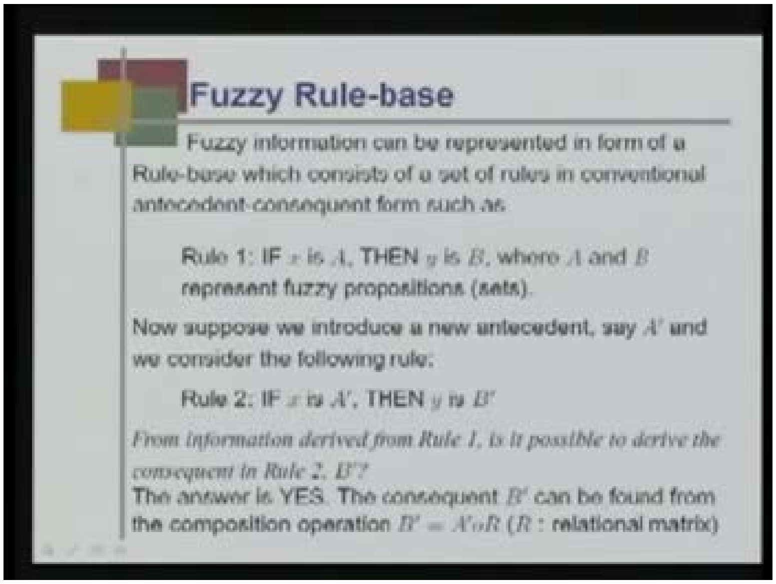

This is a fuzzy rule base. Any worldly knowledge can be expressed in form in the form of a rule base. Now, when I talk about fuzzy rule base, fuzzy information can be represented in the form of a rule base, which consists of a set of rules in conventional antecedent and consequent form such as if x is A, then y is B, where A and B represent fuzzy propositions (sets). Suppose we introduce a new antecedent say A dash and we consider the following rule if x is A dash, then y is B dash, from the information derived from rule 1, is it possible to derive the consequent in rule 2, which is B dash?

This is the question that we are trying to answer in a fuzzy rule base. A fuzzy rule base consists of a set of rules. From these rules, if I know rule 1, if I know what A is, what B is, if I have derived them, and if I have the knowledge of A dash, can I compute what is B dash? This is a very simple way. Now, I am presenting the problem to all of you, that is, we know rule 1. We know what is the set A and set B. In rule 2, we know only A dash but not B dash. Can we infer B dash? This is the question. The answer is yes. The consequent B dash in rule 2 can be found from composition operation B dash equal to A dash… this is called the compositional rule of inference, the compositional operator with R.

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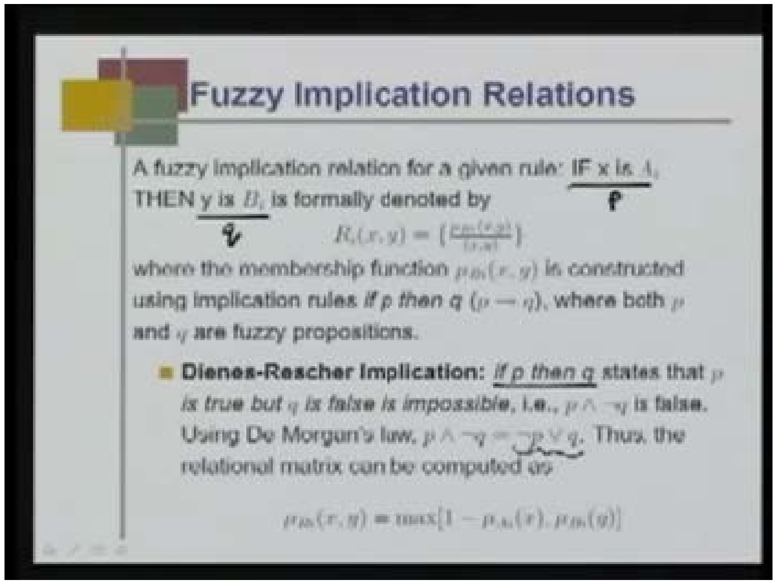

We talked about fuzzy rule base. It consists of a set of rules. Now, how do we infer knowledge from this rule base? There are certain steps. First, we will understand what fuzzy implication relations are. A fuzzy implication relation for a given rule if x is A_i, then y is B_i is formally denoted by R_i. x, y is the relation matrix whose elements are given by mu R_i x, y and this mu R_i x, y is constructed or is computed by various implication rules. There are various kinds of implication rules. Now, we will understand them.

What is the implication rule? If p then q. This is called implication rule, where both p and q are fuzzy propositions. If x is A_i, then y is B_i, but p also can be multiple propositions. If x_1 is A_1, x_2 is A_2 and so on, then what is y? Let us take this simple relation, fuzzy rule. If x is A_i, then y is B_i. I will say this is p (Refer Slide Time: 18:56) and I will say this is q. If p, then q.

Now, we will talk about Dienes–Rescher Implication. In this, if p then q states that p is true but q is false is impossible, that is, in this proposition, in this rule, if I say if p is true, then to say that q is false is impossible. That means if p is true, then q is false is a false statement, it is not possible. This is one argument. What does it mean? That means p which is true and not q (q is false, so not q is true) is false. Using De Morgan's Law, we can show that p and not q is the same as not p or q. Thus, the relational matrix can be computed for this particular relation not is mu Ri x, y is maximum. This is maximum because or means maximum and not p means 1 minus mu$_{Ai}$ x, p is if x equal to A_i. Obviously, the membership function given a crisp value is mu$_{Ai}$ x, where x is the specific crisp value. mu$_{Ai}$ x is the fuzzy membership function of p. not p is 1 minus mu$_{Ai}$ x and similarly for q, the membership function is mu$_{Bi}$ y. The maximum value comes because of the or operation – this or this (Refer Slide Time: 21:29). Obviously, the relation from A to B is defined by this expression.

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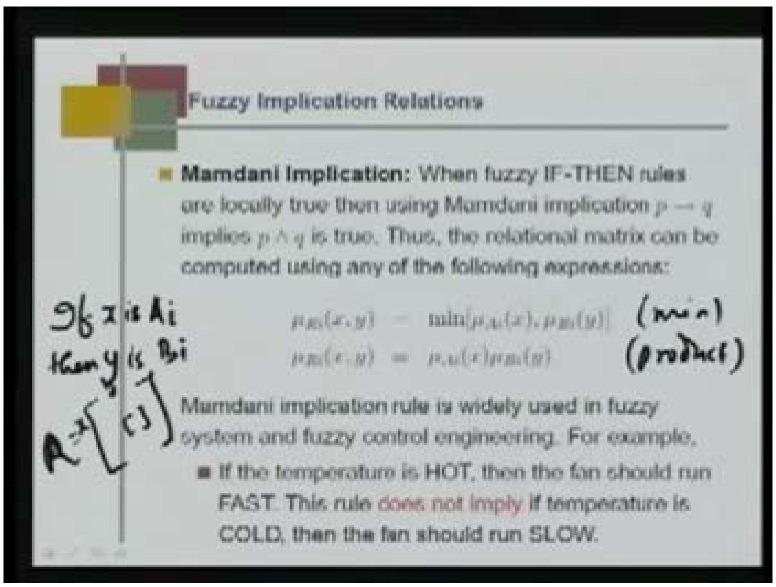

We will go to another type of implication relation, which is very popular in control engineering as well as fuzzy systems. It is called Mamdani implication. In this, when fuzzy IF-THEN rules are locally true, then using Mamdani implication, p implies q implies p and q is true. This is the AND operation; p and q are both simultaneously true. That is because we say that each rule is locally true. If p is true, then q is true. We do not ascribe any other means by which q can be true. Thus, the relational matrix can be computed using any of the following expressions.

We have already explained the first one in the relation class that this is min operation or product operation. This is a min or product. What you are seeing is that I find out the element of the relation matrix associated with the rule p implies q is that the minimum of mu$_{Ai}$ x and mu$_{Bi}$ y using Mamdani implication…. If x is A_1, then y is B_1 or here, we have written A_i and B_i. If x is A_i, then y is B_i. The relation matrix associated with this A which is R, the relation matrix we can put like this where… This is my x (Refer Slide Time: 24:19), this is my y, so any typical element associated with x_i is minimum of mu x in fuzzy set A_i and mu y in fuzzy set B_i. Each element in R is computed as using computed using either this formula (Refer Slide Time: 24:43) or this formula. The Mamdani implication rule is widely used in fuzzy system and fuzzy control engineering. In this, if the temperature is hot, then fan should run fast. This rule does not imply if temperature is cold, then fan should run slow; this means each rule is independent, they are locally true, we cannot infer.

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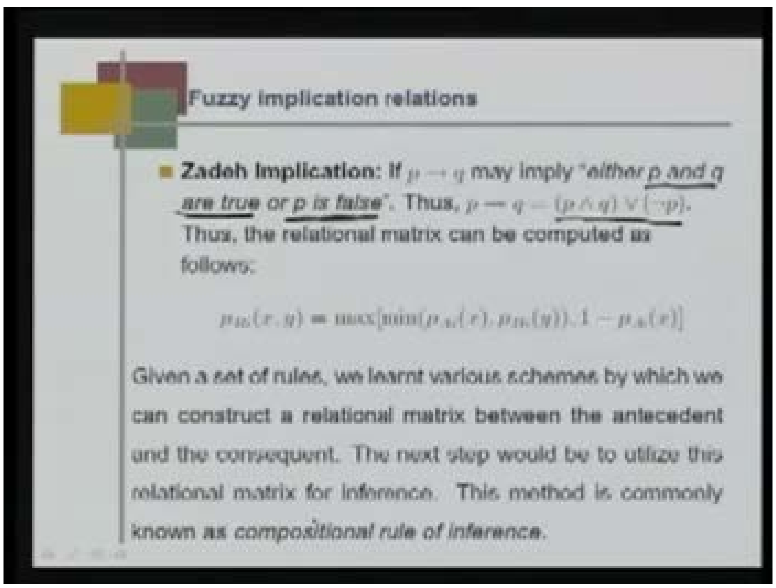

A fuzzy implication relation is another category, which will call Zadeh implication. This is if p implies q may imply either p and q are true or p is false. What we are saying is that just like a local Mamdani rule, we say p and q are true imply either p and q are true or p is false. Thus, p implies q means…. p and q are simultaneously true, which is Mamdani local rule or if p is false, then p implies q has no meaning or p is false. This has taken an extra logic that is p and q or not p.

Thus, the relational matrix can be computed as follows. If I look at this, what is p and q? p and q means minimum of mu_A x and mu_B y. What is not p? 1 minus mu_{Ai} x. This entire thing (Refer Slide Time: 26:50) has to be maximum of minimum of these and this, which is this statement. mu, the relational matrix elements are computed using this particular expression. Given a set of rules, we just learnt various schemes by which we can construct a relational matrix between the antecedent and the consequent. The next step would be to utilize this relational matrix for inference. This method is commonly known as compositional rule of inference, that is, associated with each rule we have a relational matrix. So, given a rule means given a relational matrix and given another antecedent, how can I compute a consequent? This is the prime question that we are asking.

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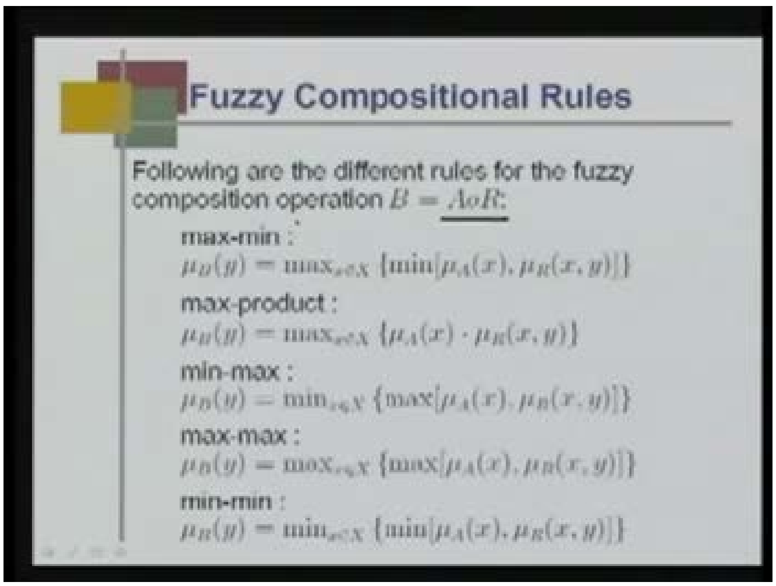

This is derived using fuzzy compositional rules. The following are the different rules for fuzzy composition operation, that is, B equal to A composition R. R is the relational matrix associated with a specific rule, A is a new antecedent that is known, R is known, B is the new consequent for the new antecedent A. I have to find out what is B for this new A, given R. That is computed by A composition R and we have already discussed in the relation class that there are various methods and max-min is very popular.

First, we compute min and then max. Similarly, max-product: instead of min, we take the product and compute what is the maximum value. Similarly, min-max: instead of max-min, it is min-max. First, max and then min. Next, max-max and min-min. One can employ these looking at the behavior of a specific data.

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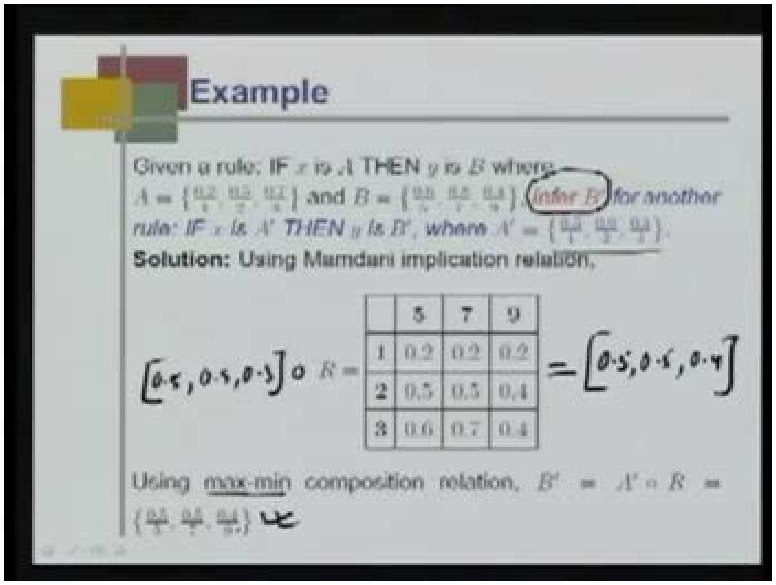

Now, we will take an example. We are given a rule if x is A, then y is B, where A is this fuzzy set: 0.2 for 1, 0.5 for 2, and 0.7 for 3. This is a discrete fuzzy set. B is another fuzzy set that defines fuzzy membership 0.6 for 5, 0.8 for 7, and 0.4 for 9. The question is infer B dash for another rule if x is A dash, then y is B dash, where A dash is known. A is known, B is known, and A dash is known. What we have to find out is what B dash is. Infer B dash is the question that is being asked (Refer Slide Time: 30:21). Hope you understand.

Using Mamdani implication relation, first we will find out between A… the first rule, that is, if x = A, then y is B. The relational matrix associated with this rule is…. For R, how do we compute? A elements are 1, 2, and 3 and B elements are 5, 7, and 9. We have to find out now for 0.2. Here, we compare with all the elements in point B and with each element, we found what the minimum is. The minimum is always 0.2. Hence, the maximum of that is always 0.2. I have to find out the relational matrix between A and B.

The Mamdani principle means minimum, so between 1 and 5, 1 is associated with 0.2, and 5 is associated with 0.6, so the minimum is 0.2. Similarly, 1 is associated with 0.2, 7 is associated with 0.8, so for 1 and 7, the minimum is 0.2. Similarly, 1 is associated with 0.2, 9 is associated with 0.4, so from 1 to 9, the minimum membership is 0.2. Similarly, you can see that from 2 to all the elements 5, 7, 9, the minimum are 0.5, 0.5, and 0.4. Similarly, from 3 to 5, 7, and 9, we have 0.6, 0.7, and 0.4. These are the minimum fuzzy memberships between an element in A to element in B. That is how we compute the relational matrix.

Once we compute the relational matrix, then we use max-min composition relation to find out what is B dash, which is A dash (which is 0.5, 0.9, and 0.3) composition R and you can compute. This is my R. I have to find out my matrix. This is 0.5, 0.9, and 0.3. So this composition R (Refer Slide Time: 33:09) is… you can easily see I take this row vector, put along the column matrix and I see what is the minimum for each case. You can easily see 0.2 will be minimum here, 0.5 will be minimum here, 0.3 and maximum is 0.5.

The first element is 0.5. Again, I take this place in parallel with this column and then, I find first minimum here is 0.2, here 0.5, here 0.3 and then maximum is again 0.5. Again, I take the same row vector, put along this column vector and then, I find here the minimum is 0.2, here minimum is 0.4, here minimum is 0.3 and the maximum is 0.4. This is the relation, this is the answer (Refer Slide Time: 34:05). This is our B dash. Given A, this is my B dash using fuzzy compositional principle or relation.

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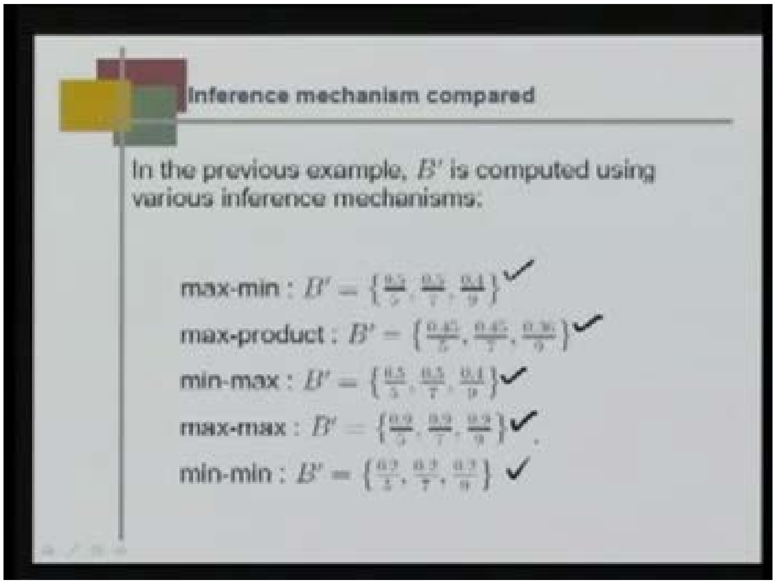

There are other mechanisms also that we discussed. For the same example, if you use max-min, you get B dash; for max-product, you get another B dash; for min-max, you get another. Min-max and max are same for this example. Then, for max-max, you see that all the fuzzy membership are the maximum values and for min-min, they are the minimum values here. Now, the question is what is the approximate reasoning? Approximate reasoning means given any logical system, we do not have, it is very difficult to make an exact result. That is why from engineering perspective, we are more liberal. We do not want to be so precise. As long as our system works, we are happy; if our control system works, we are happy.

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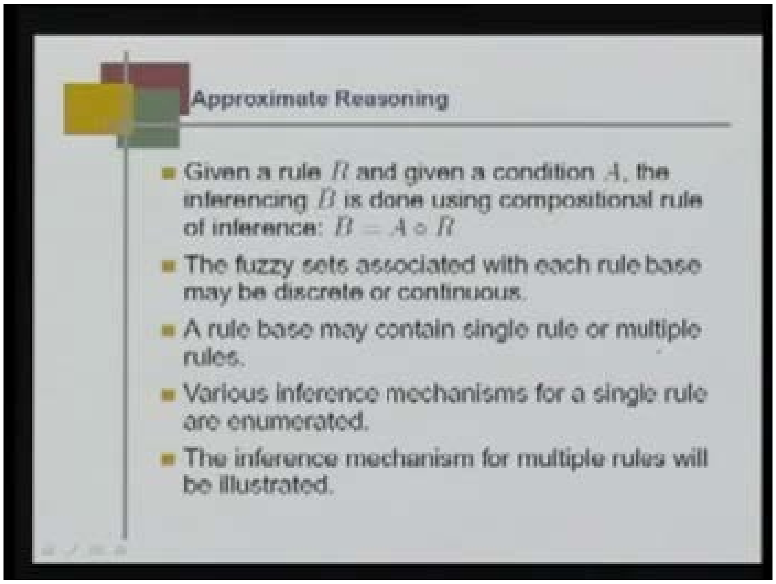

Approximate reasoning. We have set up rules so we use a specific compositional rule of inference and then we infer the knowledge or the consequence. Given a rule R (R is the relational matrix associated with a specific rule) and given a condition A, the inferencing B is done using compositional rule of inference B equal to A composition R. The fuzzy sets associated with each rule base may be discrete or continuous, that is, A may be discrete or A and B may be discrete or continuous.

A rule base may contain a single rule or multiple rules. If it is continuous, I cannot define what the R relational matrix is. It is very difficult because it will have infinite values. R is not defined. That is why for continuous, we apply compositional rule of inference but the method to compute is different. A rule base may contain single rule or multiple rules. Various inference mechanisms for a single rule are enumerated. Various mechanism means we talked about min-max, max-min, max-max, min-min and so on. The inference mechanism for multiple rules….

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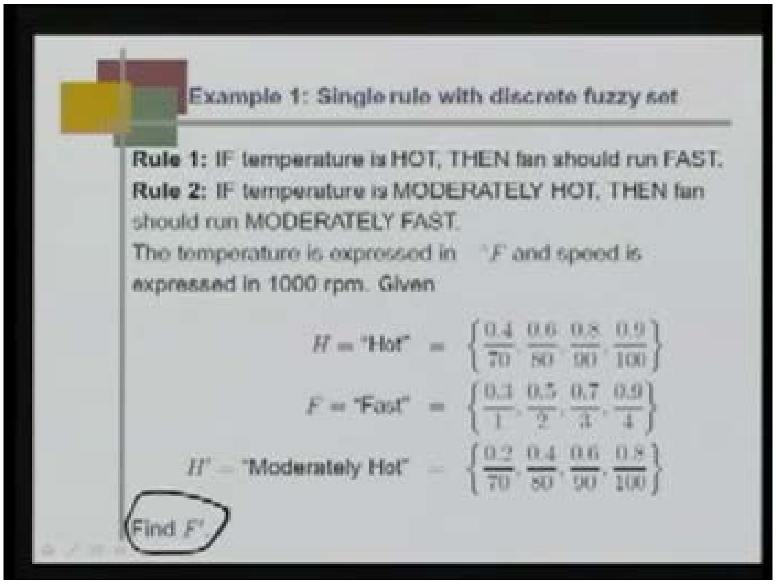

Single rule. Now, we will take the examples one by one. Single rule with discrete fuzzy set. We talked about a fuzzy set that may consist of a single rule or multiple rules. It can be discrete fuzzy set or a continuous fuzzy set. We will try to understand how to make approximate reasoning for such a rule base using the methods that we just enumerated. For each rule, we compute what is the relational matrix if it is discrete fuzzy set and then we use compositional rule of inference to compute the consequence given an antecedent. That is for discrete fuzzy set. We have already talked about this but again, for your understanding, I am presenting another example for single rule with discrete fuzzy set.

Rule 1: If temperature is hot, then the fan should run fast. If temperature is moderately hot, then the fan should run moderately fast. In this example, we are given the temperature is in degree Fahrenheit and the speed is expressed as 1000 rpm. The fuzzy set for hot H is for 70 degree Fahrenheit, 80 degree Fahrenheit, 90 degree Fahrenheit, and 100 degree Fahrenheit, the membership values are 0.4, 0.6, 0.8, and 0.9. Similarly, for the fuzzy set F, for which the fan should run fast, the fuzzy set is for 1000 rpm, the membership is 0.3, for 2000 rpm, the membership is 0.5, for 3000 rpm, the membership 0.7, and for 4000 rpm, the membership is 0.9.

Given H dash, which is moderately hot, to be for 70… moderately hot means it is a little more hot. So, same temperature obviously and their corresponding membership values will reduce, because if I am describing moderately hot, they will have the same temperature but the membership values will be less. You can easily see here that for 70, instead of 0.4, now it is 0.2; for 80, instead of 0.6, it is 0.4; for 90, instead of 0.8, it is 0.6; for 100, instead of 0.9, it is 0.8. This is moderately hot. Now, the question is find F dash.

I hope you are clear with this question. The question is very simple. We are given rule 1, we have defined what is the fuzzy set hot and fuzzy set fast by these two statements and in the second rule for moderately hot, we know the fuzzy set. We do not know what the fuzzy set is corresponding to moderately hot, that is, moderately fast. We do not know (Refer Slide Time: 40:22) moderately fast. Find out F dash. If H, then F. If H dash, then F dash. Find out F dash. First, what do we do?

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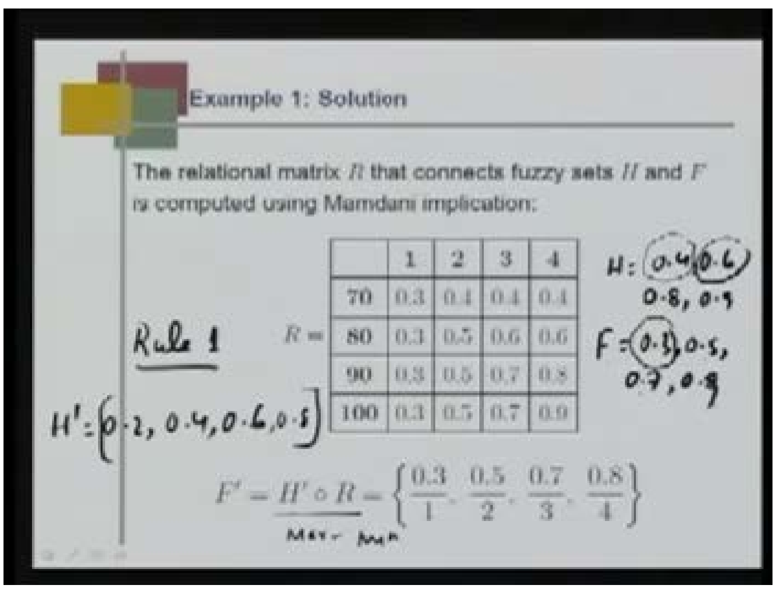

Corresponding to rule 1, we found out what is R. This is for rule 1. We knew that the membership functions for H were 0.4, 0.6, 0.8, and 0.9, and for fast, the membership functions where 0.3, 0.5, 0.7, and 0.9. If you look at this, these are my H values, the crisp values: 70 degree Fahrenheit, 80 degree Fahrenheit, 90 degree Fahrenheit, and 100 degree Fahrenheit. This is my speed: 1000 rpm, 2000 rpm, 3000 rpm, and 4000 rpm.

Between 70 and 1000 rpm, the entry would be minimum of these two (Refer Slide Time: 41:57), which is 0.3. Similarly, between 0.4 and 0.5, the minimum would be again 0.4 and then between 0.4 and 0.7, it will be 0.4, and for 0.4 and 0.9, it is 0.4.

Similarly, we go to the next one, which is 0.6. For 0.6, 0.3 minimum 0.3, for 0.6 and 0.5, the minimum is 0.5, for 0.6 and 0.7, minimum is 0.6, for 0.6 and 0.9, it is 0.6. Similarly, you can fill all other cells here with their values: 0.3, 0.5, 0.7, 0.8, 0.3, 0.5, 0.7, and 0.9. This is my relation matrix associated with rule 1: if H, then F. Now, what I have to do is I have to find out F dash given H dash, using the fuzzy compositional rule of inference, which is represented like this.

F dash is H dash compositional rule of inference with R. This is max-min composition operation. First, we take the min and then compute. H dash is given as 0.2, 0.4, 0.6, and 0.8. This is my H dash (moderately hot) and I have to do compositional inference between H dash and R. Again, I am repeating so that you understand how to compute it. You put this row vector along this column vector first (Refer Slide Time: 44:03). For each element, you find out what is the minimum. You see that here it is 0.2, 0.3, 0.3, and 0.3 and the maximum of that is 0.3.

Similarly, you take again these values and put them here vertically. Here, the minimum is 0.2, here 0.4, here 0.5, here 0.5, and maximum is 0.5. I am sure you will see here it is 0.7, but in this case, you find that if you take this here, it is 0.2, here 0.4, here 0.6, here 0.8, and maximum is 0.8. F dash is 0.3, 0.5, 0.7, and 0.8. That is how we infer or we do approximate reasoning for a rule base. This is a very simple case.

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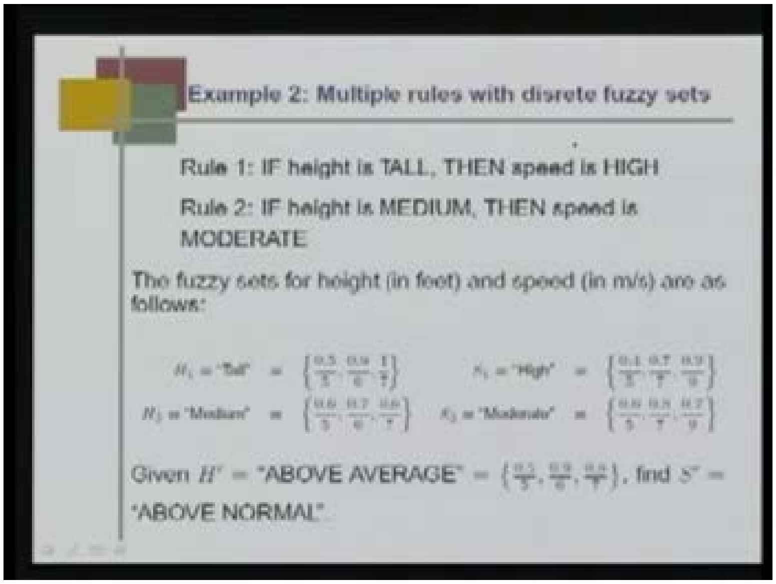

We will go to a more difficult one, which is multiple rule with discrete fuzzy sets. There are two rules now. Rule 1 is if height is tall, then speed is high. Rule 2: if height is medium, then speed is moderate. This is describing a rule for a person as to how fast he can walk. Normally, those who are tall can walk very fast and those who are short, naturally their speed will be less. This is one fuzzy rule that expresses the speed of a person while walking. If height is tall, then speed is high and if height is medium, then speed is moderate. For this, the fuzzy memberships are defined as tall, high, medium, and moderate.

Tall is 0.5, 0.8, and 1 for various feet like 5, 6, and 7. For speed is high, for 5 meter per second, 7 meter per second, and 9 meter per second, the corresponding membership values are 0.4, 0.7, and 0.9. For H_2, which is medium height, the corresponding fuzzy membership… you can easily see that when I say medium in this fuzzy set, 5 has 0.6, 6 has 0.7, and 7 has 0.6. The moderate speed is 0.6 for 5 meter per second, 0.8 for 7 meter per second, and 0.7 for 9 meter per second. If this is the fuzzy set given, now the question is given H dash, which is above average, and the corresponding fuzzy set is 0.5, 0.9, 0.8 for three different heights, find S dash, the speed above normal. I hope the question is very clear to you.

We have two rules. If height is tall, then speed is high; tall is defined and high is defined. If height is medium, then speed is moderate. I have already defined the fuzzy sets for both medium as well as moderate. They are all discrete fuzzy sets. Now, you are presented with new data and what is that new data? You are presented with a data called above average, which is 0.5, 0.9, and 0.8 for three different heights for 5, 6, and 7. Then, find S dash equal to above normal, that is, if height is above average, then the speed should be above normal. How do we do it?

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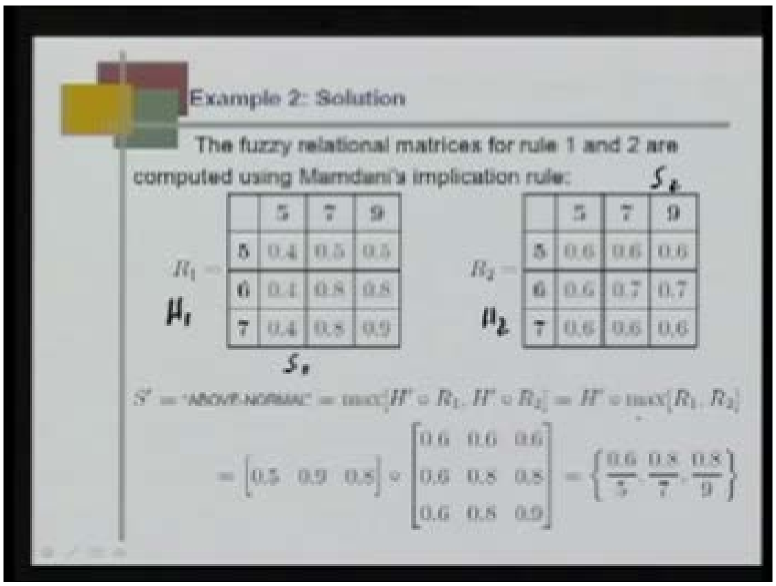

This is the solution of this example. We have two rules. Naturally, we will have two relational matrices: R_1 for rule 1 and R_2 for rule 2. I will not go in detail of how we compute. You simply you go the antecedent and consequent, look at the membership function, find the minimum for each entry. Here, these are the heights and these are the speeds; 5, 6, 7 feet is the height and 5, 7, and 9 meter per second are the speeds of the individuals.

Now, you check the fuzzy sets and corresponding to each fuzzy set, find out what is the minimum membership function. For 5, 5, you will find the membership function is 0.4, minimum 0.5, 0.5, 0.4, 0.8, 0.8, 0.4, 0.8, 0.9. You can verify this. Similarly, R_2 can be found out. Taking the minimum membership entry between these two fuzzy sets, that is, if I say this is H_1 (Refer Slide Time: 49:45) and S_1 and this is H_2 and S_2. Look at these two fuzzy sets, find out what the minimum entries are for each relation and then, how do we compute S dash above normal? We have now two relational matrices. It is very simple. We do two composition operations: H dash composition with R_1 (this one) and again, H dash composition R_2 and then, we take the maximum of that, maximum of these two.

You can easily see that the maximum of H dash composition R_1, H dash composition R_2. You can easily see that because H dash is common, this particular expression is the same as H dash composition max of R_1 and R_2. This is R_1 and R_2. We look at all those entries wherever it is the maximum: for 0.4 and 0.6, the maximum is 0.6; for 0.5 and 0.6, the maximum is 0.6; for 0.5 and 0.6, the maximum is 0.6. You see the last element here 0.9 here and 0.6, so this is 0.9. Like that, for all entries of R_1 and R_2, whatever the maximum values, you put these values here (that is called maximum R_1 and R_2) and take a composition with H dash. So H dash composition max of R_1 and R_2. H dash is already given as 0.5, 0.9, and 0.8. If you do this composition, you get 0.6, 0.8, and 0.8. I hope this clears your concept of how we compute or we do approximate reasoning in a rule base. Similarly, if there are multiple rules, we have no problem and we can go ahead with the same principle.

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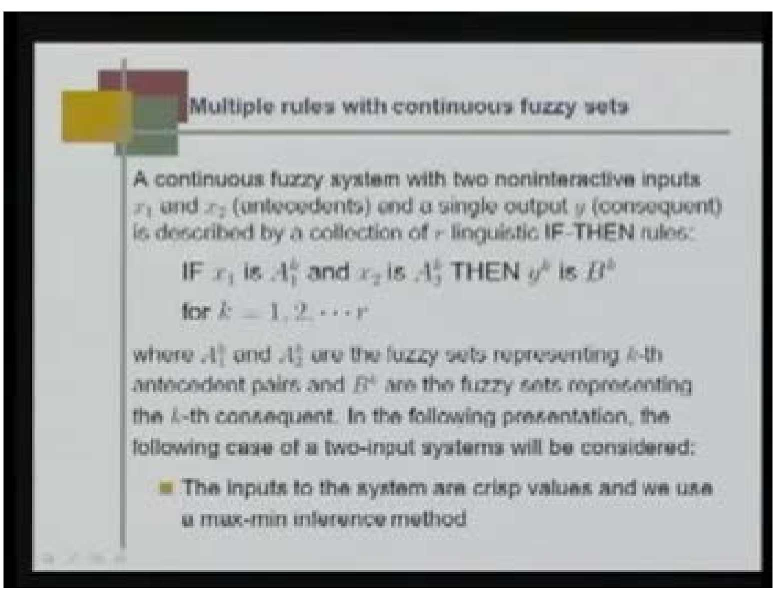

The last section that we will be covering today is the multiple rules with continuous fuzzy sets. We talked about discrete fuzzy set, but if it is continuous fuzzy sets, how do we deal with that? Normally, a continuous fuzzy system with two non-interactive inputs x_1 and x_2, which are antecedents, and a single output y, the consequent, is described by a collection of r linguistic IF-THEN rules Where the rule looks like this: If x_1 is A_1 k and x_2 is A_2 k, then y k is B k, where k is 1, 2 up to r. This is the k th rule. Similarly, we can have rule 1, rule 2, rule 3, up to rule r.

In this particular rule, A_1 k and A_2 k are the fuzzy sets representing the k th antecedent pairs and B k are the fuzzy sets representing the k th consequent. In the following presentation, what we will do now is we will take a two-input system and two-rule system just to illustrate how we infer from a rule base where the fuzzy sets are continuous. The inputs to the system are crisp values and we use a max-min inference method.

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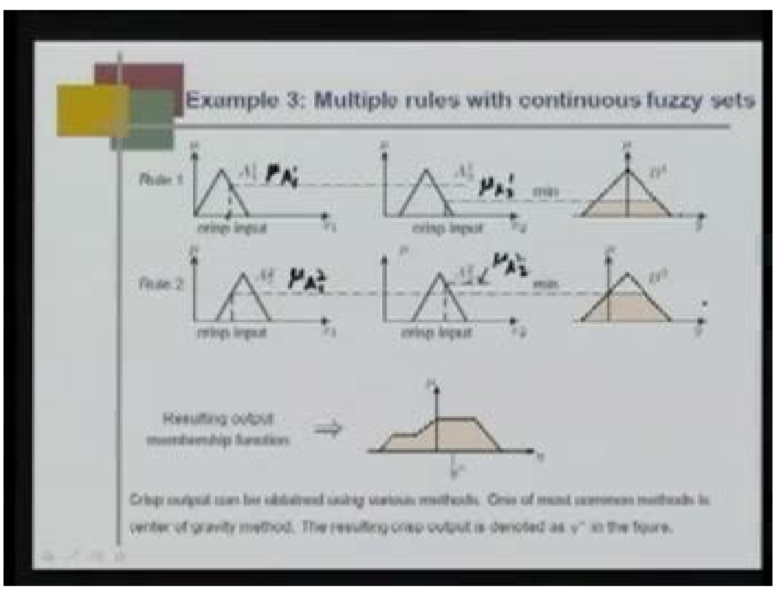

Kindly pay attention, because this is a very important concept that we will be presenting. We have two rules here. You can easily see that we have represented graphically. You can see there are two variables x_1 and x_2. There are two fuzzy variables and for each rule, we have a consequent y. The first rule says that if x_1 is A_1 1 and x_2 is A_2 1, then y is $B_{1.}$

Similarly, if x_1 is A_1 2, x_2 is A_2 2, then y is B_2. Now, how do we infer? Given a crisp input, a new input is given, crisp input in the domain of x_1 and another crisp input in the domain of x_2. There can be a system whose two variables can be temperature as well as pressure. You can easily think x_1 to be the temperature and x_2 to be the pressure. For example, for a particular given system, you found out the temperature to be 50 degrees centigrade and pressure to be some value. Given these two quantities, crisp quantities, how do we infer what should be my y?

To do that, I will just explain how to do this. The crisp input is given – temperature. Now, you find out corresponding membership values here. Corresponding to this crisp input, we get the membership value in rule 1 as mu_{A1} 1 and for the same crisp input, this rule 2 will provide you mu_{A1} 2. Now, in the second fuzzy variable, given crisp input, rule 1 will compute… you see that this one will compute mu_{A2} 1 and for the second one, the second rule, the same crisp input would give this one, which is mu_{A2} 2. Once we find out these membership values, what do we do? We graphically see which is minimum between mu_{A1} 1 and mu_{A2} 1. The minimum is mu_{A2} 1. We take that and we shade these areas in consequence.

Now, we take the second rule. We find between mu_{A1} 2 and mu_{A2} 2, the minimum is mu_{A1} 2. We take that minimum and shade the area and consequent part of this rule 2. Now graphically, we add these two taking the maximum. First, min and then max. You can easily see that when I overlap this figure (Refer Slide Time: 57:14) over this figure, I get this particular figure. You overlap this second figure on the first figure or first figure on the second figure and take the resultant shaded area. After taking this resultant shaded area…. Once you find this shaded area, the next part is to see what is y given a crisp value. There are many methods, but we will focus in this class or in this course on only one method, that is, center of gravity method.

Obviously, if I take this figure and find out what is the center of gravity, it is this value y star. The crisp output can be obtained using various methods. One of the most common method is the center of gravity method. The resulting crisp output is denoted as y star in the figure. This is y star. What we learnt in this is given a crisp input 1 and crisp input 2 and given two fuzzy rules, how do we infer correspondingly a crisp output? Our data is crisp, but we are doing fuzzy computation. Hence, rules are fuzzy. We take this data to the fuzzy rule base and then fuzzify them through fuzzification process. Graphically, we find what is the net shaded area using the max principle. We found out the shaded area for each rule in consequent taking the min principle. Taking the max principle, we found out the resultant area and then, y star is the center of gravity of these areas.

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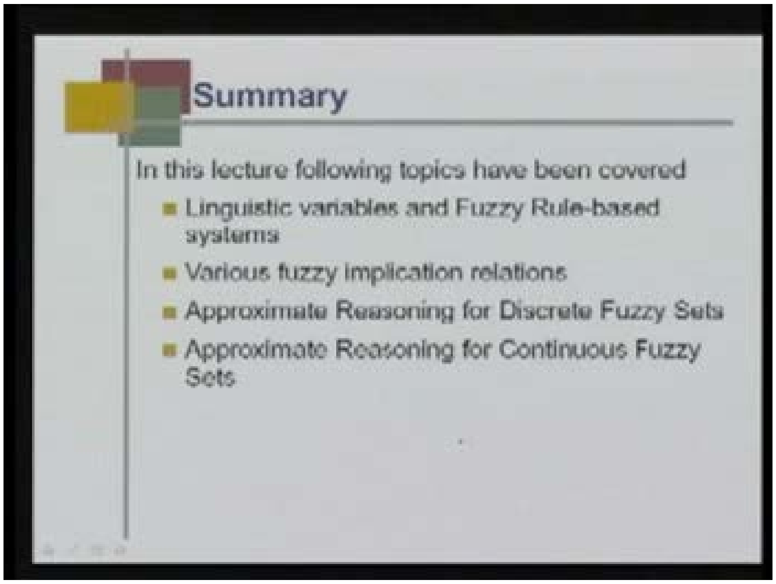

Finally, the summary. In this lecture, we covered the following topics: linguistic variables and fuzzy rule-based systems, various fuzzy implication relations, approximate reasoning for discrete fuzzy sets and then finally, approximate reasoning for continuous fuzzy sets. Thank you very much.