

**Advanced Electric Drives**  
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**Lecture - 8**

Hello. In the last lecture we were discussing about the modeling of a 3 phase symmetrical synchronous machine, we will start from that. We have already seen the expression of inductances for the stator, when we have a salient pole synchronous machine the air gap is non-uniform. So, we have variation of inductance in the d and q-axis, and we have also seen the inductance between phase a, and phase b of stator; the self-inductance of a particular phase, and these are actually function of the rotor position that is theta r.

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Mutual inductance between the stator and rotor of a synchronous Machine

$$L_{af} = (L_{g0} + L_{g2}) \frac{i_f}{i_f} \frac{N_f}{N_s} \cos \theta_r$$

$$= (L_{g0} + L_{g2}) \frac{N_f}{N_s} \cos \theta_r$$

$$L_{bf} = (L_{g0} + L_{g2}) \frac{N_f}{N_s} \cos(\theta_r - 2\theta_s)$$

$$L_{cf} = (L_{g0} + L_{g2}) \frac{N_f}{N_s} \cos(\theta_r + 2\theta_s)$$

$$L_{a \times d} = (L_{g0} + L_{g2}) \frac{N_{kd}}{N_s} \cos \theta_r$$

The diagram shows a synchronous machine with a salient pole rotor. The d-axis is the field axis, and the q-axis is the quadrature axis. The rotor angle is  $\theta_r$ . The stator windings are labeled a, b, and c. The field winding is labeled f. The mutual inductance between the stator and rotor is shown as a function of the rotor position.

Now we will briefly review the inductance expression for the stator and the rotor, the mutual inductance between the stator and the rotor, and we will start from that. Now if you see this is actually a synchronous machine a 3 phase synchronous machine where we have phase a, phase b and phase c, and the rotor is salient pole rotor. And this is our d-axis, and this is the q-axis, and we are now trying to evaluate the mutual inductance between the rotor and the stator. The rotor has got three windings; two windings are in the d-axis, we can see this is field winding.

So, this is the field winding in the d-axis, and this is the damper in the d-axis. And then we have one winding in the q-axis rotor that is the damper winding in the q axis. So, right now we will try to evaluate the inductance between the d-axis field winding and the stator winding that is phase a, phase b phase c of the stator. Now if we take the inductance of the field winding with phase a that is  $L_{af}$  that is given as  $L_{g1} + L_{g2} \frac{N_f}{N_s} \cos \theta_r$ , because inductance by definition is flux linkage for ampere. So, the flux linkage is  $L_{g1} + L_{g2} \frac{N_f}{N_s} \cos \theta_r$  by  $N_s$  into  $i_f$ . Since  $L_{g1} + L_{g2}$ , this expression we have already seen in the last lecture, are referred from the primary side. We have to multiply suitable number of turns and the number of turns is  $N_f$  by  $N_s$ .

And if we simplify this this is  $L_{g1} + L_{g2} \frac{N_f}{N_s} \cos \theta_r$  that is  $L_{af}$ , the inductance between phase a of the stator and the field winding. Similarly, we can evaluate the inductance between phase b of the stator and the field winding. It is basically a matter of phase shift. So, we can see that this  $L_{bf}$  that is this inductance is given as  $L_{g1} + L_{g2} \frac{N_f}{N_s} \cos(\theta_r - 120^\circ)$ , because phase b is shifted from phase a by  $120^\circ$ . And that is why we can see that we have a factor of  $\cos(\theta_r - 120^\circ)$  coming into picture here. And similarly we can evaluate  $L_{cf}$  that is  $L_{g1} + L_{g2} \frac{N_f}{N_s} \cos(\theta_r + 120^\circ)$ , because phase c is further  $120^\circ$  ahead of phase b. And hence we have  $\cos(\theta_r + 120^\circ)$ .

We have basically the mutual inductance between the field winding and that of the phase a, phase b phase c of the stator. Similarly, we can evaluate the mutual inductance between the d-axis damper and the stator winding; that is say for example, if you have  $L_{kd}$  the expression will be something similar, because the field winding and the damper winding in the d-axis are in the same axis. So, so far as the angle is concerned we have similarly we will have  $\cos \theta_r$  for a phase and damper in the d-axis,  $\cos(\theta_r - 120^\circ)$  for phase b and the damper in the d-axis and so on.

But what is different here is that the damper winding number of turns will be different from that of the field winding number of turns. In fact, in case of a synchronous machine there is no physical damper winding; they are basically damper bar. So, we have to effectively calculate the effective number of turns of the damper winding, and those

number of turns will be taken into account for calculation of this mutual inductance. So,  $L_{akd}$  is given as  $L_g + L_{g2}$  into  $N_k d$  by  $N_s$  into  $\cos \theta_r$ .

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The image shows a whiteboard with the following equations:

$$L_{bkd} = (L_{g0} + L_{g2}) \frac{N_k d}{N_s} \cos(\theta_r - 2\pi/3)$$

$$L_{ckd} = (L_{g0} + L_{g2}) \frac{N_k d}{N_s} \cos(\theta_r + 2\pi/3)$$

$$L_{akq} = -(L_{g0} - L_{g2}) \frac{N_k q}{N_s} \sin \theta_r$$

$$L_{bkq} = -(L_{g0} - L_{g2}) \frac{N_k q}{N_s} \sin(\theta_r - 2\pi/3)$$

$$L_{cqk} = -(L_{g0} - L_{g2}) \frac{N_k q}{N_s} \sin(\theta_r + 2\pi/3)$$

$$L_{md} = \frac{3}{2} (L_{g0} + L_{g2}) ; \quad L_{mq} = \frac{3}{2} (L_{g0} - L_{g2})$$

Similarly  $L_{bkd}$  the mutual inductance between phase b and the damper winding in the d-axis is  $L_g + L_{g2}$  into  $N_k d$  by  $N_s$   $\cos \theta_r$  minus  $2\pi$  by  $3$ .  $L_{ckd}$  is equal to  $L_g + L_{g2}$  into  $N_k d$  by  $N_s$  into  $\cos \theta_r$  plus  $2\pi$  by  $3$ . And similarly we can also evaluate the inductance of the q-axis damper winding with phase a, phase b and phase c. There will be a negative sign because of the appropriate phase angle. So, that will be  $\cos \theta_r$  plus  $2\pi$  by  $3$ , and  $\cos \theta_r$  plus  $2\pi$  by  $3$  is minus of  $\sin \theta_r$ . So, we have  $L_g$  minus  $L_{g2}$  because the q-axis for me it is something different and that correspond to an inductance of  $L_g$  minus of  $L_{g2}$  into  $N_k Q$  by  $N_s$   $\sin$  of  $\theta_r$ .  $L_{bkQ}$  is equal to minus of  $L_g$  minus  $L_{g2}$   $N_k Q$  by  $N_s$  into  $\sin$  of  $\theta_r$  minus  $2\pi$  by  $3$ .

Similarly  $L_{ckQ}$  is equal to minus of  $L_g$  minus  $L_{g2}$  into  $N_k Q$  by  $N_s$  into  $\sin$  of  $\theta_r$  plus  $2\pi$  by  $2$ . And these are actually the various inductances between the stator and the rotor, but they are actual inductances. Now we should remember that when we are talking about the parameters of a machine, it is always convenient to refer all variables from the primary side. It is basically of practice; we have seen in case of a transformer, in case of an induction machine. And we can also apply the same principle for synchronous machine that all parameters should be referred from the primary side.

Now when we refer from the primary side we have to multiply appropriate number of terms. So, that has to be taken into account when we refer all parameters from the primary side.

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Transformation to d-q equations

$$\Psi_{abc_s} = [L_{ss}] i_{abc_s} + [L_{sr}] i_{pr_r}$$

$$\Psi_s = [L_{ss}] i_s + [L_{sr}] i_{pr_r}$$

$$C_s \Psi_s = C_s [L_{ss}] C_s^T i_s + C_s [L_{sr}] i_{pr_r}$$

$$\Psi_{dq_s} = [L_{dqs}] i_{dq_s} + [L_{dqr}] i_{pr_r}$$

And furthermore when we are talking about a d Q system we are transforming a b c into d Q, and in case of a synchronous machine the stator is a 3 phase stator. And we have chosen a rotor reference frame; it means the reference frame is house than the rotor. So, if the reference frame is house and the rotor, just like this we can have a reference frame with d-axis here and q-axis here. And it is attached to the rotor rotating with the rotor speed that is equal to omega r.

We have to transform all variables of the stator physical stator; that is phase a, phase b and phase c of the stator to d and q-axis. Of course, these axes are hypothetical; I mean the winding corresponding to d and q-axis are hypothetical, but we have to transform the physical a b c variables voltages and currents to the d and q-axis current house then the rotor, because our reference frame is a rotor reference frame. We will have the following expression.

So, this is what we have here that the flux linkage in this case is psi a b c s, and if you have the flux linkage in the stator that is contributed by the stator self flux L s s into i a b c s plus the contribution from the rotor side. So, this is L s r; L s r we have already evaluated in terms of the various inductances. So, if we put those inductances that will be

$L_{sr}$ , and then we have the rotor currents; they are the field winding current, the d-axis damper and the q-axis damper. And we can pre-multiply this equation by  $C_s$ , because we want to transform this into d Q variable, and to transform into d Q variable we have to have the transformation matrix. And the transformation matrix in this case is  $C_s$ , and  $C_s$  involves  $\theta_r$  in the rotor position

So, if we pre-multiply by  $C_s$  in this case we have got  $C_s$  into  $\psi_s$ .  $\psi_s$  is same as  $\psi_{abc}$ ; that is equal to  $C_s L_{ss}$  is the inductance of the stator  $C_s$  inverse into  $c_s$ . So, this we are actually having the  $C_s$  inverse into  $C_s$  because this will be the same I matrix. There is no change of the value of the equation here into  $i_s$ .  $i_s$  is the stator current which are  $i_a, i_b, i_c$ . This is the vector plus  $C_s$  and  $i$  into a log  $s_r$ . Now we can convert this  $s_r$  into  $s_r$  prime by suitably multiplying the number of turns into  $i_{fkdkq}$  prime. So, when we multiply by suitable number of turns we can convert the rotor currents into the referred currents from the primary side; this is what we have.

Now if we see this will be giving us  $\psi_{dq0s}$ . This will be the d-axis q-axis and the 0 sequence component flux of the stator, and this will be  $L_{dq0s}$ , the inductance of the stator in the d Q frame. And then we have  $C_s$  into  $i_s$ ; we have  $i_{dq0s}$ . This will be  $i_{ds}, i_{qs}$  and  $i_{os}$  plus we can have this as  $L_{fkdkq}$  prime into  $i_{fkdkq}$  prime. So, this is what we have here, and we should be interested to know what are these matrices? These matrices are very special matrices, unlike the previous matrices that is  $L_{ss}$  which is the function of  $\theta_r$ . This  $L_{dq0s}$  and  $L_{fkdkq}$  they are independent of  $\theta_r$ ; they are basically constant matrices. We will see the expression of these matrices.

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$$[L_{dqs}] = C_s [L_s] C_s^{-1}$$

$$= \begin{bmatrix} L_{al} + \frac{3}{2}(L_{gp} + L_{g2}) & 0 & 0 \\ 0 & L_{al} + \frac{3}{2}(L_{gp} - L_{g2}) & 0 \\ 0 & 0 & \underline{L_{al}} \end{bmatrix}$$

$$L_{ds} = L_{al} + \frac{3}{2}(L_{gp} + L_{g2}) = L_{al} + L_{md}$$

$$L_{qs} = L_{al} + \frac{3}{2}(L_{gp} - L_{g2}) = L_{al} + L_{mq}$$

$$L_{rs} = L_{al}$$

So, we can see here first  $L_{ds}$ , and how did we obtain this? We obtained this by pre-multiplying with  $C_s$  and post multiplying with  $C_s^{-1}$ . Now this matrix will have this expression. This is  $L_{al}$  plus  $\frac{3}{2}$  times  $L_{gp}$  plus  $L_{g2}$ . Here we have zeros and zeros, and then the next row is  $0$   $L_{al}$  plus  $\frac{3}{2}$  times  $L_{gp}$  minus  $L_{g2}$ ,  $0$   $0$  and  $L_{al}$ . This matrix is the stator inductance matrix of the  $d$   $Q$  reference frame of a synchronous machine. Now if you see this, the stator inductance  $L_{ds}$  in the  $d$ -axis is given as  $L_{al}$ . It is the leakage inductance of the stator plus  $\frac{3}{2}$  times  $L_{gp}$  plus  $L_{g2}$ .

Now we see that this is independent of  $\theta_r$ . So, this is interesting to note that this inductance is not a position dependent. This is independent of  $\theta_r$ . It is a constant inductance, okay, and we can call this in this particular fashion  $L_{al}$  plus  $L_{md}$ .  $L_{md}$  is equal to  $\frac{3}{2}$  times of  $L_{gp}$  plus  $L_{g2}$ . Similarly, the second inductance  $L_{qs}$  is equal to  $L_{al}$  plus  $\frac{3}{2}$  times of  $L_{gp}$  minus of  $L_{g2}$ , and that is equal to  $L_{al}$  plus  $L_{mq}$ . This is  $L_{mq}$ . This is basically the  $q$ -axis magnetizing inductance which is again constant which is not a function of  $\theta_r$ .

So, that is the beauty of the transformation, when we transform the variable to the rotor side or the rotor reference frame we see that by the rotor reference frame when we transform a  $abc$  variable into  $dQ$  variables the inductances are constant inductances, and  $L_{al}$  here is the  $0$  sequence inductance. So, we can say that  $L_{os}$  is equal to  $L_{al}$ . The  $0$

sequence inductance is just the leakage inductance which is a small quantity. It is not a large value; it is a small value, and it is again constant.

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The image shows a whiteboard with handwritten mathematical equations. At the top, a 3x3 matrix is defined as:

$$[L'_{fkdq}] = \begin{bmatrix} \frac{3}{2}(L_g + L_p) & \frac{3}{2}(L_g - L_p) & 0 \\ 0 & 0 & \frac{3}{2}(L_g - L_p) \\ 0 & 0 & 0 \end{bmatrix}$$

Below the matrix, several equations are written, separated by a vertical line. On the left side of the line, the rotor currents are transformed:

$$i'_f = \frac{2}{3} \frac{N_f}{N_s} i_f$$

$$i'_{kd} = \frac{2}{3} \frac{N_{kd}}{N_s} i_{kd}$$

$$i'_{kq} = \frac{2}{3} \frac{N_{kq}}{N_s} i_{kq}$$

$$V'_f = \frac{N_s}{N_f} V_f$$

On the right side of the line, the rotor resistances are transformed:

$$r'_f = \frac{3}{2} \left( \frac{N_s}{N_f} \right)^2 r_f$$

$$r'_{kd} = \frac{3}{2} \left( \frac{N_s}{N_{kd}} \right)^2 r_{kd}$$

$$r'_{kq} = \frac{3}{2} \left( \frac{N_s}{N_{kq}} \right)^2 r_{kq}$$

$$L'_{lf} = \frac{3}{2} \left( \frac{N_s}{N_f} \right)^2 L_{lf}$$

Now if you see the rotor we can see this rotor inductances  $L'_{fkdq}$ . Now this inductance could be of this form. So, we have in this case 3 by 2 into  $L_g$  naught plus  $L_g$  2, and then we have here also 3 by 2  $L_g$  naught plus  $L_g$  2, and then we have 0. And then we have in the d-axis also we can find out that this for the q-axis 3 by 2  $L_g$  naught minus  $L_g$  2, 0 0 and 0. So, this inductance is the inductance in the d Q reference frame between the stator and the rotor. Now by this special transformation the currents of the rotors also get changed; the voltage of the rotor also gets changed, because we are referring everything from the primary side in the d Q reference frame.

Now the rotor currents will be  $i'_f$  is equal to 2 by 3 times of  $N_f$  by  $N_s$  into the actual  $i_f$ . You can see that we have a factor of 2 by 3 coming, because the rotor is a 2 phase rotor, and the stator is a 3 phase stator, and when we transforming into the stator side that is a 2 by 3 factor coming into picture here. Similarly,  $i'_{kd}$  referred from the primary side is equal to 2 by 3 of  $N_{kd}$  by  $N_s$  into  $i_{kd}$ .  $i_{kd}$  is the actual damper winding current in the d-axis. When we refer that in the primary side it will be  $i_{kd}$  prime; that is equal to 2 by 3 times of  $N_{kd}$  by  $N_s$ .

Similarly we have  $i'_{kq}$  that is equal to 2 by 3 times of  $N_{kq}$  by  $N_s$  into  $i_{kq}$ . These are the currents which are referred from the primary side, and similarly we can also have the

expression for the resistance. The voltage of course,  $V_f$  referred from the primary side is given as actual  $V_f$  into  $N_s$  by  $N_f$ . And if we see the various parameters the referred value of resistance is given as  $N_s$  by  $N_f$  whole square into the actual resistance, and this would be multiplied by a factor of 3 by 2. Similarly,  $r_{kd}'$  is equal to 3 by 2 times of  $N_s$  by  $N_{kd}$  whole square actual damper winding resistance in the d-axis, and  $r_{kq}'$  is equal to 3 by 2 times of  $N_s$  by  $N_{kq}$  whole square into  $r_{kq}$ . And  $r_{kq}$  is the damper resistance in the q-axis.

The leakages are also transformed in the same way say for example if I say that  $L_{lf}$ , the leakage of the field winding inductance that is equal to 3 by 2 times of  $N_s$  by  $N_f$  whole square into the actual leakage inductance of the field winding. So, these are actually the parameters are reflected from the primary side in the d Q reference frame. Now what we actually do in the equivalent circuit we do not have access to the actual parameters of the machine. What we actually use, we use the referred parameters since from the primary side, and hence we use  $r_f'$ ,  $r_{kd}'$ ,  $r_{kq}'$ ,  $L_{lf}'$ ,  $L_{lkd}'$  and  $L_{lkq}'$ .

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The image shows a whiteboard with handwritten equations for flux linkages and stator voltage equations in a d-q reference frame. The equations are as follows:

$$\begin{aligned} \psi_{ds} &= L_{ls} i_{ds} + L_{md} (i_{ds} + i_f' + i_{kd}') \\ \psi_{qs} &= L_{ls} i_{qs} + L_{mq} (i_{qs} + i_{kq}') \\ \psi_{f'} &= L_{lf}' i_f' + L_{md} (i_{ds} + i_f' + i_{kd}') \\ \psi_{kd}' &= L_{lkd}' i_{kd}' + L_{md} (i_{ds} + i_f' + i_{kd}') \\ \psi_{kq}' &= L_{lkq}' i_{kq}' + L_{mq} (i_{qs} + i_{kq}') \end{aligned}$$

Stator Equation

$$\begin{aligned} v_{ds} &= r_s i_{ds} + p \psi_{ds} - \omega_r \psi_{qs} \\ v_{qs} &= r_s i_{qs} + p \psi_{qs} + \omega_r \psi_{ds} \\ v_{rs} &= r_s i_{rs} + p \psi_{rs} \end{aligned}$$

So, we can write down the flux linkage expression.  $\psi_{ds}$  is the flux linkage in the d-axis; that is equal to  $L_{ds} i_{ds}$  plus  $L_{md}$  into  $i_{ds}$  plus  $i_f'$  plus  $i_{kd}'$ . Similarly we have  $\psi_{qs}$  equal to  $L_{ls}$  into  $i_{qs}$  plus  $L_{mq}$  into  $i_{qs}$  plus  $i_{kq}'$ . This is the leakage inductance. So, we have  $L_{ls}$  or  $L_{ls}$ ; this is the same thing. This is



the leakage inductance of the stator. The field flux are referred from the primary side; that is equal to  $L_{lf} i_f' + L_{md} i_d + i_f' + i_{kd}'$ . The damper winding flux linkage seen from the primary side that is equal to  $L_{lk} i_{kd}' + L_{md} i_d + i_f' + i_{kd}'$ .

Damper winding flux in the q-axis are referred from the primary side; that is equal to  $L_{lq} i_{kq}' + L_{mq} i_q + i_{kd}'$ . So, these are the various flux linkages with the winding of a synchronous machine. Now these flux linkages will be used to determine the voltage equation. Now if you remember we have chosen a rotor reference frame.

The rotor reference frame is revolving with the rotor at a speed of  $\omega_r$ , and hence in the stator equation we will have the speed induced  $t_m f$  appearing on the stator equation in addition to the statically induced  $t_m f$  we will also have speed induced  $t_m f$ . So, we will now write down the stator equations in the d Q reference frame. So, if we write down here  $V_d$  that is equal to  $r_s i_d + p \psi_d - \omega_r \psi_q$ .

The expression of  $V_d$  and  $V_q$  are given. These are the stator equations. We can have  $V_q$  equal to  $r_s i_q + p \psi_q + \omega_r \psi_d$ , and the expression of  $\psi_d$  is already given here. And the 0 sequence equation we can write  $r_s i_o + p \psi_o$ . The 0 sequence is neither coupled to d-axis nor to q-axis, and hence the 0 sequence will not have any rotationally induced  $t_m f$ .

The plane of the 0 sequence winding is orthogonal to both d and Q axis in a different plane. It can be considered in the z-axis. If the d-axis is the x-axis, and the q-axis is y-axis, the 0 sequence component can be the z-axis. And hence the 0 sequence component does not have any coupling with d and q-axis, and it does not also take part in torque production. And hence we did not solve the 0 sequence current expression.

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Handwritten equations and matrix representation:

$$\begin{aligned}
 U'_f &= r'_f i'_f + p \psi'_f \\
 U'_{kd} &= r'_{kd} i'_{kd} + p \psi'_{kd} \\
 U'_{kq} &= r'_{kq} i'_{kq} + p \psi'_{kq}
 \end{aligned}$$

$$\begin{aligned}
 L_{ds} &= L_s + L_{md} \\
 L_{qs} &= L_s + L_{mq} \\
 L'_f &= L'_f + L_{md} \\
 L'_{kd} &= L'_{kd} + L_{md} \\
 L'_{kq} &= L'_{kq} + L_{mq}
 \end{aligned}$$

$$\begin{bmatrix} U_{ds} \\ U_{qs} \\ U'_f \\ U'_{kd} \\ U'_{kq} \end{bmatrix} = \begin{bmatrix} r_s + L_{ds} p & -\omega_r L_{qs} & L_{md} p & L_{md} p & -\omega_r L_{mq} \\ \omega_r L_{ds} & r_s + L_{qs} p & \omega_r L_{md} & \omega_r L_{md} & L_{mq} p \\ -L_{md} p & 0 & r'_f + L'_f p & L'_{kd} p & 0 \\ -L_{md} p & 0 & L'_{kd} p & r'_{kd} + L'_{kd} p & 0 \\ 0 & L_{mq} p & 0 & 0 & r'_{kq} + L'_{kq} p \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i'_f \\ i'_{kd} \\ i'_{kq} \end{bmatrix}$$

$p = \frac{d}{dt}$

Similarly, for the rotor we can say  $V_f$  prime that is equal to  $r_f$  prime  $i_f$  prime plus  $p$  psi  $f$  prime. The rotor will not have any rotationally induced  $t m f$ , because the rotor and the reference frame are stationary to each other. There is no relative velocity between the rotor and the rotating reference frame, and hence the rotor equation will not have any rotationally induced  $t m f$ . We can similarly have  $V_{kd}$  prime equal to  $r_{kd}$  prime  $i_{kd}$  prime plus  $p$  psi  $kd$  prime, and  $V_{kq}$  prime is equal to  $r_{kq}$  prime  $i_{kq}$  prime plus  $p$  psi. And these equations can be written in a matrix form. If you write this in a matrix form the equation will appear as follows.  $V_{ds}$ ,  $V_{qs}$ ,  $V_f$ ,  $V_{kd}$ ,  $V_{kq}$ , all refers in the primary side, and here we have the various currents, and the currents are  $i_{ds}$ ,  $i_{qs}$ ,  $i_a$ ,  $i_{kd}$  and  $i_{kq}$ .

Now this is a 5 by 5 matrix, and we can fill up the various elements of this matrix, and the elements are as follows. We have the various rows in this case third row, the fourth row and the fifth row. So, this is the first column, second column, third column, the fourth and fifth column; this is a 5 by 5 matrix. We can fill up this by inspection. Now this is the stator resistance flux plus the self inductance flux  $L_{ds}$ , and  $L_{ds}$  is equal to  $L_s$  plus  $L_{md}$ . And we will have the coupling with the d-axis rotor. It is  $L_{md} p$  with the rotor,  $L_{md} p$  with the d-axis damper, and then we have the rotationally induced  $t m f$ . This will be minus of  $\omega_r$  into  $L_{mq}$ ; this is minus of  $\omega_r$  into  $L_{mq}$ .

Similarly, the second row would be  $L_s$  plus  $r_s$  plus  $L_{qs}$ . Now we have the coupling with the q-axis. This is  $L_{mq}$ , and then we have the speed induced  $t_m f$  or the rotationally induced  $t_m f \omega L_{ds} \omega r L_{md} \omega r L_{md}$ . So, these two rows are for the stator, and the three remaining rows are for the rotor. And the rotor equations and the corresponding element can also be written similarly. This is the field resistance  $r_f$  plus  $L_f$ . Now this  $L_{qs}$  is the q-axis inductance of the stator; that is  $L_{ls}$  plus  $L_{mq}$ . And the  $L_f$  is the field winding inductance that is equal to  $L_{lf}$  plus  $L_{md}$ .

Similarly, we can have  $L_{kd}$  that is the d-axis damper inductance that is equal to  $L_{ld}$  plus  $L_{md}$ , and  $L_{kq}$  is the q-axis damper inductance that is equal to  $L_{kq}$  plus  $L_{mq}$ . So, we were writing the third row. In the third row we have  $r_f$  plus  $L_f$ , and this will be having the coupling with the d-axis damper. We have  $L_{md}$  and the d-axis stator  $L_{md}$ , and there is no rotationally induced  $t_m f$  in the rotor. So, we have 0 sequence. Similarly, in the d-axis damper we have  $r_{kd}$  plus  $L_{kd}$  is the d axis damper inductance, and similarly we can have the coupling of this with the field winding and also with the stator d-axis, and there is no rotationally induced  $t_m f$ . So, we can could this equal to 0.

The fifth row would be for the q-axis damper  $r_{kq}$  plus  $L_{kq}$ .  $P$  is the derivative operator as we know, and this will have the coupling with the stator winding that is  $L_{mq}$ , and there is no rotational induced  $t_m f$ . So, we can could this equal to 0. So, this is a 5 by 5 matrix and this five equation will describe the electrical behavior of a synchronous machine. So, if we can stimulate this five simultaneous differential equation we can solve for  $i_{ds}$ ,  $i_{qs}$ ,  $i_f$ ,  $i_{kd}$  and  $i_{kq}$ . So, we can solve the expression for the currents of the synchronous machine. And with this we can find out the expression for the torque also, because in this case we have to identify a  $G$  matrix.

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$$T_e = \frac{3}{2} \frac{p}{2} \dot{i}^T [G] \dot{i}$$

$$[G] = \begin{bmatrix} 0 & -L_{qs} & 0 & 0 & -L_{mq} \\ L_{ds} & 0 & L_{md} & L_{md} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$T_e = \frac{3}{2} \frac{p}{2} [\dot{i}_{ds} \dot{i}_{qs} \dot{i}'_{fd} \dot{i}'_{kd} \dot{i}'_{kq}] [G] \begin{bmatrix} \dot{i}_{ds} \\ \dot{i}_{qs} \\ \dot{i}'_{fd} \\ \dot{i}'_{kd} \\ \dot{i}'_{kq} \end{bmatrix}$$

$$= \frac{3}{2} \frac{p}{2} [L_{ds} \dot{i}_{qs} - L_{qs} \dot{i}_{ds} \quad L_{md} \dot{i}'_{fd} \quad L_{md} \dot{i}'_{kd} \quad -L_{mq} \dot{i}'_{kq}] \dot{i}$$

$$= \frac{3}{2} \frac{p}{2} [L_{ds} \dot{i}_{qs} \dot{i}_{ds} - L_{qs} \dot{i}_{ds} \dot{i}_{qs} + L_{md} \dot{i}'_{fd} \dot{i}'_{kd} + L_{md} \dot{i}'_{kd} \dot{i}'_{fd} - L_{mq} \dot{i}'_{kq} \dot{i}'_{kq}]$$

So, we know that the expression for the torque  $T_e$  is equal to  $\frac{3}{2} \frac{p}{2} \dot{i}^T [G] \dot{i}$ . This is actually as per as the primitive machine equation; the equations of the torque primitive machine give us expression for the torque. And since we have a 3 phase machine and we have taken a transformation with power phase power invariance, when we calculate the total torque we have to multiply 3 by 2 to find out the torque of the actual 3 phase machine. So, we have 3 by 2 times, the  $\frac{p}{2}$  is the pole pair  $\dot{i}^T$  transpose.  $G$  matrix is the matrix associated with the speed term into  $\dot{i}$ ;  $\dot{i}$  is the vector, and  $\dot{i}$  consist of the stator current and the rotor current.

So, let us see what is this  $G$  matrix? This  $G$  matrix we can find out from the previous expression. From this we can find out the  $G$  matrix and the  $G$  matrix will have these elements, the speed terms. All the speed terms will give us the  $G$  matrix. So, if we see here this matrix  $G$  can be  $0$  minus  $L_{qs}$   $0$   $0$  minus  $L_{mq}$ , and then we have  $L_{ds}$   $0$   $L_{md}$   $L_{md}$   $0$ . Remaining three rows are for the rotor, and the rotor does not have many rotational induced t m f, and hence the speed term are absent in the rotor. So, we have this is the  $G$  matrix. Now if we pre-multiply with  $\dot{i}^T$  and post-multiply with  $\dot{i}$  we get the torque.

$T_e$  is equal to  $\frac{3}{2} \frac{p}{2} \dot{i}^T [G] \dot{i}$ . If I simplify this I will get the expression for the torque, let us see. So, we can simplify this, and we will get the expression for the

torque. So, this we can pre-multiply. So, we have got 3 by 2 into p by 2 if we pre-multiply this.

So, we have  $L d s i q s$  minus  $L q s i d s$ . Then the third row will give us  $L m d i q s$ , and the fourth row is also  $L m d i q s$ . And the fifth one will give us minus  $L m q i d s$ , and that we will multiply by this equation i. So, if we simplify this we get 3 by 2 into p by 2. So, we have  $L d s i q s i d s$  minus  $L q s i d s i q s$  plus  $L m d i q s i f$  prime plus  $L m d i q s i k d$  prime minus  $L m q i d s i q$  prime. So, this is the expression for the torque, and this torque we can further simplify, and we can separate this into three different torques.

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$$= \frac{3}{2} \frac{p}{2} \left[ \underbrace{(L_{ds} - L_{qs}) i_{ds} i_{qs}}_{\text{Term-1}} + \underbrace{L_{md} i_{qs} i'_{f}}_{\text{Term-2}} + \underbrace{L_{md} i_{qs} i'_{kd} - L_{mq} i_{ds} i_{qs}}_{\text{Term-3}} \right]$$

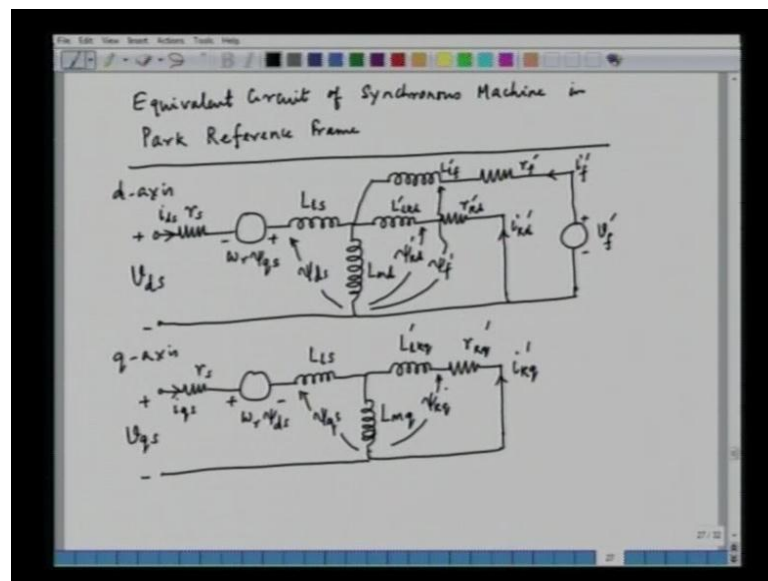
Reluctance Torque
Field Torque
Damper winding

So, this part we can write in the following fashion 3 by 2 into p by 2. So, the first part is  $L d s$  minus of  $L q s$  into  $i d s$  into  $i q s$ . Now this is called term one. So, we can call this to be term 1. And this torque is called the reluctance torque, because this torque is a function of  $L d s$  minus  $L q s$ . So, if  $L d s$  minus  $L q s$  is equal to 0 this term vanishes. And the second term we can take as  $L m d$  into  $L q s$  into  $i f$  prime,  $L m d$  into  $i q s$  into  $i f$  prime. Now this is the term 2. And this is called field winding torque; because this is basically by virtue of the field winding the torque is produced. If  $i f$  prime equal to 0 if there is no field winding current this torque also vanishes. So, this is called the field winding torque.

And what about the third term; we can see the third thing  $L m d i k q i k d$  minus  $L m q i d s i k q$  minus  $L m q i d s i k q$ . So, this is the third term here, and this is called the term

3, and this is called the damper winding term. This is called the field torque, and this is called the reluctance torque. So, the synchronous machine will have three different torque, the reluctance torque, the field torque and the damper winding torque. So, we will now draw the equivalent circuit of the synchronous machine in the d Q reference frame. We have seen that actual machine can be transformed into a hypothetical machine in d Q frame attached with the rotor, and this is for our convenience of simulation. So, we can have the equivalent circuit of the synchronous machine in rotor reference frame, and this will be used for the simulation of synchronous machine.

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So, we can do that equivalent circuit of synchronous machine in Park reference frame. So, we can first draw the d-axis equivalent circuit, and we will expect that in the stator we will have a rotationally induced t m f; in the rotor we should not have any rotationally induced t m f. So, we have the resistance here, and then we have the rotationally induced t m f, the stator leakage, the magnetizing inductance, and then we have the damper winding. And the damper winding is usually short circuited. We have the field winding, the field leakage inductance, the resistance, and we have the field apply group. So, this is our  $V_d s$ , the applied voltage in the d-axis stator. It is the stator resistance, and this is  $i_d$  the stator current, and this is the rotationally induced t m f.

This is the leakage inductance of the stator; this is the magnetizing inductance in the stator d-axis, the damper leakage inductance  $L_{kd}$  referred from the primary side

damper resistance referred from the primary side, the field leakage inductance referred from the primary side, the field resistance referred from the primary side, the field voltage referred from the primary side. We will now write down the expression for the rotationally induced t m f in the stator. So, this will be function of  $\omega_r$ , and in the d-axis the rotationally induced t m f will be because of the q-axis flux  $\omega_r$  into  $\psi_{qs}$ , and this will be helping the applied voltage.

So, this will be minus here and plus here. In a similar fashion we can draw the q-axis equivalent circuit, resistance, the back e m f in the q-axis, the leakage inductance, the magnetizing inductance in the q-axis. And then we will have the rotor, and the rotor is the damper winding only. So, this is our q-axis equivalent circuit;  $V_{qs}$  is the stator q-axis voltage. The resistance is  $r_s$ , and this is  $i_{qs}$  is the q-axis current. We have rotationally induced t m f; this is the stator leakage inductance, the magnetizing inductance in the q-axis, the damper leakage inductance  $L_{leq}$  and  $r_{eq}$ . And this rotationally induced t m f will be  $\omega_r$  into  $\psi_{ds}$ . This will be in fact opposing the applied voltage, and hence the sign will be plus here this side minus in the right hand side and various fluxes here.

We have this is basically the stator flux linkage in the d-axis  $\psi_{ds}$  which includes the mutual flux and the stator d-axis leakage flux. Similarly this one is  $\psi_{qs}$  the q-axis stator flux linkage which consists of the mutual flux and the leakage flux. This is the damper winding flux linkage  $\psi_{kd'}$  as seen from the primary side which consist of the damper leakage flux; this is your damper current  $i_{kd'}$ , and this is the field flux linkage  $\psi_f$ . This is  $i_{f'}$  the field current. So, we have shown the various flux linkages in the stator as well as in the rotor in the d-axis.

Similarly in the q-axis this flux linkage is  $\psi_{kq}$  which consists of the q-axis damper leakage flux. This is the damper current  $i_{kq'}$  the leakage flux plus the magnetizing flux in the q-axis. So,  $\psi_{kq'}$  as referred from the primary side is the flux linkage in the q-axis damper. So, we have the complete equivalent circuit in front of us. So, we can use this equivalence circuit to solve the various current expressions, and of course, we know that we have two systems. Here we have an electrical system also we have a mechanical system. In the mechanical system we have to solve for electromechanical equation.

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$$T_e = \frac{J}{p} \frac{d\omega_r}{dt} + \frac{B\omega_r}{p} + T_L$$
$$\theta_s + \int \omega_r dt = \theta_r$$

That is the torque  $T_e$  is given as  $J \frac{d\omega_r}{dt} + \frac{B\omega_r}{p} + T_L$ .  $T_e$  we already know this is the torque of a synchronous machine, and we will be equating that to the inertial torque, the viscous section torque and the load torque. Now if we solve this equation we will get the expression for the rotor speed that is  $\omega_r$ , and we can integrate this  $\omega_r$  to get the expression for the  $\theta_r$ . So, this is what we have here. We can have some initial constant the initial position. So, the integration of  $\omega_r dt$  will give us  $\theta_r$ , and the  $\theta_r$  can be used to transform the actual voltages  $V_a, V_b, V_c$  into  $V_d$  and  $V_q$ . We can stimulate the machine in the  $d-q$  equations in the Park reference frame.

Evaluate  $i_d, i_q, i_f, i_kd$  and  $i_kq$ , and this has to be an iterative process, and we have to take the help of a numerical integration technique to solve the current expression for a synchronous machine. Now you must have noted that the synchronous machine is more complex than an induction machine, and induction machine had 4 currents  $i_d, i_q, i_r$  and  $i_f$ .

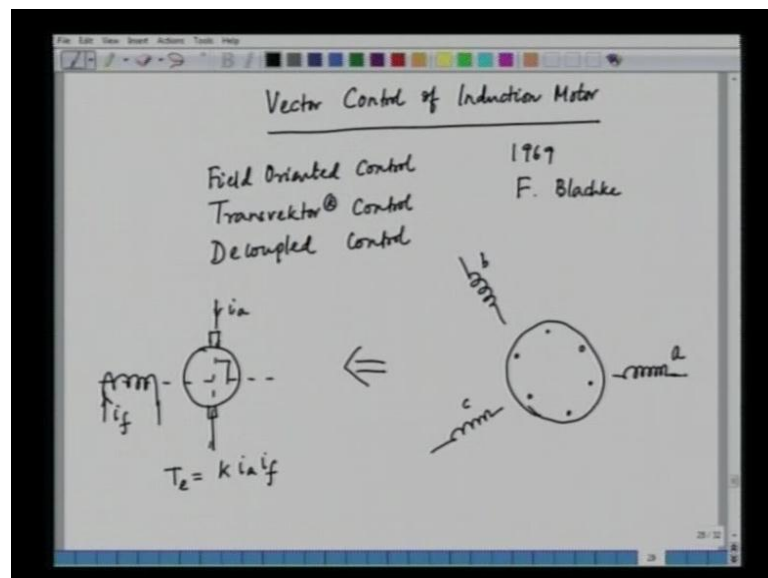
On the other hand a synchronous machine has five currents  $i_d, i_q, i_f, i_kd$  and  $i_kq$ . So, we have in addition to the field winding current we also have the damper winding current, and that is why it is more complex. And again we have to also apply a voltage from the field side, and hence it is more complex than that of our induction machine, and



the simulation will take more time than an induction machine. So, these are the equations. So, we can use this equation to simulate a synchronous machine.

So, with this we come to an end of the modeling of a synchronous machine, and with this also we finish the modeling of dc machine, induction machine and synchronous machine. So, the next part of this particular course will be the control of electric machine. The title of the course is advanced electric drives. We have seen how to model various electric machines. We have the models available with us, and we can use this model to think of innovative control technique to improve the torque and speed response of ac machine. So, the control will primarily focus on the ac machine. The first machine that we are going to control is an induction machine.

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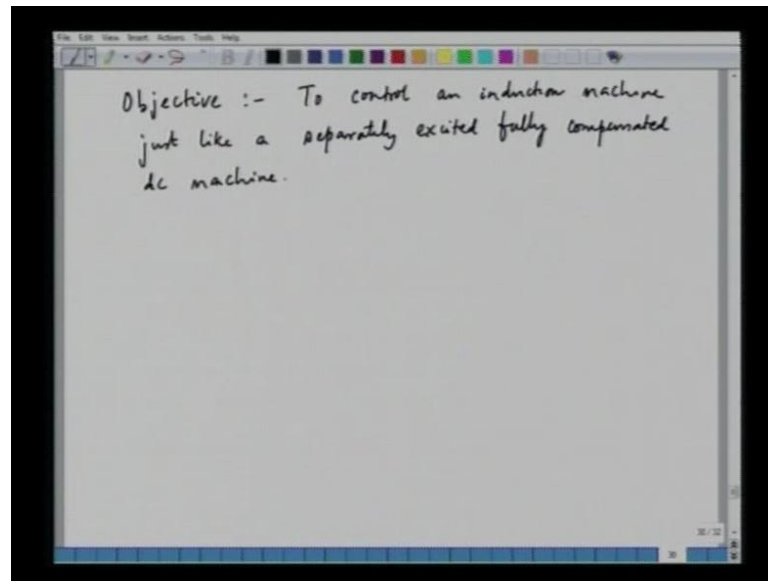
So, we will talk about a control that is called vector control of induction machine. This vector control is not a new way of controlling induction machine. In fact, the very concept the principle was given sometimes in 1969 by a scientist call Felix Blaschke. The Blaschke gave an idea to a control of ac machine called field oriented control, this work also known as vector control, because we are controlling the amplitude, the frequency and the phase angel of the currents in the stator. So, this is known in various names. One is a vector control, it is also called field oriented control, it is also called transvector control, and sometimes people also call this as decoupled control or decoupling control.

Usually our induction machine is having three winding phase a, phase b and phase c. So, if you see an induction machine this is the rotor, and we have three different windings phase a, phase b and phase c. And the rotor could be a squirrel-cage rotor. We have the rotor conductor, and this machine is a highly non-linear machine. And this can be can be only controlled by some advanced technique, and we can control this machine like a separately excited fully compensated dc machine using vector control.

So, what we are trying to do here, the objective is to control an induction machine just like a separately excited dc machine. So, this control has to be done just like a separately excited dc machine. We have a dc machine here, armature, under field winding just have a comparison in this case. We have the armature current  $i_a$ , and we have the field current  $i_f$ , and these two currents are orthogonal to each other.

This is the axis, and we can call this to be d-axis, this is q-axis, and this angle is 90 degree. And we know that the torque is given by simple expression like  $k i_a i_f$ . If we ignore saturation we have flexibility to control the torque either by controlling the armature current or by controlling the field current. But usually the field has a higher inductance, and hence the field current cannot be controlled with fast rate. So, we choose to control the torque by controlling the armature current. So,  $T_e$  equal to  $k i_a i_f$ . So, this is the expression for the torque of a dc machine. So, the similar control structure is not available with an induction machine. So, what we do we use a control to transform this just like a dc machine and this has to be in a rotating reference frame.

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The objective is to control an induction machine just like a separately excited fully compensated dc machine. So, in this case what we mean by fully compensated machine is that we can add a compensating winding here, and the compensating winding can neutralize the effect of armature reaction. If the armature flux is in this direction this will have an opposite  $m$   $m$   $f$  or flux here to cancel the armature flux, and hence we can avoid the effect of armature reaction. Usually the armature reaction has a detrimental effect on the field winding. We can say that the two fluxes are orthogonal to each other. We have a field flux, and we have an armature flux.

Although, they are orthogonal to each other, they are cross magnetizing due to the saturation of the field we will have a demagnetizing effect of the armature current on the field winding. So, to avoid the demagnetizing effect of the armature current on the field winding or the field we can use a compensating winding to compensate for armature reaction.

So, the objective of vector control is to control an induction machine just like a separately excited and fully compensated dc machine. And this has to be done in a special reference frame. So, what we do here we take a rotating reference frame. We take a rotating reference frame, and when we view the synchronous machine from a rotating reference frame we will see the induction machine will appear just like a dc machine. So, in the next lecture we will be discussing how to choose this special reference frame which is a

rotating reference frame, how to derive the various expressions for the current to control an induction machine just like a dc machine.