Advanced Electric Drives
Prof. S. P. Das
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 5

Welcome to this lecture. The equations of induction machine in phase variable; that is in a b c variables, a b c variables of the stator and a b c variables of the rotor. We have also derived the expression for the torque in a b c variable which was little complicated. Now if you see the equation for the torque in a b c variable.

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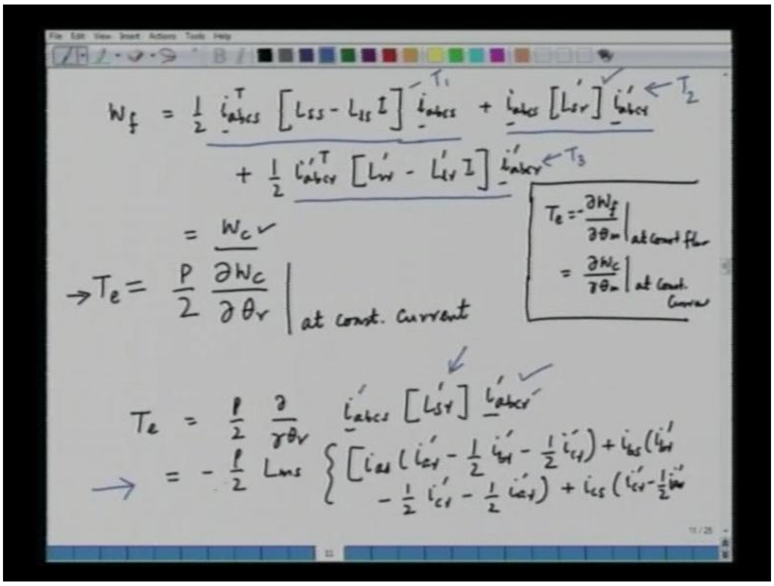

Let us see this slide. We have the energy stored in the field W f is half i a b c s transpose L s s minus L l s I in to i a b c s plus i a b c s L s r prime in to i a b c r prime plus half i a b c r prime transpose of L r r prime minus L l r prime into i matrix multiplied by i a b c r prime. So, this equation is the equation for energy stored in the coupling field. Now, we have assumed at the very beginning that the system is linear. It means the magnetic circuit is a linear magnetic circuit; there is no saturation. Hence, we can assume that the energy is equal to the co-energy. So, we can say that W f is the energy stored in the magnetic field is same as W c is the co-energy of the magnetic field.

And we also know that if you want to find out the torque; the torque is the derivative of the co-energy with respect to theta r that is given by this particular expression; that T e is the torque generated by the machine is equal to p by 2 is multiplied, because we have assumed theta r is the electrical speed. W c is expression that we are already seen, and it is d w c by d r at constant current. So, we can assume that all the current of the stator and the rotor are keep constant. When we are doing the differentiation; this is a partial derivative with respect to theta r. Now if we differentiate this equation, this equation has got three distinct torque. One is the self inductance part; that is L s s minus L l s into i.

Now we are subtracting the leakage, because leakage flux does not help in the energy transfer. In other words the leakage flux is not present in the coupling field. The coupling field is the field due to the mutual flux; the flux that links both the stator and the rotor. So, one component of the flux is due to the primary; that is half of L l s in to L l s I i a b c s, and the other component is the flux which is the mutual flux, and the third component is the rotor self inductance flux. So, out of these three components we can see this is the first component. The stator self induction flux, and then this is the rotor self induction flux. And the third component here is the mutual flux; the energy stored due to the mutual flux, and when we differentiate these various with respect to theta r we will see that the first term, the term one and the term two term three.

These two terms are independent of theta r; they are not function of theta r, only the mutual term that is term two is the function of theta r, okay. Now this term is the function of theta r. So, the differentiation of the first term and the third term will be equaled zero. So, we will just differentiate the second term. So, this is the differentiation of the second term with respect to theta r, and then we differentiate and simplify because the currents are constant; i a b c s and i a b c r, these two currents are constant. So, you have to only differentiate the matrix that is L s r prime, and L s r prime is the mutual inductance matrix between the rotor and the stator. And this matrix is, obviously, a function of theta r as we have seen in the previous lecture. Now when we differentiate this matrix with respect to theta r and simplify we get the following expression for the torque. It is minus of p by 2 in to L m s into this expression.

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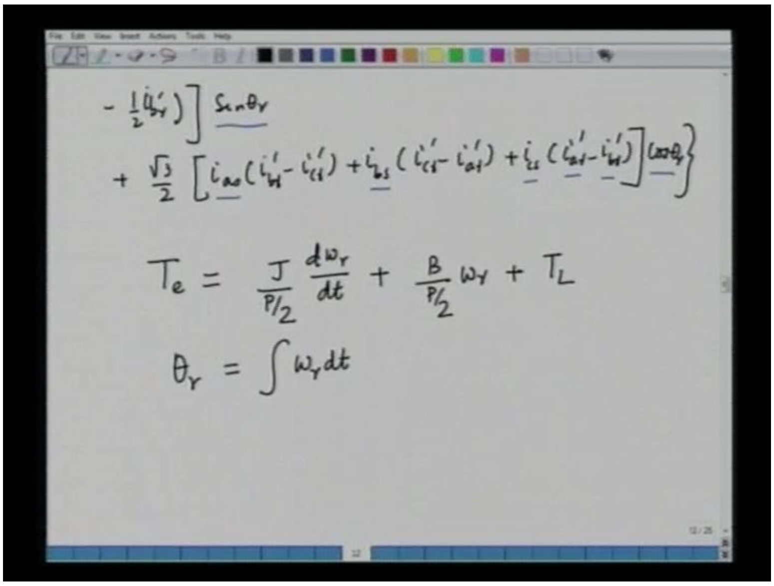

Now we see that this expression is a complex expression which involves both the stator and the rotor currents. The stator currents are i a s, i b s, i c s, and the rotor currents are i a r, i b r, i c r, and also this involves sin theta r and cos theta r. As we can see here we have sin theta r component here, we have cos theta r component here, we have i a r prime, i b r prime, i c s the stator component of current, the phase c; i b s is the phase b of the stator current, i a s is the phase d of the stator current, and similarly we can see this also contains rotor current, the stator current and this little complex. So, we have already seen that the torque expression can be derived, but it is complex. So, when we stimulate the machine in a b c frame we can do it, and we can do it exactly.

We can do it as if we are energizing a real 3-phase machine. We can apply the three voltages v a, v b, v c, and we can observe the six currents; that is the three for the stator and three for the rotor. i a s, i b s, i c s for the stator, and i a r, i b r, i c r for the rotor, and then we can derive the torque equation. And from the torque equation we can find out the equation for the speed; T e is equal to J d omega r by d t by p by 2, because omega r is the electrical speed plus B by p by 2 into omega r. B is the coefficient of viscous friction; omega r is the electrical speed plus the low torque. So, when we solved this equation we can find out the expression for the rotor speed; that is omega r. Omega r is the electrical speed of the rotor, and theta r can be obtained by the integration of omega r, and this theta r can be substituted back to evaluate the induction matrix and also the expression for the torque.

So, this basically goes in iterative process, and we have to solve these equations by numerical method using computers, and after the first iteration we can evaluate the torque, the speed and the rotor angle that is theta r. And theta r can be substituted back to recalculate the inductance matrix and the expression for the torque. So, this goes in an iterative process till we reach the final time, the time to stop the stimulation. So, this is possible, but this involves lot of complexities, and this is going to consume lot of computer CPU time.

Now to simplify the stimulation we can go for d q modeling which means we can transform the a b c variables into d q variables, and then without stimulating in actual a b c variable we can stimulate the machine in d q variables, you get the expression for the torque and the speed. So, we will go for the transformation right now. Now to be able to do that we have to transform the a b c variable in to d q variable. So, we will discuss the transformation in induction machine.

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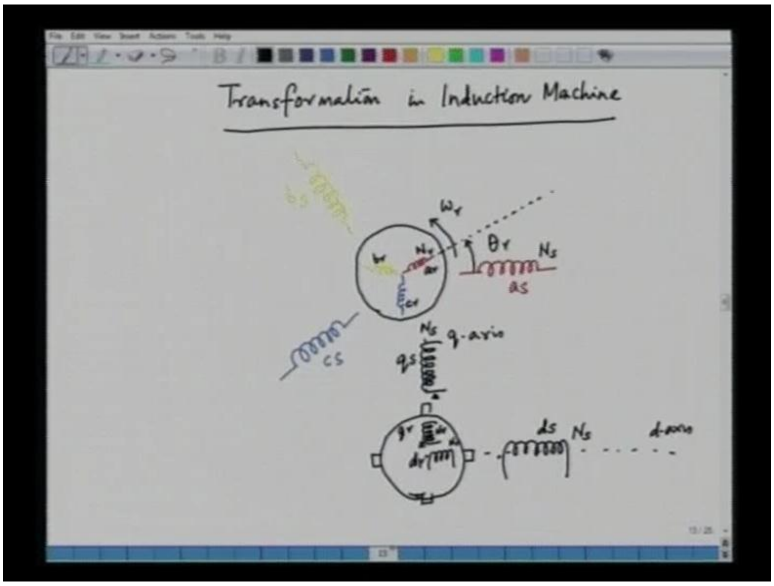

So, we will take a normal 3-phase machine with phase a, phase b and phase c in the stator. This is phase a, this is phase b, this is phase c of the stator, and in similar way you can take the rotor. This is the rotor; this is phase c of the rotor, and this angle is the rotor angle that is theta r. A rotor is rotating at a speed of omega r in the anticlockwise direction. So, this is basically our 3-phase machine that we are talking about. This is our a r, b r and c r. Now what we would like to do we would like to transfer this machine into a d q machine. This is a rotor, and we have two rotor windings. There is the pseudo stationary winding one in the d axis, and the other one in the q axis. So, this is our d axis, and this axis is the q axis.

So, we have the d axis stator, we have a d axis rotor, we have a q axis stator, and we have a q axis rotor. So, we can call this to be d s, this to be d r and these two windings are coupled. So, we can have dot here, dot here, and this is q s, and this winding is q r. This winding is coupled with this winding. So, we can show some polarity marking here. So, this is what we have, okay alright. Now in this case the objective is that we have to transform the original machine in to d q machine. So, what we do here the original machine has got the number of turns per page is N s here, and this number of turns we can call this to be also N s.

So, we can assume that the number of turns of the actual machine is same as the number of turns of the d q machine. D q machine is hypothetical machine. It does not really exist in practice but for our own convenience we are transforming the a b c machine in to a d q machine and we have equalized the number of turns N s in the stator in the 3 phase machine, and the same number of turns power phase for the 2 phase machine or 2 axis machine. So, here also we have N s in this case. The rotor is N r number of turns. Here also we have N r here and N r here. We can now write down the m m f balance equation whatever is realized by a 3-phase machine has to be realized by 2 phase machine. So, first of all what we do here we equalize the ampere turns equalize the m m f s. So, what we do here we can write down the equation of the m m f in the d axis.

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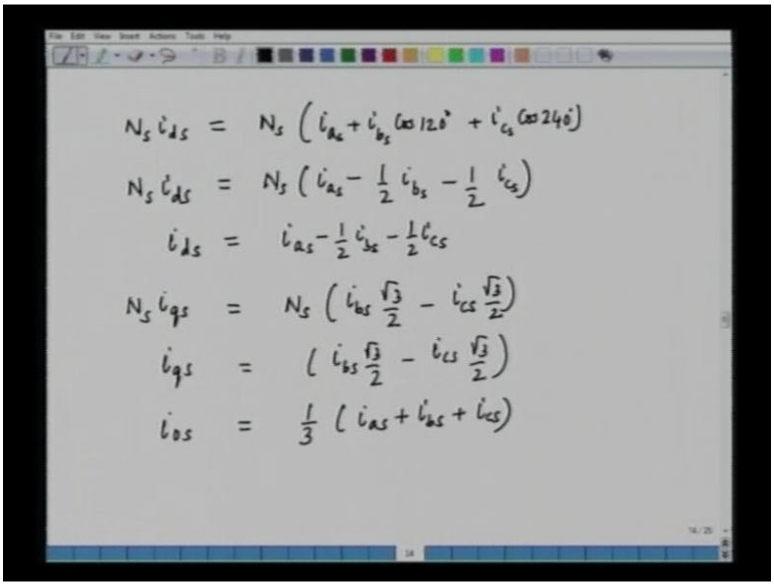

So, we can say that N s i d s is the d axis m m f produce by the d q winding in the stator, and that is equal to N s into i a plus i b cos 120 plus i c cos of 240 degree. Now if you see the m m f produced by the d axis stator will be same as the m m f produced by the a b c stator in the d axis. So, we can resolve this m m f produced by b phase and c phase onto this d axis here. This is also the d axis, and this axis is the q axis. We have defined this two axis and we are talking about stationary d q axis. This d q axes are stationary in the space. They are not moving; they are stationary with the stator. So, if we resolve this m m f this will be this current is i a s in the stator, this current is I b s, this current is I c s. So, what we do here we project the phase b m m f and phase c m m f and phase a m m f all along the d axis which is aligned along phase g of the stator, and we equalize the m m f of the d q machine and the a b c machine in the stator.

So, that is what we have written here cos 120 for phase b and cos 240 for phase c. Now if we simplify this, what we have here is the following; that is equal to N s into i a cos 120 is minus root 3 by 2 i b, and this is plus root 3 by 2 i c. So, this is N s into i d s. So, if we simplify this we can say that, yeah there is one simple thing. So, this is actually minus half and minus half, this is half and minus half. So, we can simplify this i d s equal to i a minus half i b minus of half i c. So, the stator currents are i a s, i b s I c s. So, we can make these currents are i a s, i b s and i c s. So, this is i d s. In a similar fashion we can equalize the m m f in the q axis.

So, what can we say here is that N s into i q s that is equal to N s. Now if you simplify this this will be i b s root 3 by 2 minus i c s root 3 by 2. So, what we are trying to do here we are equalizing the m m f in the q axis. So, what we can do here is that we can project all this m m f in the q axis and equalizing with that of the d q machine. So, we get the following expression that N s in to i q s is equal to N s in to i b s into root 3 by 2 minus i c s into root 3 by 2. N s and N s will be canceled. We can say that I q s equal to i b s root 3 by 2 minus i c s root 3 by 2. So, this is actually the stator transformation.

It means we are able to transform the current a b c into d q current or the d q machine, and in addition to the d q component we can sometimes also have the zero sequence component i o s, and that is equal to one-third of i a s plus i b s plus i c s. So, i o s is the zero sequence component, and here that is equal to one-third of i a s plus i b s plus i c s. So, we have i d s, we have i q s, we have i o s. Now when we transform sometimes you know we can have this transformation and transform the a b c into d q axis machine, and we can get i d s, i q s and i o s.

Now what we do in addition to that; we always aim that this transformation will not alter the parameters of the original machine. Actually the parameters are all given in the power phase of the actual 3 phase machine, and when we transform this into a b c machine the power phase parameters should not change. So, the objective here is that while transforming we are keeping the power phase exactly same. It means the power phase of the a b c machine should be same as the power phase of d q machine.

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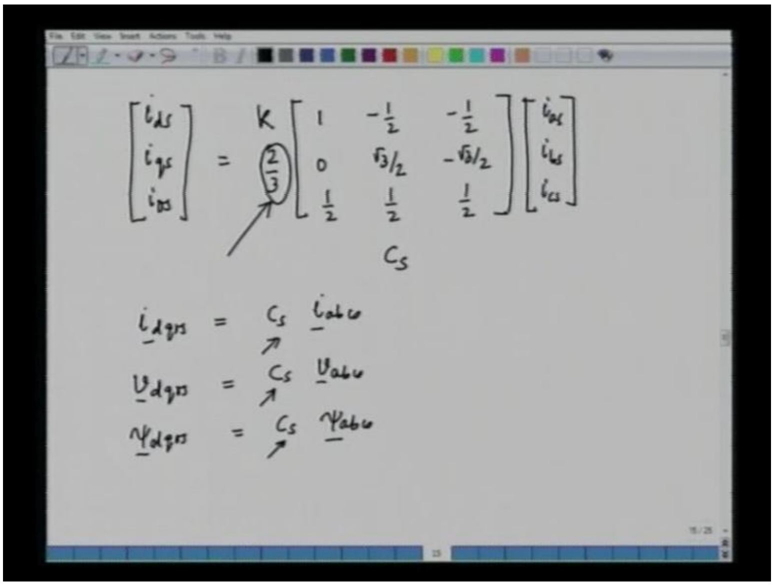

So, we can write this in a matrix expression that we have got i d s, i q s and i o s, and these are basically obtained from the original machine currents. And these currents are i a s, i b s and i c s, and what is this matrix? We assume this is 1, minus half, minus half, 0, root 3 by 2, minus root 3 by 2, and here it was one-third, one-third and one-third. So, we will just multiply here the factor of 2 by 3, and here we will have half, half and half. So, this is the transformation matrix that is C s. Now the question is that how did we obtain this 2 by 3? Now this 2 by 3 comes into picture to equalize the power phase variables. It means the power phase current and power phase voltage should be the same, and we have to use the same transformation for the current, voltage and flux linkage.

So, if this is my C s I can also write down i d q o s that is equal to C s i a b C s. So, the transformation is valid for the current. It means it can transform i a b c s into i d q s. The same transformation can be used for the voltage. So, I can use the same transformation for the voltage. I can also use the same transformation for the flux linkage. So, these are all vectors. The flux linkage, the voltage and the current are all vectors. So, we are using the transformations matrix; that is C s to transform the current voltage and flux linkage into that of the d q machine. And since we are talking about power phase power invariants and power phase variables should not change.

It means power phase impedances should be the same, power phase current should be the same, power phase voltage should be the same, and power phase flux linkage of the 3 phase machine is same as the power phase flux linkage of the d q machine. We choose this factor as 2 by 3. It is basically a constant that we are choosing, and this constant in general would have been a constant k. And when we take power phase power invariants, keeping the parameters exact to the same, this constant k turns out to be 2 by 3, okay. So, this is for the stator and what about for the rotor? Similarly we can also have for the rotor. So, for the rotor we can have a similar transformation, but that transformation will involve the rotor angle.

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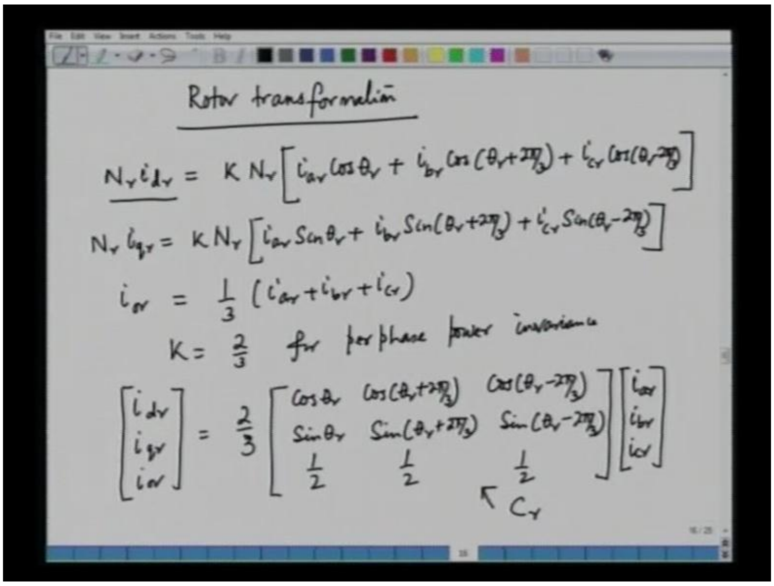

Now we can now do the rotor transformation, and similarly the rotor if you see that we are transforming this a r, b r, c r to d r and q r; again the rotor is also transformed into a 2-axis model. So, we have to project the m m f s in the d and q axis respectably and find out the d q current for the rotors as well as we have down for the stator. So, let us see how it comes out. So, we can write down here N r i d r that is equal to cos tan k into N r i a r cos theta r; we are equalizing the m m f in the d axis; that is why we have got in the left hand side N r i d r, in the right hand side we can project the m m f of a b c onto the d axis plus i b r cos of theta r plus 2 pi by 3 plus i c r cos of theta r minus 2 pi by 3.

Similarly in the q axis some cos tan k N r i a r sin theta r, here the cos will be replaced by psi i b r sin of theta r plus 2 pi by 3 plus i c r sin of theta r minus of 2 pi by 3. And further we have the zero sequence component, we can straightforward write i o r equal to one-third of i a r plus i b r plus i c r. Again we choose k equal to 2 by 3 for power phase power invariants, and we write down the matrix expression i d r, i q r, i o r to be equal to two-third of cos theta r cos theta r plus 2 pi by 3 cos theta r minus 2 pi by 3 sin theta r sin theta r plus 2 pi by 3 sin theta r minus 2 pi by 3 half half half. This is i a r, i b r, i c r.

This is the rotor transformations, so we have the rotor currents, and this matrix here is called the rotor transformation matrix that is C r. So, what we have done here we have been able to find out the transformation matrix for the stator and transformation matrix for the rotor, and having found that we can transform the equation in a b c variable into the equations in the d q variable. So, this transformation i a b c r can be transformed into i d q r.

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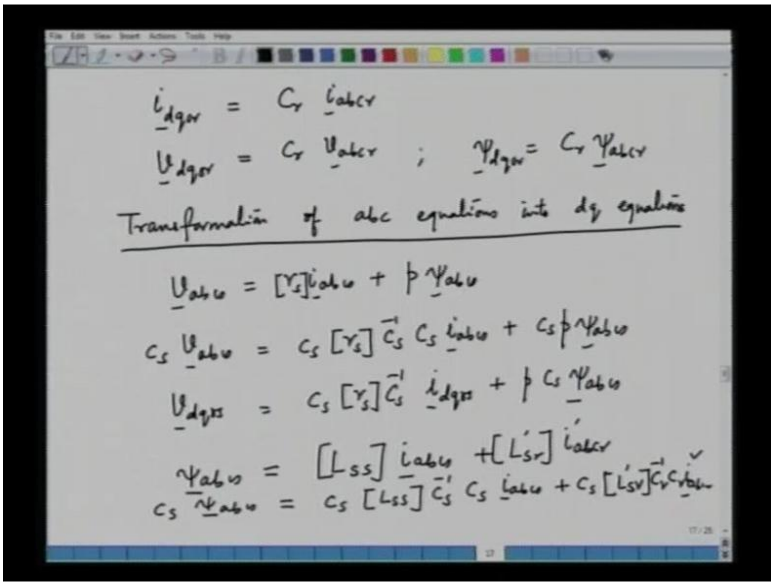

So, we can say and this is valid for the voltage as well; also for the flux linkage we can say here psi d q r also equal to C r psi a b C r. So, the same transformation can be used for the current, voltage and flux linkage; that is the uniqueness of this transformation matrix that is C r when we choose k equal to 2 by 3. Now with this help let us transform the a b c equation into d q equation. So, we start with the stator; first of all we start with the stator equations. We can write down here V a b c s that is equal to r s i a b C s; r s is the matrix plus p psi a b c s, and we can we can pre-multiply this equation with C s. So, we can say that is C s v a b C s equal to C s r s. We can have C s inverse into C s I a b c s plus C s p psi a b c s. So, we can write down this matrix in the stator equations, okay.

And the left hand side will be v d q s that is equal to C s r s C s inverse; right hand side in this case would be i d q s, because this is C s in to i a b c s that is i d q s and C s being constant, it can be taken inside the derivative torque. So, we can say here that is p C s psi a b c s. Now let us try to expand this, okay. Now what is psi a b c s? Psi a b c s is the stator flux linkages in the phase a, phase b and phase c. So, these consist of two terms; the flux linkage due to the self inductance and the flux linkage due to the mutual inductance between the stator and the rotor. So, we can write down this as follows; that is L s s the matrix into i a b c s plus L s r into i a b C r.

Now here when we write we refer everything to the primary side, and hence we have this prime factor coming to picture, because all the variables are referred from the primary side. So, when they are referred in the primary side they are multiplied by suitable number of terms, and hence we have the prime variable here. So, these have to be transformed. So, we can pre-multiply this equation by C s; C s pi a b C s that is equal to C s L s s C s inverse in the C s i a b c s plus C s L s r prime, and we can have C r inverse in the C r i a b C r.

So, this equation that we have written just now will transform the inductances of the a b c machine into the inductances of the d q machine. Now we will see very interestingly; actual a b c machine inductances some inductances are function of theta r. Now when you transform this inductance into d q machine all inductances will be constant; they are independent of the rotor position, that is the theta r. Theta r is the rotor position that all independent of the rotor position theta r.

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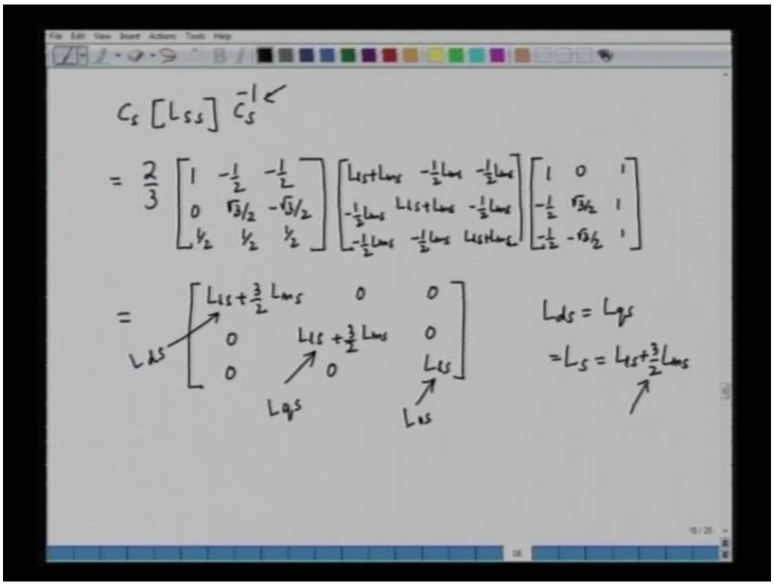

So, we will evaluate this inductance first. Now C s L s s C s inverse just evaluate this inductance; now this inductance with the stator self inductance. The stator self inductance this is independent of theta r. Now when we transform this into d q model this will be again independent of theta r, but we sum new factors coming to picture as follows. Now if you see this that is equal to 2 by 3. I can write down what is C s. C s is 1 minus half minus half, 0 root 3 by 2 minus root 3 by 2, half half and half. What is L s s? If you recall the L s s had the diagonal terms which were constant, and they also have the mutual terms, okay. The diagonal terms are L l s plus L m s, L l s plus L m s, L l s plus L m s.

Now the mutual terms are minus of half L m s, minus of half L m s, minus of half L m s, minus of half L m s, minus half L m s, minus half L m s. So, these are the mutual inductance between the two phases of the stator, stator a and stator b, and then we will post-multiply this inductance matrix by C s inverse; this is the case inverse. So, if you calculate what is C s inverse this is 1 minus half minus half, 0 root 3 by 2 minus root 3 by 2, and then what we have here is 1 1 and 1. So, if you multiply C s and C s inverse we get the i matrix, alright. So, this is what we have here, and then we can simplify this one, and when the simplify this we get the inductance matrix of the d q machine, and that is L l s plus 3 by 2 L m s 0 and 0, 0 L l s plus 3 by 2 L m s 0, 0 0 and L l s.

This is an interesting matrix in the sense that you know most of the elements are zeros here; only the diagonal elements are present. All the off diagonal elements are 0, and that is happening because there is no coupling between the d and q axis. Now if you see a d q machine in this case we have transformed the original a b c machine into d q machine, and when we find out inductance of the d matrix or the d axis inductance the d axis inductance is independent of the q axis inductance. The d and q axis are orthogonal to each other, and they are perfectly decoupled, and hence the off diagonal elements are becoming 0.

And then in addition to the d q inductance we also have the zero sequence inductance, and what is the zero sequence winding? The zero sequence winding does not have any coupling with d and q axis. It can be assumed that it is a winding in the z direction or in a plane perpendicular to the plane of the slide, okay. So, we can think like that that this is the zero sequence inductance. This is the inductance of the q axis stator. So, we can call this to be q s, this is o s and what about this? This is the inductance of the d axis stator that is L d s, okay. So, we have already seen that this is the inductance matrix of the stator in the d q model which does not have any off diagonal terms.

And we also see that L d s equal to L q s. L d s equal to L q s we can say that is equal to L s the stator inductance of the d and q axis. So, that is equal to L l s plus 3 by 2 times and L m s. Now if we see this 3 by 2 term is coming because of the fact that this is the equivalent of a 3-phase machine. So, L m s is the individual maintaining inductance of the a b c machine. When you transfer this a b c machine into a d c machine because of the coupling of a b c phase this coupling is reflected as 3 by 2 time of L m f in the d q machine. And normally when you have an induction machine we do various ways to find out the parameter.

We do the no load test to find out the parameters like magnetizing inductance, and we do the low test or block outer test to find out the inductances the leakage inductances of the stator and rotor, also the resistance of the rotor. So, this inductance which is the magnetizing inductance calculated in the no-load test it is not L m s it is 3 by 2 times L m s. This 3 by 2 times come although it is a power phase magnetizing inductance the 3 by 2 terms is coming because of the coupling between a b and c. So, this shows the effect of coupling of a b and three phases of the stator.

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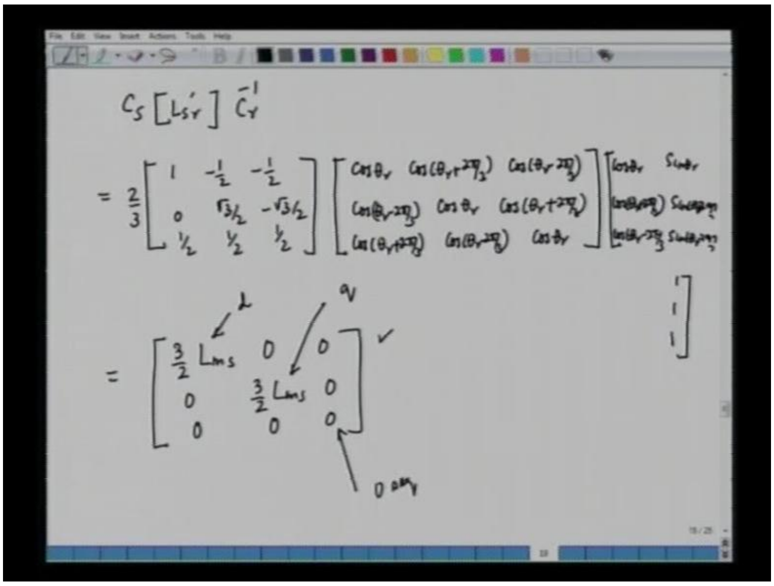

So, similarly we can evaluate the user inductance. We can transform this else C s L s r prime into C r prime. Now if we simplify this C s is 2 by 3. This is 0 root 3 by 2 minus root 3 by 2, half half and half. Now what is L s r? L s r is the mutual inductance between the stator and the rotor. Now this inductance is a function of theta r. So, if we see this inductance this inductance we have derived in the last lecture, and this induction was a function of theta r in a following fashion. So, this is cos theta r cos theta r plus 2 pi by 3 cos theta r minus 2 pi by 3, cos theta r minus 2 pi by 3 cos theta r cos theta r plus 2 pi by 3, cos theta r plus 2 pi by 3 cos theta r minus 2 pi by 3 and cos theta r. And this is post-multiplied by C r inverse.

Now what is C r inverse? C r inverse is the following. This is cos theta r cos theta r plus 2 pi by 3 cos theta r minus 2 pi by 3, sin theta r sin theta r plus 2 pi by 3 sin theta r minus 2 pi by 3, and 1 and 1 and 1. So, this is the last column of this particular matrix. Now if you simplify this, this will turn out to be the following matrix. It is 3 by 2 time L m s 0 and 0, 0 3 by 2 L m s 0, 0 0 0. So, this is the matrix which is independent of theta r; although, we have seen that originally the mutual inductance matrix between the stator and the rotor it is a function of theta r. After transformation into d q system we see that the inductance matrix that we obtained is independent of theta.

Hence, it is worthwhile to stimulate the machine in d q model, because the inductances are independent of theta r as you have seen in the following expression. So, this is the magnetizing inductance of the d axis, this is the magnetizing inductance of the q axis, and this is corresponding to the zero sequence component. And since the zero sequence component does not produce any coupling; it does not have any magnetizing inductance that is corresponding to 0. So, this is obviously coming because of the transformation, and in the d q model we have seen where the inductances are not function of the rotor position that is independent of theta r. So, with this background we can write down the equation of the induction machine in a stationary reference frame or in the stator reference frame.

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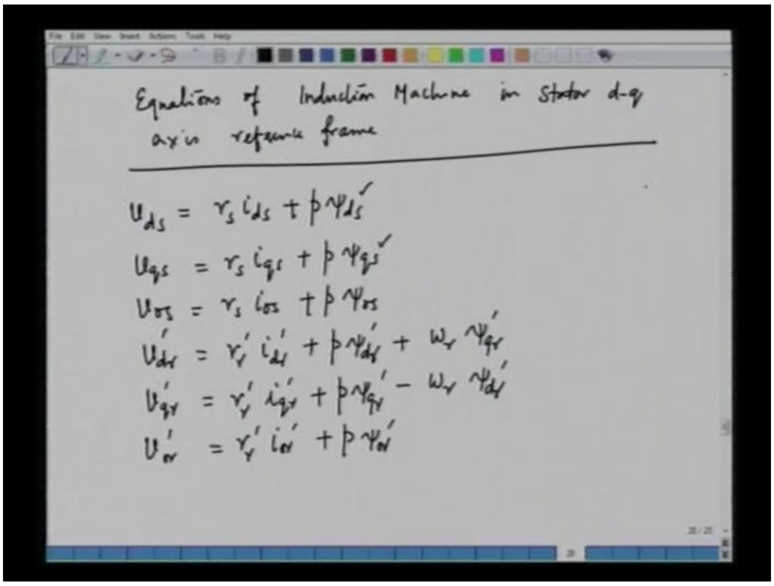

So, we can write down equations of induction machine in stator d q axis reference frame. So, we can write down the equation for the stator and equation for the rotor in the following fashion. V d s is equal to r s i d s plus p psi d s. V q s is equal to r s i q s plus p psi q s. V o s equal to r s i o s plus p psi o s. V d r prime equal to r r prime i d r prime plus p psi d r prime. All the variables are referred from the prime side plus omega r psi q r prime. V q r prime equal to r r prime i q r prime plus p psi q r prime minus omega r psi d r prime. V o r prime is equal to r r prime i o r prime plus p psi o r prime. Now these are the equation of an induction machine in stationary d q reference frame. Now if you see this flux linkages psi d s psi q s can be expressed in terms of the inductances, because we have been able to find out the inductances, and then these flux linkage also can be written in term of the inductances.

And we have seen that the rotor is rotating, and hence there will be rotationally induced c m f in the rotor. These are the rotationally induced c m f, alright, and the rotor is rotating in which direction? If we see the direction of the rotation, the direction of the rotation is from a to b to c, phase a phase b of the stator, phase c of the stator. We are assuming that the revolving field is rotating from a to b to c. So, the revolving phase field in anticlockwise direction and the rotor is trying to catch up with the rotating field, and that is rotating in the anticlockwise direction. This anticlockwise direction is in opposition to the convention that we have already set for the cross primitive machine model.

In the cross primitive machine model the rotor was rotating at a speed of omega r in the clockwise direction, alright. So, hence if you see here due to that we have flux in the d axis that is omega r into psi q r, and then we have minus in the q axis equation omega r into psi d r which was in opposition to the direction of the rotationally induced c m f of the cross due to the machine, but that does not matter. If the direction is reverse the direction of the rotational c m f will also be reversed. Nothing else will be changing, only the direction of the rotational induced c m f will change. So, we can write down the flux linkages in terms of the inductances.

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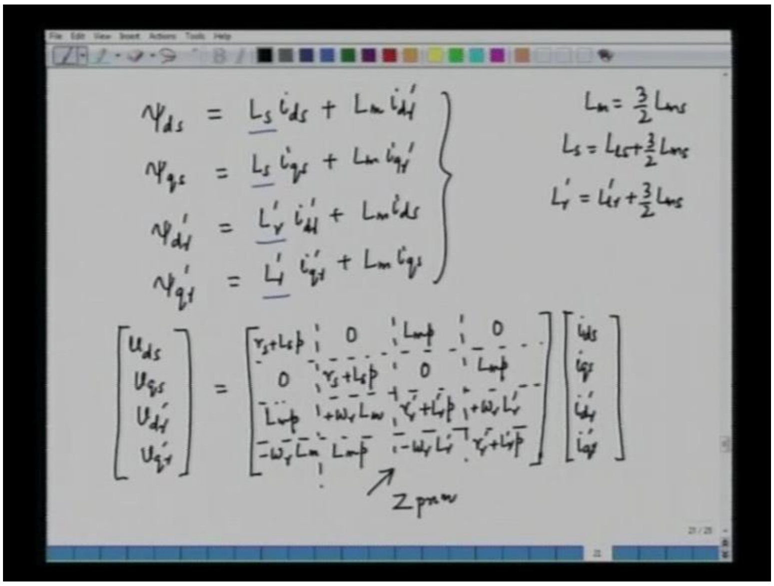

Now what are the flux linkages? We have a psi d s; now what is psi d s? Psi d s is equal to L s i d s plus L m i d r. Now what is L m? L m is 3 by 2 times of L m s, and what is L s? L s as we have already obtained that is equal to the linkage inductance of the stator plus 3 by 2 times the magnetizing inductance of the power phase of the b c machine. So, this is L f the stator inductance of the d q machine. Similarly we can write down the flux linkage in the q axis, psi q s is equal to L s i q s plus L m I q r. Psi d r is equal to L r i d r plus L m into i d s, and psi q r is equal to L r i q r plus L m into i q s.

Now here the air gap is uniform. In case of our induction machine there is no salience here. The stator is cylindrical; the rotor is also a cylindrical structure. The air gap is absolutely constant throughout the periphery of the rotor, and hence the inductance for any given position of the rotor is constant. The rotor is rotating no doubt, but inductance of the d and q axes are constant. And L d s equal to L q s due to the fact that there is no salience in an induction machine. So, we can say that we have the same inductance in the d axis rotor, the same inductance in the q axis rotor, the same inductance in the d axis stator, the same inductance in the q axis stator.

Now what about L r prime? L r prime is equal to in L l r prime plus 3 by 2 times of L m. So, we have all this inductance matrixes and from the flux linkages the flux linkages can be expressed in terms of inductance and current; as you have shown here these are basically the flux linkages. They are expressed in terms of the current and we can substitute this back into the original, and we have the 0 sequence winding in this case, the 0 sequence in the stator and the 0 sequence in the rotor. This 2 component can be neglected so far the torque production is concerned. The 0 sequence components do not take part in production of the torque, and hence the 0 sequence equations can be conveniently avoided while calculating the machine torque and machine speed.

So, we can say that is can be avoided can be ignored for torque calculations, and furthermore the zero sequence components would have been coupling with either d active or q axis. Since, the 0 sequence component do not have any coupling I mean if you ignore that particular equation you do not need i o r anywhere; we do not need i o s anywhere. So, since i o s and i o r are not required in the equations for the calculus in the torque, or otherwise we can avoid the computation of 0 sequence currents in the stator as well as in the rotor. Now why then we show them in the equation form? Now we show them in the equation form because to reconstruct back the a b c equations if you want to transform i d i Q into a b c you have to take help of the 0 sequence component. The zero sequence component although does not take part in torque production will be helpful in calculating back the a b c variables, okay.

So, we can now write down the expression for the voltages, and we can write them in a matrix form V d s, V q s, V d r and V q r. We just have four variables for the first two variable six variable in the a b c frame. So, we can just write down the currents and fill up this impedance matrix i d s, i q s, i d r, i q r. We have a 4 by 4 matrix, and we can write down this just by inspection; we do not have to remember. The first one is the stator resistance torque, the stator self-inductance torque. So, we can say it is r s plus L s p. Now the stator d axis rotor is coupled with the d axis stator. So, we have L m p, and then the stator does not have any rotational induced c m f so we can make the other terms equal.

You see that the first row has been evaluated. Similarly the second row we have r s plus L s p in the q axis stator, the resistance is the same. The inductance of the d and q axisare the same, then we have coupling with the q axis rotor. There is no rotational induced c m f. So, we can make this element equal to 0. In the rotor we have the rotor resistance torque, the rotor inductance torque, and then we have coupling with the d axis stator. Then we have to write down the elements corresponding to the rotationally induced c m f. So, this is in the d axis. So, we have omega r here into L m, omega r here into L r prime, this is q axis one is coming to picture here. So, as far as our previous equation this is positive. So, we have this positive sign here.

So, then here also we have plus sign in this case, and for the fourth row we have r r plus l r prime p. Now this is the magnetizing inductance L m p then we have the rotational induced c m f here and rotationally induced c m f here. The rotationally induced c m f is negative. So, this is the equation of an induction machine in d q model in the stationary frame. In the stationary frame the d and q axis are not moving; they are stationary with respect to the machine stator. So, from this we can also evaluate the torque equation. How do we find out the torque equation for the primitive machine? We find out the torque equation by having this p by 2 i transpose G i. So, this is my z matrix or z primitive. From this we can evaluate what is G matrix. G matrix that part of the matrix which is associated with omega r torque; so we can evaluate what is the g matrix.

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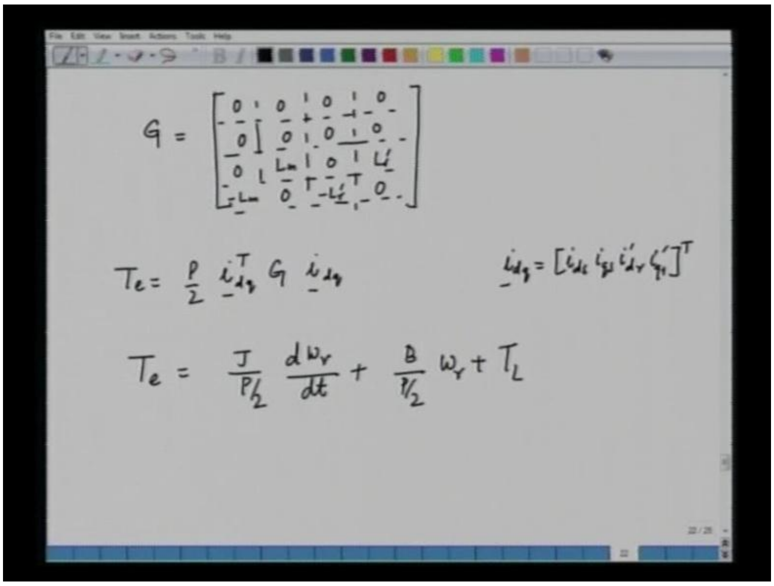

So, G matrix we can evaluate, and we can rewrite this matrix G 4 by 4 matrix. There is no speed term in the first row, no speed term in the second row; the additional speed term here, and this is L m, and this is L r. And this is minus of L m, and this is minus of L r, and these are all zeros, okay. So, we can see here we have minus L m and minus L r here, and here we have the speed terms plus L m and plus L r. So, these are present in this case plus L m plus L r minus L m and minus L r. Now we can find out the expression for the torque; T e equal to p by 2 i transpose G into i. So, if you simplify this we can calculate the expression for the torque. This is the d q equations which have d q here. So, i d q is equal to i d s i q s i d r i q r transpose. So, these are the various currents.

Now if you pre-multiply this and post-multiply this we can find out the expressions for the torque. Now when you find out the expressions for the torque we can evaluate the speed also the speed can be evaluated from the electromechanical equation, T e is equal to J by p by 2 into d omega r by d t plus B by p by 2 into omega r plus T L. So, we can simulate an induction machine in the d q reference frame. In the stationary reference frame the d q axis are stationary; they are not rotating. They are stationary, and we have the voltage equations here.

These are the voltage equations, and from this we can find out i d s i q s i d r and i q r, and then we can substitute this to find out the torque of the induction machine. And the torque can be substituted here to find out the speed, and see in the induction matrix in this case is not a function of theta r. This calculation is much faster than calculation in the a b c model. So, if you simulate an induction machine a b c, model and if you simulate an induction machine in d q model an induction machine simulated in d q model will be much, much faster than an induction machine simulated in a b c model. And after having simulated this a b c equations and in the d q model we appreciate that d q model is definitely better. And whenever we have d q currents we can transform them back into the a b c current.

It means i d and i Q can be transformed back into i a i b i c. So, we are not losing any thing in terms of the actual machine part ones, but on other hand we are able to gain some computing time, because in d q model the machine is stimulated much much faster than an a b c model. And from the a b c I mean from the d q equation we can get back the a b c equations as well. So, in this lecture we have seen how to stimulate an index machine in d q model, we have also derived the equation for the torque this expression.

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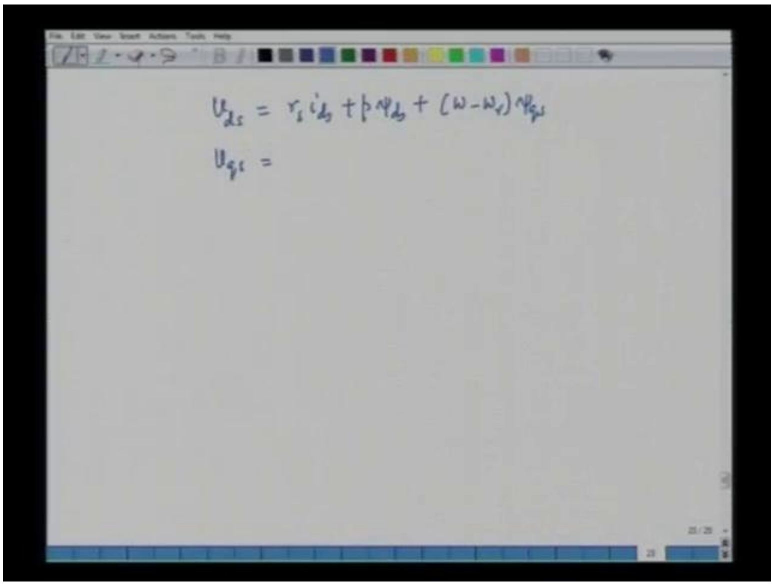

And in the next lecture we will see how our induction machine can be stimulated in other reference prime which is not stationary which will be rotating in the space at any arbitrary speed omega, omega minus omega r in to psi q s; V q s equal to this is omega s into psi q s.