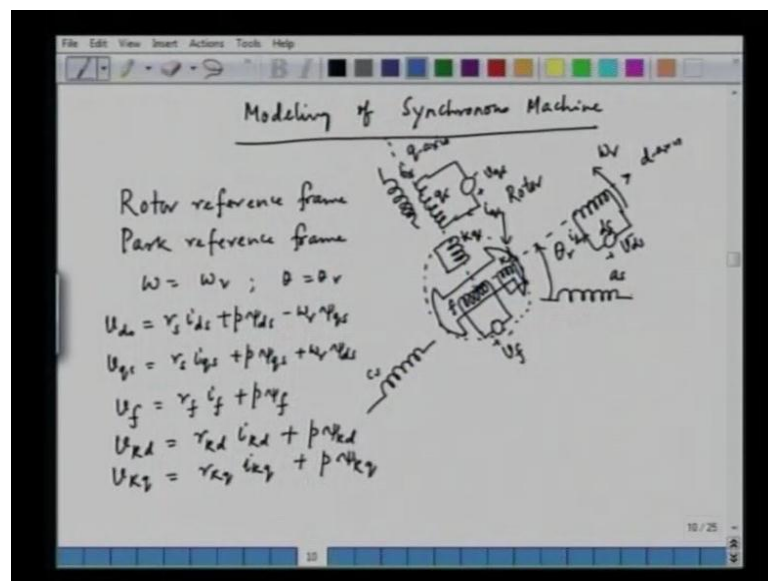


Advanced Electric Drives
Prof. S. P. Das
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture – 7

Hello and welcome to this lecture on Advance Electric Drive. In the last lecture, we are discussing about the modeling of a synchronous motor will start from that. And the modeling of a synchronous motor is more complex than the modeling of an inductance motor, because in case of a synchronous motor the router had also excitation. In fact, the router has got the field winding and the damper winding, so let see the modeling of synchronous motor.

(Refer Slide Time: 00:52)



Now, we can see here that we have a three phase machine, phase a, phase b, and phase c; it is synchronous machine and this is the stator, and the router has a saline pole structure. The saline pole structure means, the router d axis is have being lower air gap and the q axis has a higher air gap, the air gap is not uniform, and hence we call it is a saline pole synchronous machine. So, this is our d axis along this axis and the q axis is orthogonal to d axis as we already seen, this angle is 90 degree and we have the field winding in the d axis, and the damper winding in the d axis.

So, the d axis rotor we have two windings, the field winding and the damper winding similarly, in the q axis we just have one damper winding. So, we have seen in the last lecture, that if we model a synchronous motor the rotor reference frame or in park reference frame, that is a distinct advantage that the inductance is will be constant. We have a d axis inductance and we have the inductance in the q axis, and they are constant their independent of rotor position

So, we will be modeling the synchronous machine in rotor reference frame or the park reference frame, which means the reference frame is attach to the rotor, and is rotating at the speed of the rotor. Ω is equal to ω_r and ω_r is the speed of the rotor and the angle of the reference frame is same as the rotor angle θ equal to θ_r , this is the angle of the reference frame, that is θ_r . And this same at the rotor angle, the rotor is rotating at the speed of ω_r and where it is attach to the rotor, when the reference frame is attach to the rotor, we know that it is rotating with the rotor.

And hence the rotationally induce DMF will be appearing in the stator, the stator d q winding will have the rotationally induced DMF, which are present in this case, so we can write down v_d equal to $r_s i_d$ plus $p \psi_d$ minus $\omega_r \psi_q$. So, this is the rotationally induced DMF in the d axis and similarly, v_q is equal to $r_s i_q$ plus $p \psi_q$ plus $\omega_r \psi_d$, this is the rotationally induced DMF in the q axis. And the field winding is a excited independently by d c source, we have a d c source here, for the field winding, this is the field winding and this is our field current.

So, we can say here i_f , so v_f equal to $r_f i_f$ plus $p \psi_f$, this is for the field winding or for the damper we have a d axis damper and that is called k_d , k stands for the damper. So, in the d axis we can write down, v_{k_d} is equal to $r_{k_d} i_{k_d}$ the resistance drop plus $p \psi_{k_d}$ is the induced DMF. And the damper winding is usually sorted, so we can make this equal to 0, they are made for damping out the oscillation and hence they known as damper winding. Similarly, in the q axis we have the damper winding that is also sorted this, this is the q axis damper winding that sorted.

So, we can put that equal to 0 and that is equal to $r_{k_q} i_{k_q}$, the registers drop in the q axis damper plus $p \psi_{k_q}$ in the rate of change of flux linkage. And seen the reference frame is attached with the rotor, there is no relative velocity between the reference frame and the rotor and hence, the reference frame windings, the rotor windings does not have

any rotational induced DMF. And hence, we can see here that there is no rotational induce DMF in the field winding or in the damper winding, in the d axis or the damper winding in a q axis.

So, we were discussing this in the last lecture and then, when we want to transform the actual variable, the actual variables are the physical a, b, c of the stator. Now, when you take a rotor reference frame, we have to transform the variable from the physical a, b, c winding to the rotor reference frame rotating with the rotor; and hence we need the transformation matrix.

(Refer Slide Time: 05:57)

The image shows a digital whiteboard with the following handwritten content:

$$\theta = \theta_r$$

$$K_s = C_s = \frac{2}{3} \begin{bmatrix} \cos \theta_r & \cos \theta_{r1} & \cos \theta_{r2} \\ -\sin \theta_r & -\sin \theta_{r1} & -\sin \theta_{r2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\begin{cases} u_{as} = r_s i_{as} + p \psi_{as} \\ u_{bs} = r_s i_{bs} + p \psi_{bs} \\ u_{cs} = r_s i_{cs} + p \psi_{cs} \end{cases}$$

$$u_f = r_f i_f + p \psi_f$$

$$u_{kd} = r_{kd} i_{kd} + p \psi_{kd}$$

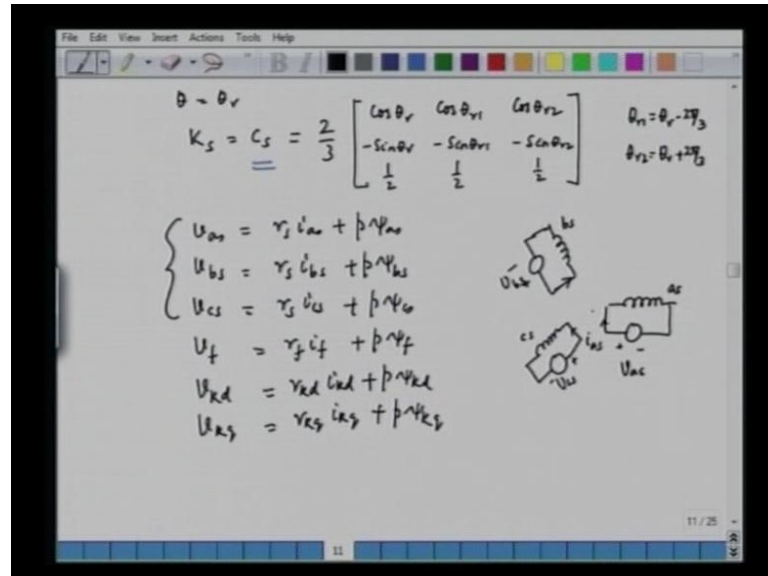
$$u_{kq} = r_{kq} i_{kq} + p \psi_{kq}$$

And the transformation matrix is C s, we have already seen in the last lecture and that is equal to cos theta r cos theta r 1 cos theta r 2 minus sin theta r minus sin theta r 1 minus sin theta r 2 half half and half. And theta r 1 is equal to theta r minus of 2 pi by 3, theta r 2 is equal to theta r plus 2 pi by 3 and if we transform using this transformation, we can get the variable in the rotor reference frame that is t q and 0 sequence component. But, before we go for the rotor reference frame, let us try to understand how the machine is simulated in the a, b, c variable.

Physically we have phase a, phase b and phase c, we have three windings in this case, these are the straighter winding and this winding this is a s, b s and c s and each winding having an applied voltage. Say for example, in phase a we can have v a s and this is i a s,

similarly we can have the voltage apply to phase b, similarly to phase c, and we can have the currents in phase b, and currents in phase c.

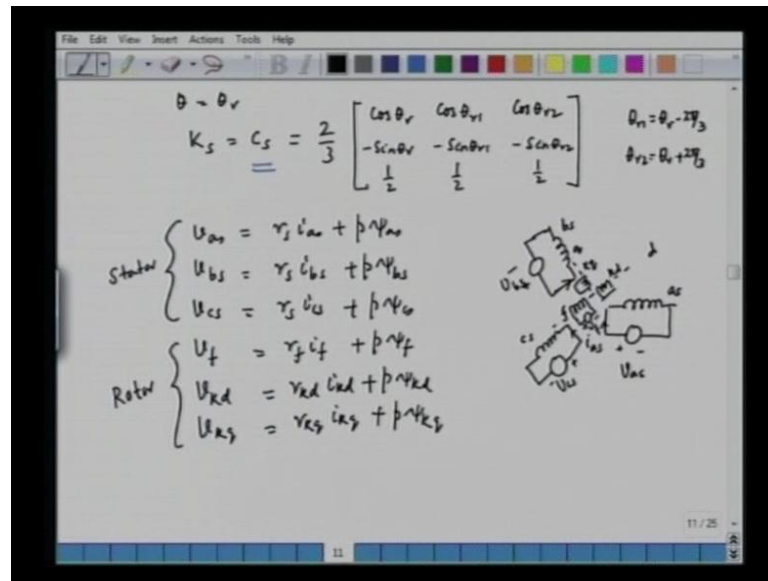
(Refer Slide Time: 07:48)



So, we can write down the three voltage equation, v_a equal to $r_s i_a$ plus $v_{\psi a}$, v_b equal to $r_s i_b$ plus $v_{\psi b}$ and v_c equal to $r_s i_c$ plus $v_{\psi c}$. And the rotor in this case has the field winding, we have a field winding this is the rotor d axis and we have a damper winding here, and in the q axis we have a damper winding. So, this is k_d the d axis damper winding and this is k_q the q axis damper winding, and that is the field winding, the field winding is applied with d c voltage that is v_r .

So, this is the schematic view of a synchronous machine we have three windings in the stator a s, b s and c s, we have three windings in the rotor, we have two windings in the d axis, the field winding and the d axis damper and we have one winding in the q axis that is k_q that is the q axis damper winding. So, we have to simulate this we can write down the six equation this is for the rotor, so we have the equation for the stator and equation for the rotor. And these are the actual equation and if we simulate this equation, these are differential equation, these are simultaneous six differential equation.

(Refer Slide Time: 09:32)



And when we solve the differential equation, we can have the values of the various currents i_a , i_b , i_c , i_f , i_{kd} and i_{kq} and when we have current we can find out the torque. And from the torque, we can solve the electromechanical equation to find out the speed. Now, before we find out the currents, we have to know the various inductances because, these flux linkages are not directly available to us, they are available by the inductance.

So, we can just write down what is the flux linkage ((Refer Time: 10:07)) ψ_a , ψ_b , ψ_c , it means ψ_a , ψ_b , ψ_c as the three components, ψ_a , ψ_b , ψ_c , ψ_a , ψ_b , ψ_c are the three flux linkages in the three windings of the stator a, b and c. And when we talk about one winding flux linkage, the flux is linked by its own current and flux is linked by the currents in the other windings also, because the whole machine is the electromagnetically coupled structure. So, if you want to find out the flux linkage in one winding, the other windings will also contribute to the flux linkage in that winding, because of the mutual coupling.

So, we can say here that this particular flux linkage can be written in this particular form, and we have i_a , i_b , i_c , i_f , i_{kd} and i_{kq} , and I can have in this case L_{aa} , the inductance of phase a with itself, the self inductance of phase a, we can call that L_{aa} . This is L_{ab} the inductance of a with b, phase b of the stator and we can say here L_{ac} ,

so inductance of phase a with phase c of the stator and then, we have the rotor and the stator mutual inductances, that is a f, the inductance of phase a with the field winding.

The inductance of phase a with the d axis damper winding, write down the other rows also $L_{ba} L_{bb} L_{bc} L_{bf} L_{bk} L_{bd} L_{bkq}$, $L_{ca} L_{cb} L_{cc} L_{cf} L_{ck} L_{cd} L_{ckq}$, now this inductance matrix is multiplied with a current vector to give of the flux linkage. So, we can divide this inductance matrix into two parts this matrix is call L_{ff} , the inductance matrix of the stator with respect to the stator, because $L_{aa} L_{ab} L_{ac}$ they are basically the inductance of the stator winding with the stator winding. But, $L_{af} L_{ak}$ L_{ad} and L_{aq} , they represent the mutual inductance between the stator and the rotor, we call this part to be L_{sr} .

So, this inductance is L_{ss} and this inductance is L_{fr} , so we can write this equation are $L_{as} L_{bs} L_{cs}$ is equal to $L_{aa} L_{ab} L_{ac}$, $L_{ba} L_{bb} L_{bc}$, $L_{ca} L_{cb} L_{cc}$ multiplied by i_{as} i_{bs} and i_{cs} plus we have $L_{af} L_{ak} L_{ad} L_{aq}$, $L_{bf} L_{bk} L_{bd} L_{bkq}$, $L_{cf} L_{ck} L_{cd} L_{ckq}$ and this we can multiply with the rotor current. And the rotor currents are field winding, the damper winding in the d axis and the damper winding in the q axis. So, we have to evaluate this to inductances, unless we evaluate this inductances we cannot to the simulation in the a, b, c variable.

(Refer Slide Time: 14:48)

The image shows a whiteboard with handwritten mathematical equations. The top equation is:

$$\underline{\Psi}_{abcs} = \begin{bmatrix} \Psi_{as} \\ \Psi_{bs} \\ \Psi_{cs} \end{bmatrix} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & \vdots & L_{af} & L_{ad} & L_{aq} \\ L_{ba} & L_{bb} & L_{bc} & \vdots & L_{bf} & L_{bd} & L_{bq} \\ L_{ca} & L_{cb} & L_{cc} & \vdots & L_{cf} & L_{cd} & L_{cq} \\ & & & L_{ss} & & & \\ & & & & & L_{sr} & \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \\ i_f \\ i_d \\ i_q \end{bmatrix}$$

The bottom equation is:

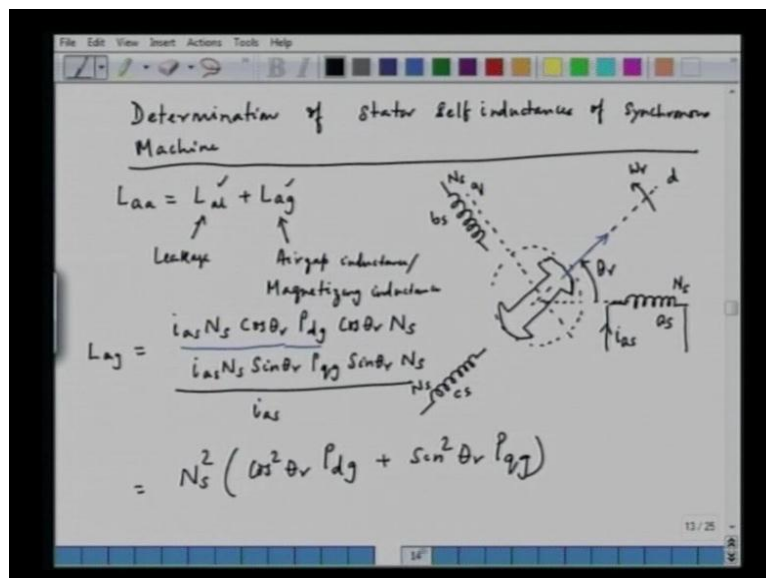
$$\begin{bmatrix} \Psi_{as} \\ \Psi_{bs} \\ \Psi_{cs} \end{bmatrix} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \\ & & L_{ss} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \begin{bmatrix} L_{af} & L_{ad} & L_{aq} \\ L_{bf} & L_{bd} & L_{bq} \\ L_{cf} & L_{cd} & L_{cq} \end{bmatrix} \begin{bmatrix} i_f \\ i_d \\ i_q \end{bmatrix}$$

So, we have to know what is this L_{ss} matrix? And how does it vary with the rotor position and what is this L_{sr} matrix and how does this vary with the rotor position, let

us see. Now, this we will analyze in two parts, when we talk about L_{ss} we have the diagonal components like this, this and this, this diagonal component are the self inductance of the individual pages. The self inductance of a with a, the self inductance of b with b and the self inductance of c with c, and there are some of diagonal elements like this once.

Now, this represents the mutual inductance between the 2 pages, the mutual between phase a and b the mutual inductance between phase a and c and so on. So, we will first consider the self inductance turn, like this stocks and then, we will talk about the mutual inductance turn which represent or which means, the coupling between two stator phases of the synchronous motor.

(Refer Slide Time: 15:56)



So, determination of stator self inductances of synchronous machine, now we can find out the self inductance and to be able to do that, let us first try to understand how the inductance is calculated. We have three different phases, phase a, phase b and phase c and the rotor is a salient pole structure, so we have the salience in this case this is the rotor pole, and this is the d axis and this axis is the q axis. And these are the various stator pages a s, b s and c s, now we can assume that the stator each page of the stator has effective number of turns of N_s , N_s is effective number of turns of a stator phase.

This also has got the same number of turns N_s , this also has the same number of turns N_s , now let us assume that this winding is carrying a current, that is i_{as} . Now, when we

talk about the self inductance, the self inductance is the flux linkage in the winding due to its own current for its own unit current. So, we can first find out the flux linkage of phase a due to its own current, and if we divide that by the current i_a we get the self inductance of phase a. Now, this rotor is rotating at a speed of ω_r this means rotating at a speed of ω_r , it is not necessary.

And the angle it obtains with phase a is θ_r , this is the phase a and this angle θ_r is the rotor angle, so that when the rotor is rotating phase a is alternatively seeing the d axis and q axis, the d axis and the q axis like this. So, the air gap seen by phase a is constantly changing and if we want to find out the inductance, we have to define inductance in some fixed position. So, we know the inductance in the d axis, we know the inductance in the q axis, so these two inductances are known or in other words we know the inductances along two axes, inductance along d axis is known and the inductance along q axis is known.

So, we know the inductances along this axis and we also know the inductances along the q axis, so if we want to find out the flux linkage. So, we can start with the following this L_a , the inductance of phase a with respect to itself, it consists of two parts, a linkage part L_{aa} and mutual path or the mutual path is L_{ag} , this is called the leakage inductance. And this is called the air gap inductance, or the magnetizing inductance, and leakage is almost constant the leakage is independent of rotor position, so we can assume that leakage remains constant.

So, ((Refer Time: 20:17)) this part will remain constant, this part will be a function of the rotor position θ_r , so you would like to evaluate what is L_{ag} , so we can find out the flux linkage due to its own current. So, L_{ag} is given as follows it is $i_a N_s$, so this is actually the MMF produced by phase a, and MMF is along the phase a axis and we can resolve this MMF along the d axis, so we can multiply here by $\cos \theta_r$. Now, this MMF is along the d axis and we know the inductance along d axis, that is $p_d g$ and we can again multiply that with $\cos \theta_r$.

And again multiply with N_f that when we multiply this $\cos \theta_r$, this is the MMF along the d axis multiplied by $p_d g$ with the flux along the d axis. And then, the flux along this axis we can again resolve this back in to the phase a, because we are trying to find out the flux linkage in phase a, so when it is again brought back to phase a we have

again a factor $\cos \theta_r$ here. And then, this is the flux and if we talk about the flux linkage multiply by the number of N_s once again, and this is the contribution of d axis. We know that we have two distinct axis, one is the d axis and other is the q axis and the flux linkage is contributed both by the d axis and the q axis.

So, what we are trying to do here the MMF is produce by phase a, we are resolving the MMF along d axis multiplying by the Formian along the d axis, we are finding out the d axis component of the flux. We are again projecting that flux on phase a axis, and finding out the component and multiplying by the number of turn will give of the flux linkage due to the d axis flux. Similarly, the MMF produce by phase a can be resolve on the q axis and then, we can again project it back on the phase a, that be the contribution of the flux due to the q axis multiplied by the number of turns of phase a, will be given the flux linkage in phase a due to the q axis component of the flux.

So, this is the d axis component and we can again say that is multiplied by $i_a N_s$ into $\sin \theta_r$ into $P_d g$, we are talking about the q axis component along this axis, we are again multiplying with $\sin \theta_r$ and N_s to find out the flux linkage in phase a. So, this is the total flux linkage in phase a and it is own current is i_a , so we can divide this by $i_a N_s$, so this is the inductance of the phase a with respect to itself that is L_{ag} . So, if we simplify this what happens here is a following, we can this N_s^2 square common and then, we have $\cos^2 \theta_r$ into $p_d g$ plus $\sin^2 \theta_r$ into $P_q g$; and this we can again simplify, we can convert this into $\cos^2 \theta_r$.

(Refer Slide Time: 24:17)

The image shows a handwritten derivation of the inductance variation \$L_{ag}\$ and its average value \$L_{aa}\$. The derivation is as follows:

$$L_{ag} = N_s^2 \left[\frac{P_{dg}}{2} (\cos 2\theta_r + 1) + \frac{P_{qg}}{2} (1 - \cos 2\theta_r) \right]$$

$$= N_s^2 \left[\frac{P_{dg} + P_{qg}}{2} + \frac{P_{dg} - P_{qg}}{2} \cos 2\theta_r \right]$$

$$= N_s^2 \left[P_{g0} + P_{g2} \cos 2\theta_r \right] = L_{g0} + L_{g2} \cos 2\theta_r$$

Below the equations is a graph of \$L_{ag}\$ versus \$\theta_r\$. The graph shows a sinusoidal wave oscillating around a constant value \$L_{g0}\$. The amplitude of the oscillation is \$L_{g2}\$. The x-axis is labeled \$\theta_r\$ and has markings at \$\pi/2\$, \$\pi\$, and \$2\pi\$. The y-axis is labeled \$L_{ag}\$. The average value \$L_{g0}\$ is indicated by a horizontal dashed line. Below the graph, the average value is given as:

$$L_{aa} = L_{al} + L_{ag} = L_{al} + L_{g0} + L_{g2} \cos 2\theta_r$$

And then, we can simplify like this L_{ag} equal to N_s^2 into p_{dg} by 2 $\cos 2\theta_r$ plus 1 plus q_{g} by 2 $1 - \cos 2\theta_r$, and that we can simplify again N_s^2 square p_{dg} plus P_{qg} by 2 plus $P_{dg} - P_{qg}$ by 2 \cos of $2\theta_r$. And that is equal to N_s^2 square into p_{g0} plus p_{g2} in the \cos of $2\theta_r$ and that is equal to L_{g0} plus L_{g2} into \cos of $2\theta_r$. So, L_{ag} have the two component, one component is constant that is L_{g0} , and other component varies with \cos of $2\theta_r$ that is L_{g2} .

So, if we plug this inductance variation, against θ_r this is L_{ag} we will see that this L_{ag} will have a constant term, that is L_{g0} and that will be a variable term which is L_{g2} into $\cos 2\theta_r$. So, in fact this will be changing in the following fashion, this is x axis we have π here ((Refer Time: 26:20)), this is π by 2 and this is 2π in radian. And then, this will be L_{g2} is amplitude is L_{g2} , because this is L_{g2} into $\cos 2\theta_r$ and the best thing is L_{g0} this is the average value that is L_{g0} . And if we find out what is L_{aa} , L_{aa} is L_{al} plus L_{g0} , L_{al} is a linkage inductance, so we can say here L_{al} plus L_{g0} plus L_{g2} into \cos of $2\theta_r$. So, this L_{aa} is the self inductance of phase a, similarly we can find out the self inductance of phase b.

(Refer Slide Time: 27:21)

$$L_{bb} = L_{al} + L_{g0} + L_{g2} \cos 2(\theta_r - 2\pi/3)$$

$$= L_{al} + L_{g0} + L_{g2} \cos(2\theta_r + 2\pi/3)$$

$$L_{cc} = L_{al} + L_{g0} + L_{g2} \cos(2\theta_r - 2\pi/3)$$

Calculation of Mutual Inductance between two phases of the stator.

$$L_{ab} = \frac{N_s^2 \left[N_s i_{as} \cos \theta_r P_{d1} \cos(\theta_r - 120^\circ)_{bs} + N_s i_{as} \sin \theta_r P_{q1} \sin(\theta_r - 120^\circ) \right]}{i_{as}}$$

$$= N_s^2 \left[-\frac{1}{2} \frac{P_{d1} + P_{q1}}{2} + \dots \right]$$

L_{bb} that is equal to L_{al} plus L_{g0} plus $L_{g2} \cos$ of $2\theta_r - 2\pi/3$ and that is equal to L_{al} plus L_{g0} plus $L_{g2} \cos$ of $2\theta_r + 2\pi/3$. And for the self inductance of phase c, similarly we can calculate L_{al} plus L_{g0} plus $L_{g2} \cos$ of $2\theta_r - 2\pi/3$, so we have been able to find out the self inductance of the three phase, phase a, phase b and the phase c.

Now, after finding out the self inductance of the stator, we can find out the mutual inductance between two phases of the stator phase a with phase b, phase b with phase c and phase c with phase a. So, we will now discuss calculation of mutual inductance between two phases of the state, so what we have here is that, we have similarly phase a, phase b and phase c. And the rotor is a salient pole rotor, is a field winding or the field the rotor field, this is the d axis and this one is the q axis, we have a s, b s and c s, this angle that is substance with phase a, this angle is θ_r .

Now, the definition of mutual inductance is this, it is the flux linkage of phase a current with phase b due to it is own phase a current, so we are talking about the flux linkage with phase b due to the current in phase a, so this show the mutual between the two phases. So, what we find out here is the following, if you phase a that is L_{ab} , we can write down the expression each one is having a number of turns of N_s, N_s, N_s , it is $N_s i_{as}$ into \cos of θ_r , this is carrying a current of i_{as} . Now, we have to find out how much of this class that is by phase a is linking phase b.

So, the flux is produced by phase a, we can resolve this component along the d axis and find out the flux here, resolve this component along the q axis and find out the flux here. And this flux is again resolved back on to phase b, and this flux is also resolved on to phase b, and we can find out how much flux is linking phase b due to the unit current in phase a. So, that is what we can find out here, $N_s i_a \cos \theta_r$ in this case is the d axis flux due to the current in phase a multiplied by $\frac{P_d}{2}$ into, we are talking about the projection of this along this axis.

So, this angle is 120° minus θ_r , so for the cos is concerned \cos of minus θ_r and \cos of plus θ_r are the same, so we can just write down in this case ((Refer Time: 31:45)) it is \cos of θ_r minus 120° . And then, plus $N_s i_a \sin$ we can again find out the sign component for the q axis, $\sin \theta_r$ in the P q g into \sin of θ_r minus 120° , and that is multiplied by the number of turns of phase b. Phase b is having a number of turns of N_s divided by the current of phase a that is i_a . So, if we solve this expression, we will get the mutual inductance between phase a and phase b of the stator. So, if we simplify this, we can simplify this expression and let me just give you the final value here that is equal to N_s^2 into minus of half $\frac{P_d}{2}$ plus $\frac{P_q}{2}$ plus, I can write down in the next slide.

(Refer Slide Time: 33:02)

The image shows a digital whiteboard with the following handwritten equations:

$$\left[\frac{P_d - P_q}{2} \cos(2\theta_r - 120^\circ) \right]$$

$$L_{ab} = -\frac{1}{2} L_{g0} + L_{g2} \cos(2\theta_r - 2\pi/3)$$

$$L_{bc} = -\frac{1}{2} L_{g0} + L_{g2} \cos 2\theta_r$$

$$L_{ca} = -\frac{1}{2} L_{g0} + L_{g2} \cos(2\theta_r + 2\pi/3)$$

$\frac{P_d}{2}$ minus $\frac{P_q}{2}$ into \cos of $2\theta_r$ minus of 120° or $2\pi/3$, so if we further simplify this we can write down the expression for L_{ab} the mutual between phase a and

b, that is equal to minus of half L_g plus L_g into \cos of $2\theta_r$ minus of 2π by 3 same as 2π by 3 . On out of symmetry, we can write down the inductance of phase b and c that is equal to minus of half L_g plus L_g , and what I can do is that I can replace this θ_r by θ_r minus 120 and simplify that, so I will have here just $\cos 2\theta_r$.

Similarly, for phase c, L_{ca} that is equal to minus of half L_g plus L_g into \cos of $2\theta_r$ plus 2π by 3 . So, we see here that again in the mutual inductance also we have a function of θ_r or $2\theta_r$, we see that L_{ab} is the function of $2\theta_r$, L_{bc} is also a function of $2\theta_r$, L_{ca} is also a function of $2\theta_r$. So, when we simulate a synchronous machine in a, b, c variable we have to update this inductance for every rotor position. So, that becomes a complex job, because all this things have to be done in iterate process and this is going to take a lot of computer time.

So, instead of simulating in a, b, c frame or a, b, c system, if we convert this into a d q system host and the rotor, taking the help of rotor reference frame we can have inductance matrix which will be almost constant. So, this is basically the self and the mutual inductance and similarly, we can derive the expression for the mutual inductance between the stator and the rotor. So, right now we will be discussing about the mutual inductance between the stator and the rotor of a synchronous machine.

(Refer Slide Time: 35:42)

Mutual inductance between the stator and rotor of a Synchronous Machine

$$L_{af} = \frac{(L_{g0} + L_{g2}) l_f N_s N_f \cos \theta_r}{l_f}$$

$$= (L_{g0} + L_{g2}) \frac{N_s N_f}{N_f} \cos \theta_r$$

$$L_{bf} = (L_{g0} + L_{g2}) \frac{N_s N_f}{N_f} \cos(\theta_r - 2\pi/3)$$

$$L_{cf} = (L_{g0} + L_{g2}) \frac{N_s N_f}{N_f} \cos(\theta_r + 2\pi/3)$$

$$L_{aKd} = (L_{g0} + L_{g2}) \frac{N_s}{N_{Kd}} \cos \theta_r$$

The diagram shows a synchronous machine with a rotor and stator. The rotor is at an angle θ_r relative to the stator axis. The rotor has a field winding with N_f turns and a field current i_f . The stator has a field winding with N_s turns. The rotor is labeled 'rotor' and the stator is labeled 'stator'. The rotor axis is labeled 'd-axis' and the stator axis is labeled 'a-axis'. The rotor is also labeled 'rotor' and the stator is labeled 'stator'. The rotor is labeled 'rotor' and the stator is labeled 'stator'.

So, what we have here is this, that the rotor has got three winding, one is the field winding and the other two windings are the damper winding, one is in the d axis and other is in the q axis. So, we have to find out the mutual inductance between the stator winding and the three rotor winding. So, we have phase a, phase b and phase c of the stator, this is the rotor and this is the field winding, this one is the damper winding k_d , this is the damper winding in the q axis, we can call this to be k_q .

And the field winding is having an applied voltage of v_f and this is the stator winding we can call this to be a s, this is b s and this is c s. Now, what we will do here is this that we will try to evaluate the mutual inductance between the stator winding a s, and the field winding f. So, this is the d axis and this angle that is obtains with the axis of phase a is θ_r and the orthogonal axis with the q axis, so when we find out the mutual between a and a f, we can write down this expression L_{af} . Now, what is L_{af} , L_{af} is given us L_{g0} plus L_{g2} , now this is the inductance in the d axis, we have already see this.

That if you see this expression ((Refer Time: 38:26)), in this case that we have a maximum inductance and we have a minimum inductance, these are the inductance of phase a, the mintage inductance of phase a when it is alternately seeing the d axis and q axis. When it is seeing the d axis the inductance in maximum, so this is actual the maximum inductance, and this is corresponding to the d axis and that is L_{g0} plus L_{g2} , and this inductance is corresponding to the q axis inductance that is L_{g0} minus L_{g2} .

So, the d axis inductance is L_{g0} plus L_{g2} and the q axis inductance is L_{g0} minus L_{g2} , so we have to remember that, we have two definite inductances, one in the d axis and other in the q axis. So, this we can substitute in this that in the d axis, if I say that this inductance is in this particular axis, so I want to find out the mutual between this and this. So, this is my inductance and the flux linkage will be into my i_f with the flux produce by this, but this flux has to link this winding, so I have to multiply by factor of N_s by N_f and the component of this flux along phase a will be \cos of θ_r .

So, I have to multiply here \cos of θ_r divided it is by the unit current that is i_f , so this show the mutual between the field and phase a. So, this we can simplify that is equal to L_{g0} plus L_{g2} into N_s by N_f into \cos of θ_r , now similarly we can find out the inductance between phase b and the field winding, phase b is here. So, we have

something quite similar to this, so this will be L_{g0} plus L_{g2} into N_s by N_f into \cos of θ_r minus of 2π by 3 . Because, b is shifted from a by 2π by 3 b is shifted from a by 2π by 3 and hence, we have \cos θ_r minus of 2π by 3 .

The angle between the phase b and the d axis 2π by 3 minus of θ_r and for the \cos , \cos of minus θ is same as \cos of flux θ \cos being an even function, so I can write down \cos of θ_r minus 2π . Similarly, we can have c f that is equal to L_{g0} plus L_{g2} N_s by N_f \cos of θ_r plus 2π by 3 , so this is the field winding. What about the damper winding, in a similar way I can write down L_{akd} is equal to L_{g0} plus L_{g2} in N_s by N_{kd} into \cos of θ_r .

(Refer Slide Time: 42:05)

The image shows a whiteboard with the following handwritten equations:

$$L_{bkd} = (L_{g0} + L_{g2}) \frac{N_s}{N_{kd}} \cos(\theta_r - 2\pi/3)$$

$$L_{ckd} = (L_{g0} + L_{g2}) \frac{N_s}{N_{kd}} \cos(\theta_r + 2\pi/3)$$

$$L_{akq} = -(L_{g0} - L_{g2}) \frac{N_s}{N_{kq}} \sin\theta_r$$

$$L_{bkq} = -(L_{g0} - L_{g2}) \frac{N_s}{N_{kq}} \sin(\theta_r - 2\pi/3)$$

$$L_{ckq} = -(L_{g0} - L_{g2}) \frac{N_s}{N_{kq}} \sin(\theta_r + 2\pi/3)$$

$$L_{md} = \frac{3}{2} (L_{g0} + L_{g2}) ; \quad L_{mq} = \frac{3}{2} (L_{g0} - L_{g2})$$

And similarly, we can have L_{bkd} that is equal to L_{g0} plus L_{g2} into N_s by N_{kd} into \cos of θ_r minus of 2π by 3 and L_{ckd} that is equal to L_{g0} plus L_{g2} into N_s by N_{kd} \cos of θ_r plus 2π by 3 . This about the mutual inductance between the stator winding and the d axis is damper and similarly, we can evaluate the mutual inductance between the stator winding and the q axis damper that is L_{akq} , so will see what is this inductance. ((Refer Time: 43:04)) We are finding out the inductance between a and kq , this is my phase a and this is kq , and multiply by factor of \cos θ_r plus π by 2 .

And \cos θ_r plus π by 2 is minus of \sin θ_r , so the expression will be multiplied by minus of \sin θ_r for phase a and similarly for phase b and c , we can change θ_r appropriately, to θ_r minus 2π by 3 and θ_r plus 2π by 3 respectively. So, we

can see what is this inductance the mutual inductance, that is $L_{g0} + L_{g2}$ into N_s by $N_k q$ into \sin of θ_r with negative sign, L_{bkq} is equal to $-(L_{g0} + L_{g2}) N_s$ by $N_k q$. Here actually that is a very interesting since you note that the inductance in the q axis is not $L_{g0} + L_{g2}$.

It is in fact if you see little before what we have seen in this case, is that ((Refer Time: 44:23)) the inductance in the q axis is in fact, this is the inductance and this inductance is $L_{g0} - L_{g2}$. And we will see that this $L_{g0} + L_{g2}$ is equal to there is a inductance call the d axis nitaging inductance, we can call that L_{md} and little later we will show you that, this L_{md} is equal to $L_{g0} + L_{g2}$ and this is factor of 2 by 3. And similarly, $L_{g0} - L_{g2}$ is the q axis nitaging inductance and that is equal to 2 by 3 times L_{mq} , L_{mq} is the nitaging inductance in the q axis.

So, when we are talking about the q axis inductance, we have to take $L_{g0} - L_{g2}$, so in this case what we have here is ((Refer Time: 45:21)), we have in this case $L_{g0} - L_{g2}$, and L_{bkq} is equal to $-(L_{g0} - L_{g2}) N_s$ by $N_k q \sin$ into $\theta_r + 2\pi$ by 3, and we have already seen that L_{md} can be given as 3 by 2 $L_{g0} + L_{g2}$, this is the d axis nitaging inductance refer to the primary side. And similarly, L_{mq} is 3 by 2 times of $L_{g0} - L_{g2}$, now this L_{md} and L_{mq} can be obtain by suitable transformation. So, these are the mutual inductance, it looks little complicated, because this is the function of θ_r , now when we transform this inductance into the rotor reference frame, the two axis rotor reference frame we will see that this inductances will be independent of θ_r .

(Refer Slide Time: 47:04)

Transformation to dq0 equations

$$\psi_{abc} = [L_{ss}] i_{abc} + [L_{sr}] i_{fkdq}$$

$$\psi_s = [L_{ss}] i_s + [L_{sr}] i_r$$

$$C_s \psi_s = C_s [L_{ss}] C_s^{-1} i_s + C_s [L_{sr}] i_r$$

$$\psi_{dq0} = \begin{bmatrix} L_{md} & 0 & 0 \\ 0 & L_{mq} & 0 \\ 0 & 0 & L_d \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{0s} \end{bmatrix}$$

Ld →

$$L_{ds} = L_{al} + L_{md} = L_{al} + \frac{3}{2} (L_{p0} + L_{p2})$$

$$L_{qs} = L_{al} + L_{mq} = L_{al} + \frac{3}{2} (L_{p0} - L_{p2})$$

So, we will see that, now we can say in the following fashion that we have this inductances and we can transform the equation, so what we have here is the following, we have psi a b c s that is equal to L s s is the matrix into i a b c s plus L s r into i f k d k q. Or we can write down in the following fashion, psi s is equal to L s s into i s plus L s r into i r, i r is the rotor and the rotor has got three components of current the field winding, the d axis damper and the q axis damper. In the stator has got three components phase a current, phase b current and phase c current, the psi s is also having three components.

So, we have already seen this earlier we can see this ((Refer Time: 48:17)) that this is the component that is L s s and this is the component that is L s r, the mutual between the stator and the rotor. Now, we will try to transform this into the d q equation, so if you want to transfer this into the d q equation, we will pre-multiply this the transformation matrix that is C s, C s the transformation matrix and this also we have seen this matrix is the transformation matrix, which is in the rotor reference frame and this is the function of cos theta r cos theta r 1 cos theta r 2 minus sin theta r minus sin theta r 1 and minus sin theta r 2 half half and half 2 by 3.

And this 2 by 3 is coming to picture to half for fit power invariant, it is a transformation where the power phase power is maintained in the three phase system same as that of the two phase system. So, when we use the transformation, we can transform this fluxes into

the d q fluxes, so we have $C_s L_{ss}$ this L_{ss} we already evaluated L_{ss} has got the self inductance and the mutual inductance between the stator, two phases of the stator, so this we have evaluated this also we have.

So, this is C_s inverse and C_s into i_s vector plus $C_s L_{sr}$ into i_r , now what is C_s into ψ_s this a, b, c will transform into the d, q , so we can say here this is d, q, o, s . And then, this matrix after transformation will have the following values and then, when we transform the C_s into i_s that will be i, d, s, i, q, s and i, o, s and this matrix $C_s L_{ss}$ and C_c invert. If you calculate this particular matrix this matrix will have the following form, so we have L_{md} here 0 and 0 , L_{md} plus L_{al} in this case we have $0, L_{mq}$ plus L_{al} 0 and $0, 0$ and L_{ll} .

So, we see that the d axis inductance, so we can this L_{ds} is the d, s inductance is L_{al} plus L_{md} and what is L_{md} , L_{md} is evaluated plus 3 by 2 times L_{g0} plus L_{g2} . And L_{qs} is equal to L_{al} plus L_{mq} and that is equal to L_{al} plus 3 by 2 times of L_{g0} minus L_{g2} . So we see that the inductance matrix in this case, we can call this inductance matrix to be L_{dqo} matrix and ((Refer Time: 51:50)) this L_{dqo} matrix is fortunately is a constant matrix, unlike the matrix L_{ss} , which is a function of θ_r after pre-multiplying with C_s and fourth multiplying with C_s inverse, there is lot of simplification involve. And this matrix that we are obtaining here is L_{dqo} that matrix is have being constant inductances, like L_{md} . ((Refer Time: 52:28)) Because, when we are transforming into the rotor reference frame we have seen that the inductance here is constant inductances.

(Refer Slide Time: 53:33)

Transformation to dq0 equations

$$\Psi_{abc} = [L_{ss}] i_{abc} + [L_{sr}] i_{fr}$$

$$\Psi_s = [L_{ss}] i_s + [L_{sr}] i_r$$

$$C_s \Psi_s = C_s [L_{ss}] C_s' i_s + C_s [L_{sr}] i_r$$

$$\Psi_{dq0} = \begin{bmatrix} L_{md} & 0 & 0 \\ 0 & L_{mq} & 0 \\ 0 & 0 & L_d \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{0s} \end{bmatrix} + \begin{bmatrix} L_{md} & L_{mq} & 0 \\ 0 & 0 & L_{mq} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{fr}' \\ i_{kr}' \\ i_{kr} \end{bmatrix}$$

$$L_{ds} = L_{al} + L_{md} = L_{al} + \frac{3}{2} (L_{p0} + L_{p2})$$

$$L_{qs} = L_{al} + L_{mq} = L_{al} + \frac{3}{2} (L_{p0} - L_{p2})$$

Last this matrix also if you simplify, this is also will be appearing in the following fashion the currents will also be transform, so we have the transformed currents here i f prime, i k d prime and i k q prime here. And this matrix would be L m d L m d and 0 and then, 0 0 L m q and then, we have 0 0 and 0, so we have the inductance matrix referred in the rotor reference frame.

(Refer Slide Time: 53:23)

$$\Psi'_{ds} = L'_{ds} i'_{ds} + L'_{md} (i'_f + i'_{kd})$$

$$\Psi'_{qs} = L'_{qs} i'_{qs} + L'_{mq} i'_{kq}$$

$$\Psi'_f = L'_f i'_f + L'_{md} (i'_{ds} + i'_{kd})$$

$$\Psi'_{kd} = L'_{kd} i'_{kd} + L'_{md} (i'_{ds} + i'_f)$$

$$\Psi'_{kq} = L'_{kq} i'_{kq} + L'_{mq} i'_{qs}$$

So, we can write down the various flux linkages, the flux linkages in the d axis is given as L d s into i d s plus L m d into i f, the referred current plus i k d. And the flux linkage

in the q axis ψ_{qs} is equal to $L_{qs} i_{qs}$ plus $L_{mq} i_{kq}$, the flux linkage in the field referred in the primary side is equal to $L_f i_f$ plus $L_{md} i_{ds}$ plus i_{kd} prime. The flux linkage in the d axis damper is equal to $L_{kd} i_{kd}$ prime plus $L_{md} i_{ds}$ plus i_f prime and the flux linkage in the q axis damper referred in the primary side is $L_{kq} i_{kq}$ plus $L_{mq} i_{qs}$.

So, these are the five flux linkages in the five windings, because we know have five winding, two winding in the stator, we have d and q winding in the stator. And three windings in the rotor, two windings in the d axis the field winding and the damper winding in the d axis and one winding in the q axis that is the q axis damper; and these are the ((Refer Time: 55:08)) five flux linkage equation referred in the primary side.

When we draw the equivalent circuit, we have to understand that all the variables are referred in the primary side and hence, we have to multiply by appropriate number of turn to get the variable in the primary side. And this variables which referred to the primary side are designated, the rotor variable referred to the primary side as shown as the prime variable like, ψ_f prime ψ_{kd} prime and ψ_{kq} prime. So, we are now gradually developing the model of an inductance machine in the part reference frame in d q equations. In the next lecture, we will see that how this model is to derive the expression for the torque. And from the torque how we can find out the speed, and how we can simulate a synchronous machine in d q equivalent circuit to get the response of the machine.