Intelligent Systems and Control

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Module 2 Lecture 1

Fuzzy Sets: A Primer

Welcome to the second module of Intelligent Control course. In today's class, the topic is fuzzy sets: A primer. That is a very introductory version of fuzzy sets that are minimally necessary to understand fuzzy control system.

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Topics to be covered: Brief review of conventional sets, introduction to fuzzy sets, membership functions and operations on fuzzy sets.

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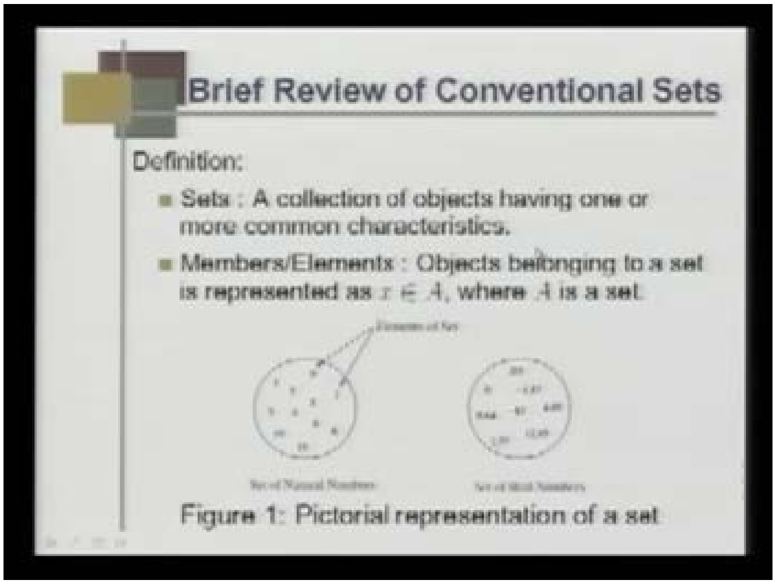

Brief review of conventional sets: These are fundamental concepts that you are already aware of. I will just revisit them.

The first set: A collection of objects having one or more common characteristics. For example, set of natural number, set of real numbers, members, or elements. Objects belonging to a set is represented as x belonging to A, where A is a set.

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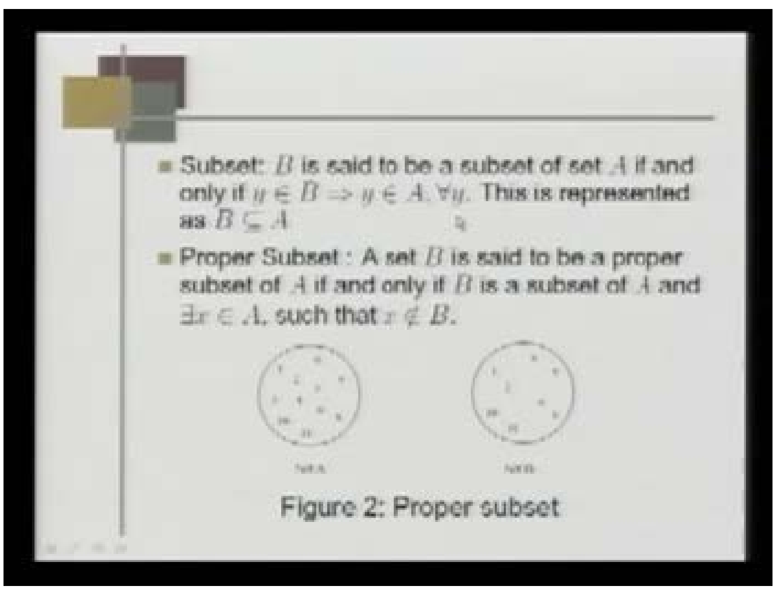

Subset B: here, subset B is a subset of A. B is said to be a subset of set A if and only if y belongs to B implies y belongs to A, for all y.

Proper subset: A set B is said to be a proper subset of A, if and only if B is a subset of A and for all A, there exist x belonging to A such that x does not belong to B. For example, in this case, B is a proper subset of A.

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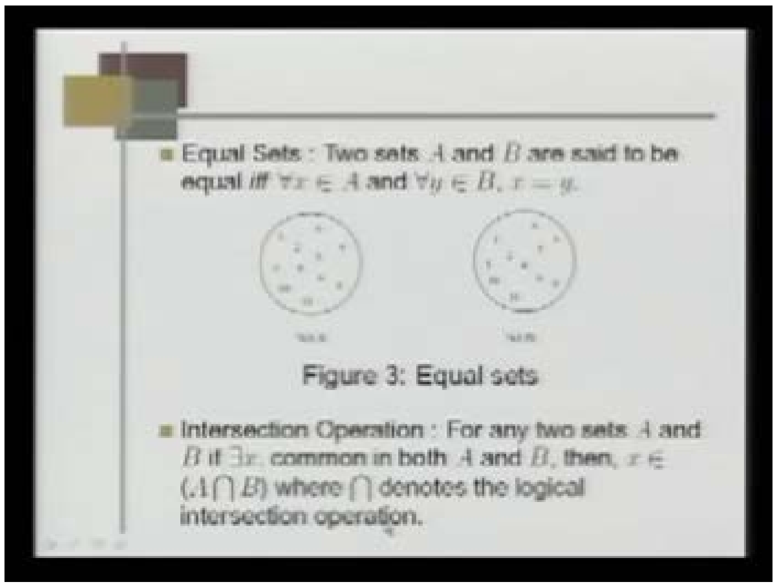

Equal sets: Two sets A and B are said to be equal, if for all x belonging to A and for all y belonging to B, x is equal to y. For example, set B and set A are equal sets.

Intersection operation: For any two sets A and B, if there exists x common in both A and B, then x belonging to A intersection B, where intersection denotes the logical intersection operation. For example, 7 and 8; these two elements belong to intersection of set A and set B.

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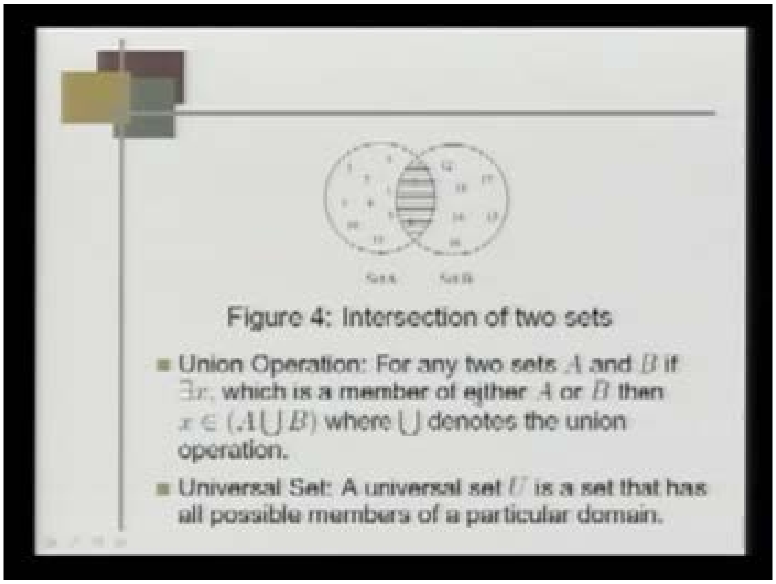

Similarly, union operation for any two sets A and B, if there exists x, which is a member of either A or B, then, x belongs to A union B, where union denotes the union operation.

Universal set: A universal set U is a set that has all possible members of a particular domain.

I will give you an idea. Before I talk, in these slides you will see, we were talking about concept of a fuzzy number.

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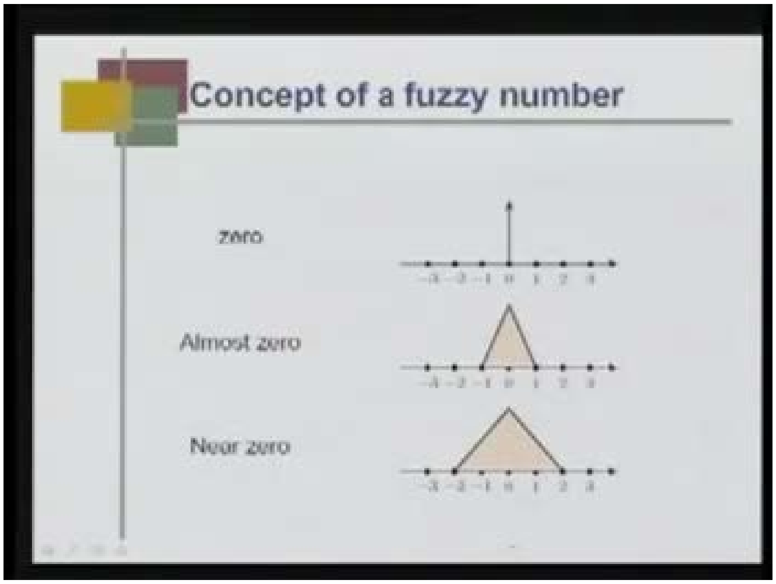

Before I talk about this, why fuzzy is used? Why we will be learning about fuzzy? The word fuzzy means that, in general sense when we talk about the real world, our expression of the real world, the way we quantify the real world, the way we describe the real world, are not very precise.

When I ask what your height is, nobody would say or nobody would expect you to know a precise answer. If I ask a precise question, probably, you will give me your height as 5 feet 8 inches. But normally, when I see people, I would say this person is tall according to my own estimate, my own belief and my own experience; or if I ask, what the temperature is today, the normal answer people would give is, today it is very hot or hot or cool. Our expression about the world around us is always not precise. Not to be precise is exactly what is fuzzy.

Fuzzy logic is logic which is not very precise. That is the best way I would say. Normally, I would always ask what is fuzzy. Fuzzy logic means that which is not very precise. Since we deal with our world with this imprecise way, naturally, the computation that involves the logic of impreciseness is much more powerful than the computation that is being carried through a precise manner, or rather I would say precision logic based computation is inferior; not always, but in many applications, they are very inferior in terms of technological application in our day to day benefits, the normal way.

Fuzzy logic has become very popular; in particular, the Japanese sold the fuzzy logic controller, fuzzy logic chips in all kinds of house hold appliances in early 90's. Whether it is washing machine or the automated ticket machine, anything that you have, the usual house hold appliances, the Japanese actually made use of the fuzzy logic and hence its popularity grew.

We would be seeing in this class, how the imprecise way of looking at things and manipulating them is much more powerful than precise way of looking at them and then manipulating them. I think that a background is enough. I did not speak much here, because, we discussed something about fuzzy concepts in our earlier introductory lecture. Now, we will go ahead with the concept of fuzzy number.

As I said, fuzzy means from precision to imprecision. Here, when I say 0, I have an arrow at 0, pointing that I am exactly meaning 0 means 0.00000 very precise. When I say they are all almost

0, I do not mean only 0, rather in the peripheral 0. I can tolerate a band from minus 1 to 1, whereas if I go towards 1 or minus 1, I am going away from 0, the notion of 0. That is what is almost 0, that is around 0, but in a small bandwidth, I still allow certain bandwidth for 0.

I will not be talking about membership now, but a notion is that I allow some small bandwidth when I say almost 0. When I say near 0 my bandwidth still further increases. In the case minus 2 to 2, when I encounter any data between minus 2 to 2, still I will consider them to be near 0. As I go away from 0 towards minus 2, the confidence level how near they are to 0 reduces; like if it is very near to 0, I am very certain. As I progressively go away from 0, the level of confidence also goes down, but still there is a tolerance limit. So what I am saying is, here I am precise, I become imprecise here and I further become more imprecise in the third case.

This concept to be imprecise is fuzzy or to deal with the day to day data that we collect or we encounter and representing them in an imprecise manner like here almost 0, near 0, or hot, cold, or tall; if I am referring to height, tall, short medium. This kind of terminology that we normally talk or exchange among ourselves in our communication actually deals with imprecise data rather than precise data. Naturally, since our communications are imprecise, the computation resulting out of such communication language, the language which is imprecise must be associated with some logic.

The father of fuzzy logic is Lotfi Zadeh who is still there. The notion of fuzzy should be now very clear to you. We are now looking forward to a tool that can manipulate those kinds of data which are imprecise. That is why we must have a mathematical tool and that is exactly fuzzy logic. As I said, the father of this fuzzy logic is Lotfi A. Zadeh from U C Berkeley. Way back in 1965, he pioneered the research in fuzzy logic.

Now, we will be talking about fuzzy sets. We had a classical set. When I talked about classical set, we had classical set of the numbers that we know, like we talked about the set of natural numbers, set of real numbers. What is the difference between a fuzzy set and a classical set or a crisp set? The difference is that the members, they belong to a set A or a specific set A or B or X or Y, whatever it is, we define them; but the degree of belonging to the set is imprecise. If I say, a universal set in natural numbers, all the natural numbers fall in this set. If I take a subset of this natural number, like in earlier case, we put 1 to 11 in one set. When I ask, whether 12 belongs to set A, the answer is no; 13 belongs to set A? The answer is no; because, in my natural number set, only 1 to 11 are placed. This is called classical set and their belongingness here is one. They all belong to this set.

But in a fuzzy set, I can have all the numbers in this set, but with a membership grade associated with it. When I say membership grade is 0 that means, they do not belong to the set, whereas a membership grade between 0 to 1, says how much this particular object may belong to the set. I give you an example: a set of all tall people. Tall if I define, classically I would say above 6 is tall and below 6 is not tall; that is, 5.9, 5 feet 9 inches is not tall and 6.1, 6 feet 1 inch is tall. That looks very weird; it does not look nice to say that a person who is 6 feet 1 inch is tall and 5 feet 9 inches is not tall. This ambiguity that we have in terms of defining such a thing in classical set, the difficulty that we face can be easily resolved in fuzzy set. In fuzzy set, we can easily say both 6.1, 6 feet 1 inch as well as 5.9 inches as tall, but level this difference; they are tall, but with a membership grade associated with this. This is what fuzzy set is.

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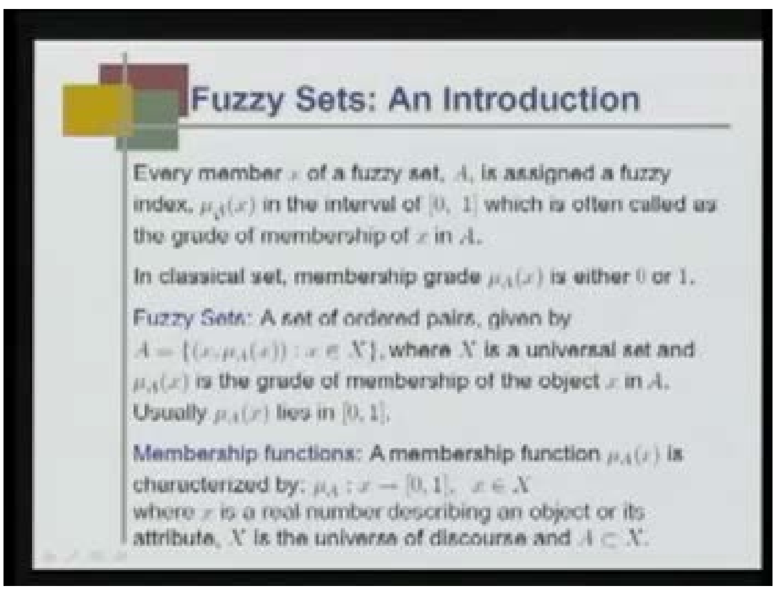

Every member x of a fuzzy set A is assigned a fuzzy index. This is the membership grade mu $_A$ x in the interval of 0 to 1, which is often called as the grade of membership of x in A. In a classical set, this membership grade is either 0 or 1; it either belongs to set A or does not belong. But in a fuzzy set this answer is not precise, answer is, it is possible. Like I say, it is belonging to set A with a fuzzy membership 0.9 and I say it belongs to A with a fuzzy membership 0.1; that is, when I say 0.9, more likely it belongs to set A. When I say 0.1, less likely it belongs to set A. Fuzzy sets are a set of ordered pairs given by A. The ordered pair is x, where x is a member of the set. Along with that, what is its membership grade and how likely the subject belongs to set A? That is the level we put, where x is a universal set and mu x is the grade of membership of the object x in A. As we said, this membership mu $_A$ x lies between 0 to 1; so, more towards 1, we say more likely it belongs to A. Like if I say membership grade is 1, certainly it belongs to A.

Membership function - a membership function mu $_A$ x is characterized by mu $_A$ that maps all the members in set x to a number between 0 to 1, where x is a real number describing an object or its attribute, X is the universe of discourse and A is a subset of X.

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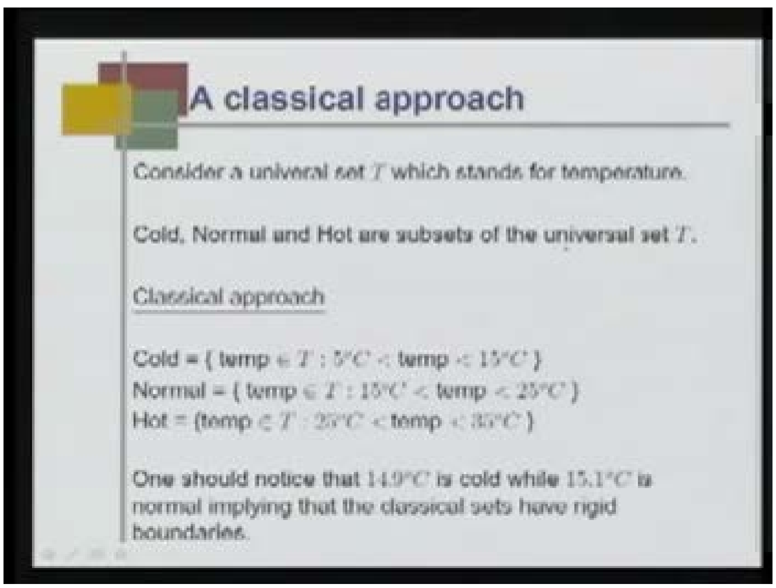

We will compare the classical approach and fuzzy approach. Let us say, consider a universal set T which stands for temperature. Temperature I can say cold, normal and hot. Naturally, these are subsets of the universal set T; the cold temperature, normal temperature and hot temperature they are all subsets of T.

The classical approach, probably, one way to define the classical set is cold. I define cold: temperature T; temperature is a member of cold set which belongs to the universal set T such that this temperature, the member temperature is between 5 degree and 15 degree centigrade.

Similarly, the member temperature belongs to normal, if it is between 15 degree centigrade and 25 degree centigrade. Similarly, the member temperature belongs to hot set when the temperature is between 25 degree centigrade and 35 degree centigrade. As I said earlier, one should notice that 14.9 degree centigrade is cold according to this definition while 15.1 degree centigrade is normal implying the classical sets have rigid boundaries and because of this rigidity, the expression of the world or the expression of data becomes very difficult. For me, I feel or any one of us will feel very uneasy to say that 14.9 degrees centigrade is cold and 15.1 degree centigrade is normal or for that matter, 24.9 degrees centigrade is normal and 25 degree or 25.1 degree centigrade is hot. That is a little weird or that is bizarre to have such an approach to categorize things into various sets.

In a fuzzy set, it is very easy to represent them here. How do I do it? This is my temperature axis and this is my membership grade.

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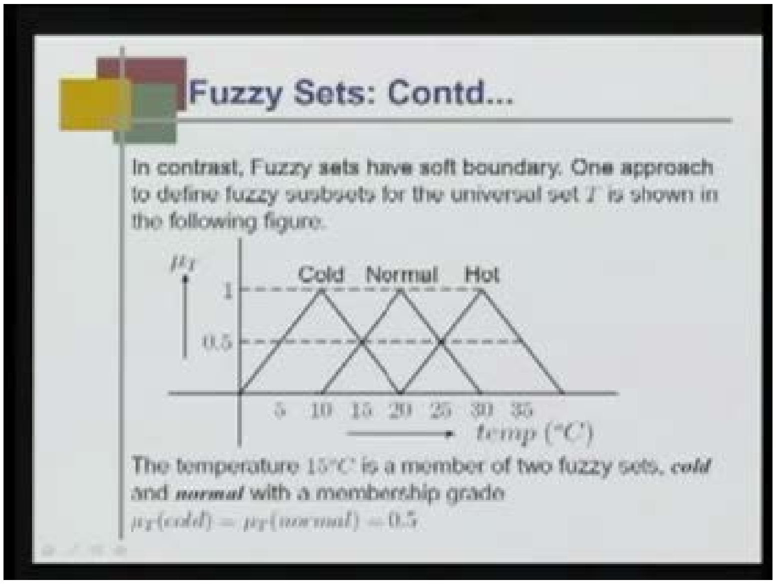

How do I define that? If the temperature is around 10 degree centigrade, it is cold; temperature is around 20 degrees centigrade, it is normal and when temperature is around 30 degree centigrade it is hot. In that sense, they do not have a rigid boundary. If you say here, 25 degree centigrade, the 25 degree centigrade can be called simultaneously hot as well as normal, with a fuzzy membership grade 0.5. 25 degrees centigrade belongs to both normal as well as hot, but when I say 28 degree centigrade, this is more likely a temperature in the category of hot, whereas the 22 degree centigrade is a temperature that is more likely belonging to the set normal. This is a much nicer way to represent a set. This is how the imprecise data can be categorized in a much nicer way using fuzzy logic. This is the contrasting feature, why the fuzzy logic was introduced in the first place.

Fuzzy sets have soft boundaries. I can say cold from almost 0 degree centigrade to 20 degree centigrade. If 10 degree has a membership grade 1 and as I move away from 10 degree in both directions, I lose the membership grade. The membership grade reduces from 1 to 0 here, and in this direction also from 1 to 0. The temperature, As I go, my membership grade reduces; I enter into a different set simultaneously and that is normal. You can easily see, like temperature 12, 13, 14, 15 all belong to both categories cold as well as normal, but each member is associated with a membership grade; this is very important.

In a classical set, you saw only members in a set. Here, there are members in a set associated with a fuzzy index or membership function.

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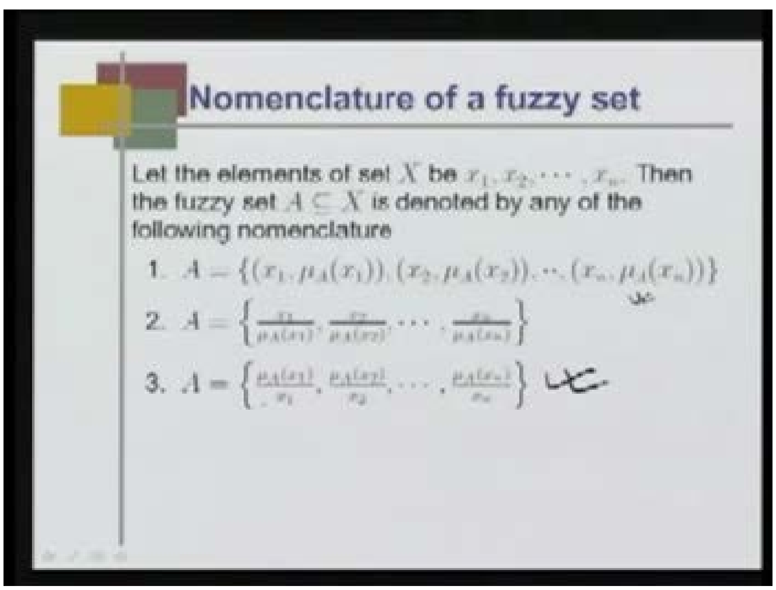

Normally, the nomenclature of a fuzzy set - how do we represent a fuzzy set there? One way is that let the elements of X be x_1, x_2, up to x_n; then the fuzzy set A is denoted by any of the following nomenclature. There are other nomenclatures also. I have taken only 3. Mostly, we will be using either this or the first one, where you see the ordered pair x_1 mu $_A$ x_1; x_1 is member of A and x_1 is associated with a fuzzy index and so forth, x_2 and its fuzzy index, x_n and its fuzzy membership. The same thing, I can also write x_1 upon mu$_A$ x_1. That means x_1 is the member and this is the membership. The other way is here, in the third pattern. I put the membership first and in the bottom, I put the member x_1 with a membership, x_2 with membership and x_n with membership.

These are various nomenclature of the fuzzy set. We can select any one of them. Once we talk about each member in a fuzzy set associated with membership function, you must know how to characterize this membership function. There are many methods. We will now discuss typical membership functions.

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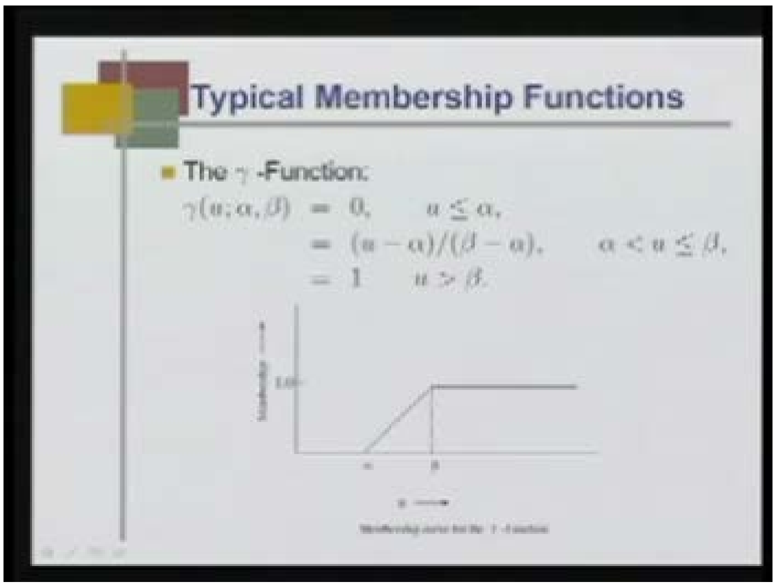

The first is gamma function. Gamma function means that, In gamma function, what happens is linear function between alpha to beta, and this is my variable u. I am defining a fuzzy index or a fuzzy membership for a variable u and this is the membership. A membership maximum value is always 1 and minimum value is 0. Between alpha and beta, the membership is linear and after beta, the membership is constant which is equal to 1. Then, this is gamma membership, gamma function.

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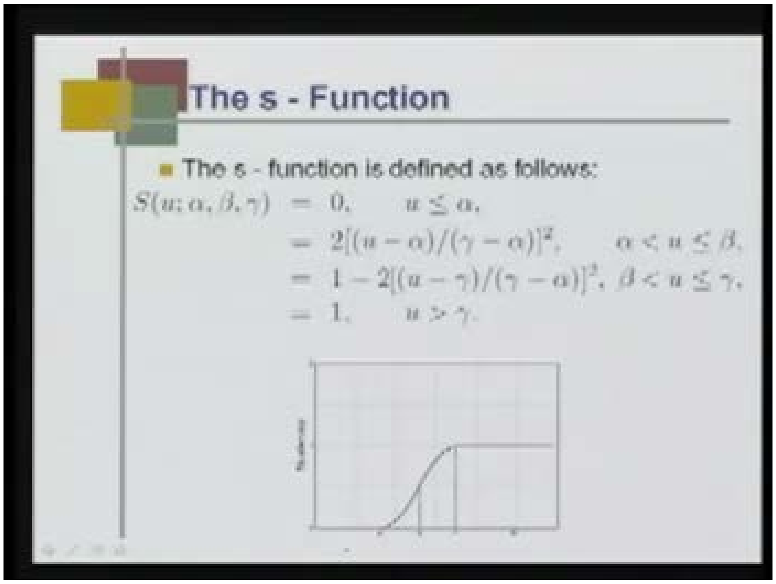

The gamma function was linear, whereas the s function is non-linear; in the sense, from alpha to gamma here, this is my u. u is my actual variable. u can be temperature, u can be height, u can be age, u can be length; whatever is u, I am defining a fuzzy index for that category. From alpha to gamma, this is a function which is nonlinear. This is defined by this (Refer Slide Time: 27:19 min) and after gamma the membership is 1, just like gamma function and before alpha the membership is 0. When u is less than alpha, it is 0; when it is alpha and beta then this is 2 u minus alpha by gamma minus alpha whole square; when it is gamma then 1 minus 2 u minus gamma minus alpha whole square, and when it is beyond gamma, it is 1.

This is a very widely used membership function which is called triangular membership function and this particular function is mostly voiced.

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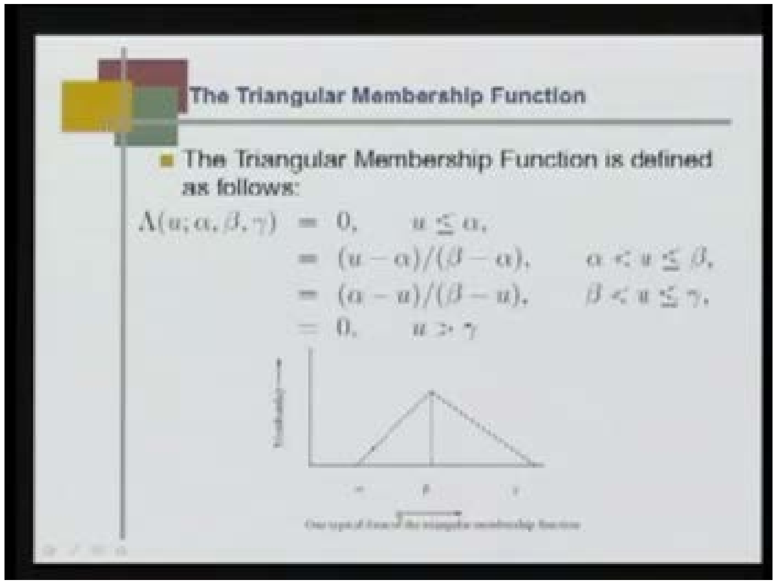

Particularly, in fuzzy logic control, the triangular membership function is pretty much used. In this triangular membership function, you can easily see that before alpha, when u is less than alpha, membership is 0; between alpha and beta, the membership function linearly increases, and between beta and gamma this linearly decreases. This is the triangular membership function.

This is another membership function, which is pi function. pi function, it is actually a kind of a trapezoidal function.

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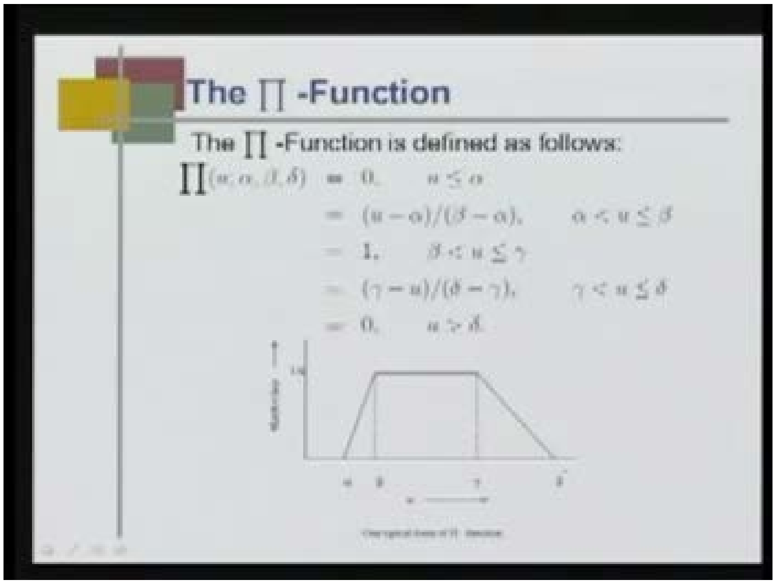

Between alpha and beta it linearly increases, between beta and alpha, it remains constant at 1, and again from gamma to delta, it linearly decreases. Such kinds of membership functions are also used. This is another membership function, Gaussian membership function. All of you know that in Gaussian function, the Gaussian function has two parameters that is mean and sigma, the variance.

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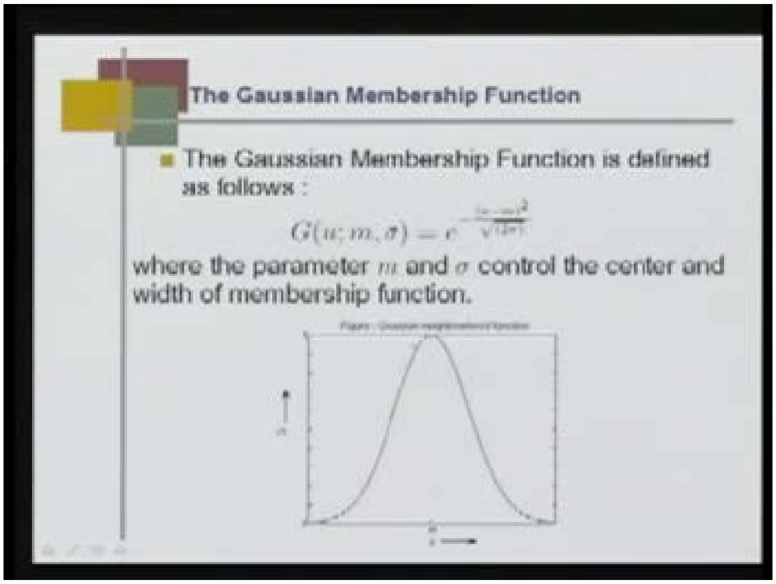

Using these two parameters, the nature of the function can be changed. This Gaussian membership function also is very widely used; in particular in fuzzy neural network and so forth. This is because, this is pretty useful in terms of system identification using fuzzy neural network.

We talked about various ways in which we can define or we can assign a membership function or membership index or membership grade to a member in a fuzzy set. Next is the operation on fuzzy sets. The main features of operation on fuzzy set are that unlike conventional sets, operations on fuzzy sets are usually described with reference to membership function. When I say operation, I do not do with the member itself, but I manipulate. When I say operation, I manipulate the membership of the members in a set; members are not manipulated, rather the membership function of the member is manipulated. This is very important; that is, x and mu x. In classical set what is manipulated is x.

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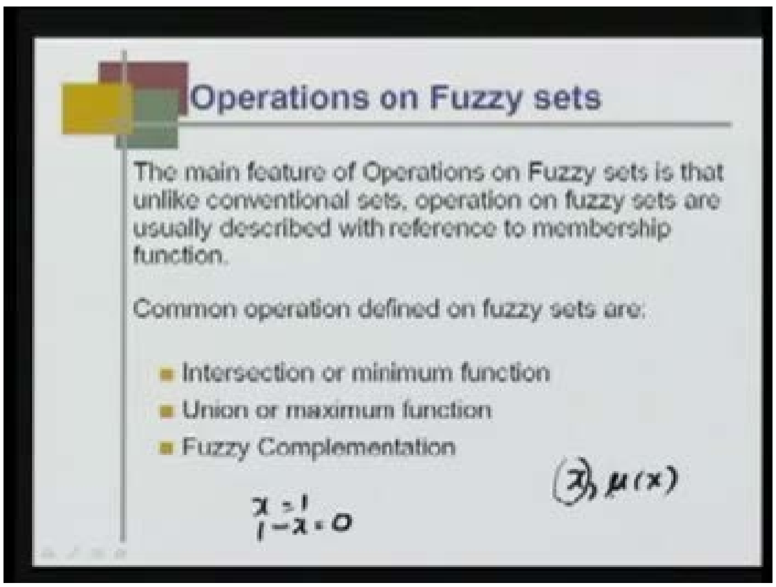

If I say, x is 1 In classical set when I say x is 1 then, I would say 1 minus x is 0. In this, the manipulation concerns with the member; whereas any kind of manipulation in fuzzy set does not involve with x; rather it involves mu x. In just a moment, we will clarify that. Common operation; there are many other operations, but in this class, I will not discuss all of them. It is not necessary also. But these three are sufficient for you to understand what the different operations on fuzzy set are. One is intersection which we say is the minimum function, union, which we say is the maximum function and then fuzzy complementation; these three are very important operations.

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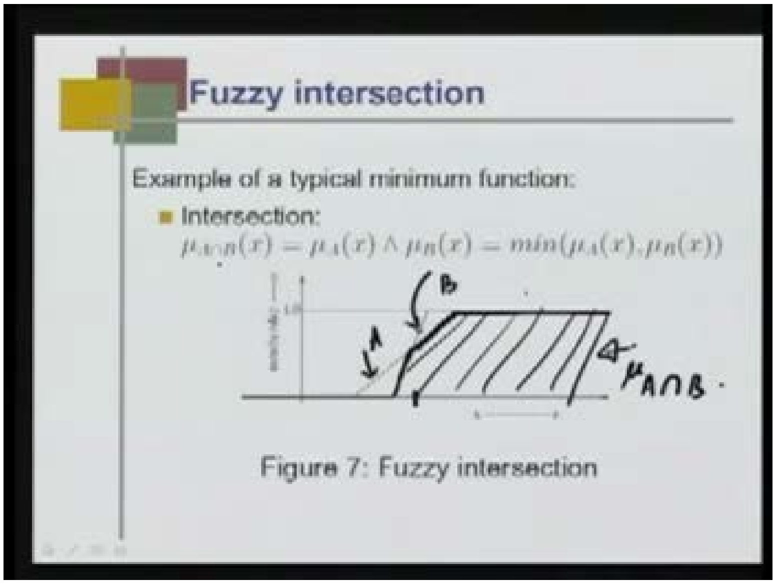

We will now explain what fuzzy intersection is. I have two different fuzzy sets here. I have defined that for the members of a universal set where x belongs to capital X. This is my one set, this particular thing, and there is another set, this one. If I say this is my fuzzy set A and this one is my fuzzy set B, what is happening here? The intersection is that which is common to both A and B; this one and this would be mu A intersection B. I always consider the minimum of two memberships. What I am trying to manipulate is the membership function and not the members. For example, if I select any value of x like this x, mu $_B$ will give this path membership function and mu A will give a membership function according to this. So, according to this, this is the membership function.

What is the overall membership function? It is the minimum of these two. My boundary is this one. We can easily see that, the membership of A intersection B is all the members that belongs to, that is common between A and B. Their membership will follow these particular curves. There are two things we are doing. We have 2 sets. One is set A and the other is set B. Classically, what we see is the common members between A and B. We are not only seeing the common members, here we are also seeing, what is their membership function. The membership function is computed minimum; that is, mu $_A$ intersection B is minimum of mu $_A$ x and mu $_B$ x. That is the membership function. When there is a common member between A and B, the membership function wherever is minimum that is retained and the other one is thrown away. The member is retained; what is changing is the membership function.

Similarly, the fuzzy union which we say maximum function; in this case again, I can easily say that this one is A and this one is set B.

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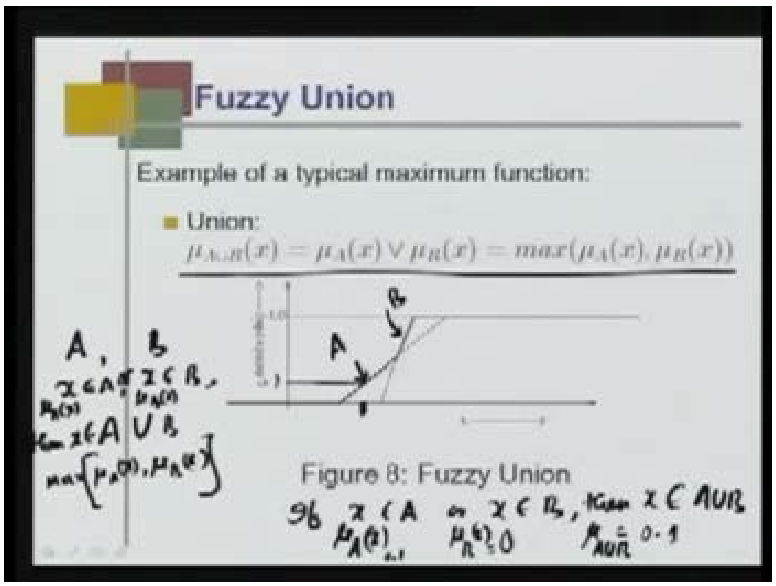

This one is A, this one is B, fuzzy set B; the members in A. What is the meaning of this? When I say this is A fuzzy set, it means all the members in set A has a membership function defined by this gamma function; whereas in B, all the members in B are assigned a membership function in this manner. That is the meaning of these two curves that we have and then we are trying to find out what the fuzzy union is. I have to find out In this the members are both belonging to A and B. But their membership is maximum of both. if I have common members. I have set A and I have set B; A union B is my union set. If x belongs to A and x belongs to B, then x also belongs to A union B. But in fuzzy set, here this is mu $_A$ x and here it is mu $_B$ x and in this case, this is maximum of mu $_A$ x and mu $_B$ x; the membership function. That is the way it is assigned. I again repeat, because I do not know whether you follow it.

I am just repeating the meaning of fuzzy union operation. This gamma function represents the way all members in set A are assigned fuzzy membership. Similarly, this curve represents the membership function by which all the members in B are assigned membership function. When I say A union B set, I am talking about a set that is A union B. How do I assign the membership towards this set? This is the question. I am saying if x belongs to A and x belongs to B, then x belongs to A union B. If x belongs to A or x belongs to B, to be very clear, here I am writing if x belongs to A or x belongs to B, then x belongs to A union B. But here when x belongs to A in this fuzzy set, the membership function that is associated with x is mu $_A$ x, like for example, this one. In this case, if my x is this value, (Refer Slide Time: 40:01) this belongs to A and it has a membership function. This value is say 0.1; here it is 0.1. But this member does not belong to set B and hence the membership function is 0 here; so, mu $_B$ x for this particular candidate is 0.

This candidate, when it comes to A union B what I do is, I take these two values, find the maximum which is 0.1 and assign here, which is 0.1. This is, mu $_A$ union $_B$ is 0.1. This is the meaning. This sentence says all that I discussed. I hope this is very clear to you. This is a very important operation that we do. When we have two different fuzzy sets, the operations are classical. The manipulation is among the membership functions; otherwise, the notion of the classical fuzzy operation also remains intact, except that the associated fuzzy membership gets changed.

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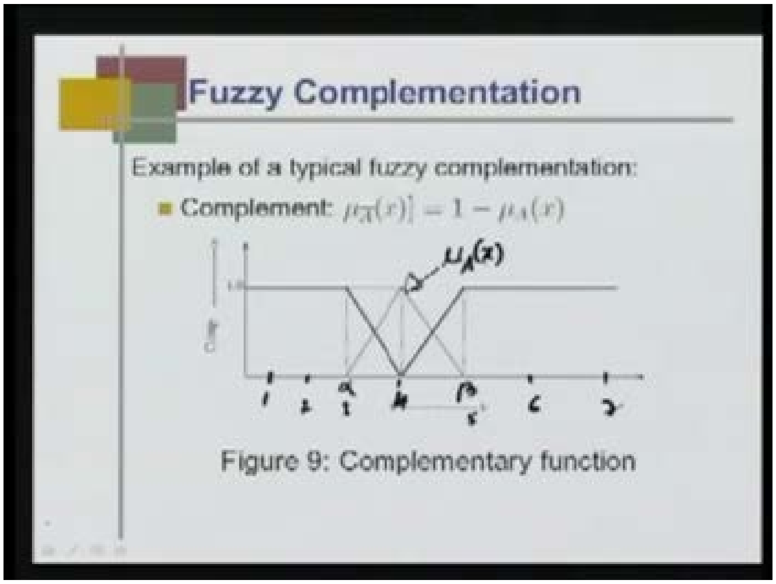

We talked about fuzzy union and fuzzy intersection; now it is fuzzy complementation. What is complement? This one, this particular triangular function is my set A; fuzzy set A. With this, and The way the membership functions were defined is according to this triangular function. That is if I say this is my alpha and this is my beta, before alpha, all the members in my set A have 0 membership function associated. Similarly, all the members beyond beta in set A are all associated with 0 membership function, whereas when I say complement of the set A, then what happens? The complement is like this; just inverse. What is 1 minus mu $_A$ x; meaning 1 minus mu $_A$ x. This is 0 and this becomes 1 (Refer Slide Time: 42:12 to 43:16) This was 0, and here it is 1 and this is a decreasing slope, linear line with decreasing slope.

Now, this is increasing slope, but the complement has decreasing slope. This was the inverted triangle and the 0 side have become inverted to 1. That is meaning of 1 minus mu x. What you are seeing is that the members remain intact in the set A, whereas the associated membership functions got changed. To let you know that, if I put here various values 1, 2, 3, 4, 5, 6 and 7, you can easily see that 1, 2 and 3 had 0 membership associated in fuzzy set A. In complementation, they all have membership 1 and you see that when it is 4, in the fuzzy set A, all the 4 is associated with 0 memberships, whereas in complementation, it is 1. Similarly, 5, 6, and 7 in fuzzy set A, the associated membership is 0, whereas in complementation they are all 1.

We talked about three very important operations on fuzzy set union which is maximum function intersection, minimum function and complementation is 1 minus the original membership function, membership value. The other operations that we know for classical sets like De Morgan’s law, the difference also can be used for the sets like De Morgan’s law.

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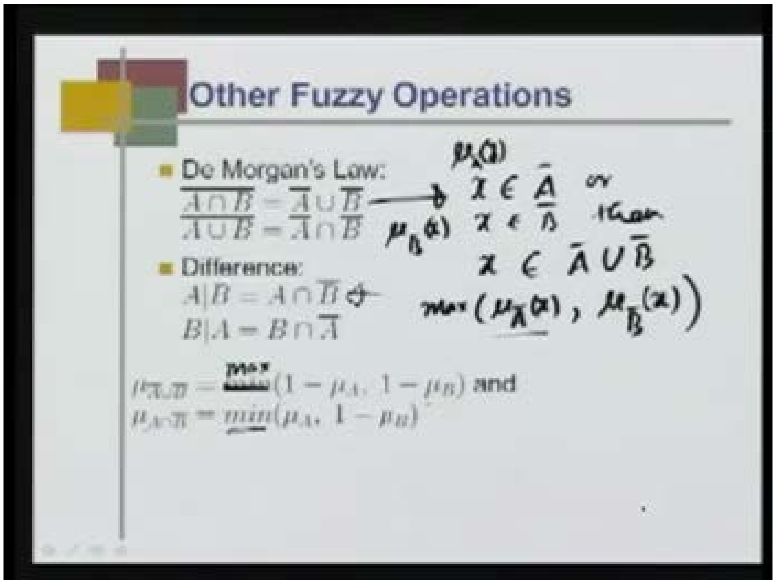

Let us take the first one; this one. In this case, we do the same thing; that is we take the A complement, B complement and then we see, if x belongs to A complement and x belongs to B complement or x belongs to B complement, then x belongs to A complement union B complement. This means if x belongs to A complement or x belongs to B complement, then x belongs to A bar union B bar. But here this I would say, mu $_A$ bar x, the associated membership function. Here it is mu $_B$ bar x, the associated membership function. Then what should be the associated membership function with this x?

Maximum that we know with this is maximum of mu $_A$ bar x and mu $_B$ bar x; bar means complement that we talk about. This has been written like this; you see here; that is the membership function with A bar union B bar is, this is maximum; it is not minimum, this is maximum. mu $_A$ bar x is 1 minus mu $_A$ that we have learnt, this is a complementation. Similarly, mu $_B$ bar x is 1 minus mu $_B$. Among these, whatever is maximum is the membership associated with A bar union B bar.

Similarly, let us take the difference case, which is this one; mu $_A$ intersection B bar. When I say intersection, this has to be minimum and associated with A is mu $_A$ and membership associated with B bar is 1 minus mu $_B$. Among these two, which is their minimum is the membership associated with A union B bar.

These are the properties of fuzzy sets.

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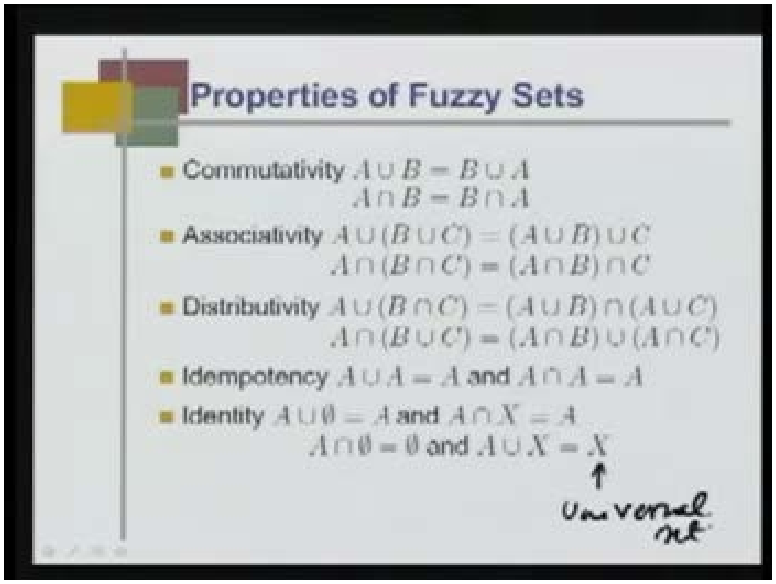

They are commutative. A union B is B union A; A intersection B is B intersection A. It is like classical sets; fuzzy sets equally hold. Associativity; A union B union C is A union B union C. Similarly, A union bracket B union C is A intersection B intersection C is A intersection B combined with intersection C.

Distributivity: you can easily see that A union B intersection C is A union B intersection A union C which is here. Similarly, here A intersection B union A intersection C. So, this is distributivity.

Idempotency which is A union A is A and A intersection A is A.

Identity: A union null set is A, A intersection universal set is A, A intersection null set is null and A union universal set is universal set X; here, X represents universal set.

We will finally give an example to illustrate how fuzzy operations are done.

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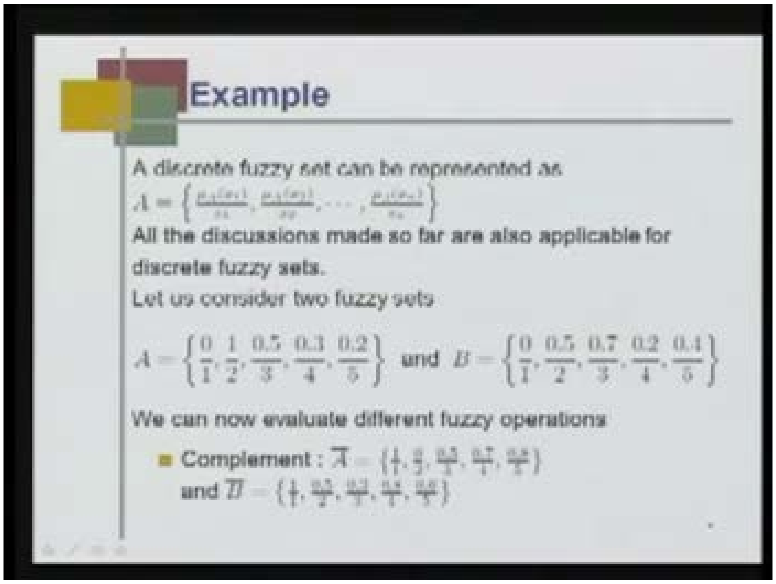

We will take discrete fuzzy set, because, that will be easier for us. That means the set has a finite member; like earlier, when I showed the fuzzy set operations they were for continuous sets, but here it is finite sets. It is easy also for us to understand and appreciate the operations that we did or we learnt in this class. Here you see that we have n members in this set A and each member is associated with a membership function; mu $_A$ x_1, mu $_A$ x_2, mu $_A$ x_n. The fuzzy set is A. It has n membership and each member is associated with a membership function. Like that we have 2 sets. One is A and another is B. In A we have five members 1, 2, 3, 4, 5. Similarly in B also we have members 1, 2, 3, 4, 5. But the difference is, here the associated membership is different and here the associated membership is different. Here, the weight is given say 0 1. These are the membership function; these are also the membership function.

Now, let us evaluate all the operations that we did. First is complementation. Given A, A bar; here 1 is associated with 0 membership; that means ideally we do not keep 1 inside A, because, it does not belong to A. But in complement, 1 belongs to complement, because with membership function 1. Similarly 2 here have a membership function 1 and hence 2 do not belong to A complement; that is, membership function is 0. But, these are all fuzzy. 3 belongs to A complement with membership 0.5, whereas here also it is 0.5. 4, it is 0.7, here it is 0.3. 5, it is 0.8 here and it is 0.2. This is complement; that means whatever the membership here, you just subtract that from 1 and you get the membership in A complement. Similarly, B complement;

you see here, whatever is the membership function, they are all subtracted from 1 and we write it here.

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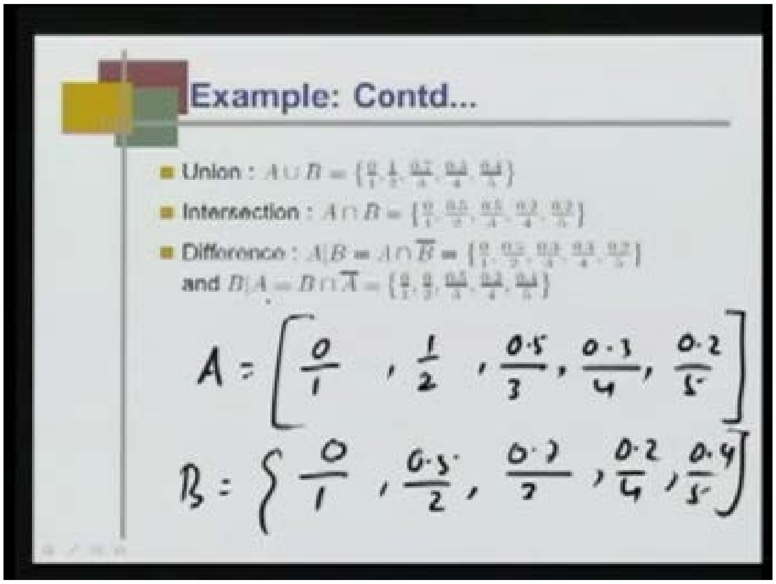

Similarly, A union B; let me write down here, because I must show you one example so that you appreciate it. 0 by 1, 1 by 2, 0.5 by 3, 0.3 by 4, 0.2 by 5; this is my fuzzy set A. Fuzzy set B is 0 by 1, 0.5 by 2, 0.7 by 3, 0.2 by 4 and 0.4 by 5. These are my 2 sets. When I find A union B, what is the maximum? In both 1 and 1, they have 0 memberships; so, in A union B, 0. Here, it is 1 and for 0.5, so, I take 1 which is our maximum. For 3, the maximum is 0.7 so, I take 0.7. For 4, the maximum is 0.3 so, I take here 0.3. For 5 it is 0.4 and I take 0.4.

In case of intersection, A intersection B, you see that I have to take minimum now. Minimum means 1, they have same, so 0. In this case 2, the minimum is 0.5 so, I take 0.5. 3 I have minimum is 0.5 so, I take 0.5. For 4, the member, I have the minimum is 0.2 so, I take 0.2 and for the member 5, the minimum is 0.2. So, I take 0.2.

Similarly, you can also evaluate the difference, A difference B or B difference A. Like that whatever operations we learnt, we can apply to this example. Through this example, I hope that you understood how to apply the various fuzzy operations on two different sets or on a set. This is very important because, they would be useful as we learn further.

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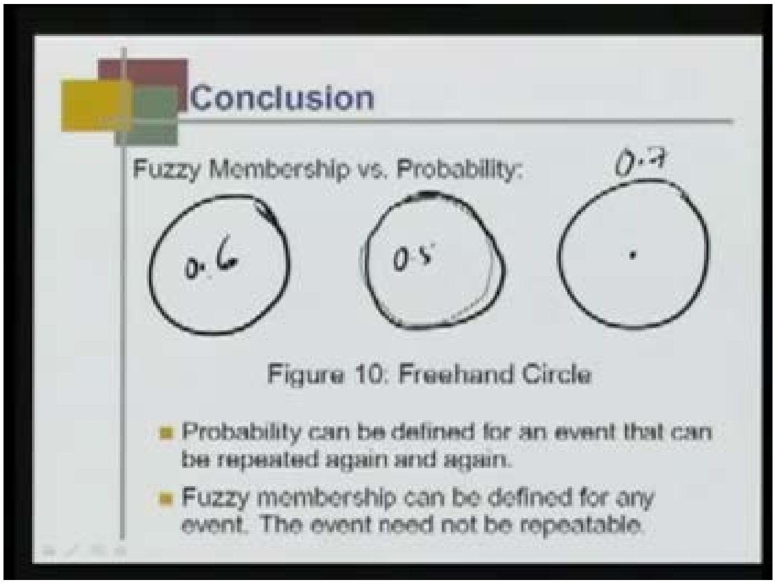

Finally the conclusion; earlier I told that fuzzy logic was introduced to deal with imprecise data that we encounter in our day to day life. Here, I give you another very interesting example as to why fuzzy is very powerful? Here, somebody has drawn a circle in hand like this or I can draw any number of circles. I would like to draw them. These are free hand circles. But as many times I may try, very rarely I would find that I can actually draw a circle in free hand, because circle means from the center all points in this curve should be equidistant. Then only it will be a circle.

If I ask a question: what is the probability if I draw once on a board, You go to the black board and draw a circle and ask a question, what is the probability that this is a circle. The answer is, I cannot define probability here, I have drawn the circle only once because I have drawn the curve only once. I have to repeat it millions of times; probably, I may not be able to still draw a circle. In this case, I draw a free hand circle on the blackboard. They appear to be circle, but they are not circles; rigidly, mathematically they are not circles. Hence, using probability, First of all if I draw a circle once, I cannot define probability. Probability means I should be able to repeat that experiment. Probability can be defined for an event that can be repeated again and again. But now, I am asking a question that I have drawn a free hand curve, I am asking a question whether it is a circle or not? What is your answer?

Using probability theory, I cannot give an answer. But using fuzzy logic, I can always say this is a circle with membership function say 0.7. This is a circle say 0.5 membership function and this is a circle say 0.6; it all depends. What I am trying to do is that now I am able to say that they are circles with a possibility index 0.7 or 0.5 or 0.6 or 0.2, depending on the looking at the curve.

You draw a free hand circle and anybody can assign some kind of belief or confidence level on this. How good it is as a circle? I can get an answer. But using probability theory, I cannot get an answer for such a question. Fuzzy membership can be defined for any event. The event need not be repeated, because I can draw a free hand circle on the black board and still assign a fuzzy index for it.

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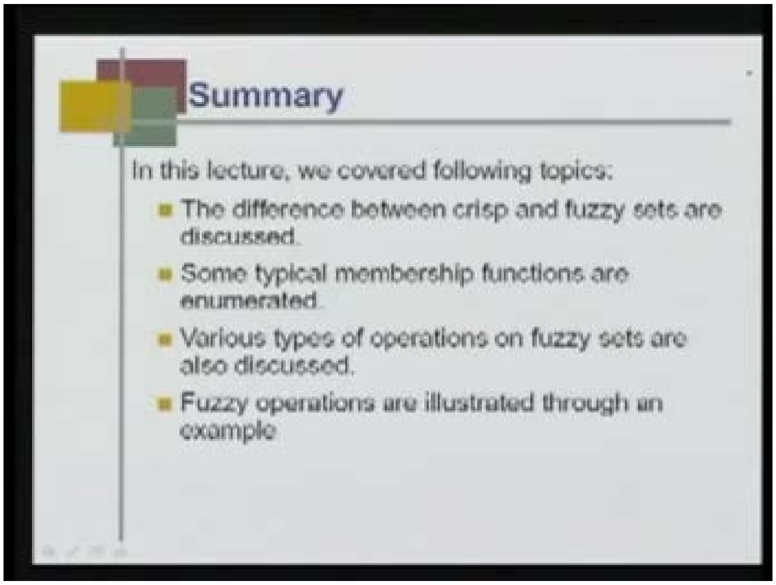

Finally, what we discussed in this lecture is the difference between crisp and fuzzy sets. In this, you saw that in crisp, we have only members. In fuzzy sets, along with members, associated membership function, some typical membership functions are enumerated. We had Gamma function, s function, pi function, Gaussian functions and we also discussed various types of operations on fuzzy sets. Primarily, three operations are: fuzzy union, fuzzy intersection and fuzzy complementation and any other fuzzy set operations that we know can also be applied on a fuzzy set.

Fuzzy operations are illustrated through an example. In the last example, we took a discrete fuzzy set. We showed how we can compute fuzzy complementation, fuzzy union and fuzzy intersection. With that, thank you. We will meet in the next class to discuss fuzzy relations.