Intelligent Systems and Control

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Module 2 Lecture 2

Fuzzy Relations

This is lecture 2 of module 2 in our course on intelligent control. This is the module that covers the fuzzy logic concepts. Today, we will be discussing fuzzy relations. In the last class, we discussed fuzzy sets and some introductory ideas of fuzzy sets. It was different from crisp set- the concept of membership function fuzzy operations. We gave some examples to illustrate fuzzy operation.

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Today, we will be talking about fuzzy relation.

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Topics that we will be covering today are the extension principle of fuzzy sets, fuzzy relations, projection of fuzzy relations and cylindrical extension of fuzzy relations, fuzzy max min and max product composition operations.

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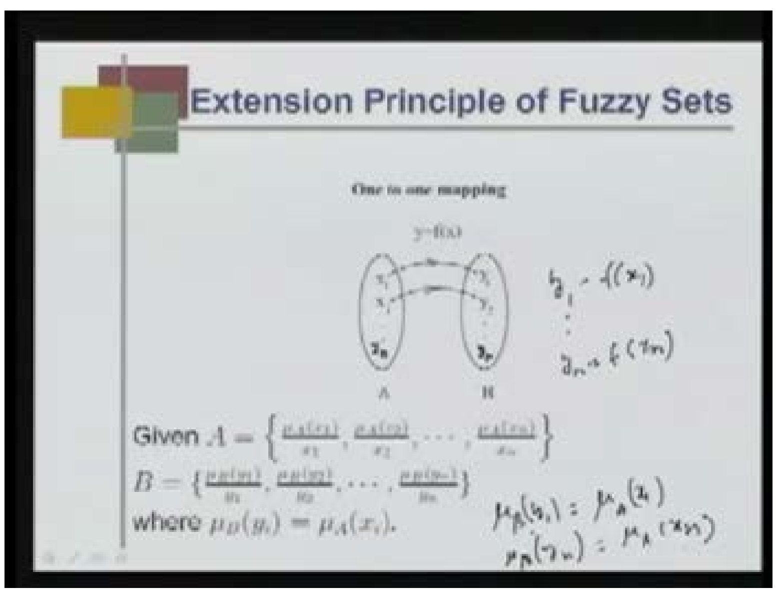

This is the extension principle of fuzzy set. This is our fuzzy set A. As usual, in the last class, we said that fuzzy set is always an ordered pair, where there are member and associated membership function. There are n elements here; x_1 x_2 up to x_n and similarly, y_1 y_2 up to y_n. There is a one to one mapping from set A to set B; that is, y_1 is f of x_1 and on until y_n is f of x_n. This relationship is also valid. That means there is a relationship between the members of the set B with members in set A in the form of y equal to f of x. If that is true, that is the case now. We know that A is a fuzzy set and B is a fuzzy set. The normal way they represent A is mu $_A$ x_1 upon $x_{1,}$ mu $_A$ x_2 upon $x_{2;}$ that means, the members and associated fuzzy membership function. Then similarly, B also; their associated membership function is mu $_B$ y_1 mu $_B$ y_2 mu $_B$ y_n. Then the question is that, do we have to again find what B is, if A is given? No, because we know already y has a relationship with x in terms of y equal to f of x.

If we know, what fuzzy set A is, fuzzy set B is already known. How? y_1 y_2 y_n are f of x_1 until f of x_n. We know what x is; x_1 until x_n from $x_{1,}$ x_2 until x_n. We know y is y_1 up to y_n. All that we have to do is, once we compute what is $y_{1,}$ y_2 up to $y_{n,}$ because, we already know the information; that is, between A and B there is one to one mapping. All that we have to do is that after computing what is y_1 y_2 up to $y_{n,}$ instead of mu $_B$ $y_{1,}$ we just have to replace mu $_A$ $x_{1;}$ that is, here mu $_B$ y_1 is same as mu $_A$ $x_{1,}$ because this is one to one mapping. This is how we compute the fuzzy set B from A given the knowledge of fuzzy set A and the mapping from A to B, if you already know what should be the fuzzy set B.

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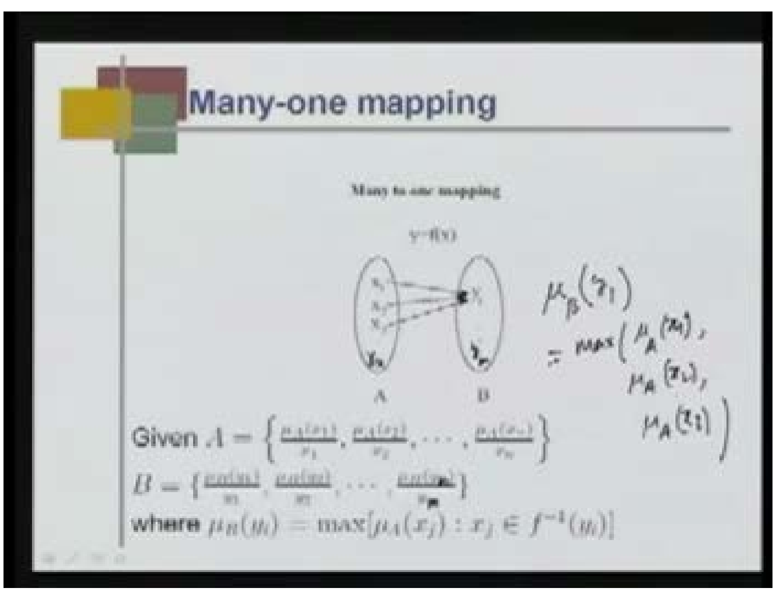

Similarly, many to one mapping: In many to one mapping, there is $x_{1,}$ x_2 up to x_n. They are all mapped to y_1 and so on. That means there are many members in x_1 and they are mapped to the same member in B. Many members in A are mapped; like here, three members in A are mapped to a single member in B. There is many to one mapping through the mapping y equal to f of x. Simple example is y equal to x square. In that case, minus 1 will map to same point in B minus 2 plus 2 will map to same point in B. That is how many to one mapping is. Given A mu $_A$ x_1 up on x_1 mu $_A$ x_2 upon $x_{2,}$ until you know we have the last element x_n in A. This is our fuzzy set A. Similarly, fuzzy set B would be.

If we have an element or let us put y; the membership function associated with $y_{1,}$ y_2 until up to y_n is computed by the relationship y equal to f of x. When the associated member function is computed, what is the maximum membership function associated with corresponding x here; that is, maximum mu $_A$ $x_{j,}$ where x_j is f inverse belongs to f inverse y_i. For example, in this, if I want to find out what mu $_B$ y_1 is, in this case is maximum of mu $_A$ x_1 mu $_A$ x_2 and mu $_A$ x_3. These are the three members in A. They map to same element in B. Hence, the associated membership function with y_1 is the maximum membership function associated with its counterparts in A; that is, in counterpart of y_1 in A is not 1; It is many, because many to one mapping.

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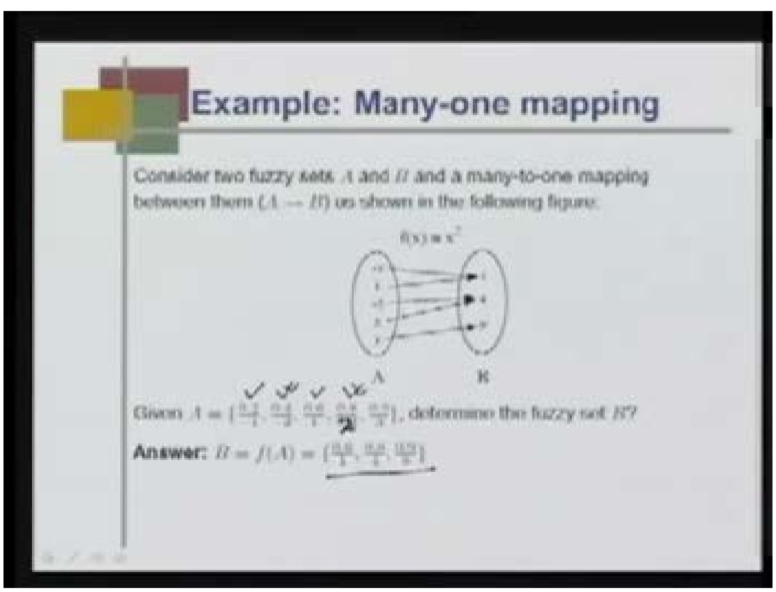

This is the example; like here, as I said f of x equal to x square is a good example of many to one mapping, where minus 1 maps to 1 minus 2 maps to 4 and 3 maps to 9. In this case, if A fuzzy set is given like this, where minus 1 has membership function associated 0.2 minus 2 0.4 1 0.6 0.2. This is not 0.2, this is 2.2 has 0.8 and 3 has 0.9; that is, the membership function B whose members are 1 4 9. Membership function would be like for 1 the maximum membership minus 1. The maximum is 0.64 minus 2 which is 0.4 and plus 2 is 0.8. Maximum is 0.8, and then 9 of course is single one to one mapping. Whatever is here, will come here. 0.9 will come here. Then the answer is this; B equal to the fuzzy set B is 0.6 by 1.8 by 4.9 by 9. That means it has 3 members 1 4 9 and associated membership functions are associated membership index or indices are 0.6 0.8 and 0.9.

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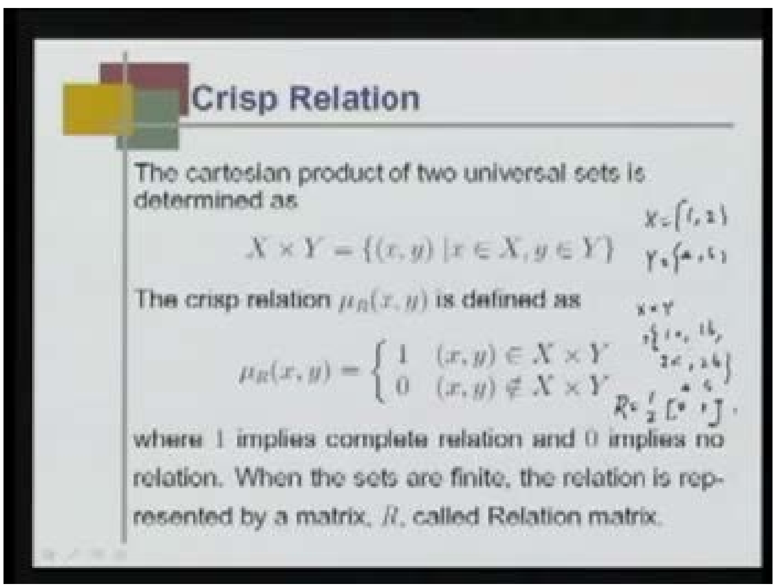

Now, we will be talking about crisp relations. When we talk about relation, that means it is more than one set; that is, two or more than two sets are involved in this process.

Let us take a simple case. You know, we can have multiple sets, but we will first talk about crisp relation before we talk about fuzzy relation. In crisp relation, the Cartesian product of two universal sets is determined; X times Y is this. This Cartesian product is an ordered pair. An ordered pair x and y, where x belongs to X, capital x, and y belongs to capital Y. The crisp relation mu $_R$ x y is defined as this. Crisp relation is actually defined on the Cartesian product space. mu $_R$ x y is either 1 or 0. This is 1 if x y belongs to the Cartesian product and 0 if x y does not belong to the Cartesian product space. What is the meaning of 1? 1 implies complete relation and 0 implies no relation. When the sets are finite, the relation is represented by a matrix R, called relation matrix. I will give you a simple example. Let us say, x is 1 and 2, and y is a and b. Relation means the Cartesian space is of course x times y is a_1 b_2 and this is your Cartesian product space.

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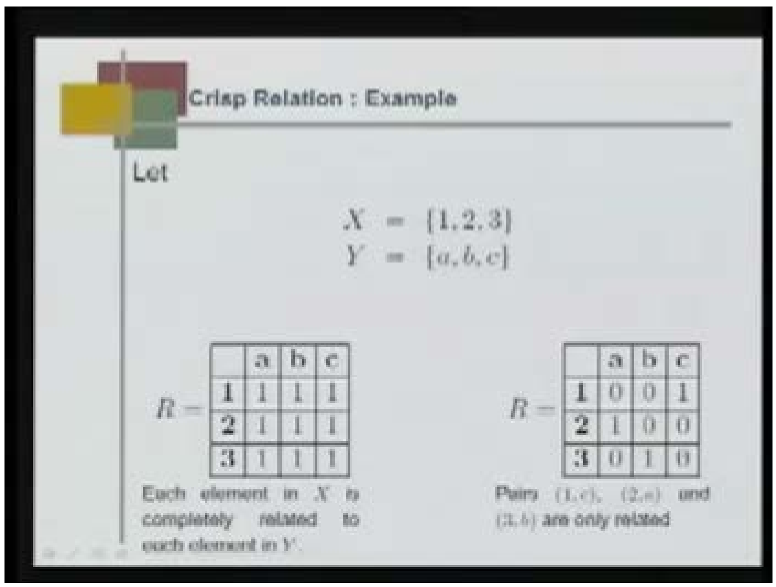

Now, when I say relationship, that is, if 1 has no relationship with a, then the membership function mu R x y is the relation. They are associated with this 0 because, that means 1 is not related to a, but if 1 is related to b and if the relationship is there, then we associate with this 1; that is, in the relations space, the matrix R is the relation matrix which is x is equal to 1 2 and y is equal to a and b.

When I say the relation 1 and a are not related, this means 0, 1 and b are related. 1, 2 and b are not related 0; then this is our relation matrix and because we have finite number of elements in x and y, we can always represent a relation by a matrix. But if it is a continuous set or a set having infinite points, then we have difficulty in terms of representing a relation in terms of matrix. This is a crisp relation; X is 1 2 3 and Y is a b c because, I said earlier which are 2 by 2 two elements in X and 2 element Y and now it is 3 element, x_3 element in Y. If I represent R, then if R is 1, it implies all the elements in X are related with all the elements in Y.

Each element in X is completely related to each element in Y; that is 1. a1 b1 c1, c2 a2 b2, c3 a3 b3 are all completely related; whereas, in case of this particular relationship where we have 0 0 1 1 0 0 0 1 0. It implies 2a 1c and 3b. They are only related and others are not related. So, this is the meaning of crisp relation. A crisp relation means either the relationship index is 0 or 1. There is no in-between relationship. That is the classical relationship matrix.

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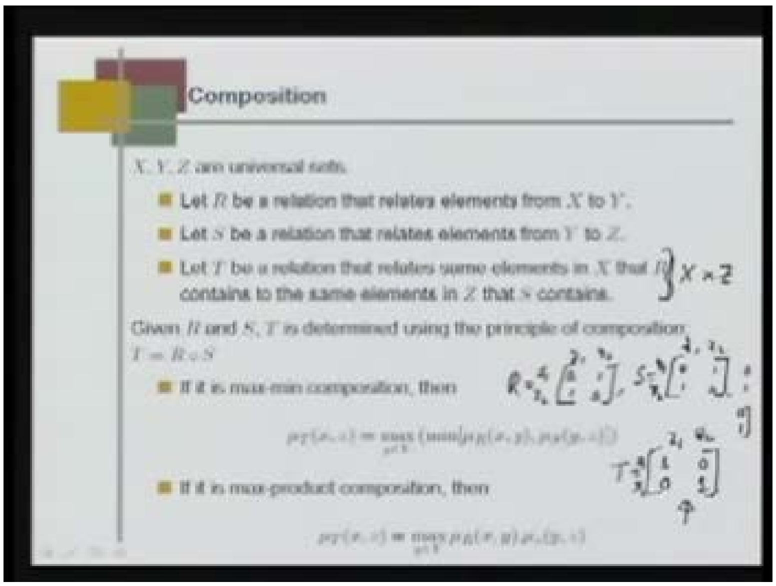

Now, the question is that if we have multiple Cartesian product space and we know the relationship, then we can infer the relationship; for example, here x y z are universal sets. Let R be relation that relates elements from x to y in the Cartesian product space of x and y. R is defined. Similarly, in the Cartesian product space of y and z, S is defined; the relation S is defined. Similarly, T is defined in the relationship. The Cartesian product relationship x and R. T is the relation that relates same elements in x that R contains to the same elements in z. S contains the Cartesian product space. Here, this is x times z. This we can do, if R is given and S is given.

Can I compute T? This is the normal question you would ask. If I know the relation in the Cartesian product space, x y as well as Cartesian product space y and z, can I infer the relation in the Cartesian space x and z? Because x y is known and y z is known, can I know x and z? That is the inference; knowledge inference, information inference. Yes. How do we do it?

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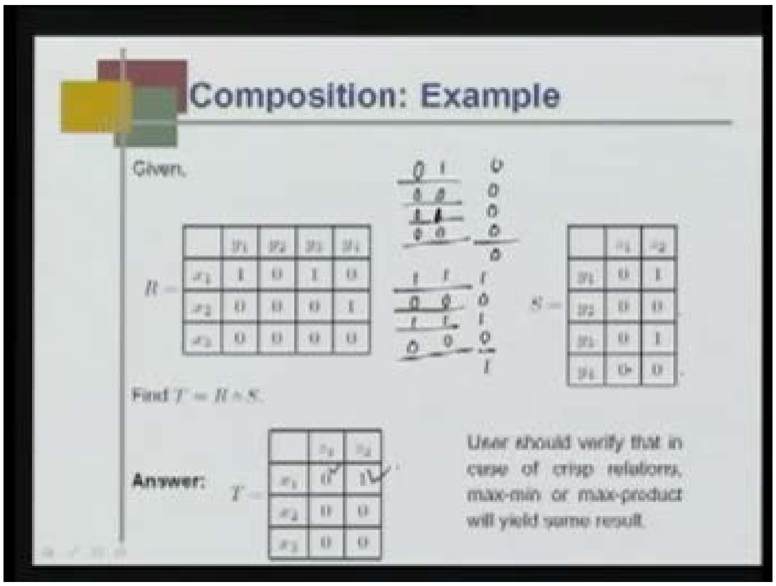

The given R and S, T are determined using the principle of composition, where T equal to R composition S. This composition can be found out either by method one which is max min product composition. In max min composition, the associated membership function muT is computed as a maximum of minimum. mu R x y mu S y z, I will give you a simple example. This is my R and this is my R S and then I have to find out what is T. Let us say R is x x_1 x_2 and y is y_1 y_2. The relationship is 0 1 1 0. Similarly S is y and z y_1 y_2 z z_1 and z_2. Let us say again the relationship is 0 1 1 0. That is the case, now, I have to find out what the relation is between x and z_1 z_2. How do I find out in this max min composition? I want to find out now, what is x_1 and $z_{1,}$ final x_1 and z_1.

How do I find x_1 and z_1? 0 1 you bring here 0 1. It is like a matrix multiplication, we do this row. You put parallel to this column over y_1 and y_2. Then 0 0 is the minimum and this is 0 mu R x y; mu R x y is 0 and mu S y z is 0 and so minimum of that is 0. The first minimum is 0 and then second overall y; similarly, x_1 y_2 x_1 y_2 and y_2 $z_{1.}$ That is 1, x_1 y_2 is 1 and y_2 z_1 is 1. You find that it is very simple. The basic principle is that this 0 1. You put parallel at 0 1 and find the minimum in each case. What I can do is that for 0 1, I have another 0 1 here in parallel, it comes from here and finds the minimum from 0, 0 is 0 and 1 one is 1 and find the maximum. From this, you put, x_1 $z_{1.}$ Similarly, I have x_1z_1. Similarly, x_1 $z_{2.}$ How do I find? Again 0, 1. We put it here 0 1 0 1. Obviously, minimum is 0 minimum is 0 maximum is 0. Similarly, x_2 $z_1$1 0 1 0; this is 0, similarly 1 0 1 0 this is 1.

The max product composition will also lead to the same thing, because, this is crisp relation. In crisp relationship, whether I follow max product or max min, both will come and yield the same result. What is product? This is multiply 0 1 0 1 multiply 0 0 is 0 1 1 is 1 and max maximum of that is 1 here. Similarly, 0 1 1 0, here 0, if you multiply and then take the maximum, that is 0. Similarly, 1 0 0 1, you get 0 and 1 0 1 1. If you multiply, one element is 1, another element 0. The maximum is 1. This is called a composition. The principle of composition - Given two relations R and S, I can compute what T is, using these two rules. How to find out in this case? This is max min composition, this is max product, this dot means product. I multiply mu R x y into mu S y z.

The example again I showed you, is little more elaborated. We see that R is a Cartesian product, where x is 3 dimensional, y is 4 dimensional, S is 4 dimensional; y and z, where y is 4 dimensional and z is 2 dimensional. Obviously, T has to be a relation from x to z which is 3 dimensional, x and 2 dimensional z. Using the compositional rule, you can easily find out that if it is 1 0 1 0, you put here 1 0 1 0. Obviously, in every case minimum is 0 you get 0. Here, the maximum of that is 1 1 0 1 0. You put here 1 0 1 0.

Obviously, this will be 1 here and 0 0 0 1 0 0 0 1. Obviously, in this case, again all the 1 is there, minimum is 0, the total maximum is 0. What we are trying to do is that, you take this row and take this column, you put them like here. This column is 0 0 0 0 and this row is 1 0 1 0. To find out the relationship between x_1 and $z_{1,}$ what I do I take this row which is 1 0 1 0 and this column 0 0 0 and find for each 1 what is minimum.

The minimum in this is 0. Overall, what is maximum? That is 0. This 0 entry comes. Now, I want to find out how this 1 entry came. This 1 entry, to find out these 1 entry x_1 and z_2 x_1 means you take this row which is 1 0 1 0 and z_2. z_2 means the corresponding column that is 1 0 1 0. Now, you individually find out what is the minimum of this. This is 1, this is 0, this is 1, this is 0 and what is the maximum of all these elements? That is 1. This is called max min composition and you can do all max product composition. You will get the same answer. The user should note and verify that in case of crisp relation max min or max product will yield same result.

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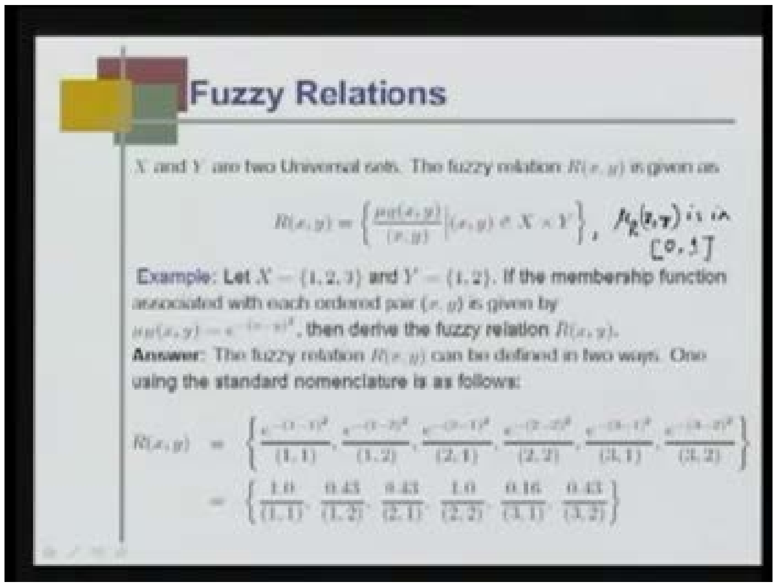

Now, we will be talking about fuzzy relation. If x and y are two universal sets, the fuzzy sets, the fuzzy relation R x y is given. As this is all ordered pair, mu R x y up on x y for all x y, belonging to the Cartesian space x, you associate mu R x y with each ordered pair.

What is the difference between fuzzy and crisp relation? In fuzzy this is missing, where mu R x y is a number in 0 and 1. mu R x y is a number between 0 and 1. This is the difference between crisp relation and fuzzy relation. In crisp relation, it was either 0 or 1. It is either completely connected or not connected, but in case of fuzzy, connection is a degree; that is, it is from 0 to 1.

The example is, let x equal to 1 2 3. Then x has three members, y has two members 1 and 2. If the membership function associated with each ordered pair is given by this e to the power minus x minus y whole squared. You can easily see, this is the kind of membership function that is used to know, how close is the members of y are from members of x. Because, if I relate from 1 to 1 using this, then you can see 1 minus 1 is 0 that is 1 and 1 very close to each other; whereas, 2 and

1 is little far and 3 1 one is further far. This is a kind of relationship we are looking between these two sets.

Let us derive fuzzy relation. If this is the membership function, fuzzy relation is of course all the ordered pairs. We have to find out 111 2 2 1 2 2 3 1 and 3 2. These are all the sets of ordered pairs and associated membership functions. You just compute e to the power minus x minus y whole square. Here, 1 1 1 minus 1 whole square, 1 2 1 minus 2 whole square, 2 1 2 minus 1 whole square, 2 two 2 minus 2 whole square, 3 1 3 minus 1 whole square, 3 2 3 minus 2 whole square and if you compute them, you find 1 0.4 3 0.4 3 1 0.1 6 0.4 3. This is your membership function. This is one way to find relation.

Normally, I know, it is easier to express the relation in terms of a matrix instead of this continuum fashion, where each ordered pair is associated with membership function. It is easier to appreciate the relation by simply representing them in terms of matrix. How do we do that? This is my x 1 2 3 y is 1 21 the membership function associated was 1 1 2 membership is 0.4 3 2 1 0.4 3 2 2 1 3 1 0.1 6 and 3 2 is 0.4 3 that you can easily verify here 1 3 0.4 3 0.1 6 and 1.

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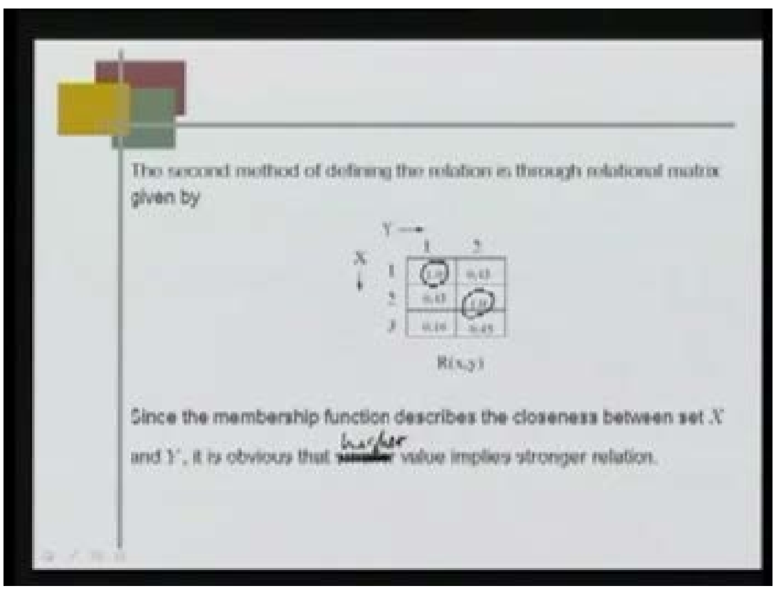

The membership function describes the closeness between set x and y. It is obvious that higher value implies stronger relations. What is the stronger relation? It is between 1 and 1, and they are very close to each other, and 2 and 2; they are very close to each other. Closeness between 2 and

2, between 1 and 1 is actually 1 and 1. They are very close to each other; similarly, 2 and 2. If I simply say numerical closeness, then 2 and 2 are the closest, and 1 and 1 are the closest. That is how these are the closest. Higher value implies stronger relations.

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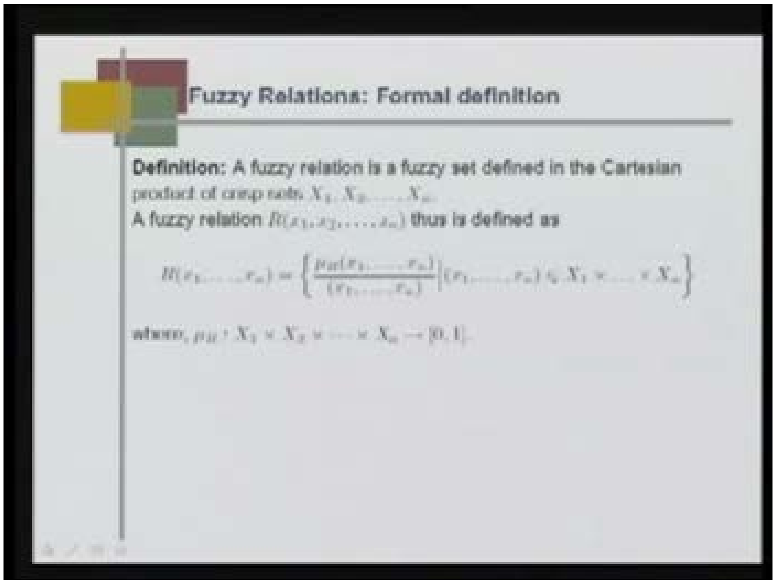

This is a formal definition of fuzzy relation; it is a fuzzy set defined in the Cartesian product of crisp sets; crisp sets x_1 x_2 until $x_{n.}$ A fuzzy relation R is defined as mu R upon x_1 to $x_{n,}$ where x_1 to x_n belongs to the Cartesian product space of x_1 until x_n; whereas, this mu R the fuzzy membership associated is a number between 0 and 1.

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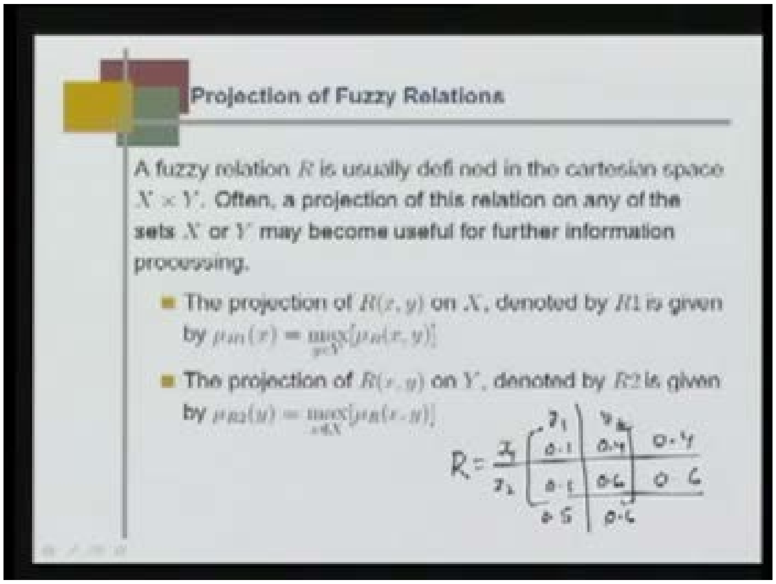

We will be talking about projection of fuzzy relation. A fuzzy relation R is usually defined in the Cartesian space x and x and y. Often a projection of this relation on any of the sets x or y, may become useful for further information processing.

The projection of R x y on x denoted by R 1 is given by mu R 1 x is maximum. So, y belongs to y mu R x y. The meaning is that if I have R, this is x_1 and x_2 and this is y_1 and y_2, and this is 0.1 0.4 and this is 0.5 0.6. If these are the membership functions associated with x_1 y_1 x_2 y_2 is 0.4 x_2 y_1 is 0.5 x_2 y_2 is 0.6.projection, which means for x projection, I find out what the maximum is. Overall, y in this case maximum is 0.4 and for x_2 the max maximum projection is if I took it here, 0.6. Similarly, if I make projection of R, x, y over x, what is the maximum? This is 0.5 and this is 0.6. This is called x projection and y projection of a relation matrix R.

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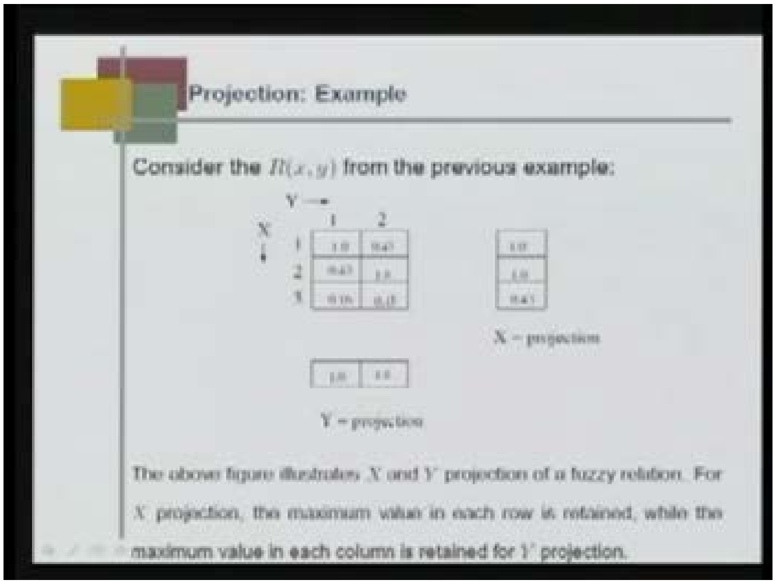

This is again for your benefit. We repeat another example. We have x as 3 components 1 2 3, y has 2 components 1 and 2. This is the previous example that we had 1 0.4 3 0.4 3 1 0.1 6 0.4 3. x projection would be 1 0.4 3 maximum 1 0.4 3 1 maximum 1 0.1 6 0.4 3 maximum 0.4 3. This is x projection. Similarly, y projection; look here, this maximum is 1 and here maximum is 1. This is my y projection of relation R and this is my x projection.

Above figure illustrates x and y projection of fuzzy relation. For x projection, the maximum value in each row is retained. What is the maximum value in each row? Here, x projection maximum value in each row is retained, while the maximum value in each column is retained for y projection.

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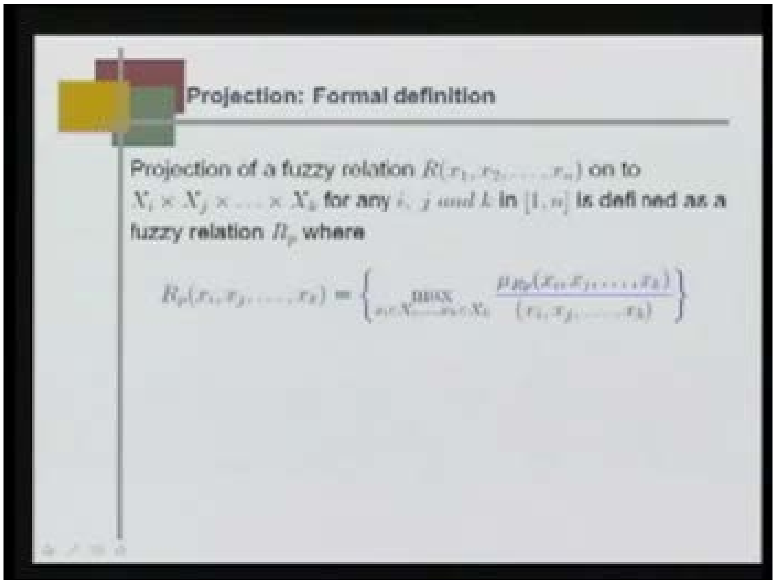

This is our formal definition of a fuzzy relation, projection of a fuzzy relation R on to any of its set in the Cartesian product space; that is in the Cartesian product space. This is our Cartesian product space and for that, we can map this one to any of these i or j or k; whatever it is, for any value, then is defined as a fuzzy relation Rp, where Rp is defined as maximum over Xi until X_k, where this is our X_i X_j X_k and this is mu Rp.

First, we talked about fuzzy relation projection of fuzzy relation. Once we have projection of fuzzy relation, we can extend the projection to again infer what should be the relation. This kind of technique may be useful in coding the information, where we have a huge number of information and we want to transfer such a kind of projection and from projection to extension would be beneficial for coding operation.

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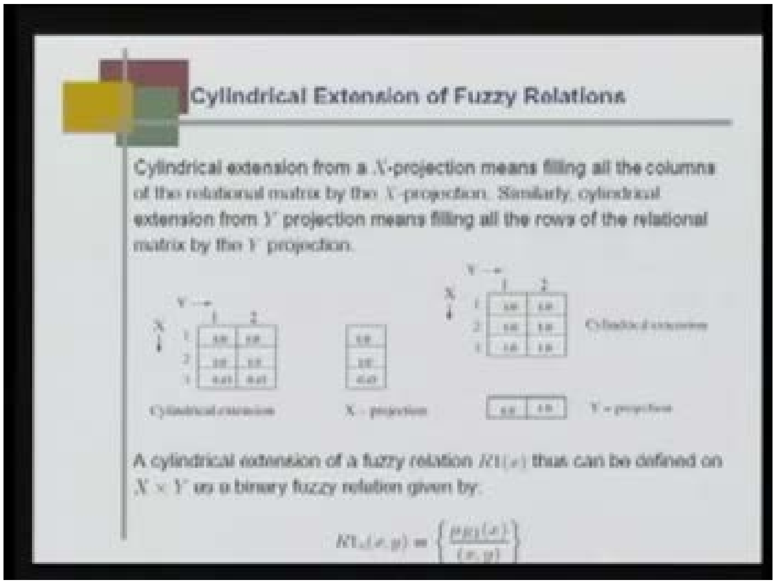

Now, let us understand what is the extension of fuzzy relation cylindrical extension, from an X projection? It means filling all the columns of the relational matrix by the X projection. Similarly, cylindrical extension from y projection means filling all the rows of relational matrix by the X projection. This is my X projection in the previous example 1 1 0.4 3. What I do is I create a relational matrix between X and Y, then I fill all the rows with 1. Here, all the rows with this 1, then all the rows here 0.4 3; whereas in Y projection, if I extend to the relation, then from Y, I fill all the columns here. This one is filled with all 1. That is how we have actual relational matrix projection.

Again from projection, obviously, this is not a very good kind of extension, but this is known as cylindrical extension. Maybe other methods of how to extend from projection to the actual relational matrix is a different question altogether. What is the relation we learnt about projection and from projection to extension?

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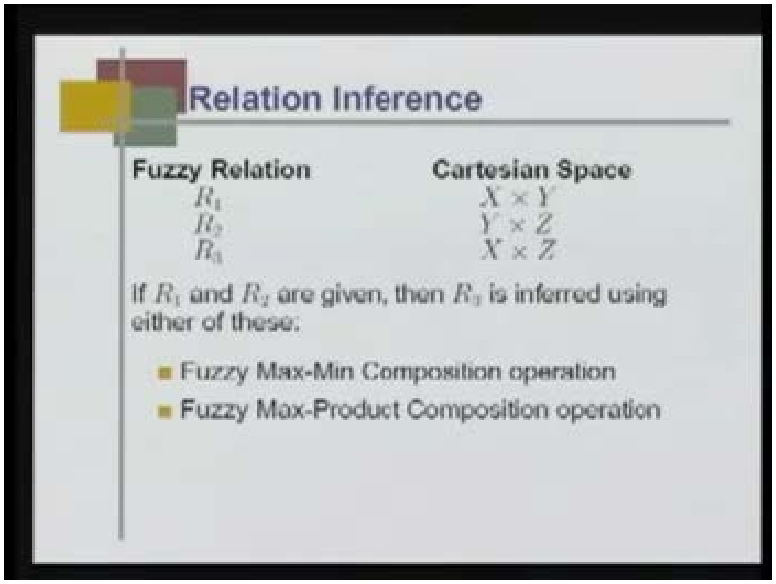

We will be talking about given two relations; just like we learnt in crisp relation that given two relations, the third relation can be inferred through composition rule max min. Let us define R_1 over Cartesian space X and Y, R_2 over Cartesian space product space Y and Z, R_3 over Cartesian product space X and Z. If R_1 and R_2 are given, then R_3 is inferred using either max min composition, as we have already discussed our max product composition.

The same principle - I will not explain this again, because the only thing is earlier this mu R used to be in crisp relation. They used to be either 0 1, whereas in fuzzy, it can be any number between 0 and 1.

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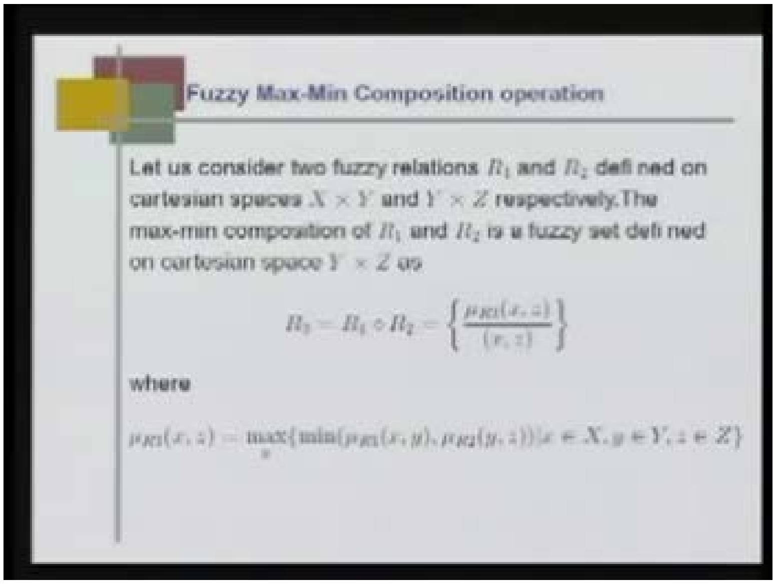

We will now explain, max min composition operation using an example that makes things much more clear. This is my matrix, relational matrix R_1 relating x and y and R_2 relating y and z. I have to find out the relational matrix from x to z using fuzzy rule of composition. We normally write R_3 is R_1 composition $R_{2.}$ Using max min composition, how do we compute R_3? Very simple.

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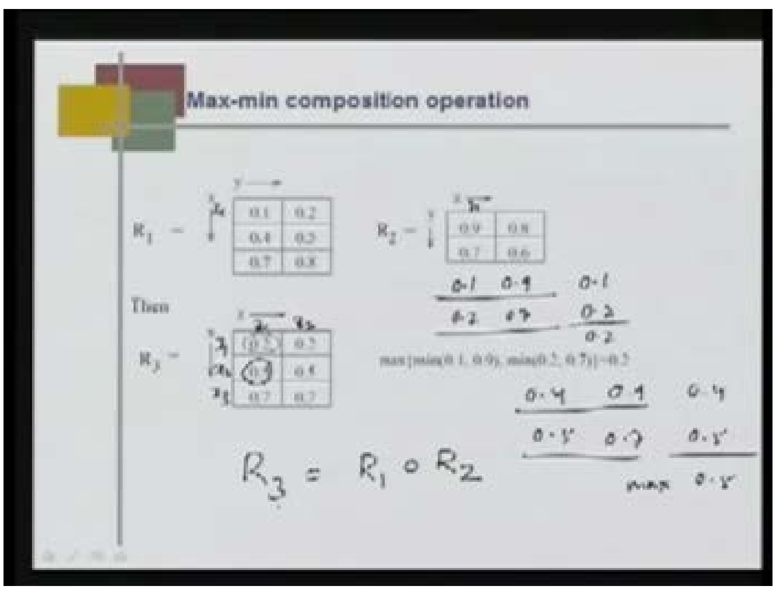

I want to now build a relationship between x and this. This is the membership associated with x_1 and z_1. Let me put it very precise, x_1 x_2 x_3 z_1 and z_2; if you look at what we will be doing here, This is my x_1 row and this is my z_1 column. What I do, x_1 row and z_1 column; I put them parallel and find out what is minimum. Here, minimum is 0.1 and here minimum is 0.2. After that, I find out what is the maximum, which is 0.2. This is what maximum of minimum 0.1. 0.9 is minimum 0.2. 0.7 is 0.2. This is how we found out. The easiest way if I want to find out is this one; this x_1 x_2 and z_1. x_2 means this row which is 0.4 and 0.5 and x_2 and $z_{1.}$ z_1 is again 0.9 and 0.7. I will find out. Minimum here is 0.4, minimum here is 0.5 and maximum here is 0.5. You get this 0.5

Similarly, we can compute all the elements in R_3 using a max min composition operation. As usual, any max min composition can follow certain properties, associative and distributive over union. That is P fuzzy composition Q union R is P composition Q union P composition R.

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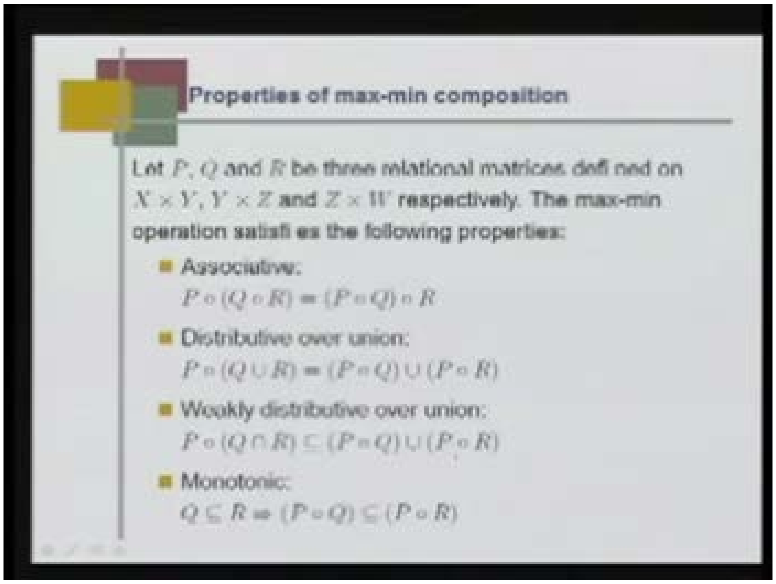

Similarly, weekly distributed over union is P composition, Q intersection, R is a subset of P composition. Q union P composition R monotonic Q is a subset of R implies that, P composition Q is a subset of P composition R.

Fuzzy max product composition operation, earlier we talked about max min. We will talk about max product as you know that, in case of crisp set, both max product and max min yield the same result. In this case, it will not and that we will see now. Max product means how we do? Now, again, the same example we have taken R_1, R_2 and R_3. Now, I want to find out from R_1 and R_2, what R3 using max product composition is.

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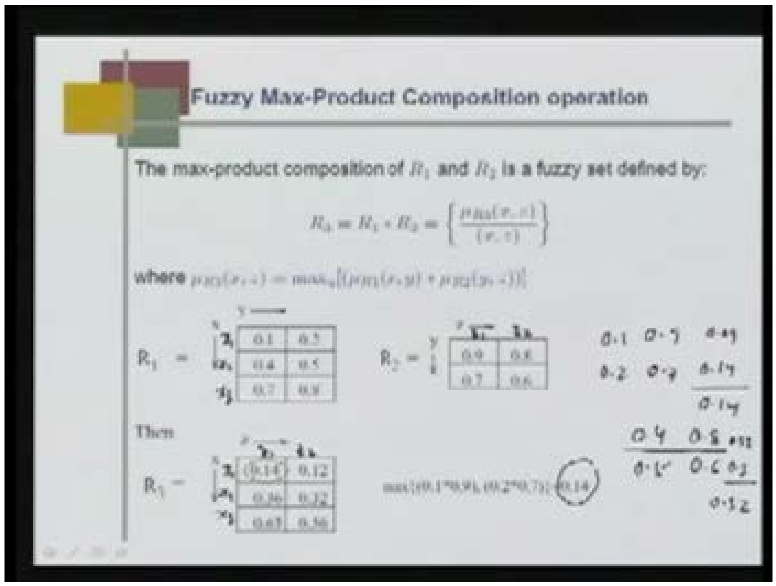

Again I do the same thing. Let us say, this is x_1 x_2 x_3 z_1 z_2 z_1 z_2 and this is x_1 x_2 x_3 for x_1. I take this row which is 0.1 0.2 and finding the relation the fuzzy membership associate x_1 and z_1. I take the column from z_1 which is 0.9 0.7 and I multiply them here 0.1 0.9 is point 0 9 0.2 0.7 is 0.1 4 and find out what is the maximum. This is the maximum 0.1 4.

I take another example. Let us find out the relationship between x_2 and z_2; for x_2 the row is 0.4 0.5 and z_2 the column is 0.8 0.6. Corresponding to this, if I multiply I get 0.4 0.8 is 0.3 2 0.5 0.6 is 0.3. Maximum is 0.3 2. This is 0.4 3 0.3 2. This is where it is 0.1. The answer is here, the R_3 and if I go back, if I look, R_3 here is different.

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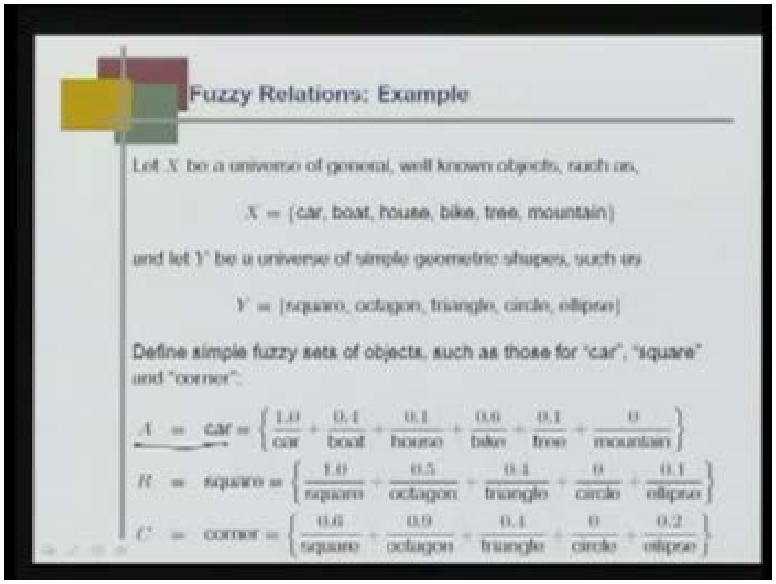

Now, we will consider another example of fuzzy relation. A little more application oriented. Let us think of 2 different sets of objects. 1 set of objects are images collected from various objects like car, boat, house, bike, tree and mountain. We have such images and now the geometry of these images can be compared to various known geometries like square, octagon, triangle, circle, and ellipse like that.

Now, the question we may ask is that, if I have an image of any of these objects like car or boat or house or bike or tree or mountain, how close these images are to an image of a car? This is a question. How close an image of car is to those of either a car or a boat or a house or bike or tree or mountain? I create a fuzzy set and I call that fuzzy set to be a car. My fuzzy set A is a car fuzzy set. Obviously, when I compare the image, we look at a car image. Obviously, the membership is 1, because car and boat may not look like a car, but the fuzzy index is 0.4. Obviously, a house will not very less likely to look like a car. The fuzzy index is 0.1.

A bike may look like a car with fuzzy index 0.6 or more; whereas, the tree is similar like house very little resemblance with a car with a fuzzy index 0.1, but a mountain may not have any similarity with the car, so 0 fuzzy indexes. When I want to look at all these objects and infer whether they are cars, the associated membership functions are 1 0.4 0.1 0.6 0.1 and 0.

Similarly, I look at the various geometrical features and I want to infer whether this geometry is a square. Obviously, if I am looking at a square, actual square the fuzzy membership is 1, if it is octagon then it is 0.5, triangle 0.4, circle 0, ellipse 0.1.

Now, the third is that this geometry is an object; whether they have corners? The square will have 4 corners, octagon has 8 corners, triangle has 3 corners, a circle has no corner and ellipse has no corner. All the 8 do not have exactly corners, but it may appear like to have some corners. When I look at the fuzzy set of a corner, then square will have fuzzy membership 0.6, octagon has maximum number of corners 0.9, triangle has 30.4 circle has no corners 0 and ellipse 0.2. This is how we specify the set car, square, and corner.

Now, we would like to establish the relationship between car and square. Square and corner and then infer the relationship between car and corner using composition rule.

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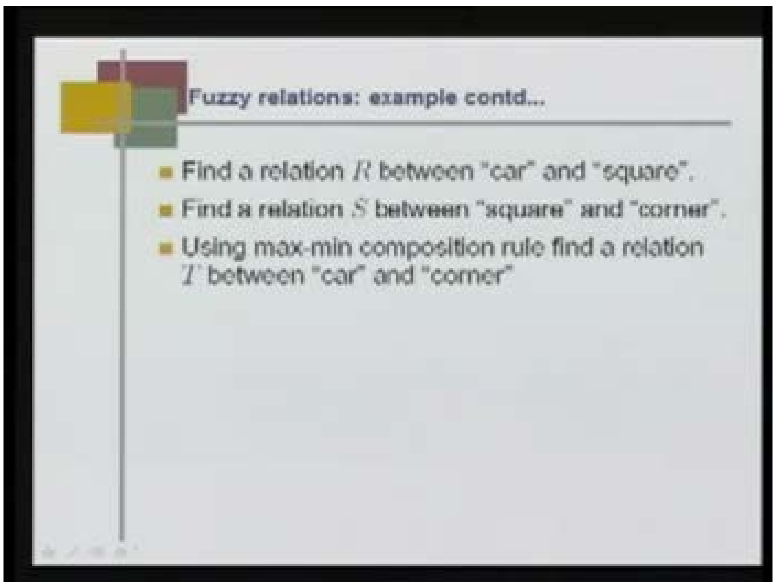

Find a relation R between car and square? Find a relation S between square and corner using max min composition rule? Find a relation T between car and corner? This is the question. If I adjust the relationship matrix between fuzzy set A and B, what was A? A was the set which is car and we defined this fuzzy set. It will be this way. Car was 1 by car, 0.4 by boat, 0.1 by house, 0.6 by bike, 0.1 by tree, and 0 by mountain. This is my car subset fuzzy set and B which is my square subset is 1 upon square. Octagon has 0.5 and triangle 0.4 and circle has probably the membership 0 because, circle will never look like a square. Ellipse is 0.1.

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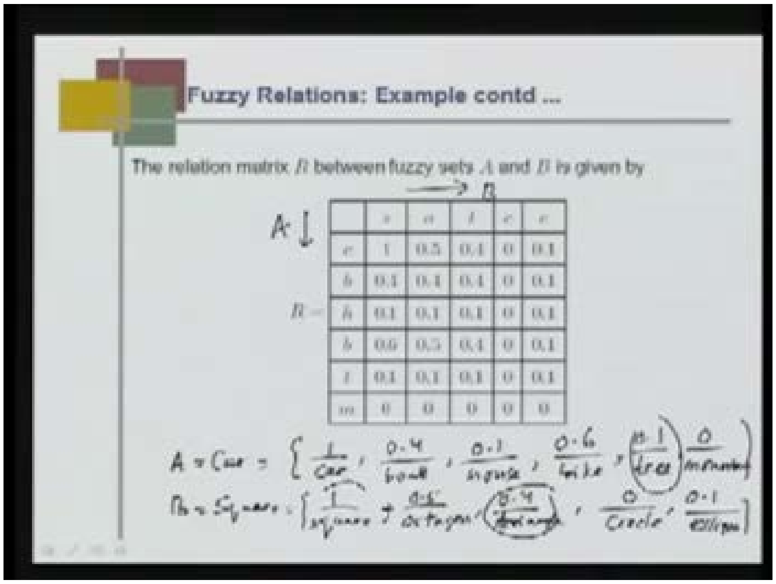

How do I find the relationship R? You can easily do so here. Here this is square; S stands for square, octagon, triangle, circle and ellipse in this. This is my B, B set and this is my A set. In A set, the elements are car, boat, house, bike, tree and mountain. How do I find these members here?

All that I have to do is that each element, because, I have to create an ordered pair and take the minimum membership function, the car will be made ordered pair with the square octagon triangle, circle and ellipse, car, circle, octagon, triangle, circle and ellipse and with car when I say square, the minimum membership function is 1. Car with octagon minimum membership is 1, between 1 and 0.5 is 0.5. Similarly, car and triangle the minimum membership is between 1 and 0.4. Similarly, car and circle the minimum between 1. 0 is car and ellipse is between 1 and 0.1 minimum is 0.1.

Similarly, I would find out the relationship between B and all other elements in the boat in set A with all the elements in B. Boat has a membership function 0.4 when I compare a square, obviously, the minimum is 0.40.4 and then with octagon 0.5, because this is 0.4 0.5. I have 0.4 is 0.1. Like that I can fill all these entries.

What is the method? The method is that in each ordered pair, I find out what is the minimum of the membership function. For example, if I make an ordered pair between this and this. The minimum is 0.4 and 0.1 is 1 0.1 0.4 0.1 0.4 0.1 is 0.1. You can easily see, this is triangle and this is tree. This is tree triangle and this is tree and the relationship is 0.1. This is how we find out the membership function between A and B. Similarly, this is the relationship between B and C. This is a set B and this is C set. C set B is a square and set C is the corner.

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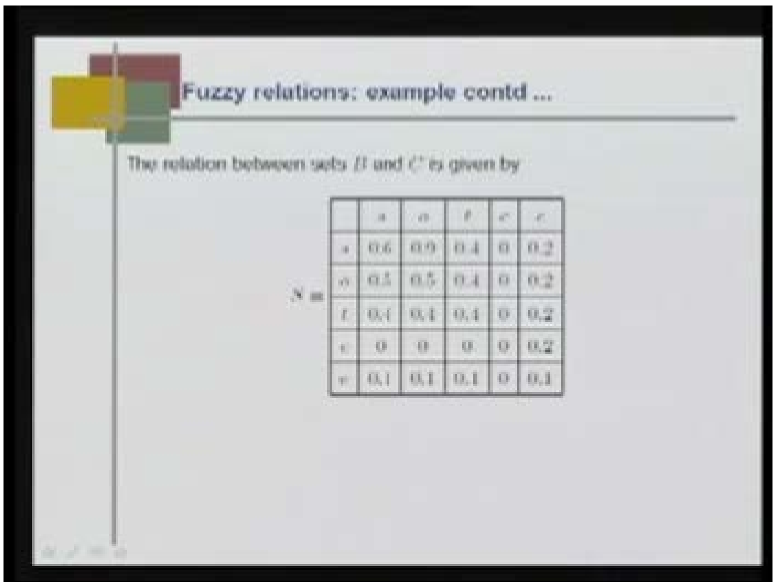

Once I found what is R and S, the next is to find out the relation T. The relation T between car and corner can be found out using inference mechanism which is composition rule. We have used this as max min composition. This has been found out using max min composition and as I explained earlier how to find out max min composition, what I would like to show you here is the same matrix. That is the relation matrix that I have got; I can also get directly from this. This is my A and this CA stands for car.

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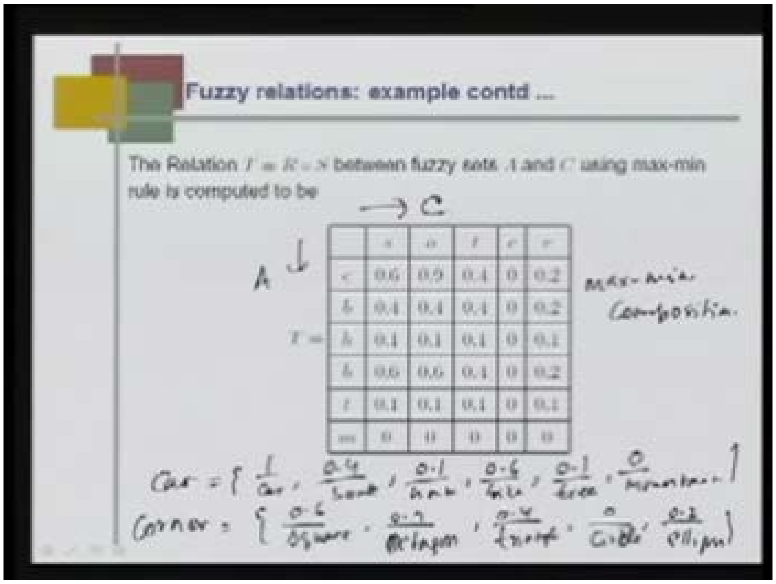

My car was 1 and 0.4 by boat, and 0.1 by house, 0.6 by bike, 0.1 by tree, 0 mountains. This is my car and C is corner which shows the corner. The square is a 0.6. Octagon has maximum number of corners 9 0.9. Then triangle has 3 corners 0.4 circle has no corner and finally ellipse may look like have some corners. 0.2.

Although we found by max min composition principle, the same thing also, you can find out simply looking at these 2 sets; car and corner. You can easily see that when I relate car with this these things, you should say because car has membership function 1. All these membership function always will be minimum with respect to 1. They will be there 0.6 0.9 0.4 0 and 0.2. Next, when I go to boat 0.40.4 is minimum here, 0.4 is minimum here, 0.4 is minimum here. Past 3 will be 0.4 and then 0 minimum, here 0.2 minimum here. Exactly, whatever I got from inference mechanism, the same mechanism and same thing I will get, using directly looking at car and corner.

Finally, the summary: what we discussed today, we extended the function mapping concept to fuzzy sets. The crisp relation and fuzzy relation; the difference was discussed today, where we showed that crisp relation; the index is either 0 or the membership grade is either 0 or 1; that is, either complete relation or no relation. Whereas, in fuzzy the relation has a grade from 0 to 1. Fuzzy composition rule; max min composition max product composition unlike in crisp relation, where both max min and max product gives you the same answer; whereas in fuzzy composition, max min and max product will give two different answers and fuzzy projection and cylindrical extension. We discussed and also we illustrated all these principles through various examples.

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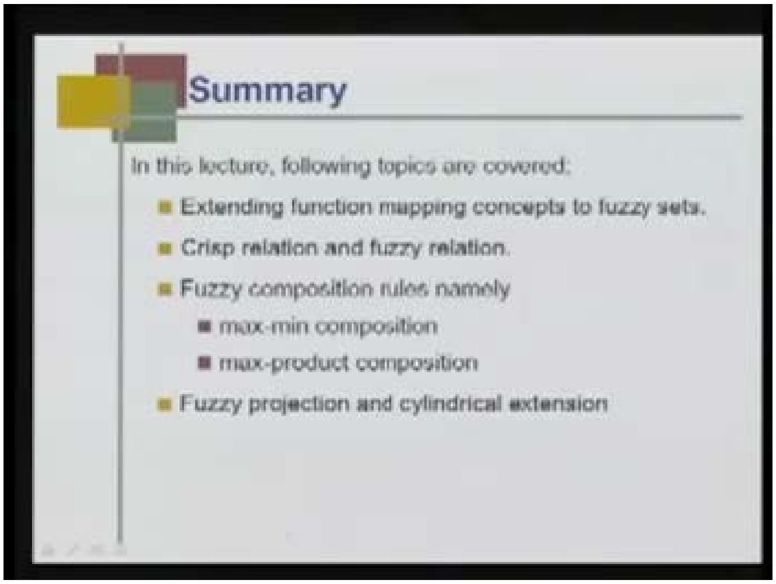

Thank you very much.