Advanced Electric Drives
Prof. S. P. Das
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 6

Hello, in the last lecture we have already seen the modeling of induction machine in stationary reference frame. We will just recapitulate what we have done in the last lecture.

(Refer Slide Time: 00:30)

Model of induction machine in stationary reference frame. Now we can write down the equation for the stator and the equation for the rotor, and then we can develop the torque equation, and from the torque equation we can find out the speed. So, this goes in an iterative process. So, let us write down the equation for the stator. So, this is V d s is equal to r s i d s plus p psi d s; the stator is stationary. Hence, the stator does not have any rotationally induced T m f, and we can express the flux linkage. This psi d s is the flux linkage in the d axis stator. So, we can express that in terms of inductance and current. So, that is equal to r s i d s plus L s p i d s plus L m p i d r prime; i d r prime is the current referred from the primary side.

And similarly we can have V q s; V q s is equal to r s i q s plus p psi q s; that is equal to r s i q s plus L s p i q s plus L m p i q r prime. So, these are the two equations for the

stator. Now we did not simulate the zero sequence equation, because the zeros sequence equations does not take part in torque production. So, for the calculation of the machine torque it is sufficient only to write down the equation for the d and q axis, and find out the corresponding currents and evaluate the torque. Similarly for the rotor we can write down the equation for the rotor here V d r prime r r prime i d r prime plus p psi d r prime, and the rotor will have a rotationally induce p m f. And the rotationally induced p m f will be a function of the speed of the rotor.

So, that is plus omega r psi q r, and we can again express this in terms of current only i d r plus L r p i d r plus L m p i d s plus omega r into L r i q r plus L m i q s. Similarly we can write down the q axis voltage; V q r prime equal to r r prime i q r prime plus p psi q r prime minus of omega r psi d r prime. This we can express in terms of the currents only; r r prime i q r prime plus L r prime p i q r prime plus L m p i q s minus of omega r L r i d r plus L m i d s. Now these four equations when we solve these are four simultaneous differential equations. Now we can saw this numerically; we cannot have any analytic solution, because the voltages are sinusoidal, but sometimes the machine is also fed from an inverter.

When the machine is fed from an inverter the voltages are not sinusoidal, and furthermore they can be any function of time. So, a closed form solution of these equations is not possible. So, we have to take the help of numerical method. One of the popular ways to simulate this deferential equation is the Runge Kutta fourth order integration technique. So, we can use Runge Kutta fourth order integration technique to find out the values of i d s, i q s, i d r and i q r, and these values will be useful for the torque calculation. Now this equation those that we have written here we can express them in the form of a matrix.

So, we can just write down these equations in the form of a matrix V d s, V q s, V d r and V q r. And here we have the impedance matrix, and then we have i d s, i q s, i d r and i q R. Now this impedance matrix is a 4 by 4 matrix, and we can fill up this matrix from the previous equations. We can also fill up by inspection; we do not have to remember the various elements here. By intuition we can fill up each and every element of this matrix; let us see. So, we have four rows here and four columns. Now this is r s plus L s p, and this one is L m p, and the stator does not have any rotational induced t m f. So, we can make this equal to 0, but the q axis stator this is r s plus L s p and this is L m p. Again we have this element 0, because in the stator we do not have any rotationally induced t m f.

In the rotor we will have the resistance drop, the rotor self inductance, and then we have the coupling from the stator side. And these two are the speed induced t m f term. So, we have omega r here and omega r, and this will be coupled to i q r. So, we have L r prime in this case, and this is L m from the stator side, and for the fourth row that is for v q r we have r r prime plus L r prime p. Then we have coupling from the stator side, then we have the rotational induced t m f minus omega r L m and minus omega r L r prime. Now this equation is the key equation for solving the differential equation of induction machine, and this we can also represent in a symbolic form

So, if we represent this is in a symbolic form we have the following equation. So, V is equal to an r matrix and i plus L is an inductance matrix. P i plus omega r the G matrix,

and then we have i. So, this matrix equation can be written in the following form. Here V is the vector; V is corresponding to this vector, i is corresponding to this vector, r is the resistance matrix which is a diagonal matrix having the elements r s r s for the stator and r r prime and r r prime for the rotor. And this L p is the matrix associated with the derivative of the current. So, this matrix can be easily valuated.

Those element which have got derivative terms they are clogged into L matrix, and those elements which has got the speed term is clogged in the G matrix; as we have already seen in a Crone primitive machine model as this G matrix can be usually identified. So, we can write down what is the G matrix here, because this G matrix is useful in calculating the torque of the machine. So, from this 4 by 4 matrix we can see that we have got this element is having omega r term, this element is also having omega term, this is having omega term and this is having omega term. So, this G matrix will have four elements. So, we can write down this G matrix, and once we evaluate this G matrix we can find out the torque. Torque is equal to p by 2 i transpose G and i.

Now we know that we have a 3 phase machine, and the d q model is a hypothetical model. It does not really exist; we are simulating a 3 phase machine by means of a 2 axis model. So, when we are finding out the torque we should be careful that we should get the torque back to the 3 phase machine torque. And we have already seen that the transformation that we have used, the transformation for the stator and the transformation for the rotor has got per phase power invariants. So, if we have 2 phase machine the power of the 2 phase machine is 3 by 2 times of; I mean if you have a 3 phase machine the total power is 3 by 2 times the power of the 2 phase machine, because the per phase power is invariant here.

So, if you take a 2 phase machine you just have 2 phases; if you take a 3 phase machine you have 3 phases. So, if you convert the power of the 2 phase machine into the 3 phase machine you have to multiply by a factor of 3 by 2. So, here we have to multiply in this case the factor of 3 by 2, because our model is a 2 phase model or a 2 axis model; actual machine is a 3 phase machine. So, if you want to convert it back into the torque of a 3 phase machine you have to multiply 3 by 2 in this case. So, the torque in this case is 3 by 2 into p by 2 is the whole pair i transpose G into i. So, we can simplify this, and if you simplify this we can pre-multiply and post-multiply. Let we just write down the final expression. The torque is given in the following form, 3 by 2 into p by 2 into L m i d r

prime i q s minus i q r prime i d s. So, it means if we have known the various currents we can evaluate the torque of the machine. So, how do you simulate this equation? Now this equation can be simulated in the following fashion.

(Refer Slide Time: 13:21)

Now we can evaluate what is this L p? L p i is given as V minus of R i minus of omega r G i. If you see this we are just trying to rearrange this equation. So, we are taking this term to the left hand side and trying to evaluate what is L p i. So, L p i is given as V minus R i minus omega r G i. Now ultimately what we need is the current. So, if you want to find out the current we should know the derivative of the current. So, p i is equal to L inverse of V minus R i minus omega r G i. This is what we have, and this can be called using Runge Kutta fourth order integration technique. So, we solve this. So, we can use Runge Kutta fourth order integration technique to solve this equation. And once you solve it we can find out the values of the various currents, and from the current we can find out the torque, and from the torque we can also find out the speed.

Usually when we simulate a machine the electrical variables vary at much faster rate than mechanical variables, and hence in this particular case the currents i d s, i r s, i d r and i q r vary at much faster rate than the speed. Speed is the mechanical variable. So, we can take this advantage of this particular property, and we can simulate the current equation separately and the speed equation separately. What we are trying to do here is this that the four equation that we have this is basically four equations we have i d s, i q

s, i d r and i q r. We can solve them simultaneously by using Runge Kutta fourth order technique, and then when we come to the speed equation we have the mechanical variable and that vary much lower than the electrical variable.

So, while simulating the electrical variable we can assume omega r to be constant, and after we obtain the torque we can simulate this equation by linear integration technique and first order is sufficient. We can just have first order linear integration which will be much accurate to give the value of the speed. So, we can find out the speed from this equation, and we know the torque and the speed. So, once we know the torque and the speed we can plot the torque-speed characteristic of an index machine. Now you will be surprised to see that the torque-speed characteristic that we obtain of an actual machine is much different than what we have studied in the previous courses in electric machines.

(Refer Slide Time: 17:11)
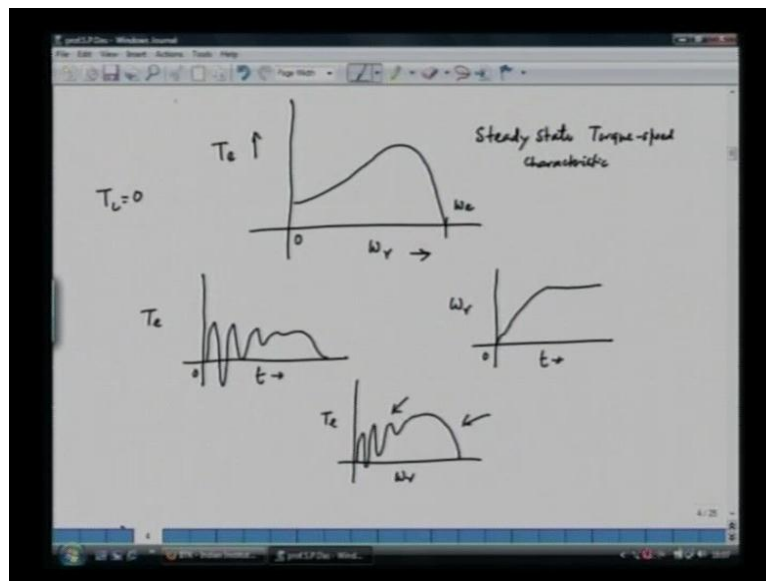
In the previous courses in electric machine if you see that if you plot the torque-speed characteristic of an induction machine T e verses omega r. You will see a curve like this. So, this is the synchronous speed, and this is your torque axis and near the speed axis. Here the speed is equal to 0, and this is the nature of torque-speed characteristic, and this has been obtained by assuming steady state equivalent circuit. The equivalent circuit which you have already studied in your previous courses in electrical machines specially in the undergraduate second year level gives the torque-speed characteristics of a steady state machine.

The machine is in the steady state condition, but here when we are simulating the machine the machine is actually; the actual machine it is basically proceeding from the tangent condition to the steady state condition. So, if you take t equal to 0 you are starting time for the simulation and take some delta t for the simulation. And you have a finish time t f, and we can simulate this equation. The respond that we obtain is the following. Now we would like to call this to be a steady state torque-speed characteristic. Now if you have an exact simulation starting from rest the torque-speed characteristic will be something different. We can first plot torque versus time, we can plot speed versus time.

You will see that the torque is not a smooth variable. It is basically a function of time. It axis like this, and finally, if you have no load condition it settle down to T e equal to 0. If you are assume that T L equal to 0 assume the machine is being started from the torque condition without any load it accelerates and reaches the synchronous speed or close to the synchronous speed it is called pre-acceleration. So, when we simulate the machine under pre-acceleration condition there will be torque oscillations initially. Then finally, it settles down to the final speed where the torque is equal to 0; in the steady state it operates under no load condition. What about the speed? The speed gradually increases and finally this is the steady state value here.

And if we plot the torque-speed characteristic under free acceleration condition it is something like this. So, in this case we have we have exactly the plot that is available as the machine is starting from rest to the full speed. Now this is because of the fact that the machine is being simulated through the transient condition. And when we energize an induction machine in addition to the a c component of the current there is a d c component of the current, and the d c component produces a d c flux and the d c flux interacts with the rotating flux to produce a pulsating torque. So, that is the reason why we have torque pulsation as the machine starts from rest.

So, this can be verified when we simulate the machine in the d q equivalent circuits in the d q model taking into account the transient condition and reaching the steady state finally. So, this is an interesting thing which can be seen in the simulation, okay. Now with this before we proceed further we would like to introduce the concept of reference frame. Reference frame is extremely important, because when we analyze the machine we take a reference frame as per our requirement. It can be a stationary reference frame

which is stationary in the space, it can be rotating reference frame which is rotating in the space, and this reference frame has not chosen arbitrarily. There are basically chosen depending upon the need of a special control theory or need of machine simulation.

(Refer Slide Time: 21:41)
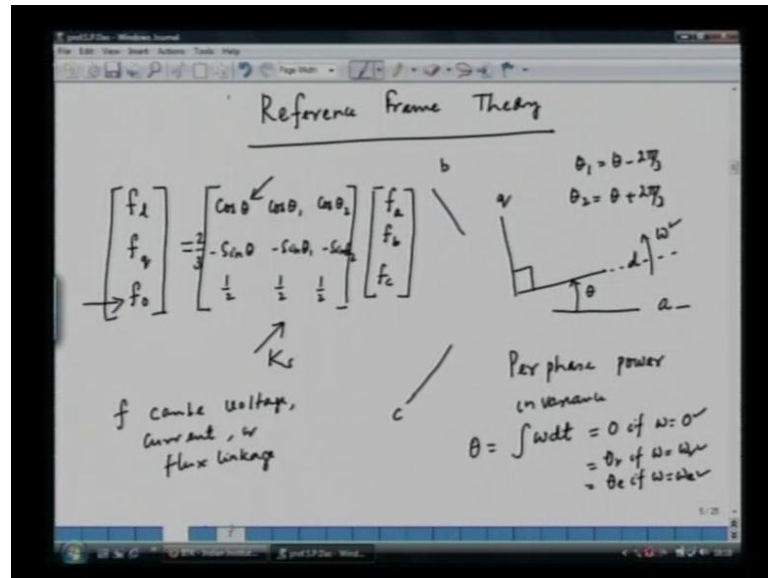
So, we would discuss in this case reference frame theory. So, what we have here is the following. We have a state of a b c system. This is phase a, phase b and phase c, and we would like to transform this into d q system, but the d q system need not be always stationary. This may be rotating at certain speed any arbitrary speed omega. So, we can say that when we convert this into a d q system, d q system is always orthogonal. This is my d axis, and this is the q axis, and this angle that it obtains with the a axis is theta. And the reference frame is rotating at a speed of omega radian per second. So, this reference frame is not stationary; it is rotating at a speed of omega radian per second, and this is called the rotating reference frame.

Now when we transform the variables from a b c into d q we get the following equation. Now we have f d, f q and f o; these are the variables that we have in the d q reference frame. F o in the 0 sequence component which can be present in general when we have an unbalanced system, there could be a 0 sequence component. Now this f d, f q and f o can be obtained from f a, f b and f c. So, we have a transformation here. So, we can use a transformation in this case, and this transformation will give us f d, f q and f o come

from f a, f b. f c. Now what are this f a, f b, f c? They could be voltage, they could be current, they could be flux linkages for an electric machine.

So, we can say that f can be voltage, can be current or flux linkage. Now when we transform this a b c into d q we will take the help of this transformation and the transformation can be easily found out. What we can do here we can project this a onto d axis, project this b onto d axis, project the c onto d axis, and calculate what is the equivalent f d. Now if we do this simple excise we will get the following result. We have cos theta cos theta 1 cos theta 2, minus of sin theta minus of sin theta 1 minus of sin theta 2, half half and half. So, we have a multiplication constant here that is 2 by 3. Now this 2 by 3 is multiplied in this case to keep the per phase power invariant.

When we are transforming the a b c system into d q system we have to be careful that this does not alter the system parameter, neither it alters the various variables of the original system, say, for example, if I have a 400 volt 3 phase system the per phase voltage is 230 volt. Now if I transform this into a d q system the per phase voltage will also 230 volt. Otherwise, you know there would be a discrepancy and extra work has to be done to calculate this transformation. Similarly if we have a 3 phase system the 3 phase current per phase current is 10 ampere in the 2 phase system where in the d q system the per phase current should be also 10 ampere.

So, we have to take such a transformation which does not change the per phase power. So, this is based on per phase power in variant, and that is why we multiply in this case 2 by 3 so that per phase of the 2 phase system or d q system is same as per phase of the 3 phase system that is a b c system. The parameters are same, the variables are also same. So, when converting from 3 phase to 2 phase we do not have to change the variables or change the parameters. So, in this case you can see that this omega r can have various values and accordingly we have so many systems.
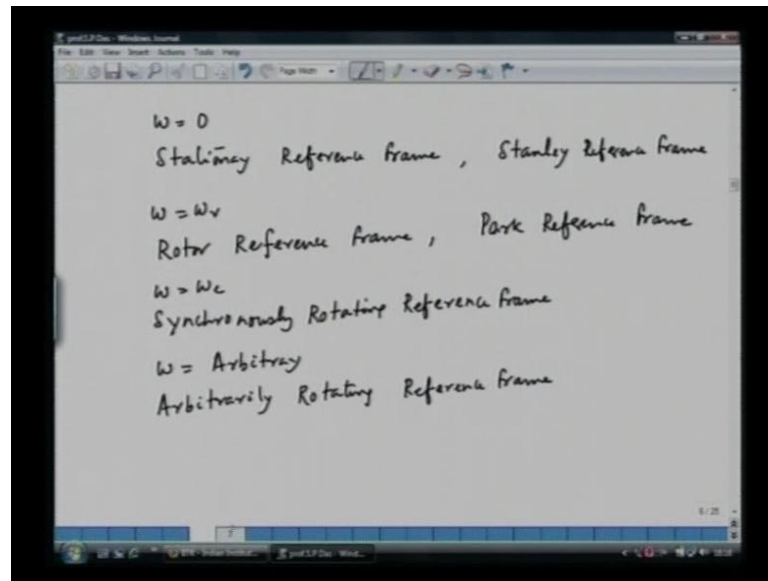
So, if omega r equal to 0 we call this to be a stationary reference frame. It is also known as Stanley reference frame by the name of a scientist called Stanley, okay. So, this can be implied in case of an induction machine. As you have seen little earlier that the induction machine was stimulated in stationary reference frame when the speed of the reference frame is 0 it was stationary in the space. Now if we take omega equal to omega r the speed of the reference frame is same as the rotor speed, and we call this to be rotor reference frame. And this actually will be useful when we model a synchronous machine the reference frame is house than the rotor, and hence we have certain advantage in the case of an induction machine, and this is also called park reference frame. So, we also call this as park reference frame.

Now when omega equal to omega e omega is the synchronous speed; we called the reference frame to be synchronous speed rotating reference frame or synchronous reference frame. So, when omega equal to omega e we call this to be synchronously rotating reference frame, or in brief we can call this to be a synchronous reference frame. And if omega is arbitrary we call this reference frame to be an arbitrarily rotating reference frame or arbitrary reference frame. If omega is arbitrary we call this to be an arbitrarily rotating reference frame. So, we can recalculate this matrix. This is our transformation matrix; we can call this to be k s or c s. Now this matrix k s is the function of theta. Now this theta 1 here is theta minus 2 pi by 3, theta 2 in this case is theta plus 2 by 3.

So, it is basically phase shifted 2 by 3 and phase shifted by 4 pi by 3 respectively. So, we know this cos theta minus 2 pi by 3 cos theta plus 2 pi by 3. Similarly we have the sin, and the third is corresponding to the 0 sequence component. So, we can always evaluate this transformation matrix. And depending upon the speed if the speed is equal to 0 theta is an integration of the speed. Basically this is the angle that it sustains the d axis sustains with phase a. Now if omega equal to 0, theta is equal to 0 and that is equal to theta r if omega equal to omega R, and that is equal to theta e if omega equal to omega e.

So, theta is integration of the speed; depending upon the speed the theta is defined. If the speed is the rotor speed, theta is the rotor angle with respect to phase a of the stationary axis. If omega is the synchronous speed theta is theta e which is the synchronous reference frame angle or the angle of the synchronously rotating flux vector. So, we can substitute the corresponding theta here depending upon whether it is a stationary frame or the rotor frame or synchronously rotating reference frame. And we can evaluate this transformation matrix that is K s, and when we have the transformation matrix we can transform the a b c variable into d q variable, okay. So, this is what we wanted to discuss here.

(Refer Slide Time: 31:55)

And if you want to take the reverse transformation we can also do that. It means if you want to calculate f a, f b, f c from f d, f q and f o, we have to take the inverse of the matrix. And in this case the inverse of this matrix is the following. It is cos theta cos

theta 1 cos theta 2, minus sin theta minus sin theta 1 minus sin theta 2, 1 1 and 1. This is the inverse of K s. It is an invertible matrix. So, if you inverse this matrix we get K s inverse and K s inverse is given as cos theta cos theta 1 cos theta 2 in the first column. Second column is minus sin theta minus sin theta 1 minus sin theta 2 and 1 1 1. We can verify that if we multiply k and K s inverse we get I matrix or the identity matrix that can be verified, okay. So, with this background we can just see if we simulate an induction machine in arbitrarily rotating reference rotating frame, what would be the basin model?

(Refer Slide Time: 33:27)
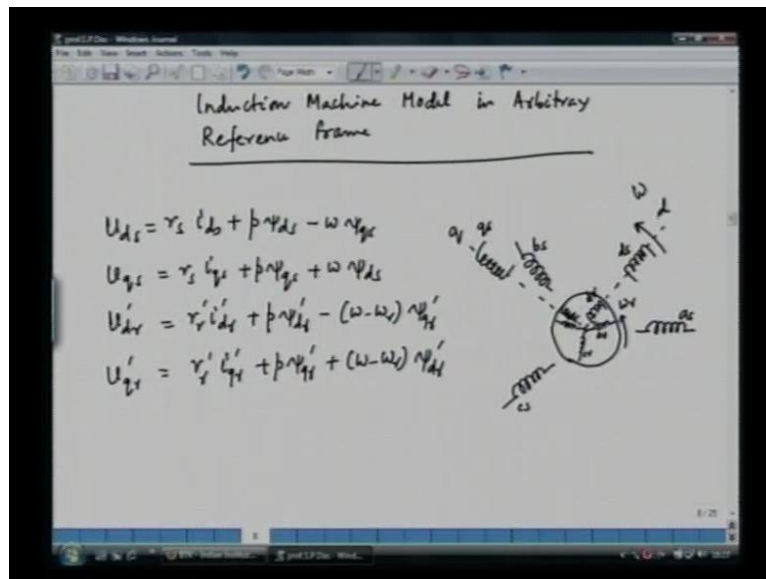
So, we can do that induction machine model in arbitrary reference frame. So, what we have here is the following. We have the rotor, we have the stator, and then we have the rotor windings. This is a s, b s, c s, a r, b r and c r and is rotating at a speed of omega r, and we have taken a reference frame which is rotating arbitrarily. It means this reference frame is having d and q axis. This is the d axis, and this is the q axis, and the speed of the reference frame is some arbitrary speed that is omega radian per second. So, if we model an induction machine in arbitrary reference frame, the stator variables will also have speed induced t m f; it means the actual machine is stationary.

We have got a s, b s and c s but the hypothetical or the virtual machine is rotating. It means this is my d s, and this is the q s winding; as per the Crone primitive machine model we have a 2 axis model in this case, but the axes are rotating at a speed of omega add radian per second. And since the axis are rotating the actual stator winding is

stationary, but the d q windings are rotating in the space, and because of the rotation of the stator winding there will be a rotationally induced t m f even in the stator which is not usually present, and the rotationally induced t m f will be at a speed of omega. Omega is the speed of the reference frame.

So, we can write down the equation of the stator that V d s is equal to r s i d s plus p psi d s, and that is not enough. In this case the stator is rotating at a speed of omega. This winding d s is not stationary; it is in the rotating reference frame, and the speed of the reference frame is omega. So, we have to have in this case a rotationally induced t m f that is omega into psi q s. In a similar fashion we can write down for the q axis V q s that is equal to r s i q s plus p psi q s plus omega into psi d s, and in the rotor we will have the difference of the rotating reference frame speed and the rotor speed, because the rotor is also rotating at a speed of omega r.

The reference frame is rotating at a speed of omega r. The differential speed in this case is omega minus omega r, and the rotationally induced t m f will be appearing at a differential speed that is equal to omega minus omega r. So, we can just write down the rotor equation V d r equal to r r i d r plus p psi d r minus of omega minus omega r psi q r. These are all referred from the primary side, and hence we have the primed variable. V q r is equal to r r i q r plus p psi q r plus omega minus omega r into psi d r. So, these are the four equations of the induction machine in arbitrary reference frame where we have the rotationally induce t m f appearing most in the stator and also in the rotor.

So, we will now just write down the equivalent circuit of an induction machine arbitrary reference frame. The equivalent circuit will be d q equivalent circuit, because we have the d axis stator and we have the d axis rotor here d r. And we have the q axis stator and also we have the q axis rotor, and the reference frame is rotating at a speed of omega as we have said before. And if we draw the equivalent circuit we have to draw the d q equivalent circuit in this case.

(Refer Slide Time: 38:33)
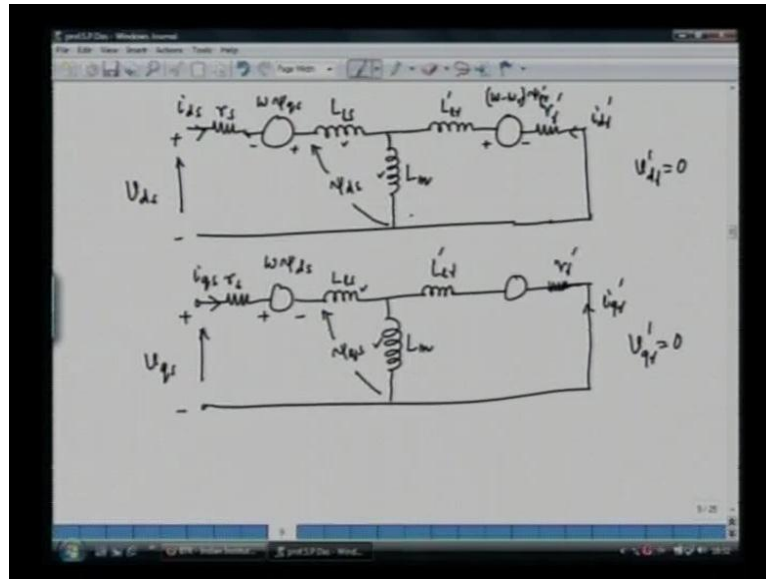
And the equivalent circuits will be of this form. So, we have this is the d axis circuit. So, what we have in this case is here we have V d s r s, and this is our magnetizing inductance L m, and this is the stator leakage inductance. This is the rotor leakage inductance, this is the rotor resistance seen from the primary side, and we have the voltage source here, and that represent the rotationally induced t m f in the stator. In the rotor also we will have a rotationally induced t m f. So, in this case also we have a rotationally induced t m f in this case. So, we have the rotationally induce t m f in this case, and then we have resistance in the rotor. So, this is the rotor resistance, and this is the stator current i d s. And this rotational induce t m f we can see here it is omega into psi q s in the d axis, and it is helping the applied voltage.

So, we can see this is the applied voltage with plus and minus. This is minus and plus here, and this is omega into psi q s, and this rotationally induced t m f in the rotor will be appearing as omega minus omega r into psi q r. And this sign we can also find out. So, this current is i d r entering this kind of thing entering the rotor circuit like this, and this will be minus here and plus here. And this is also helping the rotor applied voltage. Now in case of an induction machine the rotor is short circuited. So, we can say that V d r prime is equal to 0. The rotor is short circuited, and hence V d r prime is equal to 0. So, we have sorted the rotor in this case.

Similarly for the q axis we can write down the equivalent circuit. This is the q axis equivalent circuit; inductances are same for d and q axis because we have a cylindrical structure. In an induction machine the rotor is cylindrical, the stator is also cylindrical; the air gap is uniform, and hence the inductances of the d and q axis are the same. So, we can say here that we have the same arrows in this case. We have the same L s, we have same L r, and we have the same r r here. And the signs of the rotational induced t m f will be different here; that is omega into psi d s, and here we have V q s the q axis applied voltage.

And this is i q s the q axis current, and this current is i q r the rotor current in the q axis. And again V q r is equal to 0, and hence the rotor is short circuited. Now what are the various flux linkages? Now when we show here psi q s, this is the psi q s the flux linkage in the q axis stator. The flux linkage in the q axis stator consists of the magnetizing flux and the stator leakage. This flux is the magnetizing flux associated with the rotor I mean the magnetizing inductance of the q axis and the flux associated with the leakage inductance of the q axis in the q axis leakage flax. So, the magnetizing flux plus the leakage flux is the total flux linkage in the q axis.

Similarly in the d axis this flux linkage is psi d s; the stator d axis flax linkage which consist of the magnetizing flux which is associated with the magnetizing inductance, and the leakage flux which is associated with the leakage inductance of the d axis. So, this flux plus this flux is the total flux linkage in the d axis stator. Similarly we can have the rotor d axis flux linkages psi d r prime referred from the primary side, and then this is psi q r prime the flux linkage in the rotor q axis referred from the primary side. So, this is the equivalent circuit of an induction machine arbitrary reference frame, and if we take this equivalent circuit we can stimulate the machine both in the transient condition, also in the steady state condition without in loss of accuracy.

And we can get back the original current i a, i b and i c when we transform i d s, i q s and i o s by means of the transformation matrix into i a, i b and i c. So, this finishes the modeling on induction machine. We have seen that the induction machine modeling is very interesting, and we have seen how the generalized theory of induction machine can be applied to stimulate an induction machine in 2 axis model. And there is advantage we have already seen that the 2 axis model is stimulated with less computational effort compared to a 3 phase a b c model.

And the a b c model is much more complex compared to the 2 axis model, and hence in many stimulation method we go for the 2 axis modeling of induction machine. Induction machine is called the work horse of industry; about 60 to 70 percent of the motor are induction motors in the industry; however, for very large for application we go for synchronous motor, specially for application which are more than one megawatt or so synchronous machines are preferred because of the design economy. So, we will be discussing here the modeling of a 3 phase synchronous machine.

(Refer Slide Time: 45:27)
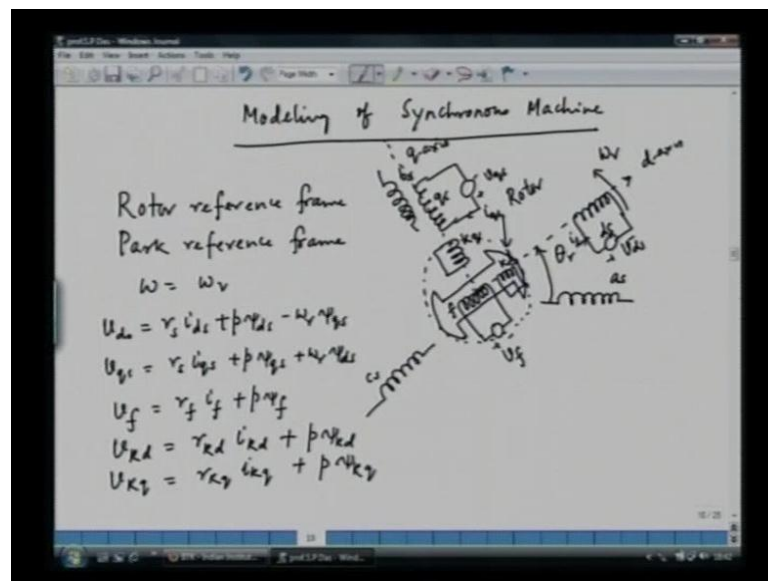
Synchronous modeling is more complex than induction machine, because it is a w excited system. We have 3 phase stator, and we also have a field winding in the rotor. So, if we see the structure of a synchronous machine, these are the stators winding a s, b s and c s. And we have the rotor here, and the rotor need not be a cylindrical rotor; here we can also have a silent pole rotor. So, we represent the rotor by means of a silent pole structure. So, this is our rotor here, and the rotor has got the natural d axis. So, we can call this to be the d axis, and perpendicular to this axis is the q axis. So, in the rotor we have windings on the rotor. The rotor has got the field windings and also the damper winding.

So, the field windings are here, and this is excited by a dc voltage source V r, and we also have the damper windings in the rotor; they are present in the d axis and q axis respectively. So, we have the d axis damper winding which is sorted by bars copper bars,

and then we have q axis damper winding which is also sorted, and they function like index motor winding. So, we have the field winding is called f, and the dumper winding in the d axis called k d; k stands for damper winding, and the damper winding in the q axis is called k q. So, we have little complex situation.

So, instead of a cylindrical rotor we have silent pole rotor, and the rotor has got dc winding as well as winding like index motors, and they are called damper winding. So, when we stimulate this machine we take the help of rotor reference frame. So, what we will do here we will choose the rotor reference frame for stimulation, and this is also called Park reference frame after the name of the scientist called Park. Now this reference frame is attached with the rotor. This is the rotor here, and the rotor is rotating at a speed of omega r radian per second. And we assume that the phase sequence is a b c is rotating from phase a to phase b to phase c, and the angle between phase a and the d axis is called the rotor angle that is theta r.

Now if you take the rotor reference frame we have certain advantage. The advantages are as follows. When you take a rotor reference frame the inductance matrix or the inductances of the synchronous machine which are space dependent becomes constant inductance, because in the d axis; this is the d axis. The air gap is constant; air gap is always constant in the d axis. Similarly in the q axis air gap is also constant, because the stator is cylindrical; rotor is silent pole rotor, but the d axis and the q axis have well-defined inductances. And we popularly say them as L d and L q or x d and x q for reactants in the d axis and the q axis respectively. So, if you take the rotor reference frame there is a natural advantage of having constant inductance in the d and q axis respectively.

So, in this case the speed of the reference frame is the rotor reference frame, and if we write down the equation in rotor reference frame we can say that V d s is equal to r s i d s plus p psi d s minus omega r psi q s. V q s is equal to r s i q s plus p psi q s plus omega r into psi d s. Now what we are saying here is that we can have hypothetical winding here. When we transform this a b c in to b q system we are basically talking about a a axis winding here and a q axis winding here, okay. So, the a b c windings are transformed into d s and q s which are rotating in the space with the speed of the rotor.

And when we find out the voltage of V d s and V q s they are given by these two equations respectively. So, this is the V d s applied voltage in the d axis stator, and this is i d s. Similarly for the q axis we can have the voltage and this is the current i q s. So, the a b c are transformed into V d s and V q s in the stator. Now what about the rotor? Rotor is already in the 2 axis model. We can see that the rotor is we have definite d axis and definite q axis, and in the d axis we have the field winding and the damper winding, and in the q axis we have only the damper winding. So, we can write down the equation directly for the field V f is in the field voltage that is equal to r f i f plus p psi f. The rotor does not have any rotationally induced t m f because the reference frame is attached with the rotor.

So, with respect to the rotor the reference frame is not moving. So, the electrical velocity between the reference frame and the rotor is 0, because we have chosen rotor reference frame. And hence the rotor does not have any rotationally induced t m f, because the relative velocity is 0. Similarly the damper winding k d is equal to r k d i k d plus p psi k d no rotational induce p m f. V k q is equal to r k q i k q plus p psi k q no rotational induced t m f. So, we have five equations. Now if you compare this with the equation of an induction machine in induction machine we just had four equations V d s, V q s, V d r and V q r. So, naturally these are simulation of synchronous machine. It is a little more complex than that of an induction machine. Now let us try to see what exactly is the transformation here?

(Refer Slide Time: 53:22)

Now if you choose in this case theta is equal to theta r. This is the transformation angle, because in this case the reference frame here is attached with the d axis. Now this is the reference frame angle, and we have assumed that omega equal to omega r which means theta; the angle of the reference frame with respect to phase a of the stator is equal to theta r. So, if we substitute that here and write down our k s matrix the matrix for transformation that is also equal to C s matrix. We can say it is 2 by 3 of cos theta r cos theta r 1 cos theta r 2 minus of sin theta r minus sin theta r 1 minus sin theta r 2, half half and half. So, this is the transformation matrix which will transform the a b c variable of the stator into d q variable which is in the rotor reference frame.

And we can also write down the flux linkages. Psi a b c s is the flux linkage in the individual phase a, phase b, phase c. Now when we simulate the machine in the d q reference frame it is quite easy, because we know that inductance matrix is constant. The inductance in the d axis and the q axis they do not involve theta r, but what about the original equations? If you simulate in the actual a b c model we can just go back to the way we did for induction machine what we did is the following that we wrote down the equation V a s equal to r s i a s plus p psi a s. This is simulation from the past principle, and similarly V b s is equal to r s i b s plus p psi b s.

And similarly we wrote down for the C phase r s i c s plus p psi c s. Now we have three phases and three equations. And we also have V f that is equal to r f i f plus p psi f v k d is equal to r k d i k d plus p psi k d, and V k q is equal to r k q i k q plus p psi k q. Now this equation represent the equation of the actual synchronous machine in 3 phase a b c, and this is little more involved, because we have one two three four five six, six equations. And we have to follow six differential equations which is more complex; not only that the flux linkage here psi a s, psi b s, psi c s will involve the inductance matrix and the currents i a, i b and i c or i a s, i b s and i c s, and the induction matrix will be a function of theta r because of the salient C.

So, we will see in the next lecture how we can transform these equations into d q equation in the rotor reference frame, and as a result of that the induction matrix which we will see in the next lecture which is a function of theta r will be independent of theta r and which is an advantage of simulating a synchronous machine in rotor reference.