Advanced Electric Drives
Prof. S. P. Das
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 4

In the last lecture, we have seen the modeling of DC machine. In that case, the armature and the field currents – both were DC currents. Now, today in this lecture, we will pick up the modeling of an induction machine, which is an AC machine and it is a 3-phase symmetrical induction machine. So, we will discuss the modeling of a 3-phase symmetrical induction machine.

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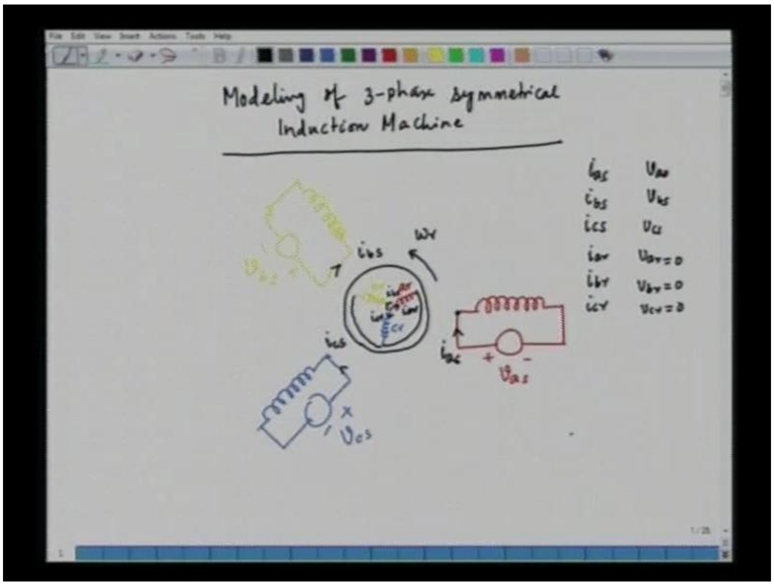

Modeling of a 3-phase symmetrical induction machine; now 3-phase machine will have 3-phase in the stator and 3-phase in the rotor. In an induction machine, the rotor is very often squirrel cage, it means the rotor does not have any windings, but have symmetrical bar distributed over the surface of the rotor embedded inside the rotor and sorted by means of end rings. However, that is also symmetrical, because the bars do not have any… are symmetrical, they do not have any fault. Each bar is of the same length, same thickness, so we can consider the rotor to be also symmetrical.

So, let us try to draw this. So, we have a rotor here and we have a 3-phase stator. This is phase a. And we excite this from a voltage source, that is, v a s. This is phase b shown in yellow color. And we excite this from voltage v b s. And we have a phase c shown in blue color. And we excite it from a voltage source, that is, v c s. And the rotor also has three windings; we can similarly show the rotor windings as a r and then b r and then c r. And the rotor is rotating in the anticlockwise direction at a speed of omega r. The phase sequence here; we can assume that the phase sequence is a, b, c and there is a revolving two fields from a to b to c; the rotor tries to rotate in the same direction as that of the revolving field. So, it is rotating in the anticlockwise direction as shown in this particular figure. And the rotor is short-circuited. So, we can show that, these three windings are sorted.

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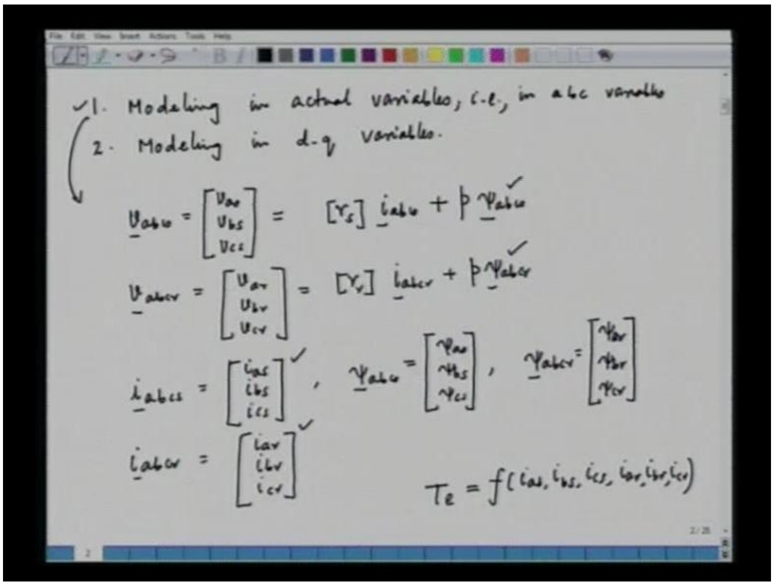

So, when we want to model this induction machine, we have got two ways out. What are the two different methods? The first one is the modeling in actual variables. Now, what are the actual variables? The actual variables here are the voltages and the currents. Here in phase a, we have got v a s; similarly, we have got i a s here; phase a current. In the phase b, we will similarly have i b s; and in phase c, we will have similarly, i c s. So, we have three currents in the stator. And similarly, we will have three currents in the rotor as well. So, we have here i a r, here i b r and here we have i c r. As per the Kron's primitive machine model, we assume that, the terminal closer to the center is positive. So, you can see that, in this winding, this terminal is close to the center compared to the other terminal. So, we have assumed that this is a positive terminal. Similarly, for the rotor. So, the currents are entering the positive terminals of the winding, that is, as per the convention of the Kron's primitive machine model.

And, here we can simulate the machine in the various variables. So, we have i a s, i b s and i c s. For the stator, we have three currents. And similarly, we have i a r, i b r and i c r for the rotor. So, we have three currents for the stator and three currents for the rotor. So, we have to essentially solve for six variables – six currents. And we have the forcing functions are v a s, v b s and v c s for the stator, and for the rotor. We have correspondingly, the voltages v a r, v b r and v c r. And for a normal induction machine, v a r, v b r and v c r are supposed to be 0, because the windings are short-circuited. So, these are 0 voltages. So, we can simulate an induction machine in the actual variables. And actual variables are i a s, i b s, i c s; i a r, i b r and i c r. And the forcing functions are v a s, v b s and v c s; v a r, v b r and v c r.

So, the second way – as you know that, we have six variables here; so the simulation is going to be complex; it is not going to be easy one. But, if we transform these variables into dq model as similar to a Kron's primitive machine model; the number of variables can be reduced. And there are some other advantages also that, the inductances will be constant. The inductances will be independent of the position of the rotor. So, that is the second advantage. And if we simulate an induction machine in dq variables, the simulation will be simpler than that of the actual variable simulation; so modeling in actual variables, that is, in a, b, c variables. And the second one is modeling in dq variables.

So, we will take up the first one first – number 1 and try to understand how we can simulate an induction machine in actual variables. So, we can write down the equations v a b c s. This is basically a vector; and this has got three components, that is, v a s, v b s and v c s. These are vectors. And we can write down the equation for the phase a, equation for phase b and equation for phase c. So, we can say that this is equal to r s. This is symmetrical machine. So, by symmetrical machine, we mean the phases are balanced. So, each phase has a resistance of r s. So, we will naturally have a resistance drop; that is, r s into i a b c s plus the rate of change of flux linkage. So, we can say here p psi a b c s. So, psi a b c s is again a vector having three components – three elements here; psi a s – flux linkage in phase a stator; psi b s – flux linkage in phase b stator; psi c s – flux linkage in phase c stator.

And similarly, we can write down the equation for the rotor – v a b c r, that is, v a r, v b r and v c r. And that is equal to r r; the rotor is also symmetrical – i a b c r plus p psi a b c r. And here we have already seen that, i a b c s has the three components: i a s, i b s, i c s. And psi a b c s is also a vector, which has got three components: psi a s, psi b s and psi c s. And psi a b c r – the rotor of flux linkage, is also a vector, which has got three components or three elements: the phase a flux linkage, phase b flux linkage and phase c flux linkage. So, this is the equation for rotor. And stator and the rotor equations will be six equations: three equations for the stator and three equations for the rotor.

And, we have to solve these equations and find out the values of the currents of the stator: i a s, i b s, i c s. And the rotor currents similarly, i a b c r, that is, i a r, i b r and i c r. So, we can solve these equations; these are the differential equations; we can solve the simultaneous six differential equations to find out the values of the currents – the stator currents: i a s, i b s and i c s; and the rotor currents: i a r, i b r and i c r. And after we find out the currents, we find out the torque, because torque is a function of the currents. And from the torque, we can find out the speed.

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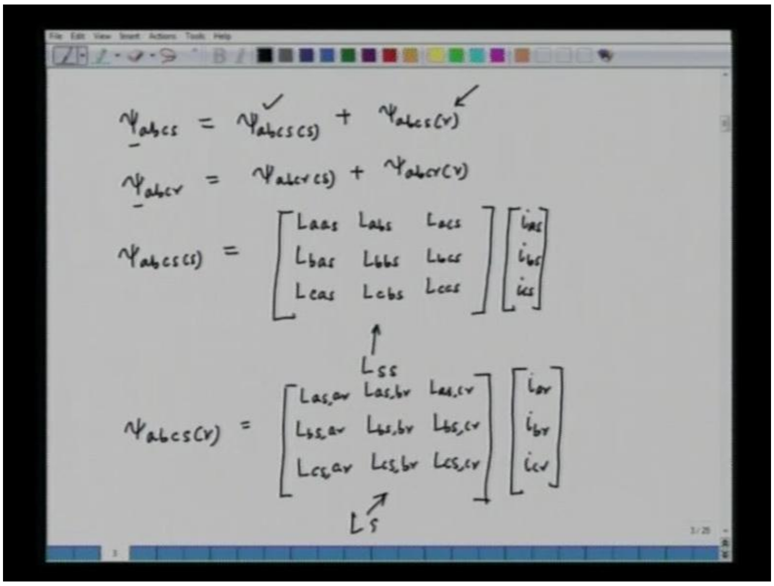

Now, with this introduction, let us try to see what is this psi a b c s? And what is this psi a b c r. We will first take up psi a b c s and see what is psi a b c s? Now, if you see psi a b c s; it is the total stator flux linkage. It is basically a vector having three elements. And we know that, the stator and the rotors are coupled together; it is not that we are only having stator, not having rotor; both are present and both are magnetically coupled. So, the flux linkage in the stator will be the flux linkage due to its own current and also the flux linkage due to the rotor current. So, we can understand that, the flux linkage in the stator will have two components. And the two components are as follows. We can say that it is psi a b c s s – the flux linkage in the stator due to its own current plus the flux linkage in the stator due to the rotor current. Similarly, in case of rotor also, psi a b c r; the flux linkage in the rotor due to the stator current plus flux linkage in the rotor due to the rotor current. So, the rotor will also have two components of the flux: the flux linkage due to its own current and flux linkage due to the stator current.

Now, we can see that, if you see this particular term – psi a b c s s – the flux linkage of the stator due to its own current; it means whenever the currents are flowing in the stator, the stator windings are producing some flux; and the flux is also linking the stator windings. Now, if we express this in terms of the inductance; because ultimately, when a machine is available to us, we do not know the flux linkage directly; we know the machine parameter. And inductances are important parameters of the machine. So, the inductances are given or known. So, the inductances are specified. And we will find out what is this inductance matrix. So, this inductance matrix will have L a a; and we have here L a a s, L a b s, L a c s. This L a a s – it means the inductance of the stator phase a with respect to the phase a; inductance of the stator phase a with the respect to the phase b is L a b s; inductance of the stator phase a with respect to stator phase c is L a c s. Similarly, we can have b a s, b b s and b c s; and also L c a s, L c b s and L c c s.

So, we can call this to be matrix L s s; it means something like a self inductance of the stator; the flux linkage in the stator due to its own current. So, we can say that, capital L s s is the self inductance of the stator. So, similarly, we can say psi a b c is due to the rotor current; this component. That also will be written in terms of the inductance and current; but instead of stator, we have the rotor currents producing the flux linkage – i a r, i b r and i c r. And here we will have a s a r, a s b r and a s c r; b s a r, b s b r, b s c r; c s a r, c s b r, c s c r. So, this actually is a matrix, which is something like a mutual inductance between the stator and the rotor. And we call this matrix to be capital L s r. So, this matrix is L s r, which is the mutual inductance between the stator and the rotor.

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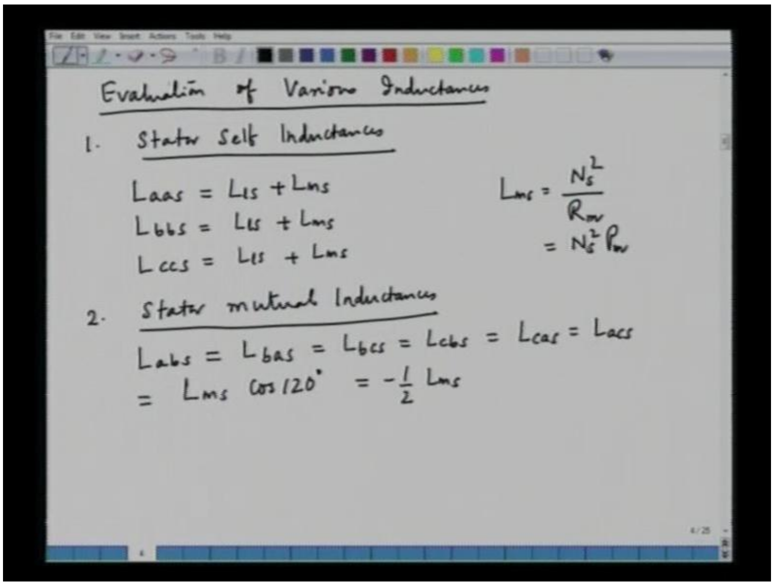

So, let us try to evaluate what is the various inductances here. So, we can evaluate the inductances. Number 1 – we will find out the stator self inductance. Now, what are the stator self inductances? The stator self inductances are L a a s, L b b s and L c c s. Now, if you go little back, we are trying to find out these inductances: this inductance, this inductance and this inductance. This is basically inductance of phase a with respect to itself, inductance of phase b with respect to itself, inductance of phase c with respect to itself. So, this is not very difficult to find out, because we know that, the self inductance will have two components. One component is contributed by the leakage flux; there is the leakage inductance. And the other component is the magnetizing inductance. So, we can write this in the following fashion L l s plus L m s; L l s is the leakage flux or the leakage inductance; L m s in the magnetizing inductance of phase a, similarly phase b – L l s plus L m s. The phases are symmetrical. So, the inductances will be the same – L l s plus L m s.

Now, when we talk about L m s; L m s can be represented in terms of the number of the turns of the stator and also the reluctance of the magnetizing part. So, we can say that, L m s can be given as N s square by reluctance of the magnetizing part. N s square is the effective number of turns of the stator. The stator is a distributed winding. And of course, whenever it is distributed, we always have the pitch factor and the winding factor. Pitch factor – so if you find out the effective number of turns, N s is the effective number of turns, which has taken into account the pitch factor and the distribution factor. So, if you multiply the actual number of turns by the pinch factor and the distribution factor, what you obtain is N s, that is, effective number of turns. And R n is the reluctance of the magnetizing part; or, we can also write down in terms of the permeance. So, N s square into P m; P m is the permeance of the magnetizing part. So, this will be same for phase a, phase b and phase c. And hence, we can say that, the inductance is the self inductances, are equal.

Now, we can find out the second, is the stator mutual inductance. Now, what are the stator mutual inductances? Now, they are a L a b s. Now, let us just go back to the earlier thing, Now, the inductance between phase a stator and phase b stator. That is the mutual inductance between phase a and phase b stator. Similarly, the inductance between phase b and phase c stator is b c s or the inductance between b and c of the stator. The inductance between c and a will be L c a f.

Now, they are all equal, because the machine is a symmetrical machine and the windings are phase-shifted by 120 and 240 respectively. We can always say that, the mutual inductance between the two phases of the stator will be the same. So, we can find out this L a b s equal to L b a s; a b and b a will be the same. That is equal to L b c s – L c b s equal to L c a s equal to L a c s. Now, what is this? Now, that is equal to… Since the flux is distributed sinusoidally, we can see that, the component of the flux produced by phase a linking phase b, will be by a factor of cos 120. So, we can see that, this is L m s into cos of 120, because this angle is 120; angle between phase a and phase b is 120. And hence, we can say it is L m s cos of 120. And that is equal to minus of half L m s. So, we can write down the matrix L s s.

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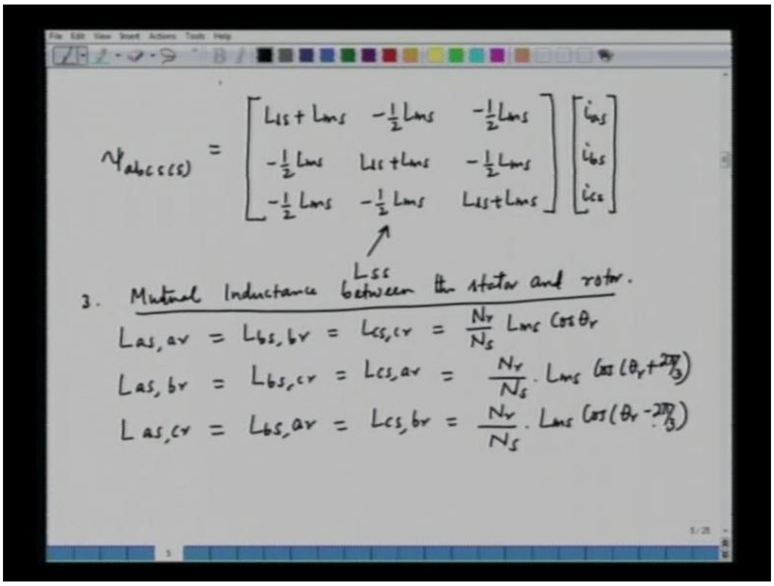

Now, we can write down this matrix L s s. So, psi a b c s due to its own current – that is equal to – this is L l s plus L m s, minus half L m s, minus half L m s; minus half L m s, L l s plus L m s and here also minus half L m s; minus half L m s, minus half L m s and L l s plus L m s. So, this matrix is L s s. We have been able to find out this – multiplied by i a s, i b s, i c s will give me the stator flux due to its own current. So, psi a b c s of s is equal L s s into i a s, i b s, i c s. So, L s s has been found out. Similarly, we can find out the mutual inductance between the stator and the rotor. So, we can say that, the third one is the mutual inductance between the stator and rotor.

Now, what is this mutual inductance between the stator and the rotor? Let us go back to our original slide. Now, if you see here; the rotor – this is a rotor axis; rotor phase a is a r and stator phase a is a s. This is the stator phase a axis. So, stator is stationary, rotor is moving away at a speed of omega r. And thereby, the angle is increasing. So, this angle between the stator and the rotor is ((Refer Time: 25:33)) theta r. Theta r is constantly changing. And that is the why, the mutual inductance between a r and a s will be changing, will be a function of theta r. So, we can find out this. If you want to find out the inductance between a r and a s, that will be a component of cos theta r.

So, we can just say that, L a s, a r – that is equal to L b s, b r. Let us see here. Now, the situation between a s and a r will be same as the situation between b s and b r. And the same thing will be happening between c s and c r. So, we can say that, these inductances will be equal; L a s, a r equal to L b s, b r equal to L c s, c r. Now, what is this inductance? This inductance is L m s into cos theta r, because angle between a r and a s is theta r. So, it is cos of theta r into L m s. But, L m s is the magnetizing inductance. But, we are talking about the mutual inductance. So, magnetizing induction is basically referred from the primary side.

Now, if you talk about the mutual inductance, the mutual inductance we will take into account the number of turns of the rotor and that of the stator also. So, in this case, we can multiply here by N r by N s. N r is the effective per phase number of turns of the rotor; N s is the effective per phase number of turns of the stator. And in a similar way, we can calculate L a s, b r, which will be same as L b s, c r – L c s, a r. And that will be equal to N r by N s into L m s cos of theta r plus 2 pi by 3.

We are talking about the interaction of a s and b r; a s here and b r here. Now, this angle between a s and b r is 120 plus theta r – 2 pi by 3 plus theta r. That is what has been shown here. Cos of theta r plus 2 pi by 3. Similarly, we can have L a s, c r; that is equal to L b s, a r; that is equal to L c s, b r. Since it is symmetrical, we can write without any difficulty; N r by N s into L m s cos of theta r minus 2 pi by 3. So, this is about the stator. Now, similarly, we can write down for the rotor. Now, what about the rotor? Now, if you see the rotor, the rotor equations are given here. This is basically the rotor equation. So, we can have similarly in the rotor, the flux linkage due to its own current and the flux linkage due to the stator current – psi a b c r of s.

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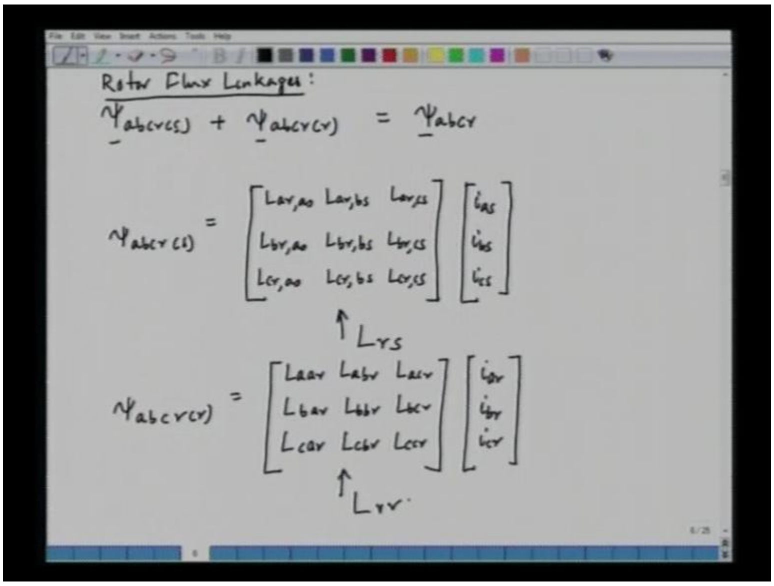

So, we can write down the rotor equation. We can say here that, psi a b c r of s – this is for the rotor flux – rotor flux linkage. So, psi a b c r of s is the flux linkage in the rotor due to the stator current. Now, this is basically a matrix plus psi a b c r of r – the flux linkage in the rotor due to its own current; this is the total rotor flux linkage a b c r. So, these are all vectors. So, we can write down the component of fluxes – a b c r of s. That is equal to – I have here the currents i a r, i b r and i c r; and here I have i a r a s, a r b s, a r c s; and b r a s, b r b s, b r c s; c r a s, c r b s, c r c s. In a similar way, we can… This is called the mutual inductance between the rotor and the stator – L r s.

Now, in a similar way, we can find out a b c r due to its own current. There is small change here; this will be the stator currents: i a s, i b s and i c s. Now, this will be the rotor currents: i a r, i b r and i c r. And this matrix will be a a r, a b r, a c r; L b a r, b b r, b c r; L c a r, c b r, c c r. Now, this matrix is a self inductance of the rotor. So, we can call this matrix to be… As we have L s s, we can call this to be L r r, so inductance of rotor with respect to itself.

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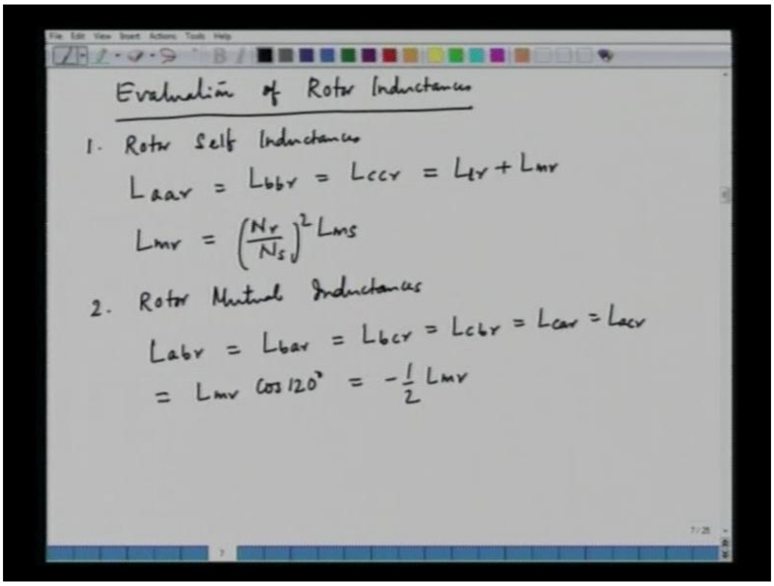

Now, we can evaluate this L r r and also L r s. As we have done in case of the stator, similarly, we can evaluate this inductance for the rotor. So, we can say that… First of all, the rotor self inductance; out of symmetry, we can say L a a r is equal to L b b r is equal to L c c r. The rotor is also a symmetrical rotor. So, we can say that, the inductance of each phase of the rotor will be the same. So, L a a r is equal to L b b r is equal to L c c r. That is equal to the leakage inductance – L l r plus L m r. L l r is a leakage inductance – per phase leakage inductance of the rotor; and L m r is the magnetizing inductance of the rotor. And there is a relationship between L m r and L m s. L m r is the magnetizing inductance of the rotor seen from the rotor side.

So, that is equal to N r by N s square into the magnetizing inductance of the stator seen from the stator side; because the rotor and the stator share the common magnetic part; there is the air gap, which is coming in between; and the rotor and the stator will have the common magnetic flux. And since they have the common magnetic flux, depending upon from which side you see, the number of turns will be multiplied. So, if you see from the rotor side, it will be L m r, which is N r by N s square into L m s. If you see the same thing from the stator side, it will be L m s. And L m s are the components of the stator number of turns, that is, N s square.

We can evaluate similarly, the rotor mutual inductance. Now, L a b r is equal to L b a r is equal to L b c r is equal to L c b r is equal to L c a r is equal to L a c r. Now, since the rotor is also symmetrical, we can write down this equation. And this is equal to L m r into cos of 120, because we know that, in the rotor also, phase a and phase b are shifted by 120 degree. So, if we want to find out the mutual inductances between a r and b r, it will be L m r into cos of 120, because a r and b r have the angle of 120; the axis of a r and axis of b r are spread apart by an angle of 120. So, that is why we can say that L m r into cos of 120 is equal to L a b r. And that is equal to minus of half L m r. So, this is what we have found out. And then we can find out the mutual between the rotor and the stator, so that…

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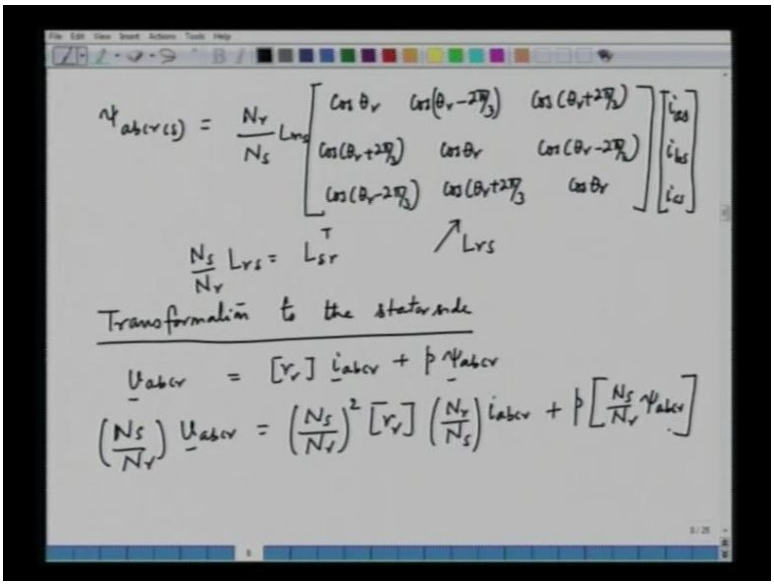

That we can see here – psi a b c r of s. Now, this if you write down in this case; this will be if you fill up this particular matrix; that will be the mutual between the stator and rotor. So, this matrix will have the following expression: cos theta r, cos theta r minus 2 pi by 3, cos theta r plus 2 pi by 3; cos theta r plus 2 pi by 3, cos theta r, cos theta r minus 2 pi by 3; cos theta r minus 2 pi by 3, cos theta r plus 2 pi by 3, cos theta r. This is multiplied by i a s, i b s and i c s. Now, we are writing down this component of flux. So, this flux is linking the rotor due to the stator current. So, this involves the mutual inductance between the stator and the rotor. And the mutual inductance is expressed in the following fashion. This is the mutual inductance between the stator and the rotor. So, we have L m s here; N r by N s into cos of theta r, cos of theta r minus 2 pi by 3, cos of theta r plus 2 pi by 3. So, this is the mutual inductance. So, we can say this to be L r s; the mutual between the rotor and stator is L r s.

Now, it is interesting to know that, L r s is equal to the transpose of L s r. This can be verified. That if you transpose this matrix, what you obtain here is that, it will be L s r; of course, there will be number of turns factor will be there. But, this will be L r s equal to L s r transpose into N s by N r. This number of turns will be multiplied; but the main matrix, which is containing cos theta r, cos theta r minus 2 pi by 3, cos theta r plus 2 pi by 3. The transpose of that will be appearing in L s r. So, this is what we can see here. We can just go back and see. This is L r s; L r s is given by this. And this multiplied by N s by N r will be the transpose of L s r. So, this actually are the equations in the actual machine variables: i a s, i b s, i c s, i a r, i b r, i c r. But, very often, whenever we have a machine, we would like to refer everything to the primary side; that means to the stator side. All the parameters that we have in an equivalent circuit are also referred from the primary side. So, there is a need to transform all these variables to the primary side, which is the stator.

So, we can see that, transformation to the stator side. So, we know we have the equation here v a b c r is equal to r r i a b c r plus p psi a b c r. Now, we can transform this equation to the stator side. Now, how do we transform? We can transform this by multiplying suitable ratio of number of turns. So, we can multiply the left-hand side with N s by N r; N s by N r into v a b c r; that is equal to N s by N r square r r N r by N s. We have some mathematical manipulation here – i a b c r. And this – the flux we can write as p of N s by N r into psi a b c r. This we can write.

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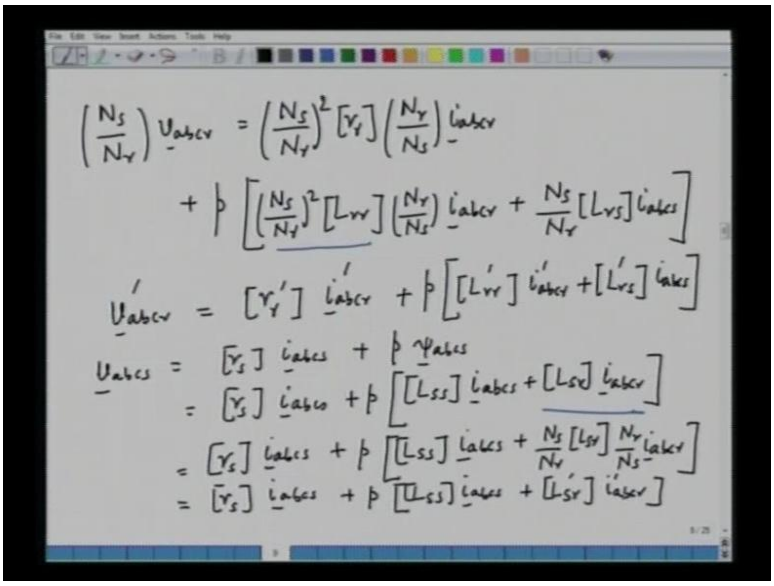

And, we can further simplify this in the following fashion. N s by N r into v a b c r; that is equal to N s by N r square r r N r by N s i a b c r plus we can expand this N s by N r whole square L r r N r by N s into i a b c r plus N s by N r into L r s i a b c r. So, this is basically equation after transformation. Now, what we have done is that, we have transformed to the stator side by suitably multiplying by the number of turns. So, the left-hand side would be i a b c r prime – the rotor voltages as referred to the stator side – i a b c r prime. And then in the right-hand side, we have N s square by N r r r. So, that is basically r r prime – the rotor resistance referred from the stator side.

And then N r by N s into i a b c r; that is, i a b c r prime – the rotor current referred from the stator side plus we have this p term – the derivative term; within that, we have L r r into N s by N r whole square. Now, that is actually L r r prime. The rotor self inductance referred from the stator side. So, we can write that as L r r prime. And then we have N r by N s into i a b c r. So, that is actually i a b c r prime – the rotor current referred from the stator side plus L r s into N s by N r; so that is, L r s prime – the mutual inductance between the rotor and the stator referred from the stator side into i a b c s. So, this is the transform equation. So, by multiplying the ratio of the number of turns, we have transformed this to the stator side.

Now, similarly, in the stator equation, we have got v a b c s; that is equal to r s i a b c s plus p psi a b c s; and that is equal to r s i a b c s plus p. We can express the flux in terms of the inductance and current. So, we can say here that is L s s into i a b c s plus L s r – the mutual between the stator and the rotor i a b c r. And here we can do some manipulation in this case, because in the stator equation, we do not have to do much transformation, because the stator is already transformed to the stator side. So, we have to just transform that part, which involves the rotor current. So, this is shown in the ordinary line term.

So, this we have to transform. And to transform that, we can do some mathematical manipulation plus p L s s i a b c s plus we pre-multiply this with N s by N r; then we have L s r here; and then post-multiply with N r by N s into i a b c r. Now, N r by N s into i a b c r is i a b c r prime. So, we can rewrite this in the final form that, r s i a b c s plus p L s s i a b c s plus L s r prime into i a b c r prime. So, this equation that we have obtained finally is basically the transformed equation seen from the stator side. So, we have been able to derive the equation for the various currents: i a s, i b s, i c s and i a r, i b r, i c r referred to the stator side.

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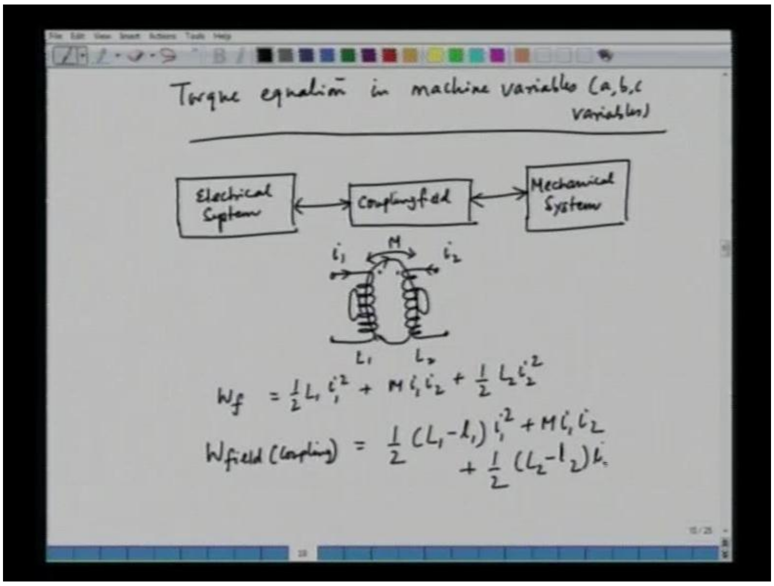

Now, let us see how we can find out the expression for the torque. Torque equation in machine variables means a, b, c variables. So, we are interested to find out the expression for the torque or the equation for the torque in a, b, c variables. Now, how is the torque produced? We have actually coupling field between the stator and the rotor. And any change of the field will result into the mechanical output.

So, what we have here; we have an electrical system and we have a mechanical system. And these systems are connected by means of a magnetic field. The rotating machine converts the electrical energy into mechanical energy or mechanical energy into electrical energy depending upon whether it is a motor or a generator. Now, the field between the stator and the rotor is the coupling field. And the coupling field is basically the mutual field or the magnetizing field, which does not have any component of the leakage flux. So, we can write down the expression for the coupling field.

Now, if you have two coils like this; one coil like this and the second coil like this; and these two coils have the self inductances as L 1 and L 2 respectively and these two are magnetically coupled. And there is a mutual inductances between these two, which is M; and the current flowing in the primary is maybe i 1 and current flowing in the secondary is i 2 as shown in the figure. And if you want to find out the magnetic field; in this case, the stored energy in the field; the stored energy is basically given by w f is half L 1 i 1 square plus M i 1 i 2 plus half L 2 i 2 square.

Now, if you are finding out the stored energy in the coupling field; the coupling field is the field that couples the primary with the secondary. Now, that field does not any leakage component. These are the leakage fluxes. The leakage fluxes do not link the coupling field. Now, this field will not have any leakage component. So, if I want to find out let us say the field – coupling field; that will be half of L 1 minus l 1; small l 1 is the leakage inductance into i 1 square plus the mutual inductance i 1 i 2 plus half L 2 minus… So, the coupling field is given by half of capital L 1 minus small l 1 into i 1 square plus M i 1 i 2 plus half of capital L 2 minus small l 2 into i 2 square; the small l 2 is the leakage inductance of the second coil. So, the coupling field does not involve any leakage flux; and hence, the inductances are suitably modified by subtracting the leakage inductance from the self inductance.

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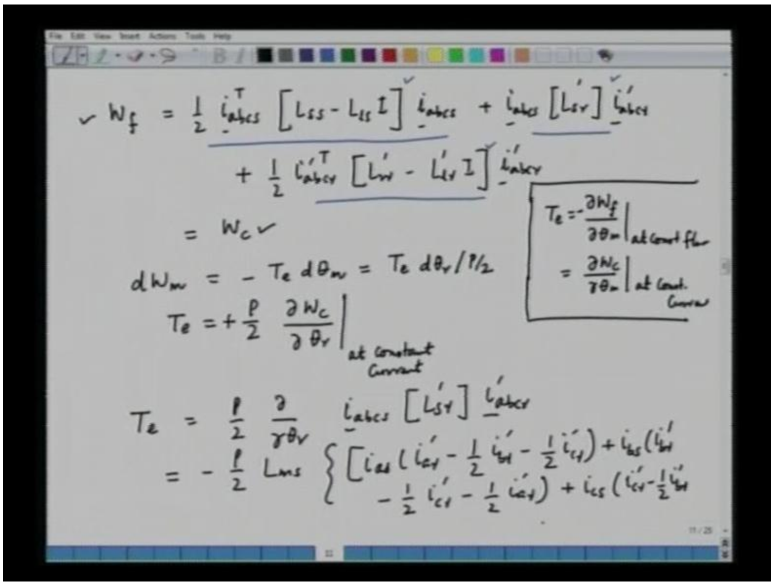

So, similarly, if you have the expression for the coupling field in an induction machine, we can write down in the following fashion: w f is equal to half of i a b c s transpose L l s minus L l s i i a b c s. This is the self inductance plus i a b c s L s r referred from the primary side i a b c r, because we referring all variables now from the primary side. So, we have this prime; prime means the variable referred from the primary side – plus half i a b c r transpose L r r prime minus L l r prime I; I is the identity matrix – into i a b c r prime. So, this is basically the field stored in the coupling field or the energy stored in the coupling field. And since we are assuming a linear magnetic circuit; we have assumed in the very beginning that, the magnetic saturation is neglected; which means that, this energy stored in the coupling field is same as the co-energy. The energy is equal to the co-energy for a linear magnetic circuit. So, we can say that, that is also equal to the co-energy w c.

And, from this co-energy, we can find out the expression for the torque. So, we can say that, the mechanical output – this is the work done is equal to minus T e into d theta m. And this is basically the work done in this case. And if you want to find out T e; T e can be calculated as the pole-pair into the derivative of the co-energy with respect to theta r at constant current. We can say that, current remains constant – at constant current. So, all currents of the stator and the rotor will be kept constant.

So, if you take this co-energy and differentiate that with respect to theta r multiplied by p by 2; that will be the torque expression. Now, why this p by 2 is coming into picture? Because this is T e into mechanical thing. So, that is equal to T e into d theta r by p by 2. And hence, if you want to find out theta r; you have to take this p by 2 to the other side. So, we can say here that, the torque is equal to the derivative of the energy with respect to d theta m – negative of that at constant flux linkage; or, that is equal to the derivative of the co-energy with respect to theta m at constant current. So, because it is the derivative of the co-energy with respect to theta m; that is why we have a positive sign here. So, we have a plus sign here. And this is p by 2, because we are talking about the electrical angle; theta e is the electrical angle; theta e is equal to p by 2 into theta mechanical; that is the mechanical angle. So, this is the expression for the torque.

And, if we see this; this co-energy is same as the energy, because we are talking about a linear magnetic system. So, out of these three terms that we have here; this is one term concerning the stator; this is the rotor; and this is the mutual. So, in this case, this will be independent of theta r; this will only be a function of theta r. So, we have to differentiate this with respect to theta r to find out the torque. So, finally, you can see that, the torque can be obtained by differentiating this part with respect to theta r. And the part is i a b c s L s r i a b c r. And since L s r is a function of theta r; when you differentiate this with respect to theta r, you get the torque, that is, T e. Now, you know that, this L s r will be a function of cos theta r, cos theta r minus 120, cos theta r plus 120. So, we can differentiate that; and thus simplifies this. That expression will be complex expression. We can finally write this expression as follows. That is equal to minus p by 2 L m s i a s i a r prime minus half i b r prime minus half i c r prime plus i b s into i b r prime minus half i c r prime minus half i a r prime plus i c s into i c r prime minus half i b r prime…

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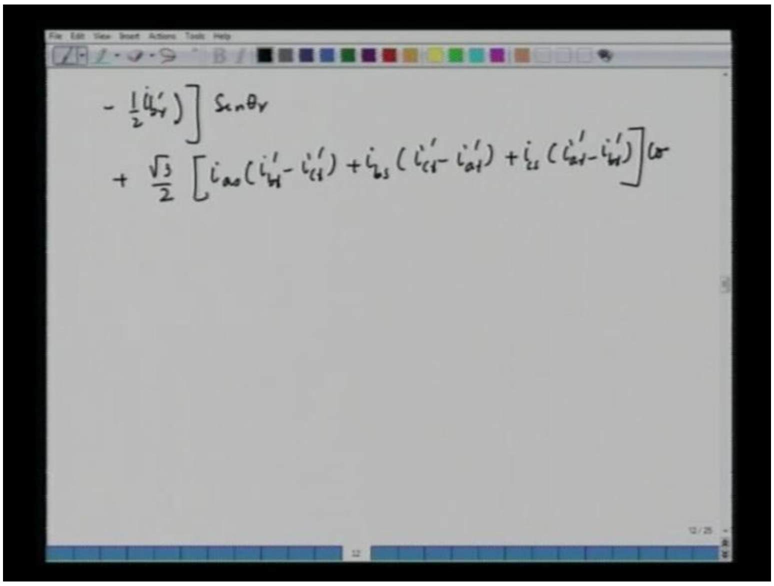

Minus half i b r prime; this is i c r prime minus half i a r prime; and then minus half i b r prime multiplied by sign of theta r plus root 3 by 2 into i a s into i b r prime minus i c r prime plus i b s into i c r prime minus i a r prime plus i c s into i a r prime minus i b r prime into cos of theta r. So, this is the expression for the torque. And in this particular lecture, we have simulated model, the induction machine in a, b, c variables. In the next lecture, we will see how we can transform these a, b, c variables into dq variables.