High Voltage DC Transmission

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Module No. # 06
Lecture No. # 06
Dynamic Stability Analysis

So welcome to lecture number 6 of this module 6. In the previous lecture we saw that various modeling aspect of AC and DC systems, and we derive the various differential equations for the different blocks, different models and that should be soft. So in this lecture, I will slightly likely I will just review the integration techniques that will be used to solve those differential equation, then I will move on the dynamic stability analysis.

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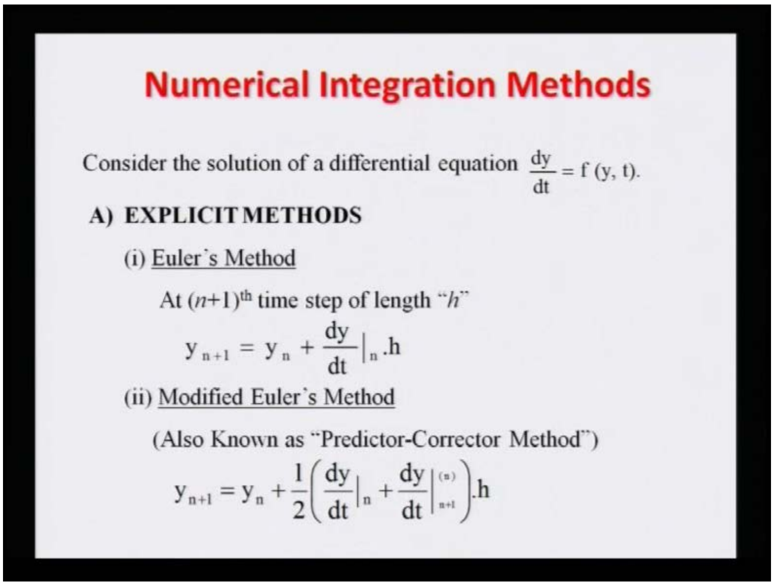

So, to solve the any differential equations there are the two types of methods are consisting, one is the explicit method, and other is implicit method. So, in the explicit methods the Euler's method as well as the modified Euler's method, Rungkutta method and so many other methods are proposed by the different scientists and the mathematicians.

So, let us see what is the Euler's method? Say in suppose you want to solve this is a differential equation dy upon dt is equal to f (y,t) and this is a function differential function with respect to y and t this differential function with respect to t this can be solved by the Euler's method. So, at any step that is n plus one th time step having the length h that can be written here y is equal to n plus 1 time will be equal to y nth plus dy upon d dt and this value should be evaluated at here this dy by dt, you can see this function.

Here, if we want to put at y n time, so this is y n here multiplied by the h. So, this is your solution and then it can be progressively it is moving starting from n is equal to 0 to n is equal to larger value, and then we can get the function tracing with respect to time. So, this we can plot the function with respect to time; time step length that is h and then the different value of y x is such value we can get from this Euler's method.

Another here in this there's method there is a lot of problems then the modified Euler's method was proposed and this is also known as the predictor and corrector method sometimes what here it is slight modification instead of the second term of the first Euler's method is now replaced by the two term one is the same that is dy upon dt at nth instant plus dy upon dt plus n plus oneth instant. So, this what we are doing here we are just calculating this two and then divide taking average multiplied by h.

So, this term here the third term now you can say that is replaced and instead of this first term second term here. So, we are having the two terms. So, here this is average of this is nth plus n plus nth term and that is calculated here.

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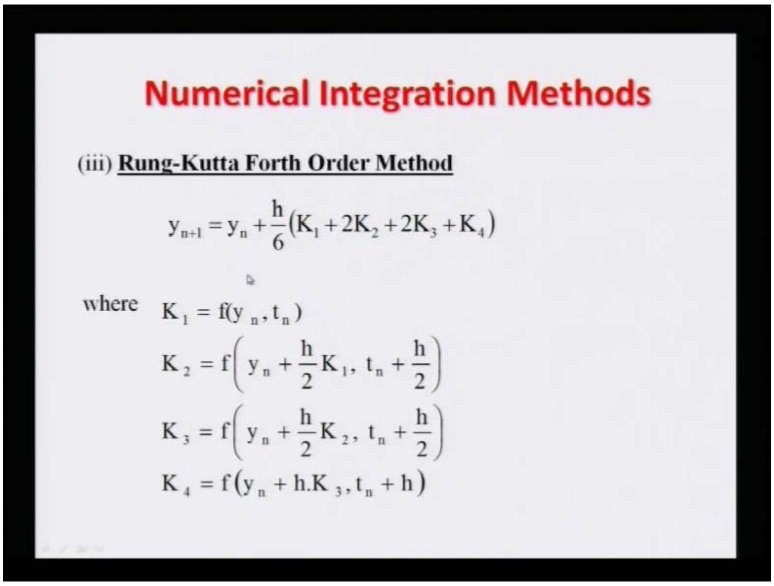

Another very popular method that is called Rung Kutta method for solving the differential equations whether is a linear non-linear differential equation for it is both where it is valid. So, thus you can see this is a equation just if you want to calculate the value of y at n plus oneth instant that will be equal to y n plus there is a term here of the various k s that is K 1, K 2, K 3 and K 4 that is having some here the K 1 plus; 2 K 2; plus 2 K 3; plus K 4 sum together then we are multiplying by h and that is divided by 6 these values are basically calculated and it was given by the rungkutta method.

The K 1 is nothing but it is the function value this function value which is just you can see this is a function value here and that value is basically at here nth instant. So, this y n t n value thus will give the K 1, K 2 is calculated using the K 1 function the K 1 value once we have calculated now you are again this a function that same function y t where we are putting this value this y and plus modified by h plus 2 K 1 and then we are adding the t n plus h by two that is h is the step length.

Similarly; the K 3 is using your K 2 and the K 4 is using K 3 and this is progressive. So, all these constants are calculated and then there putting this equation and then you can get this y nth plus. So, there is a various order here is a fourth order because you are getting the fourth K constants already two and three order also differential equation solutions of rungkutta methods are available but this is very badly used third and fourth are the rungkutta methods.

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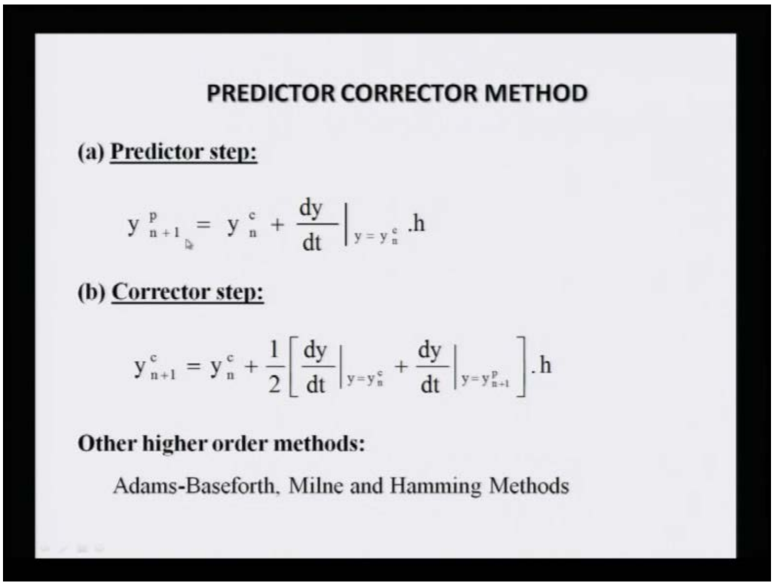

Another here method is that is called predictor and corrector method here in one step we predict and second step we cut. So, in the predictor step what we do we just calculate the value of y that is predictor value at n plus oneth instant and here that is why the p is denoted that is showing that it is a predicted value that will be equal to the corrected value obtained in the previous step and the previous step is nothing but your this nth step before because the n plus oneth step we are calculating. So, previous is your y nth the value and this is c showing the corrected value just we are using plus dy upon dt and this value is evaluated at this y is equal to y n c this is corrected value just we are calculating multiplied by h.

So, this is you are predicting but here this prediction must be corrected because there is some error and that is correction here just we are getting the n plus oneth corrected that is a final value will be equal to the corrected value of nth instant plus half of this two value one is the before the corrected step and another we are using at the predicted step

Because y n plus 1 predicted because this c value is not known because we are going to calculate here. So, this is average of the value dy upon dt at the nth corrected value plus the dy by dt at the y plus nth predicted value that should be averaged out and then it is multiplied by h that will give you this nth plus 1 instant the corrected value of your y

There are some other methods as well and that is higher order methods sometimes adamsbaseforth method Milne method hamming methods etcetera that are also proposed and that has been used for the various human applications.

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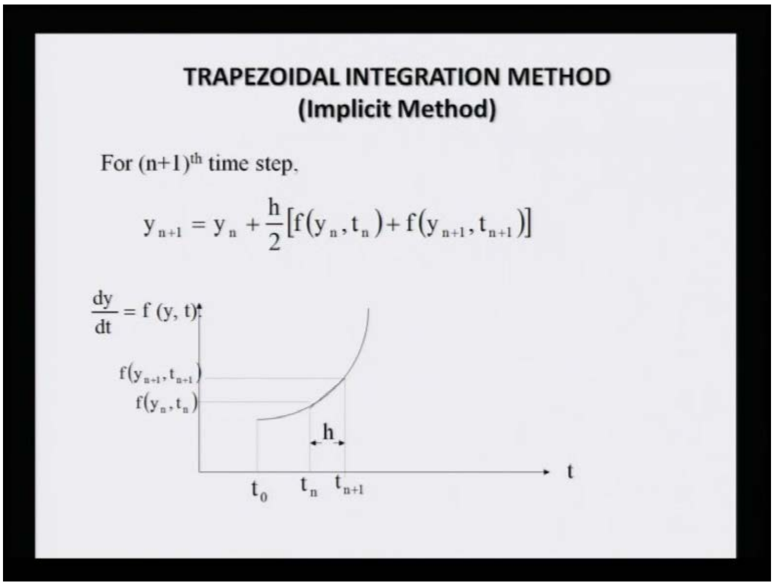

Another is very widely used that is implicit method here in this it is called the trapezoidal integration method and this is also known as implicit method here what we do we use the trapezoidal rule of integration means you can see that is the value here the y n plus at the instant n plus oneth instant will be equal to y n plus this is here if you can say this is the area of the trapezoidal that can be understood by this c if this is a curve which is dy by dt and this is a time step you can see the function value here this we are talking the function the function at the t n it is we are getting f n t n n m t plus n one instant we are getting this

So what we are doing we are just linearizing this and you can see this whole this is a trapezoidal. So, if you want to calculate the area of this trapezoidal what will happen this is here the multiplied by this and the average of this side you can say right hand side and the left hand side then you can take average it by 2 and then multiplied by s. So, you can say this value means we are taking this height that is from here to here and then we are here this second term we are adding and then we are taking by 2 multiplied by h that is this trapezoidal

So, this is a trapezoidal concept we are linearizing because we are assuming this h is very small. So, that can be taken as a linear function even though its function is non-linear but this a we can make this is linear that is why it becomes trapezoidal. So, this is also very widely used but it has some limitations. So, the people go for the rungkutta another methods because this method sometimes with nonconverging solution.

So these methods are basically can be used for the solving the differential equations and then we can predict the stability of the system that is a transient stability.

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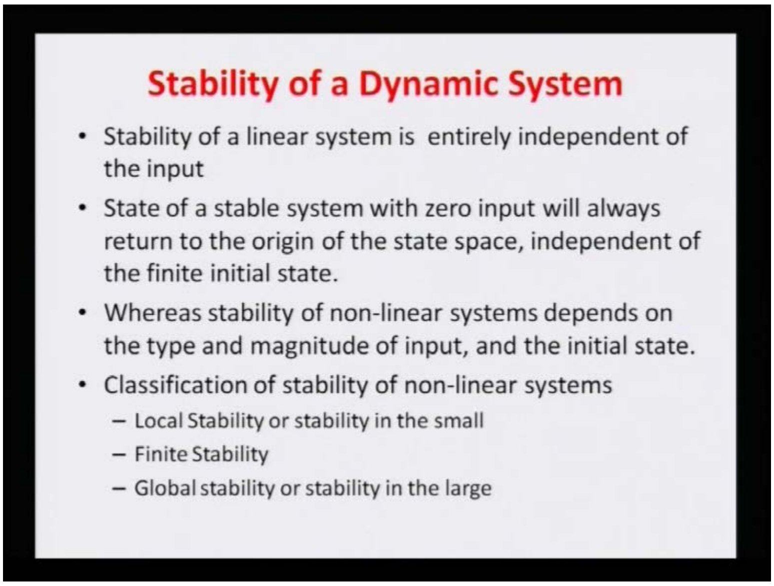

Now, I have to come on the another the main topic of this lecture that is the stability of a dynamic system here we are going to discuss that what is the dynamic stability of a system

To begin with let us see the stability again to just revise the stability of a linear system is entirely independent on the input because it is not dependent on its initial state and what is its other value but it depends upon the input the state of a stable system with the 0 input will always return to the origin of the state space independent of the finite initial state. So, here for the linear system even though this two bullets are showing for the linear system because the linear system the stability is directly related to the input it is not related to the initial as well as the system here as well.

So you can see in the linear system if your input is 0 then always it will return to its original value and it is that is a independent of its initial state where it is lying however for this non-linear systems which depends on the type and magnitude of input because it depends upon magnitude as well as the type and also at the initial condition

So these three are the governing criteria in which decide the stability of a non-linear system since the power system including HVDC system is highly non-linear. So, that we have to see this we have to take the non-linear system as well. So, the classification of stability of non-linear system can be classified in the three terms one is called the local stability or the stability in small or it is known as another is finite stability third one is the global stability or stability in the large.

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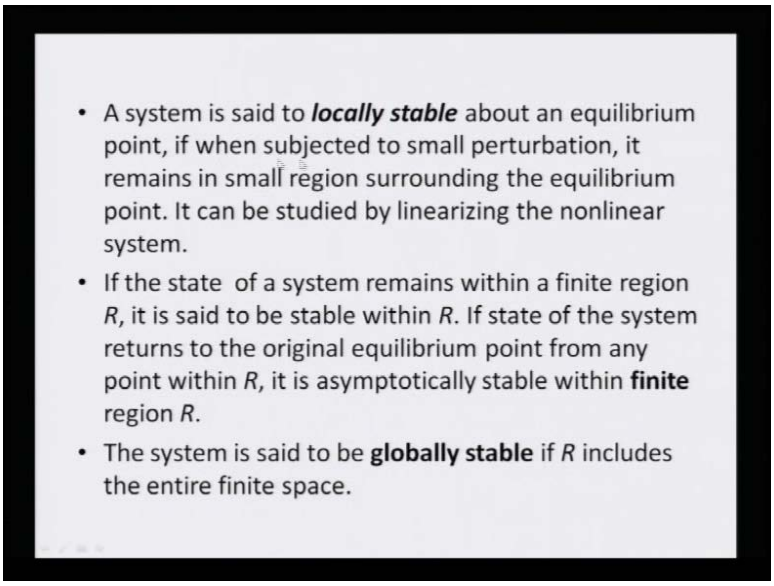

So a system is said to be locally stable about an equilibrium point if when subjected to small perturbation it remains in small region surrounding the equilibrium point it can be studied by linearizing the non-linear system what does it mean its meaning that is if you are having the your equilibrium point..

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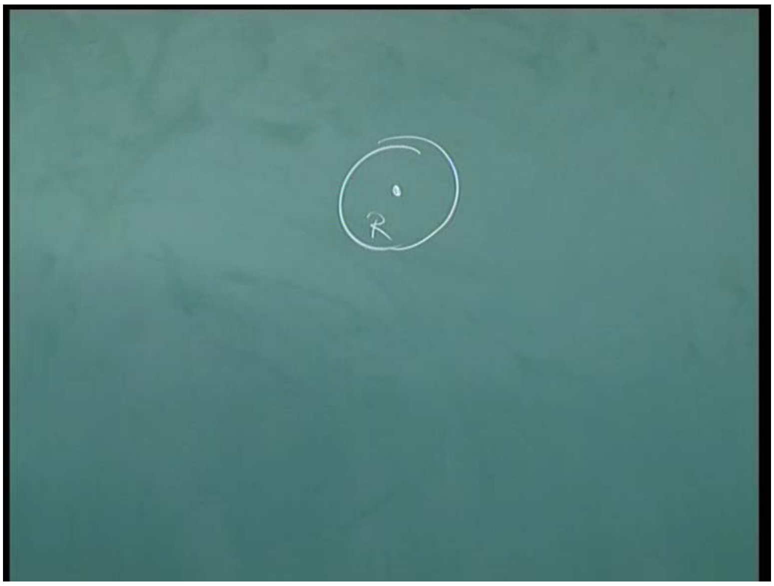

And the equilibrium point you can see from here that is if we are having the equilibrium point here and then what happens your the system can live with around the equilibrium point then we can say it is locally stable

It is not after putting some input at between some disturbance if it is deviating from here from other equilibrium point and this region it is fine coming here. So, that is why it is said the system is said to be locally stable about an equilibrium point if subjected to small perturbation it remains to small region surrounding the equilibrium point it can be studied by linearizing the system. So, non-linear system must be linearized to see the local stability linearized and then it should be studied and that we will see in this lecture

If and another way is a finite stability that is also defined in the another way if just state of a system remains within a finite region R it is said to be the stable within the region R if the state of the system returns to the original equilibrium point from any point within R it is asymptotically stable within the finite region

So this is now we are talking about this is a region R. So, it is stable in this and if we are it is coming to the original equilibrium point then it is called the asymptotically stable a system is said to be globally stable if all R which includes the entire space space. So, it is can say the globally stable for any region. So, region of R should be very large and the finite space entire So, that is called globally stable.

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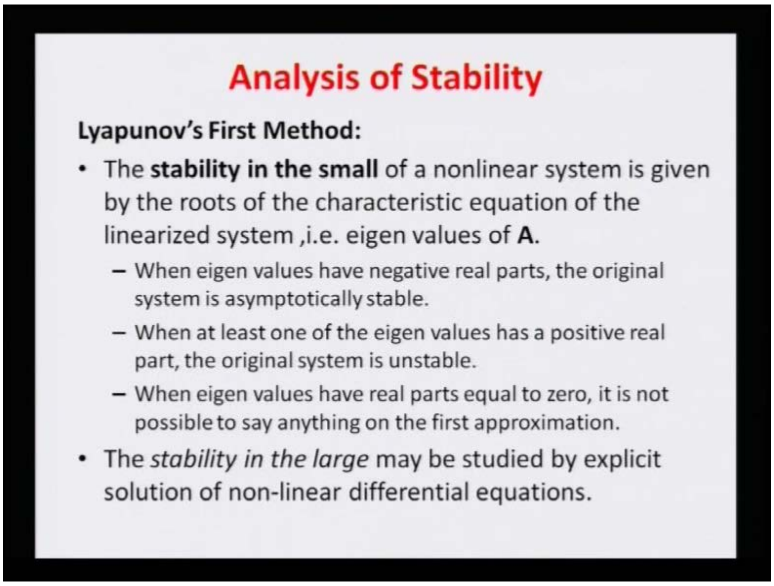

To analyze the stability of the system here this lyapunov's first stability methods that is called the lyapunov's first method in this the stability in the small of a non-linear system is given by the roots of the characteristic equation of the linearized system that is basically nothing but Eigen values of A that is the state transition matrix.

So, by looking the Eigen values first you have a non-linear differential equations you are linearizing and then you are getting the state space representation where a is your state matrix and then if you are calculating the Eigen value of a and this and that Eigen values you can derive lot of observations and those observations will tell that your system is stable or not stable,

For example if you are having these Eigen values means your all Eigen values are having the negative real parts then we can say your original system is asymptotically state if all the Eigen values are having the negative real parts we are only talking about the real parts not about the imaginary parts

Second condition is that when at least one of the Eigen value has a positive real then original system is unstable.

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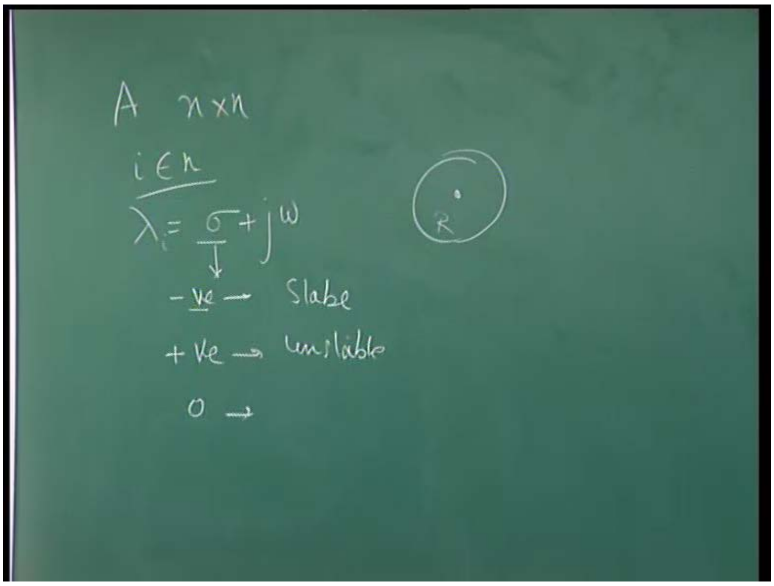

What happens if you are having this Eigen value here I can say this omega plus j here omega sigma omega in this j omega here we are talking this value is your negative then all it is negative it is not 0 we are talking negative means negative. So, it is negative then it is stable.

If positive at least one then it is unstable and if it is 0 because it can be positive it can be negative or it can be 0 as well then for the 0 you can see when Eigen values have a real part equal to 0 it is not possible to say anything on the first approximation. So, we cannot say whether system is stable or unstable it is may be absolutely it will be stable or unstable. So, just we are having the three criteria based on the Eigen value this is we are having large number of Eigen value it depends upon what is the size of a,

If you are having this A matrix here this can be your n cross n matrix then you are having this is I here is having the n values and then we can see for all the Eigen values we are talking. So, this is the K th for the stability in the small means that is a dynamic stability because where the non-linear system is just treated as the linearized system where we are linearizing around the equilibrium point and then we are trying to get the information of the Eigen values

The stability in the large may be studied by explicit solution of non-linear differential equations and that is why we went for the transient stability analysis. So, just I were talking here the stability in the large may be studied by the solution of the non-linear differential equations. So, we have to use the differential equation we have to model all the components of AC DC system and then we have to use the solution techniques for the solving these differential equation and then we can get the stability in the large by solving those and we can see the pattern whether system is stable or unstable

So the stability in the small is related to your dynamic stability means where the non-linear system is linearized and then we are trying to get the Eigen values and based on that we are protecting whether system is stable or not stable. So, this is a lyapunov's first method.

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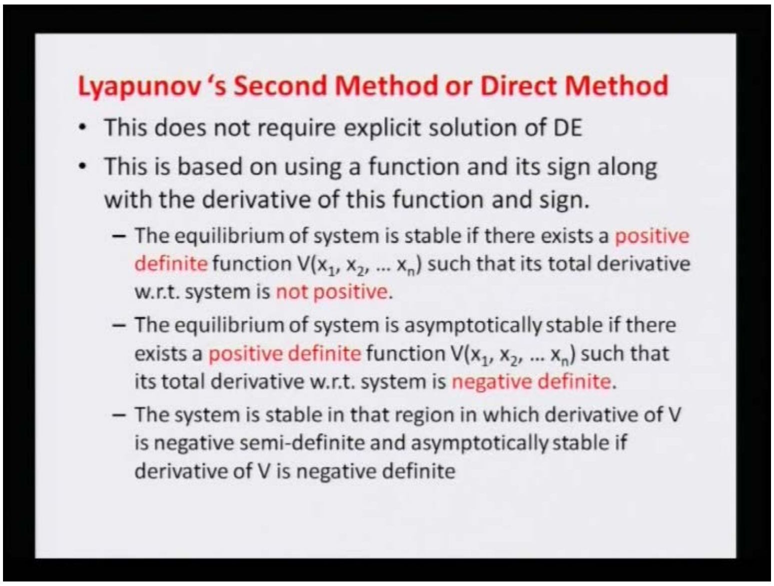

Lyapunov's second method or also it is known as the direct method here it does not require any explicit solution of the differential equations it is based on the using a function and its sign along with the derivative of the function and its sign. So, we can form a function that is called the lyapunov function and then we can see the function nature along with the sign then we can differentiate this function and then we can see the sign then this will give the information about your the non-linear differential equation that is without solving this equation

So what happens that in this or the equilibrium or the system is stable we can say the equilibrium of system is stable if there exist a positive definite function v. So, we should be very clear about what is the positive definite function what is the semi definite positive function all and what is a negative definite etcetera each will be clear

So the function V which is having thus X 1 to X n states. So, these are having the n state system because we are having a matrix that is having states n cross n a matrix is n cross n means we are having X here I can say it is X this matrix this variable it is having one this n cross one elements means it is having n elements X 1 to X n such that it is total derivative with respect to the system is non positive

So it is a stable if it is a non positive means you can be very careful non positive means it will be negative or it can be 0 but it should not be positive. So, the system will be said to be the stable if there exist a positive definite function v such that the total derivative with respect to system is non positive we can also further categorize this equilibrium of system is asymptotically stable if there exist a positive definite function V of X 1 to X n such that its total derivative with respect to system is negative definite means it is not 0 it is not positive it is always negative then we can say the system is asymptotically stable

Also the system is stable in that region in which the derivative of V is negative semidefinite and the asymptotically stable if the derivative of V is the negative definite. So, if it is negative then we can say it is asymptotic if it is negative semidefinite then it is called your stable in that region. So, these are the three criteria derived from the lyapunov's second methods or direct method So, here there's no need to solve the differential equation

We can form the function that it is called lyapunov's function and then we can see it is derivative of this function and along with this function itself we can conclude about the stability of the system but however we cannot form we cannot predict about the relative stability etcetera.

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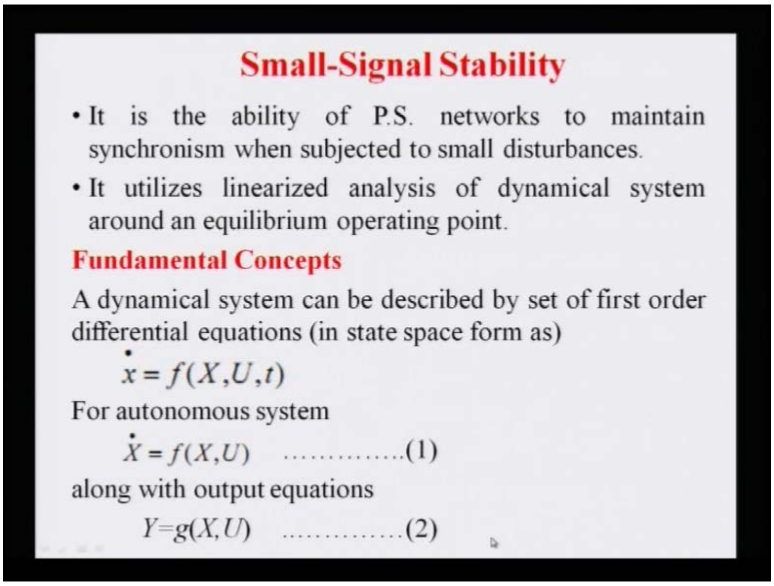

So now thus as this lecture is dedicated for the small signal or dynamic stability of the system. So, it is ability let us definite what is this it is the ability of the power system network to maintain the synchronism when subjected to small disturbances if you are talking about the large disturbance then it becomes the transient stability and in that case the linearized system will not work we have to solve the differential equation or you can use the lyapunov's methods this second method to check whether the system is stable or not.

So since the disturbance is small. So, it utilizes basically the linearized analysis of the dynamical system around an equilibrium operating point. So, we can linearize the non-linear equations around the equilibrium point and then we can go for this analysis

So to see this fundamental concept this dynamical system can be described by the set of first order differential equation here you can say X dot is equal to f X U this. So, this is the function here and this may be a non-linear function and then you can say it is in state space form we can go for this here this is for autonomous function this here t is means not varying. So, t is not there. So, for autonomous system I can say X dot is equal to a function of X and U X is your state of the system and U is your the input of the system

Along with your output equation that is why is your output that is governed by another equation another function I can say it is g which is also a function of X and U. So, here X is the states of the system and U is the input.

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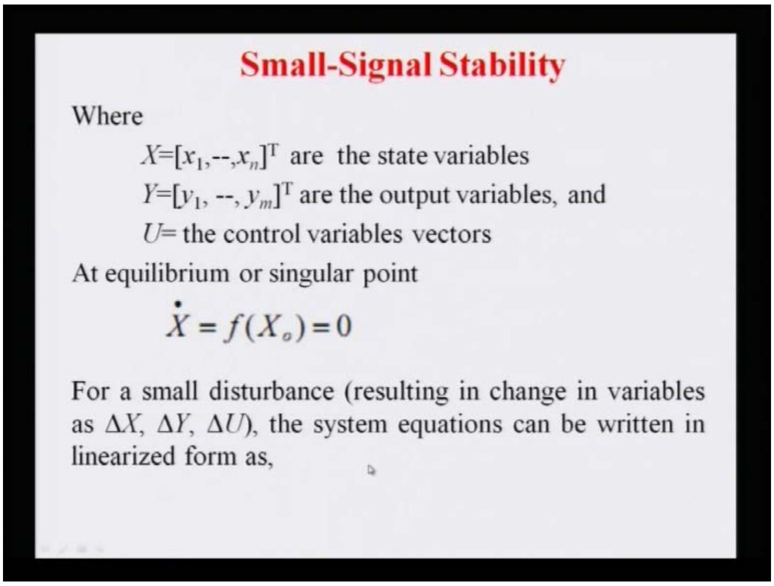

So the we are having the output this Y this you can say that is why here it is written where X is having x 1 to x n it is assumed that is there are n states variables and then output is having the mth output variables and U is the control variable vectors. So, at equilibrium point or singular point this X dot are (( )) points what happens X dot is equal to 0 means is not varying with the time. So, the function X not that will be equal to 0 here

For small disturbance resulting in the change in the variables then what will happen your the states will change from X to another state. So, the change I can say the del X there is a change in output that is called del Y and if there is a change in the input that is the del U the system equation can be written in the linearized form.

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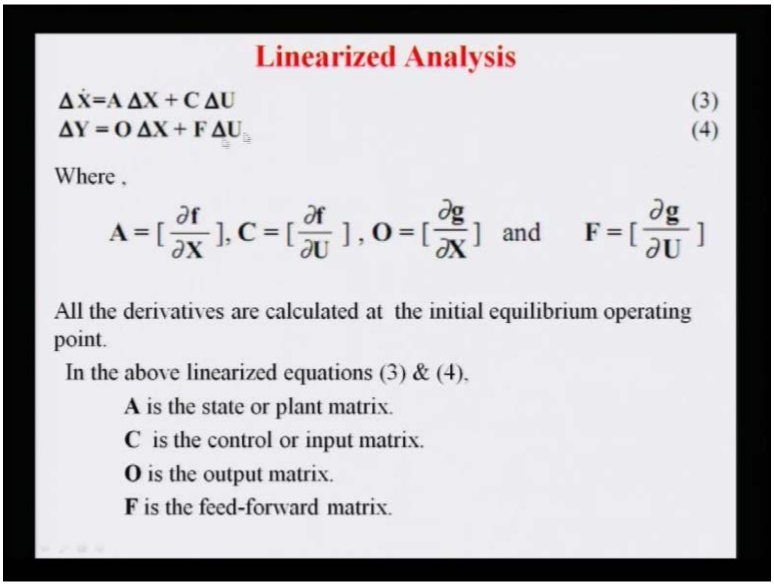

So, your this equation this first equation here, and the two can be written here in this form that is the change in X dot is equal to a multiplied by change in the X plus C multiplied by change in your input vector. And your output change will be equal to O matrix multiplied by del X plus F into del U. So, these two equations are basically the now linearized state space equations, where A is defined as the change in the function with respect to X and your C is change in your function f with respect to input however your U O and F are the change in the g function with respect to X, and your change in the g function with derivative of the g function with respect to U respectively.

So all the derivatives are calculated at these initial equilibrium points and here also this A C O F matrices are known a is known as the state or plant matrix it is also called the state transition matrix in some books C is known as the control or input matrix O is called your output matrix F is known as the feed forward matrix. So, we are having all these four matrices means there is A C and O F they are basically forming the linearized equation.

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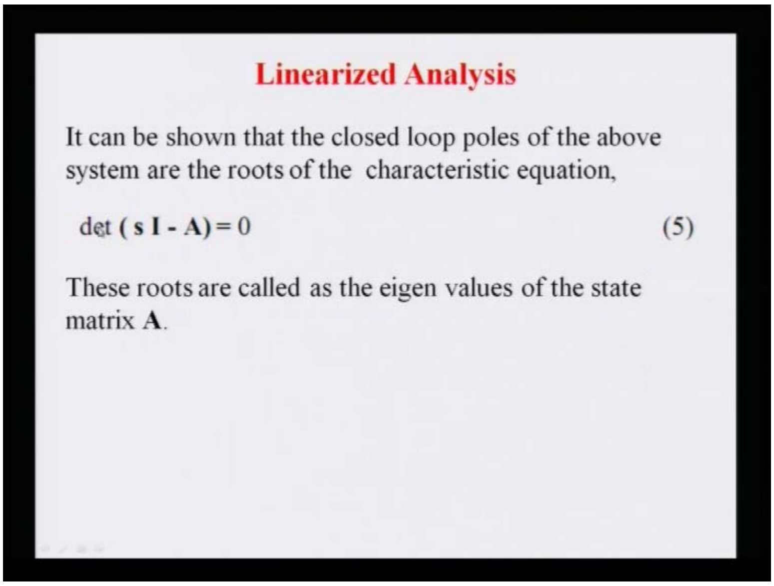

Now it can be now we can as after linearizing this now we can see the behavior of the system by nothing but we can see the characteristic equation and that is the roots of the characteristic equation will be basically giving the information. So, the it can be shown that the closed loop poles of the above system are roots of the characteristic equation and the root of characteristic equation here you can say the determinant of s that is s is the Laplace operator multiplied by your identity matrix minus A that is A transition matrix is equal to 0.

So if you can solve this means these roots are called the Eigen values of the state matrix h and this will be having the s will have equal to your the Eigen values.

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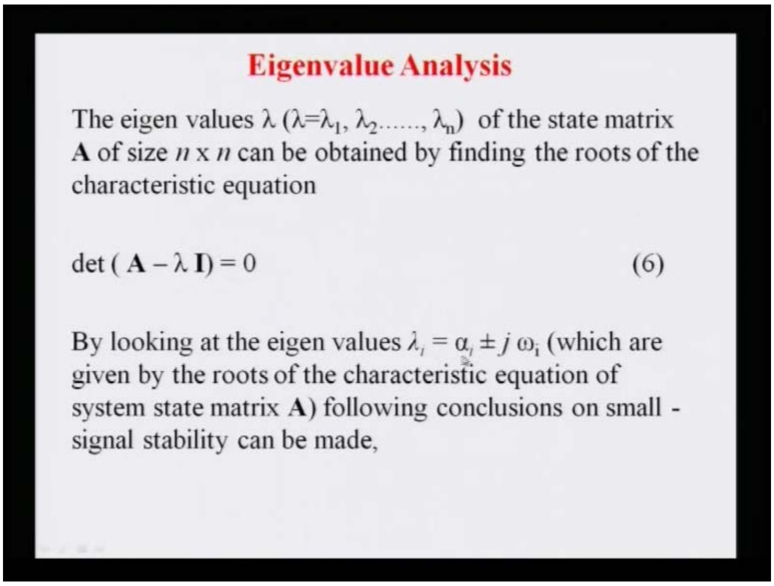

So the Eigen values here now I can replace here this earlier I used s now we can use the lambda lambda is nothing but these are the Eigen values and since your a matrix is having the size of n cross n. So, it will be having your n Eigen values. So, that is why you can see here the lambda is equal to lambda 1 lambda 2 to lambda n and these are the basically just of state matrix A can be obtained by finding the roots of the characteristic equation here A minus lambda I

So lambda that is we can get here after solving this. So, we can get the A minus lambda I also if you'll see then the previous equation I used the s I minus A both are having same means you are having A minus lambda I and you are getting the determinant that will be equal to the determinant of lambda I minus A because this is your and that is will be equal to 0 and we are solving this equation. So, it will not change

So by looking at the Eigen values and the lambda I as I said the I will be here n number because we have taken a of size n cross n. So, that can be written here alpha I plus minus j omega I which are given by the roots of characteristic equation system state matrix A following conclusion on the small signal stability can be drawn.

Here even though in the previous just I wrote here this sigma here we can also write here some in book we will find here alpha. So, for this here we can say this alpha I here omega I and this you will have always if you are having this imaginary part as well of the Eigen value So, it will always be in the complex conjugate. So, you are having this I can say if you are having the plus you will have the minus as well if you are having the imaginary part however if you are not having imaginary part. So, it can be having distinct or it can be the same. So, that depends upon here and there is on the system and as you call say the values.

So. here in general I can say this lambda that is Eigen value for the ith Eigen value will be equal to your alpha i plus minus j omega i and based on that we can draw the following information you can see here all these three informations are almost same as the lyapunov's first stability criteria.

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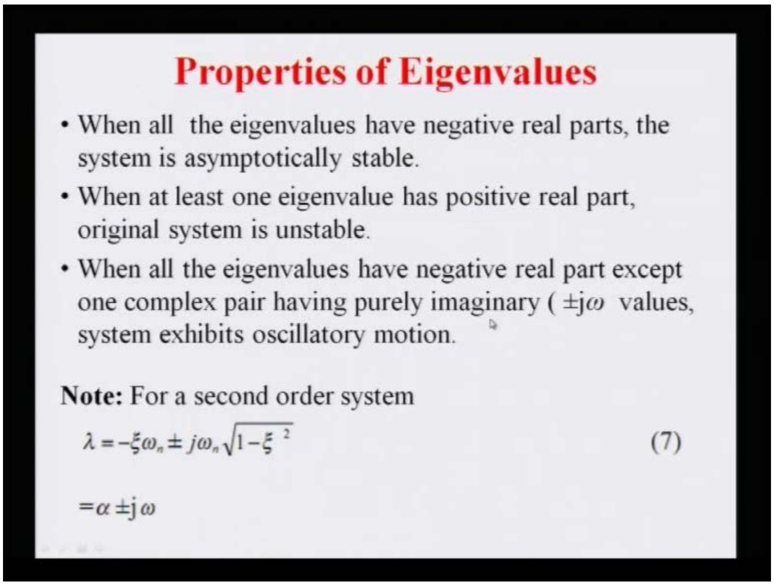

So that you can see here again I have written here when all the Eigen values have the negative real parts the system is asymptotically stable. I want to say that if all these here alphas here alpha will be the negative none of them even the 0 we are talking the negative So, all these here value will have the value and it will be negative then we can say the system is asymptotically stable if at least one Eigen value has the positive real part then original system will be unstable means at least suppose there is a n Eigen value even though one Eigen value is having positive then no need to see other one then system will be automatically unstable.

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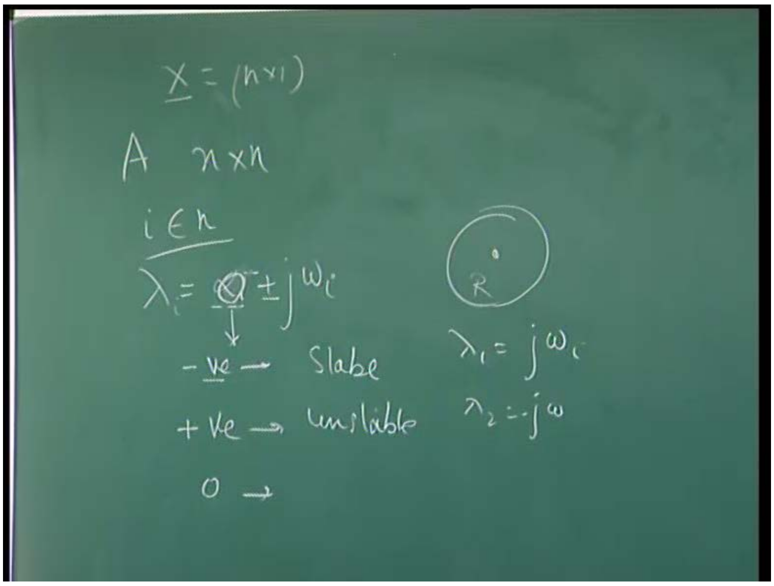

When all the Eigen values have the negative real part except one complex pair having the purely imaginary means at least you are having here 1 0 here this means you are having the two Eigen values that is one you can say lambda 1 will be your z omega i and another your lambda 2 is your minus j omega i. So, you are having the two Eigen values means they are the purely imaginary then system will exhibit the oscillation.

For second order system you can see this lambda that we can write here in terms of damping factor you can see here the lambda again I have written alpha is equal to plus minus j omega that can be represented by this another slightly deviated here omega i have written omega n under root 1 minus zeta square and this alpha is defined as the zeta omega 1 why it is written because this zeta is known as your the damping ratio.

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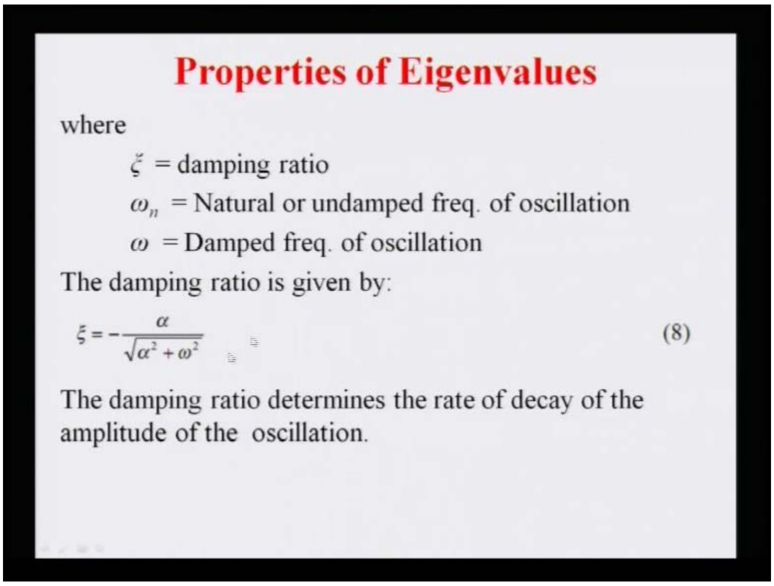

And this gives the very good information because the damping ratio determines the rate of decay of your amplitude of oscillations if there is any oscillation. So, this decide what is the damping factor if damping factor is higher it will be damped out easily and if damping factor is less then it will be damping of the long time

Omega n which is the natural or undamped frequency of oscillations and omega is called the damped frequency of oscillation and the zeta is defined as in terms of real as well as the imaginary values of your Eigen value you can say the zeta is equal to minus of alpha that is the real part of the Eigen value divided by the square of under root square root of the square of alpha square plus omega square and then it is giving you zeta.

So, this zeta. So, you can see if you are having this Eigen value is having the complex conjugate then you can have all these things you can represent in terms of zeta now if your here you can see the omega is 0 then you are having only the real part of the Eigen values and you can see if this is your omega is 0 you are having the zeta is your is unity and that is minus one. So, the damping factor is hundred percent. So, there is no oscillation.

But, if you are having some value here then it will have the less and based on that it will be the deciding factor. So, normally some of the differential equations are represented instead of here sorry see at this equation the real and imaginary part it is also equally important to represent this Eigen values in terms of the damping ratio and that also gives the lot of information about how system is your damped or critically damped or under damped that will give the information.

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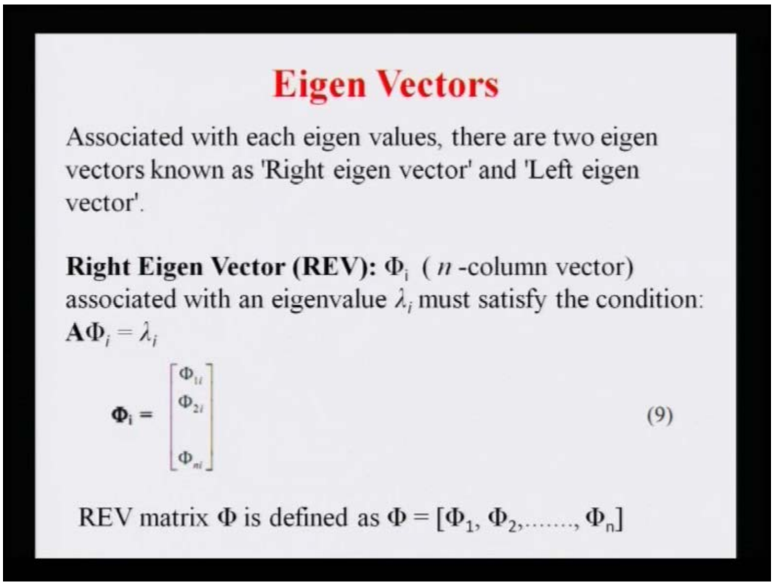

So now another first thing we saw the Eigen values based on the Eigen values we decide that your system is stable or unstable now that is not only sufficient because knowing only your system is stable is we are happy if system is unstable then the question arise what to do or if your system is stable very closely we want to make more and more stable we want to design a system to have a very robust system then we require some more information we require some more information that what states are creating the problem in the system and those states must be identified and then we can devise we can go for the some new controllers to improve the system stability.

So, the Eigen vectors are one two based on the Eigen vectors we can and of course, later on we will see the participation factors using this Eigen vectors we can derive or we can just devise some control mechanics and we can see which of the states are participating much or less in any control modes. So, that is required and to have this let us go for this Eigen vectors first.

So there’re the two Eigen vectors one is called the right Eigen vector another is called the left Eigen vector. So, REV and levs are the vectors and based on that we can form some matrix this vector matrix we can say left Eigen vector matrix and then we can say the right Eigen vector matrix and these information are used to calculate the participation factor.

Now, first let us see what is your right Eigen vectors that is rev and this is denoted by the phi and it is nothing but is a n column vector associated with the an Eigen values lambda i and which must satisfy the condition a A phi i is equal to lambda i and here it should be also phi having there is some mistake here.

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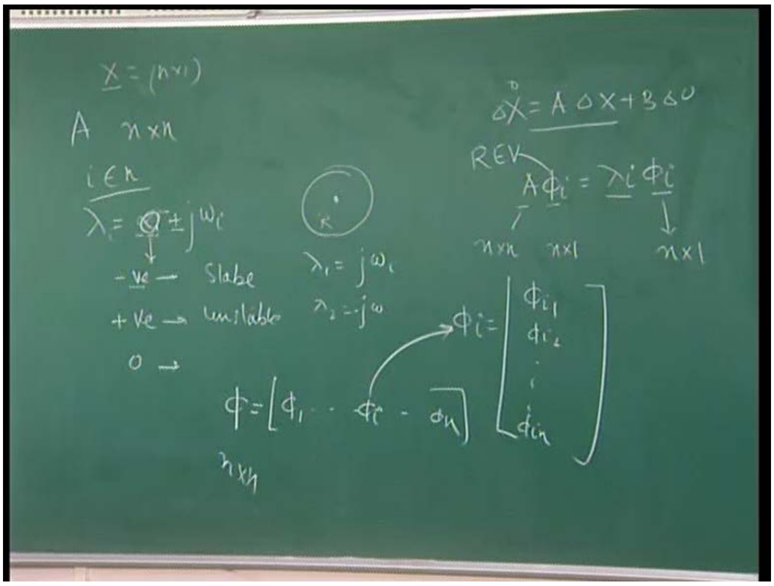

So, what we can do here we can this is defined if you know this is we are having this X dot what we are writing this change in X dot here X plus B U change in your this now we can write this is your A matrix here that has the lambda and now since you are writing this right. So, it is your phi i is equal to here phi i here writing.

So now this is a vector we are writing and this is a corresponding to one Eigen value that is I mind it here this is your vector this is your matrix this is a scalar value and this is a matrix two. So, this your a here it is n cross n matrix here this is your n cross 1 as I said is a vector here this is scalar and this is again your n cross 1. So, you can say this is a balance and is thus satisfactory equation.

Now, we can see there’s phi here we can also write this phi i is nothing but we are having the vector of n one. So, I can write phi i 1 phi i 2 to here phi i n means we are having n values for this one this phi matrix phi vector itself. So, we are getting n. So, we can solve and since you can say this is. So, that is why it is called this here this REV right Eigen vector because you can say this vector we are multiplying the right hand side of this equation. So, these value we can determine we can solve by this equation.

Only I just I want to impress such here that this phi the equations we are getting because here for we are getting this n equations because here where n values are there. So, we are having the n equation for one Eigen value. So, in this the so many equations are dependent on each other. So, what happens it your phi matrix or phi vector here is not a unit. So, you can put one value and based on that you can calculate. So, you can have the different vectors but it has a special property and that we will see later on.

So we can calculate here because you cannot directly solve it here you can have the choose one value then second or et cetera that you can calculate and you are having an equations and you can get your this phi. So, here I want to emphasize that this phi I this vector is not unique you can have the setup phi I and but again the property wise we will find the same giving the information

Now as I said here this lambda I this you are calculating for ith similarly, since you are having the n lambda. So, you are having n this phi's, so then if you can arrange in a such way that you can say this is your phi here I can say this is matrix then this is nothing but you are putting here the phi one to here your phi n, So, all these here only just you can say phi i is also this and that is why I put it here

So you are having this matrix now becomes n cross n and it is known as the right Eigen vector matrix and this is one matrix of a full matrix. So, this is going to give another information and we will see how it is going to be utilized before this let us go for the another vector that is called your left Eigen vector.

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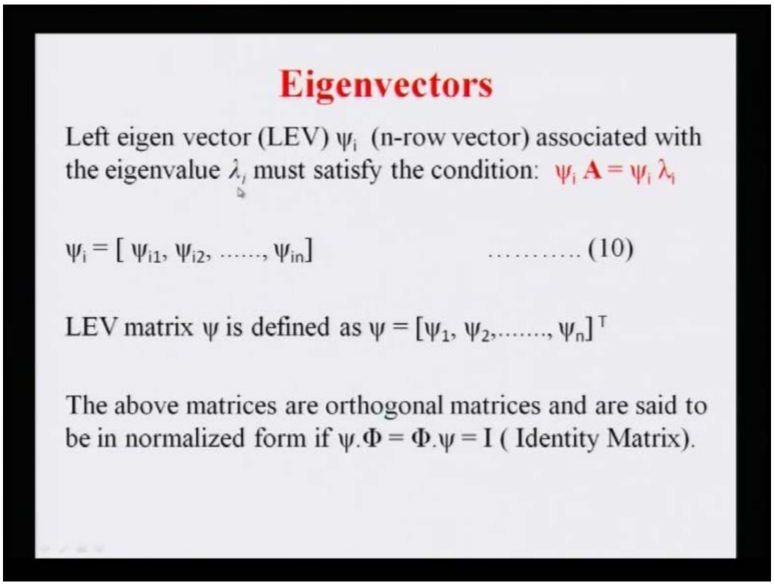

And this is represented by your phi and this is a n row vector earlier it was you can say it is a column vector and that is your row vector associated with the Eigen value here the Eigen value lambda i that must satisfy the condition here you can see this.

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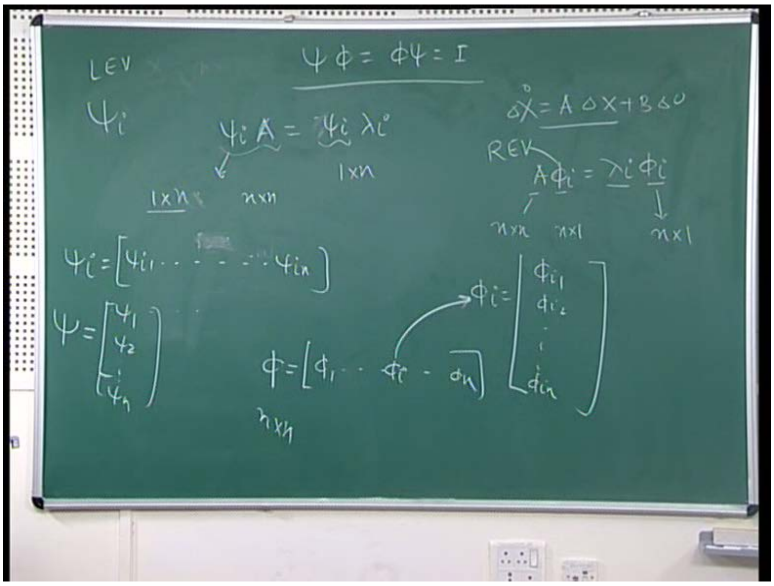

So left Eigen vector just now I am explaining as I said it is row matrix and it is denoted normally by just phi i why it is called this left Eigen vector because again we are writing here a psi i is equal to your. So, psi i here lambda i now you can see this is your 1 cross 1 cross n and this is your we are having n cross n. So, this multiplication exists, so we are going to have this 1 cross n and this is already we are having 1 cross n and this is your scale

So in this case we are going to have this phi psi i this is your nothing but psi i 1 to here psi i n means I want to tell you that before single again lambda i again the lambdas are we are having n because this matrix is having n cross n. So, we are having n Eigen values and then correspondingly we can for ith we can write this.

So we can again using all this we can have a this matrix now we can have all these values here corresponding to one. So, this is your psi 1 here you are having psi 2 and similarly, here you can have psi n and these are matrix that is we have the writing in the vectors. So, this completely we are writing this or you can write in the transpose form and this becomes a matrix.

Now in this case also as I said here some of the here since we are having the n equations here and these equations must be solved for the different value of psi i you will find some of the equations here are independent to each other means you cannot uniquely have the solution then by putting some value you can get some other values. So, this matrix or this elements are also not unique.

But after getting that you are having a matrix this your that is your left Eigen vector matrix as well as your right Eigen vector matrix they are having a unique property and in the normalized form if you are multiplying these two you are going to have the identity matrix and that you can say here if you are having this because this matrix here phi will be equal to your phi psi here and that will be equal to unit identity matrix.

So this is a normalized form here you can already I have also written here the matrix are orthogonal. So, that is why its multiplications becomes identity. So, these two matrices are basically orthogonal matrices and is said to be normalized if they are having this identity matrix otherwise this I will be not there it will be though diagonal matrix having the some constant.

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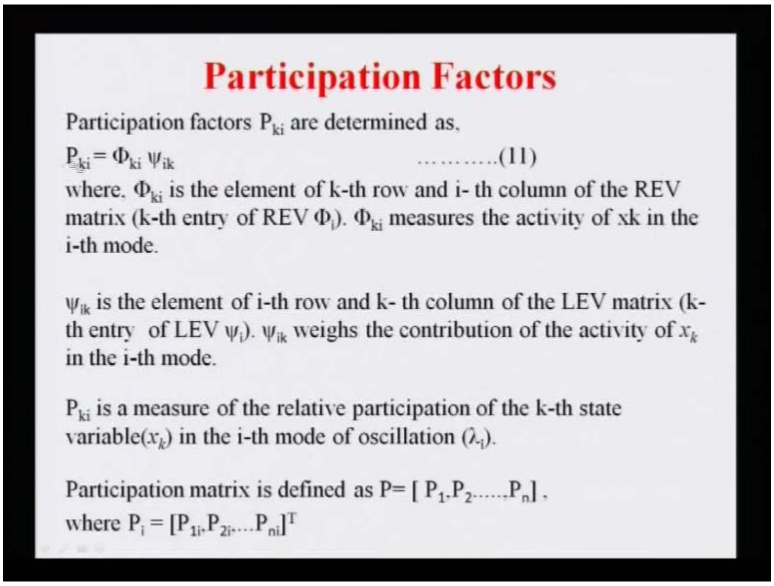

Now let us see the participation factor because the participation factor is calculated this participation factor the P ki are determined as the you can say the multiplication of your left Eigen Eigen vector the values of the kth multiplied by the psi i that is where the phi ik is the element of kth row and ith column of the right Eigen vector matrix that is nothing but the kth entry of your this right Eigen vectors phi i

The ki basically what this phi ki measures the activity of x k in the kth mode x k means x kth means just we can say this x is state as I said in the beginning here we are having the x this is nothing but your x one to your you are having x n. So, here we are talking x kth. So, this x kth here the activity measures the activity of the kth state in the ith mode.

Similarly this psi ik is the element of ith row and the kth element of left Eigen vector matrix the kth entry of this matrix and this basically what does it weighs the contribution of the activity of the kth state in the ith mode. So, this weighs the contribution and this may weigh this measures the activity.

Based on these two it gives the P ki that is the participation factor is a measure of relative participation in the kth state variable x k in the ith modes of oscillation why we are talking the ith modes of oscillation because this ith modes of oscillation will be decided by its ith Eigen value this Eigen value will have your real and imaginary part and then we can say thus what is the relative participation of this in this oscillation.

So, based on that here this is the element that we are calculating we can form the participation matrix that is defined as the P it is having the all the P 1 to P n and this P i is basically nothing but it is calculated from here. So, you can say K 1, K 2 for all the states we can have the participation matrix.

So, we can from this participation factors we can determine which state having contributing which modes of oscillation modes of oscillation means ith we are talking about the ith Eigen vector Eigen value. So, Eigen value just what is the kth kth is which states what is the value of this will be the deciding how much it is contributing in that oscillation So, this will give the information that which of the states are the problematic how much they are participating in that oscillation.

So then knowing this information we can propose we can device some of the controllers that will be useful for mitigating the oscillations or also improving the stability of the system as well.

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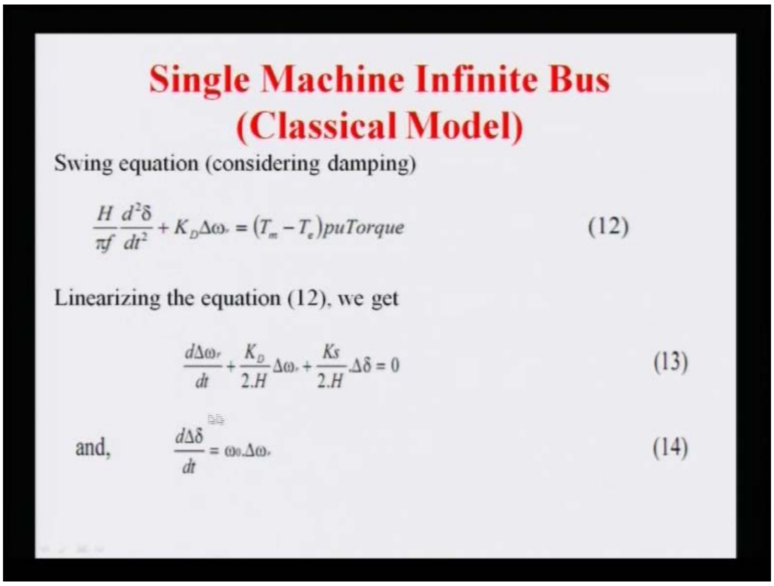

So now let us come these are the theory behind the dynamic stability or small signal stability now we can go move ahead to see how we can go for this HV AC DC system and we can determine the dynamic stability.

First this equation this classical model as you know the swing equation already I defined many times here that we can write in the per unit torque or per unit power we can write this swing equation having some damping constant to K D and that can be written in this form that is equation 12

Now since we are going to analyze the small signal stability then we have to linearize this equation because this is equation of machine and again it is a classical because we have considered. So, many thing so many assumptions are there and based on that it is written. So, it is not a detail modeling of synchronous machine.

But if you are going for the detail then you have to go for writing the detail differential equations. So, that we will see in the next slides here the equation number 12 will be linearized and then you can say linearized we can get this equation. So, after getting this equation you can say you are having the first order differential equation that is omega this del omega r and then you are having another that is equation of del delta with respect to time.

So, we are having it since is a second order differential equation you will have the two differential equations as first order to differential equation and that can be linearized around the equilibrium point and thus we are having the linearized equations here that is equation 13 and 14 and that 13 and 14 can be even though using the Laplace transform or we can have the block diagram.

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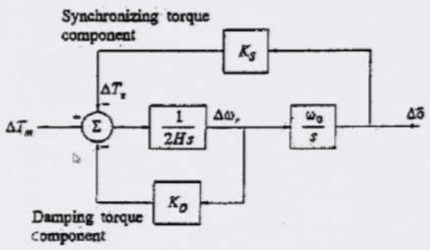

Figure-1: Linearized model of a Syn. Machine

Ks = synchronizing torque coefficient in pu torque/rad

K_D = damping torque coefficient in pu torque/pu speed deviation

H = p.u. intertia constant in MW-.s/MVA

$\Delta\omega_r$ = speed deviation in pu = (ωr- ωo)/ ωo

Δδ = rotor angle deviation in elec. Rad

s = Laplace operator

ω = rated speed in elec.rad/s = 0 2 f π

= 314 or 377 for a 50 or 60 Hz systems, respectively

And we can have this block diagram you can see using those K s K D 2 H s and omega s all these values are here the calculated you can see now here you can say this is a change in the torque looking at this linearized model of synchronous machine this is a classical model I can say. So, here the T m is your input and now you can say this is your the damping torque coefficient the K D is coming the omega term here you can see that we can write from here this omega or with this term is going there sorry and then we are just integrating this we are getting delta and this delta is coming to the K s term is also coming that is the synchronizing torque.

So you can see the K s is synchronizing torque coefficient in the per unit the K D is damping torque coefficient H is your nothing but it is a inertia constant del omega r is the speed deviation in the per unit that is defined as omega r minus omega not divided by omega not del delta is equal to the rotor angle deviation in the electrical radian s is your Laplace factor and omega is the rated speed that is 2 pi f.

So it is here we can write and then we can the solve means we can write in terms of state space equations because you can see here it is a linearized equation.

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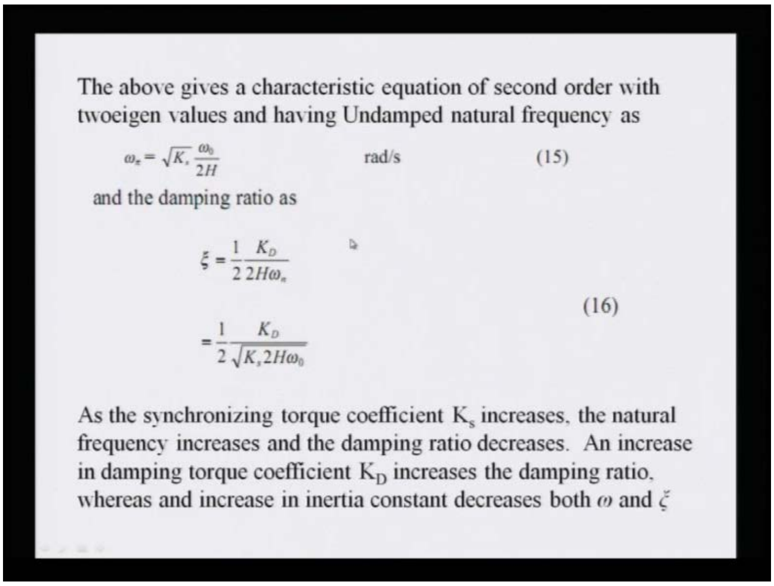

From there itself we can this can give the characteristic equation of the second order with the two Eigen values having undamped natural frequency the omega here that can be written the K s upon omega is radian and the damping ratio can be also calculated in the similar fashion based on the calculating the Eigen value.

So from that block diagram you can have thus you can form state transition matrix a and then you can calculate the the you can calculate this Eigen values and after the Eigen values you can calculate the damping as well as the natural omega frequency that you can calculate.

So from there here means from the zeta we can draw few of the information here this omega n and zeta you can see as the synchronizing torque coefficient K s increases the natural frequency increases because you can see if the K s increases omega n increases from the equation 5 15 and the damping ratio decreases because you can see in the damping ratio here the K s is there if this is increasing this is going to decrease however an increase in the damping torque coefficient K D if your K D increase you can say that increases the damping ratio that is very important. So, if your K D is higher your damping ratio is increasing and your the oscillations are damped quickly.

Whereas the increase in the inertia constant that is H decreases both omega n as well as your the zeta you can say here this H is here. So, if you are having the more H it means that it decrease your damping here as well as your this H. So, these are the classical machine based on the classical machine theory and the linearized one we can draw these conclusions.

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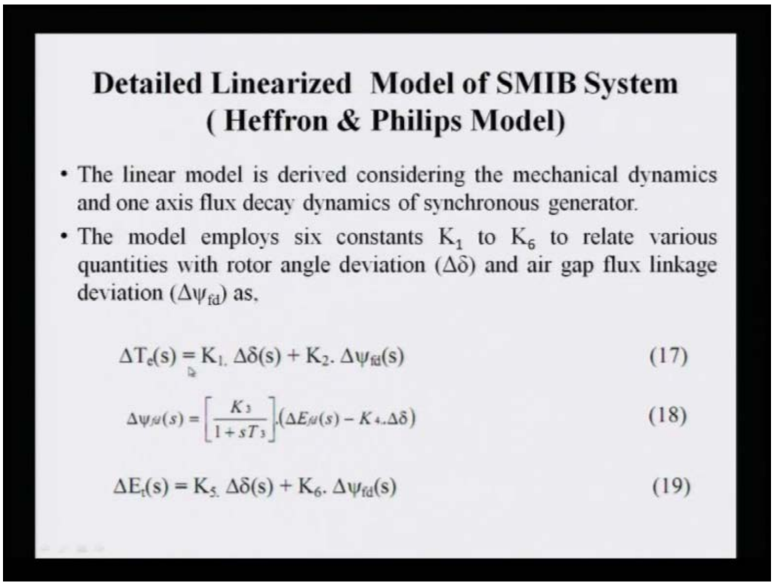

Now if you want to go for the flux decay model and other models as well then the detailed linearized model of the single machine infinite bus can normally known as the heffron Philips model in that the linear model is derived considering them mechanical dynamics and one axis flux decay dynamics of synchronous machine and the model employs the six components K 1 to K 6 to relate the various quantities with the rotor angle deviation that is change in delta and the air gap flux linkage deviation that is change in the psi f d and that can be related here

So you can say is a third order differential equation here we are having this is in s form you can say this is one s here also we can now this we can write here our this model just earlier the model.

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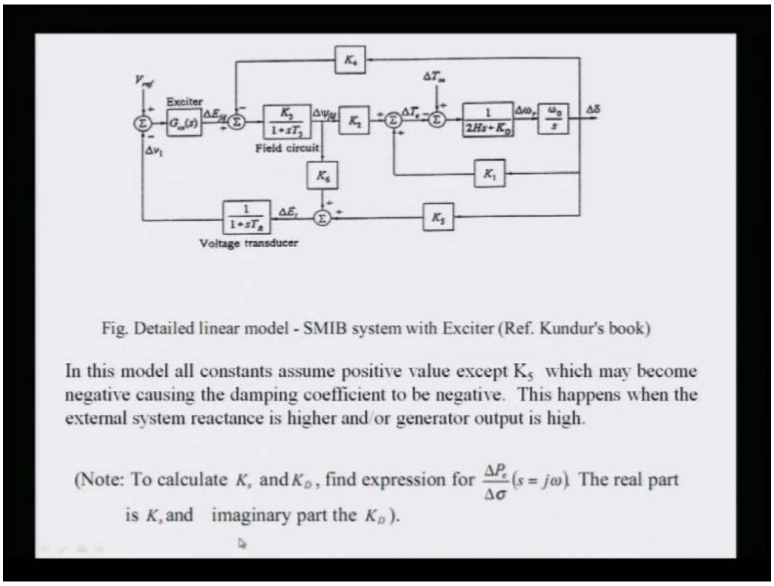

Which was here now we can have all these things as well means we are using this the exciter we are using the voltage regulation. So, and field circuit measurement etcetera with reference to voltage. So, this is basically the detailed linear model and that is it is taken from the (( )) kundur's book this power system stability and control

In this model all the constants assume the positive value except K 5. So, this K 5 can become the negative and this will cause the damping coefficient to be negative and this happens when the external system reactance is higher and or the input of the generator is output of the generator is high to calculate the K s and K D the find expression for the change in the electrical with respect to your the change in your sigma that is your real value of Eigen value then s is equal to j omega the real part is the K s and the imaginary part will be the K D. So, that can be calculated here.

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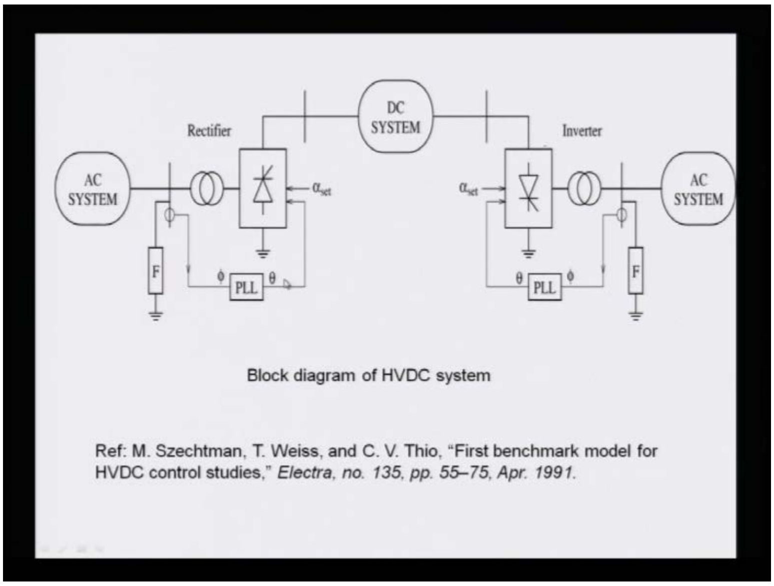

Now we have to now come as I said we can if we are going for the detailed modeling of the generator. So, then we have to write all this governing equations for the generators and your the power system and then you can linearize then we are coming for the DC system. So, your DC system requires again the differential equations then that should be linearized. So, the equations related to your the converter rectifier then here this side your inverter then you are having the DC system model already in AC side I said

You have to have the network model you have to machine model all the models you have to include then you have to include the filters both side and then you are having the phase lock loop that is basically from the phi it is calculating the theta and this is basically nothing but this block diagram HVDC system is taken from the first benchmark model for the HVDC control studies basically normally also it is known as benchmark model and that can be used each and each differential equations will be (( )) along with your other differential equations and the differential equations means that your AC as well as DC and then you consult.

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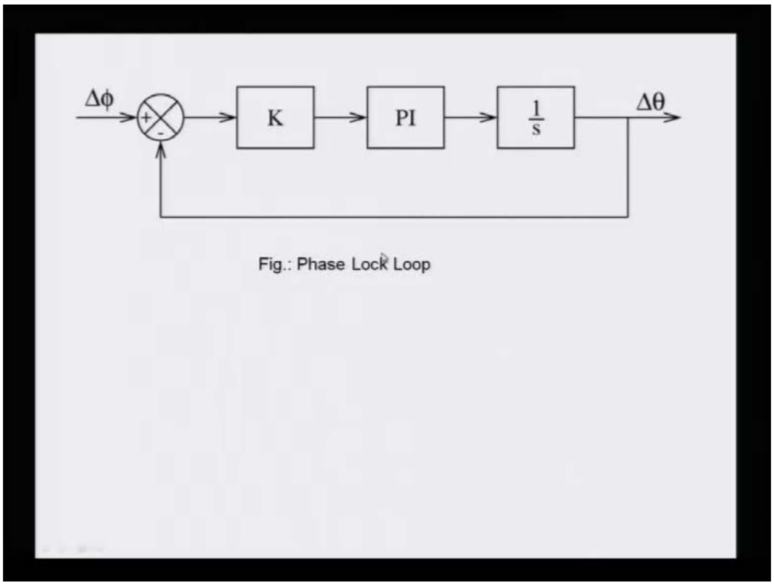

This phase lock loop you can say this is also is a simple block is shown here but it is also having some pi as well as 1 upon S. So, you can also relate here. So, based on this here we can write the various governing equations for this system as well means for your for DC system then along with the AC system we can linearize and we can just go for all your Eigen values as well as you can go for the vectors and then you can see the participation of all the Eigen value and then you can analyze the stable small signal stability of the system as well.

So in this lecture what we did we just I gave a brief introduction about the small signal stability in general then we just derive one the machine model and then the DC system as well and based on that you can calculate the Eigen values having forming the state transition matrix and then you can relate and you can find whether this system is stable or unstable or you can say go for the various analysis.

Once you know system is unstable then you can also find which modes by analyzing the participation matrix you can analyze which one is the contributing and which one is the culprit for this. So, that can be done. So, with this I can basically end up this module module 6 and this is a lecture number 6. So, in this whole total module we had the 6 lectures including from very beginning that we had your this AC DC load flow then we had this your transient stability and then also we have done stability.

So, in this now this end of this module 6, we will next lecture will go for the module number 7 and in that we will go for the various diverse topics like the topic including your HVDC light means HVDC having this your… As if it is we also see your multi terminal HVDC, and also we will see the HVDC application for the wind and renewable energy sources as well. So with this I conclude my this end of the module 6 and the lecture, thank you