

Advanced Electric Drives
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Lecture – 39

Hello and welcome to this lecture on advance electric drives. In the last lecture we were discussing about the tractive effort and the drive rating, when a vehicle is moving, when a train is moving, it requires some tractive effort to continue its motion. And the tractive effort is because of various components in the locomotive or in the train. Imagine a train is moving. When a train is moving it has to accelerate horizontally that is basically the horizontal acceleration. It is having a huge mass. So, it has to acceleratory horizontally, and then there are so many rotating parts like wheels, axels, the motors; they are also up to accelerate rotationally.

So, we need some tractive effort or the equivalent torque to accelerate the rotating mass, and then sometime the train also negotiates or overcomes a gradient. When we say that the train is having a gradient it can be half gradient, or it can be down gradient. The tracks may not be leveled; it can going off the track or may be going down that is because of the gradient. So, it has to overcome that gradient, and then again the train has to also move against the friction. The friction could be the air friction, could be the friction with the vile and the rail and the internal friction. So, we were trying to evaluate the component of tractive effect for each of this. Let us recapsulate what we have discussed in the last lecture.

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Calculation of Drive Rating
Calculation of Various Tractive Efforts

(i) To accelerate the train horizontally
$$F_{a1} = 1000 M \frac{\alpha \times 1000}{3600} = 277.8 M \alpha, N$$

 $M = \text{mass of the train in tonne}$

(ii) To accelerate the rotating parts.
 $N_x = \text{no. of axles}$
 $J_w = \text{moment of inertia of a wheel}$
 $J_r = \text{moment of inertia of the wheels} = 2 N_x J_w$
 $N = \text{no. of driving motors}$
 $\eta_1 = \text{teeth on the motor side gear}$
 $\eta_2 = \text{teeth on the axle side gear}$
$$a = \frac{\eta_1}{\eta_2} = \frac{\text{Wheel Speed}}{\text{Motor Speed}}$$

To accelerate the train horizontally we just find out the train motion in this following fashion. So, this is what we have here. Let us see the various component of tractive effort, now tractive effort to accelerate the train horizontally. So, we have the train mass in tone M, we can find out that in kg by multiplying in 1000. Then alpha is acceleration which is expressed in kilometer per hour per second. So, we can convert that into meter per second square kilometer into 1000 will be meter and hour, 1 hour is 3600 second.

So, we can convert the acceleration into meter per second square, and then what we finally have is the tractive effort in Newton and then to accelerate the rotating parts of the train. So, we have we have the wheel, the moment of inertia of the wheel, and we have N x number of axels. So, we have to N x number of wheels. So, this is the total movement of inertia of the wheels, and then we have the gear ratio a which is N 1 by N 2. N 1 is the teeth on the motor side gear, and N 2 is the teeth on the axels side gear.

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Moment of inertia of the motors referred to the wheels = $J_2 = \frac{N J_m}{a^2}$

$J =$ Total moment of inertia = $J_1 + J_2$

Linear acceleration = $\frac{\alpha \times 1000}{3600} \text{ m/s}^2$

Angular acceleration = $\frac{\alpha \times 1000}{3600 \times R} \text{ rad/s}^2$

$R =$ Radius of the wheel in m

$T_{a2} = (J_1 + J_2) \frac{\alpha \times 1000}{3600 \times R} = (2N_s J_m + \frac{N J_m}{a^2}) \frac{\alpha}{3.6 R} \text{ Nm}$

$F_{a2} = \frac{T_{a2}}{R} =$ Tractive effort for angular acceleration of the rotating parts

$F_a =$ Total tractive effort to accelerate the train horizontally including the rotating parts = $F_{a1} + F_{a2}$

So, we can take the ratio of N_1 and N_2 and that is the gear ratio, and we can convert this J_m is the moment of inertia of each motor, and we have N driving motors by a square is the moment of inertia J_2 ; that is the movement of inertia of the motors referred to the wheel side. So, if we add this J_1 and J_2 we get the total movement of inertia of the rotating mass, and this rotating mass as to accelerate rotationally. So, we have to multiply this rotating mass with the angular acceleration. And we can calculate the angular acceleration from linear acceleration by dividing the linear acceleration by r ; r is the radius of the wheel in meter. And then we can find out the equivalent torque.

The torque is the moment of inertia into angular acceleration, and then we can divide this torque by the radius of the wheel. And then that gives the tractive effect for angular acceleration of the rotating parts. And if we want to find out the total tractive effort because of acceleration, by acceleration we mean both linear and angular acceleration. So, we have two components. One component is for the linear acceleration or horizontal acceleration that is F_{a1} , the other component is for the angular acceleration of the rotating mass that is F_{a2} . If you want to find out the total tractive effort F_a , that is equal to F_{a1} plus F_{a2} , and that will be giving us an equivalent mass of the train, and this equivalent mass is calculated as follows. So, we have the total tractive effort f_a is the sum of the linear tractive effort and the tractive effort for accelerating the rotating parts F_{a2} .

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$$F_R = F_{A1} + F_{A2} = 277.8 M \alpha + \left[2N_r J_w + \frac{N J_m}{a^2} \right] \frac{\alpha}{3.14 R}$$

$$= 277.8 M_e \alpha$$

(iii) Gravity / Gradient

$$F_g = 1000 M g \frac{g}{1000}$$

$$= M g g, N$$

$$= M g, Kg$$

(iv) Tractive effort required to overcome train resistance

$$F_r = A + BV + CV^2 + \dots (N)$$
 Difficult to estimate

$$F_r = \gamma M, N \quad \gamma = 20 N / \text{tonne}$$

Diagram: A right-angled triangle representing a gradient. The vertical side is labeled "1000 M g Sin θ". The horizontal side is labeled "1000 m". The hypotenuse is labeled "1000 m". The angle θ is at the bottom. Below the diagram, it says "Sin θ = g / 1000".

And we add these 2 components, and then what we obtain is the equivalent mass in e of the train. That is not just mass in tone, but it is something higher than that; that can be calculated in the following formula. We have to know the moment of inertia of the wheels, the number of wheels, the moment of inertia of the driving motors, the number of driving motors in the gear ratio a a square here and then the radius of the wheel. So, this expression will give us the effective mass a b of the locomotive. Then the second part of the tractive effort is to overcome the gradient. The gradient could be half gradient or it could be down gradient.

Suppose the train is moving of the gradient. So, we have a component of force which is opposing the motion. We can calculate that force in the following way. We have the train moving upwards. This is the weight of the train that is 1000 mG, this angle is theta, and we have a component of this force which is opposing the motion. So, we can calculate this component, and this component is 1000 m into g into sin theta. So, sin theta here is approximately equal to g by 1000; g is the gradient, and it is usually expressed the elevation in 1000 meter. So, we can find out sin theta dividing g by 1000. So, we can substitute sin theta by g by 1000.

So, what we obtain is the tractive effort in Newton N small g into capital G, and then finally if you want to find out in kg we can divide by small g that is the acceleration due to gravity 9.81 meter per Second Square. So, we get the tractive effect in kg. And then

we go forward to find out the fourth type of tractive effort. That is the tractive effort required to overcome the train resistance. Now this tractive effort is difficult to estimate, because this resistance means it could be internal resistance, it could be external resistance, it could be resistance with the air and the frictional with air.

So, this is basically modeled as a form is equal to a plus b v plus c v square in the following way. So, if we want to find out an expression for this it will be a complicated expression as follows. So, this F r is equal to A plus B V. V is the velocity of the train plus c v square and so on, and it is in Newton, so difficult to estimation. So, this is difficult to estimate. So, instead what we do we find out this resistance as a function of train mass. If the train mass is capital M in tone we say that the train resistance is r into M; r is small r, and r is defined as the Newton per ton.

So, this requires this much of tractive effort to overcome the resistance. It means if the mass is more the resistance will be naturally more, because if you have higher mass there will be higher internal friction. So, it is quite justified that we can have approximate expression for the train resistance F r is equal to r into M. So, we can say here F r is equal to r in to M where R is approximately 20 Newton per ton. So, we can evaluate this, and this will be in Newton. So, this is the tractive effort for overcoming the train resistance.

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The image shows handwritten notes on a whiteboard. The notes are organized into sections:

- Total tractive effort**

$$F_t = F_a + F_g + F_r$$

$$= (28.3 M_e \alpha \pm M G + \frac{M V}{9.81}) \text{ , kg}$$

$$= (28.3 M_e \alpha \pm M G + \frac{M V}{9.81}) g \text{ , N}$$
- Drive Rating**

$$T_w = R F_t$$

$$T_t = \text{Total torque referred to the motor shaft}$$

$$= \frac{a R F_t}{\eta_t} \text{ Nm}$$

N = no. of driving motor

$$T_m = \text{Torque per motor} = \frac{T_t}{N} = \frac{a R F_t}{\eta_t N}$$
- Diagram**

A schematic diagram shows a motor connected to a gear on a motor shaft. This gear meshes with a smaller gear on an axle, which is connected to a wheel. The wheel is on a rail. Labels include 'Motor', 'motor shaft', 'axle', 'Rail', and 'Wheels'. The gear ratio is given as $a = \frac{n_1}{n_2}$. The transmission efficiency is denoted as η_t .

So, if you want to find out the total tractive effort F_t is equal to F_a is the tractive effort to accelerate the train horizontally and also the rotating parts of the train plus the tractive effort to overcome the gradient F_g and the tractive effort to overcome the train resistance F_r . And if we simplify this or if we write the expression for each one of this we have in kg if we write down the expression in kg. So, we have $28.3 M_e$ into alpha; the gradient could be plus minus M into capital G plus M_r by 9.81 and this will be in kg. And if you want to find out in Newton we can multiply this expression by g , small g ; small g is the acceleration due to gravity. So, we can also say that that is equal to $28.3 M_e$ alpha plus minus $M G$ plus M small r 9.81 into g in Newton.

Now after we have found out this tractive effort we need to calculate the rating of the motor. Ultimately the motor will have to drive the train to move it forward. So, when we want to find out the rating of the motor we have to find out the equivalent torque that is reflected at the motor side. So, let us say we have N motors, and we have a transmission efficiency that is η . The efficiency is not hundred percent. We have a gear in this case like this; we are discussing about drive rating. So, we have the motor here, and the motor is connected to a gear, and the gear is basically driving the axel, and we have finally the wheels. So, this is the motor, and we have the gayer, we have N_1 here and N_2 here, and the gayer ratio in this case is N_1 by N_2 , and these are the wheels.

And this is axel, and here we have found out what is F_t . F_t is the tractive effort we have the train moving on the rail here. This is the rail and the tractive effort that has to be applied here is F_t . So, if F_t is the tractive effort, and we have the radius of the wheel is R ; what is the torque that is experienced in the axel? So, we can find out that the torque at the wheel or the axel is equal to R into F_t , and the total torque on the motor shaft; here is the motor. So, we experience the reflection in this torque at the motor. So, T_t is the total torque referred to the motor shaft. This is a motor soft. So, when we try to transfer the torque to the motor shaft we have to take into account the transmission efficiency. This is the efficiency of transmission is η , and then we have the gear ratio.

So, we have a into R into F_t divided by η . So, we can call this to η_t to signify the transmission efficiency; this is in Newton meter. So, η_t here is the transmission efficiency. The transmission efficiency is a mechanical quantity. It says how much of torque is transmitted to the motor, and transmission efficiency is a quantity which is less than 1. It could be something like 0.9, 0.95 and then it is reflected is the motor side, and

the motor experience says that higher torque than actual torque that at the wheel side. So, this is what we have here and then if we have N driving motor.

So, T_m is the torque per motor is equal to T_t by N and that is equal to $a R F_t$ by $N t$ into capital N . So, this is how we find out the torque which is seen by the motor. So, we can also find out the speed equivalent speed at the motor side, because we know the gear ratio. So, we can also calculate the speed, we know the torque. So, we can evaluate the power, and once we know the torque and the speed we can pick up right kind of motor for the traction application.

So, this calculation gives us an idea how can we choose the directing of the motor, and once the motor rating is chosen we can go ahead with the $d i$. Now when we see the actual locomotive, the locomotive is driving the power from the overhead line. And the power is usually brought in a c ; the transmitted power is a c . So, what we do we step it down and then we rectify it, then feed it to the motors. The motors could be $d c$ motor or a c motors. So, let us take a look at the a c locomotive in which we bring the power from overhead 25 k v line and then feed the $d c$ motors.

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Total tractive effort

$$F_t = F_a + F_g + F_r$$

$$= \left(28.3 M_e \alpha \pm M G + \frac{M R}{9.81} \right), \text{ kg}$$

$$= \left(28.3 M_e \alpha \pm M G + \frac{M R}{9.81} \right) g, \text{ N}$$

Drive Rating

$$T_w = R F_t$$

$T_t = \text{Total torque referred to the motor shaft}$

$$= \frac{a R F_t}{\eta_t} \text{ Nm}$$

$N = \text{no. of driving motor}$
 $T_m = \text{Torque per motor} = \frac{T_t}{N} = \frac{a R F_t}{\eta_t N}$

Diagram labels: Motor, motor shaft, axle, Wheels, Rail, $a = \frac{r_1}{r_2}$, $\eta_t = \text{transmission efficiency}$

So, 25 k v 50 hertz a c traction. So, what we have here is the following. So, we first choose unload tap changer. So, we obtain this power from the overhead line that is 25 k v, and this is the transformer primary. And we have the transformer in the secondary side which is actually a step-down transformer, but this secondary side has got so many

tapings. And we have a simple diode rectifier here, a high power diode rectifier, and then we have filter, and then we have the traction motors, the dc series motors. So, let us say we have the four motors here.

So, we connect these motors in the following fashion. This is the filter inductor that is L_d , and then we have unload tap changer. This tap changer is connected to the secondary side, and we have the diode bridge here, and this is the traction motors. And here we have used the dc series motor, and this is the transformer with tap. Now when we have the transformer with tap here the objective is to control the speed. The secondary side we have so many taps and what we have in a typical tap changing transformer, we have 20 to 40 taps presented in the secondary side. And when we want to change the speed we change the speed by changing the taps. In fact, what we do to start the machine or start the motors we apply a load voltage. So, we start with the 0 voltage, and then we go on changing the taps, and thus we achieve higher and higher speed.

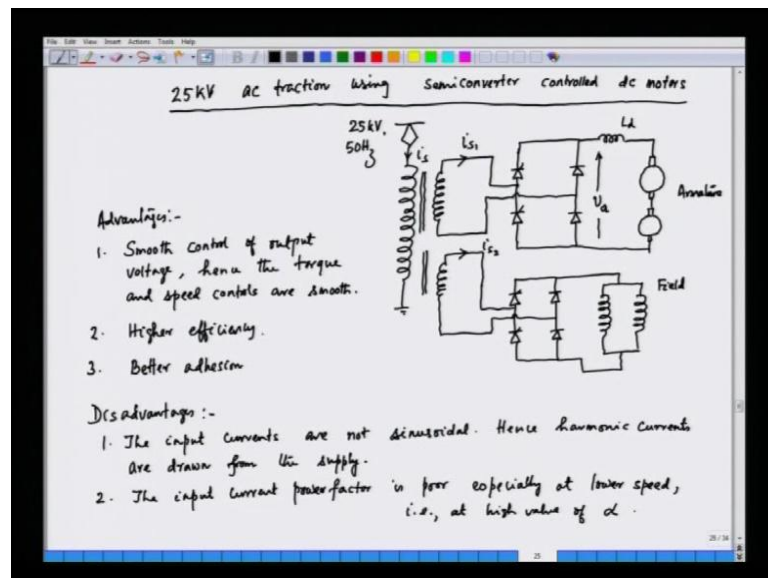
So, this is a primitive type of control which is also still in use and in this case we drive a dc motor. The tap changing transformer supplies to a rectifier bridge, the output is rectified; as we have seen here this is the rectified bridge. And then we have a filter here which filters the ripple in the dc side, and then we have the four traction motors. We have two in series and two in parallel here, and this combination is used to reduce the current bond. We can use all in parallel, but if you use all in parallel the current requirement will be higher in the ac side. We have the source current in this case. We choose to have two in series and then two combinations in parallel. So, what is the advantage? Advantage in this case is that it is an electric drives, but this topology has got lot of disadvantages also.

Now when we have a tap changing transformer, the tap changing transformer requires lot of maintenance. We have the mechanical tap changer, and they have to be emerged in oil, and they undergo wear and tear. So, they require frequent maintenance. So, one disadvantage of this arrangement is that the tap changing transformer requires periodic maintenance; tap changing transformer requires periodic maintenance. In fact, when we change the taps there is a jerky way of control. The control is not very smooth. We are in a low top we are changing to hard top; that is always a current spike. So, there is a possibility of wheel slip; the adhesion is less when we are changing the tap.

So, the second difficulty here is that we have discrete control. The control is not smooth; it is not a continuous control. So, we have discrete control, and hence there is problem of wheel slipping, discrete control possibility of wheel slippage during tap changing operation. So, instead what we do here that we go for a different kind of topology. In case we want to improve upon this situation we go for smooth control, and smooth control means we have to go for semiconductor based control. We do not use a mechanical control, because mechanical control is always cumbersome. It is having its own problem; it requires frequent maintenance. So, instead of having this mechanical control, what we do? We go for semiconductor based control. So, we have the same transformer a step-down transformer, because 25 KV is the higher voltage.

Definitely we cannot have motors for such a high voltage; we have to step it down. We bring the high voltage at 25 KV. We step it down, and then after stepping down we do not use any tap changer there. We use a half control bridge, a half controlled single phase bridge using SCR silicon controlled rectifier, and thus we get a smooth variation of the output voltage. And when we obtain a smooth variation of the output voltage the speed changes smoothly, and there is no problem of wheel slippage. The adhesion is better in case of a semi controlled semiconductor thyristor bridge. So, we will now discuss about the thyristor control of traction motors.

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So, we have a 25 KV ac traction using semi converter controlled dc motors. So, let us see the power circuit here. What we have here is the input ac, and the input ac is obtained at higher voltage; that is 25 KV 50 hertz single phase, and then what we have? We have semi controlled converter; by semi controlled converter we mean two of the devices are the SCR and two are diodes. So, this is a semi controlled converter bridge, and then we have inductor here, and here we have the dc motors. And we can use separately excited dc motor here, and let us say we have two motors connected. And we have the secondary feeding the semi controlled converter bridge like this. What about the field winding? This has the field winding as well.

So, we have two field windings, and the field windings of this motor are fed again through another semi controlled rectifier. So, we have semi controlled rectifier here, and they speed the field winding. These are the armature, and the field winding converters is also fed from a secondary. So, this is the topology in which we have connected to separately excited dc motor fed from a semi controlled converter bridge. Now what is the advantage here? The advantage in this case is that this semi controlled converter bridge can smoothly vary the output from 0 voltage to a maximum voltage. There is no changing in step; there is no distinct control, and hence we can have a seamless variation of the speed.

So, let us list down the advantages of this kind of control. So, advantages are we have smooth control of output voltage. Hence the torque and speed control are smooth. So, we have this is the output voltage. We can say that this is the voltage that is coming out of the half control converter. So, we can call this to be V_a , the armature of applied voltage. So, this V_a is controlled, and hence we have smooth control of speed and torque. And then of course, it is a highly efficiency system; we do not have any resistors. We do not have any losses here, because we have solid state devices like a SCRs, and the SCRs and the diodes have got minimal losses.

The efficiency is higher here, higher efficiency. Since, we have smooth control we have no problem of wheel slipping. The adhesion is better here, because we are employing a smooth control. And hence we do not have any sudden change in the torque, adhesion is better, better adhesion. Now we can also list down some disadvantages here. Although this is quite attractive there are some drawbacks' of this kind of topology. The drawbacks are as follows. When we are employing this semi controlled converter we have to be

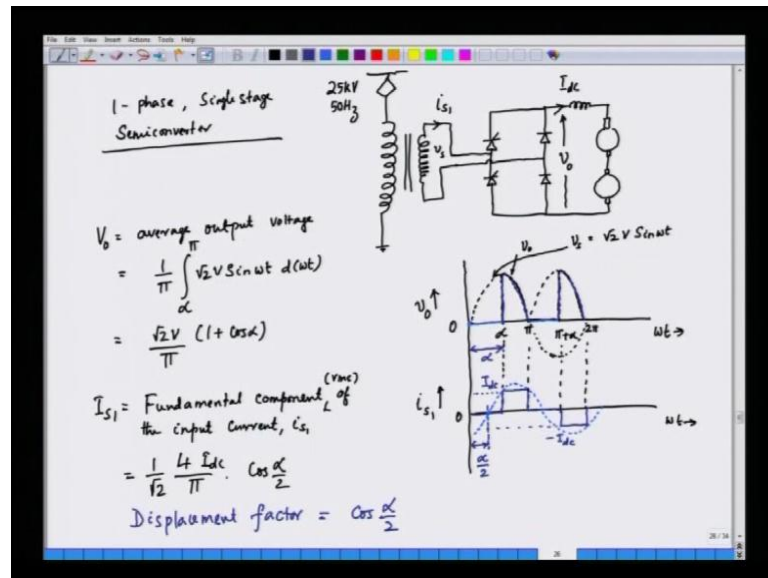
careful about the input current in this case. This is i_{s1} , and this is i_{s2} , and this current which is drawn from the supply we can call this the input current that is i_s .

Now when we employ a semi controlled converter this i_s input current contains harmonics; the input current is not sinusoidal. So, when the input current is not sinusoidal, they are basically rectangular current. The current nature is not sinusoidal, and hence they cannot be harmonics. So, the harmonics are drawn from the supply. So, number one, the input current is not sinusoidal. Hence harmonic currents are drawn from the supply. So, in fact when we draw harmonic current from the supply the supply system gets polluted, and the power system also gets polluted. So, we have the problem of power quality. And in fact when we draw harmonic currents with the transformer, the transformer gets overheated due to the increased losses because of the harmonic currents.

And what about the other drawback? The other drawback is this; that in addition to harmonic current this converter also draws reactive current from the supply. So, also have a reactive current requirement which results in poor power factor. So, in this case the second disadvantage is the input current. Power factor is poor especially at lower speed that is at high value of α ; α is the triggering element. So, when the speed is low α is very high, because when the speed is low the voltage has to be also low. And the voltage can be low when we have higher value of α ; when α is high the power factor becomes poor.

So, these are the drawbacks of this kind of although this converted is very popular, this converter is employed in many practical locomotives. But one has to be really concerned about the drawbacks of these converters. The number one drawback is the harmonics and then the current drawn is at poor power factor. Let us now see how the power factor of a single stage converter is poor, and why the current content harmonics?

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Let us try to understand that by drawing this circuit diagram. So, we have a semi controlled converter here. These are the two SCRs, and then we have the load here; we have the dc motor load, the traction motors. And this is obtained from the overhead transition line through a transformer. So, we call this to be a single phase single stage semi converter. So, we have only one stage here, and similarly we can also fill the field circuit. So, this is a step down transformer. Now if we see the input current this input current here let us say here is i_{s1} . Now if we have this input current i_{s1} , and this current is i_{dc} , and this converter is operating at a triggering angle α ; let us assume that this converter is operating at a triggering angle α ; α is the triggering delay.

So, we can draw the output voltage with respect to the input voltage. So, these are our waveforms. So, the output here is V_a let us say or V_o the voltage here, and this output voltage for certain triggering angle will be like this. This is ωt in the y axis said in the x axis, and this is V_o the output voltage in the y axis. Let us say the triggering angle here is α , and the triggering is symmetrical. So, we can say this is $\pi + \alpha$. This is π , and this is 2π . So, the output here will be of this nature. So, this is v_{naught} . So, the v_{naught} of the output voltage will be dc load out, the rectification of the input. And since we are controlling the output voltage we have the triggering angle α , and up to α the voltage remains 0.

And then from α to π voltage becomes V_s , and then from π to $\pi + \alpha$, this is ϕ to $\phi + \alpha$. So, the voltage basically remains zero from 0 to π and then π to $\pi + \alpha$. These are called the free willing interval. And then we have the output V_o will be present from α to π and $\pi + \alpha$ to 2π . Then we can also draw the corresponding current waveforms. So, we are more interested in the input current i_s or i_{s1} here; from 0 to α I_s will be 0, because there is free willing. We do not have any current in the input side, and then from α to π we have block current like this. And then again from π to $\pi + \alpha$ we have 0 current, and from $\pi + \alpha$ to 2π we have the negative half cycle.

So, what we have here is this. We have the negative current, and the amplitude of this current is the dc link current. In the dc side we have I_{dc} . So, this current is basically I_{dc} , and similarly this is minus I_{dc} . So, if we see that the input current is not sinusoidal, input current in fact contains harmonics. This is basically rectangular nature of current or called quasi rectangular current and this contain harmonics. Now if we find out the input power we have to find out the output voltage here. V_{naught} is average output voltage that is equal to is the input; this is the source voltage V_s that is here source voltage, and we can say that is equal to $\sqrt{2} V \sin \omega t$. In fact this is the minus of that. So, this is the source voltage we have. V_s equal to $\sqrt{2} v \sin \omega t$.

So, the voltage source goes like this in the negative direction also and the rectification of that we have in the positive here. So, the average output voltage will be the average of this. So, we can integrate this from α to 2π $\sqrt{2} v \sin \omega t$ and $d\omega t$, and this is over one-half cycle. So, 1 by π and if we simplify this what we obtain is the following, $\sqrt{2} v \frac{1}{\pi} [1 + \cos \alpha]$. So, this is the output voltage, the average output voltage. Now if we see the output current, the input current, the input current here is quasi rectangular nature; it contains harmonics. So, we can find out what is the fundamental component of the input current. So, in fact if we want to find out what is I_{s1} is the fundamental component of the input current I_{s1} , and we take the RMS value here, fundamental component within RMS.

So, it is $\frac{4 I_{dc}}{\pi} \cos \frac{\alpha}{2}$, and for the RMS we divide by $\sqrt{2}$. So, this is the RMS value of the fundamental component of the current. Now we say that this current is delayed from the voltage by $\frac{\alpha}{2}$. In fact, the current is lying behind the voltage, because there is a delay; we are applying a triggering delay. The angle is α ,

and the current is lagging the source voltage. Now if we draw this fundamental current here it will look like this. So, this will be a nature of the fundamental current. We take this, and this has to be symmetrical here.

So, this is the fundamental component of this if we draw this. Now this angle is alpha, and this angle is alpha by 2. So, in fact it means the current here is lagging behind the voltage. The voltage is V s which is taping from zero here, but the zero of the current is taping at alpha by 2 after the voltage 0. And hence we say that the displacement factor here is cos alpha by 2. So, we can understand the displacement factor in this case is non-unity. It is cos of alpha by 2. So, the displacement factor is equal to cos of alpha by 2.

So, in fact if we make alpha higher and higher the displacement factor will be lower and lower. It means the power factor of the current will gradually go down, and the converter will draw a lagging current from the supply. And lagging current will create difficulty in power system. So, there is a solution to this. So, instead of using a single stage converter we can go for a two stage converter. Now before we do that let us understand what is the plot between the reactive power requirement and a power unit voltage?

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The image shows handwritten mathematical derivations on a whiteboard background. The equations are as follows:

$$V_o = \frac{V\sqrt{2}}{\pi} (1 + \cos\alpha)$$

$$V_{o,max} = \frac{V\sqrt{2}}{\pi} (1 + 1) = \frac{2\sqrt{2}V}{\pi}$$

$\alpha = 0^\circ$
 $V = \text{rms of the source voltage, } V_s$

$$V_o = \frac{V_{o,max}}{2} (1 + \cos\alpha)$$

or, $2 \frac{V_o}{V_{o,max}} = 1 + \cos\alpha$

$V_{o,pu}$ → or, $2 V_{o,pu} = 1 + \cos\alpha$

or, $2 V_{o,pu} - 1 = \cos\alpha$ — (1)

$$Q = V \frac{2\sqrt{2} I_{dc}}{\pi} \cdot \cos\frac{\alpha}{2} \sin\frac{\alpha}{2} = \frac{\sqrt{2} V}{\pi} I_{dc} \sin\alpha$$

$$= Q_{max} \sin\alpha$$

$$\frac{Q}{Q_{max}} = \sin\alpha$$

$$Q_{pu} = \sin\alpha \quad \text{--- (2)}$$

From (1) & (2)

$$(2V_{o,pu} - 1)^2 + Q_{pu}^2 = 1 \quad \text{--- (3)}$$

So, we have this output voltage here. V_o is equal to V into root 2 by pi into 1 plus cos alpha; alpha is the triggering angle. So, what is V_o maximum? Now V_o maximum is equal to V into root 2 by pi 1 plus 1, and that is alpha equal to zero. So, when alpha is equal to 0 the output is maximum. So, cos alpha is 1, so that is V_o maximum, and that is

equal to $2 \text{ into } \sqrt{2} V \text{ by } \pi$. So, V is the RMS of the source voltage. V is the RMS of the source voltage that is $v \text{ s}$. So, if we express this $V \text{ naught}$ once again in terms of $v \text{ naught maximum}$, so we can say $V \text{ naught}$ is equal to $V \text{ naught maximum by } 2 \text{ into } 1 \text{ plus } \cos \alpha$, or we can say $2 v \text{ naught by } v \text{ naught maximum}$ is equal to $1 \text{ plus } \cos \alpha$ and $v \text{ naught by } v \text{ naught maximum}$ this we can say this is equal to $V \text{ naught power unit}$.

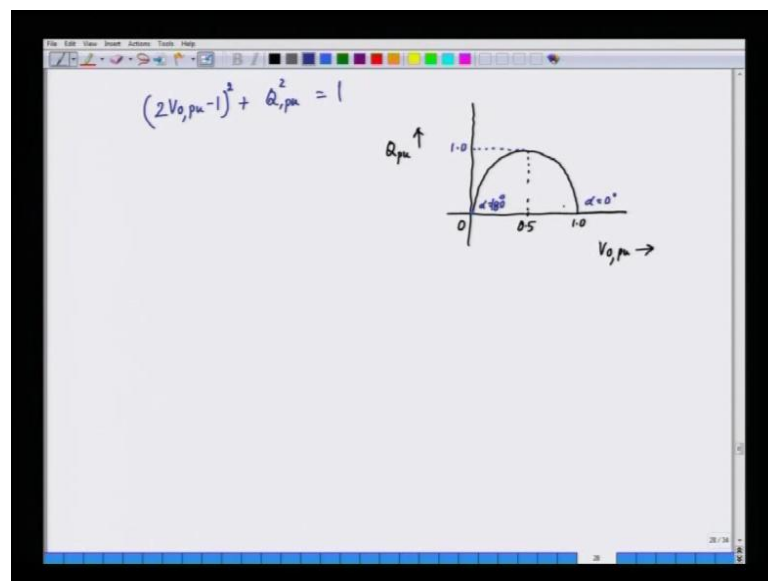
So, we will rewrite this expression as $2 \text{ into } V \text{ o power unit}$ that is equal to $1 \text{ plus } \cos \alpha$ or $2 v \text{ naught power unit minus } 1 \text{ is equal to } \cos \alpha$. So, we are trying to have a relationship between the power unit output voltage and the reactive power. Now what is a reactive power here? The reactive power is q , and q is equal to voltage into current into sign of the angle between them. So, we can evaluate q as follows. Q is equal to what is the voltage here $V \text{ RMS}$ is V . So, we have the V here into what about the input current? We will go back and see the input current here. The input current is given by this expression that is equal to $2 \text{ into } \sqrt{2} I \text{ dc by } \pi$, $2 \text{ into } \sqrt{2} I \text{ dc by } \pi \text{ into } \cos \alpha \text{ by } 2$.

So, if we take this $i \text{ s } 1$ that is the RMS component, and then we have displacement factor is $\alpha \text{ by } 2$. So, it is $2 \text{ into } \sqrt{2} I \text{ dc by } \pi \text{ into } \cos \alpha \text{ by } 2$ is the RMS value of the input current, but then we are interested in the reactive power. And the reactive power is voltage into current into sin of the angle between them, and angle between V and i is $\alpha \text{ by } 2$ here, because we have a semi controlled converter. And the current is lagging behind the voltage by $\alpha \text{ by } 2$. So, we will again multiply by $\alpha \text{ by } 2 \sin$ of $\alpha \text{ by } 2$. So, we have $\sin \alpha \text{ by } 2$ here, and that is equal to $\sqrt{2} v \text{ by } \pi \text{ into } I \text{ dc into } \sin \alpha$. And that is equal to we can say $Q \text{ maximum into } \sin \alpha$, because the reactive power here is maximum when α is $\pi \text{ by } 2$ when $\sin \alpha$ is $\pi \text{ by } 2$.

When $\sin \alpha$ is 1, it means α is $\pi \text{ by } 2$; we have the maximum reactive power. So, we can rewrite this in this following fashion. Q is equal to $Q \text{ max } \sin \alpha$, and then what about our equation? We can write down this equation in the following fashion or $Q \text{ by } Q \text{ maximum}$ is equal to $\sin \alpha$, or we can say $Q \text{ power unit}$ is equal to $\sin \alpha$. So, we have the equation 1 here; we can call this to be equation 1 let us say, and we had equation 2 here. So, equation 1 gives an expression in terms of the output voltage, and equation 2 has got expression for the input reactive power.

So, from this two expressions we can say from 1 and 2 we can say here $2 V_{\text{naught}}$ power unit minus 1 whole square plus Q_{n} power unit square; that is equal to 1, and that we can say that is equation number 3. So, we know that $\sin^2 \theta + \cos^2 \theta$ is 1. So, we have from expression 1 or the equation 1 we have $\cos \theta$. So, $2 V_{\text{naught}}$ p u minus 1 square plus Q power unit square is equal to 1, and this equation 3 can be plotted. So, we can plot this equation 3 as follows.

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So, we can again rewrite this equation 3, $2 V_{\text{naught}}$ power unit minus 1 whole square plus Q power unit whole square that is equal to one. So, we can draw this graph. So, what we will do here we will take V_{naught} power unit in the x axis and q power unit in the y axis. And this graph we can plot; this will be something like this. The reactive power in the y axis and output voltage in the x axis and here we have 0.5 and here we have 1. So, we can see in this case that the reactive power is a function of the output voltage. It means when the output voltage requirement is very low the reactive power requirement is also very low. And when we are gradually increasing the output voltage the reactive power drawn also increases.

Let us say that here when the output voltage is low it means it is close to 0; that is 0 reactive power, and this is because in this case α is in fact 0 here. We have 0 reactive power here, and then we reach half the voltage here output voltage 0.5 power unit. So, in this case we have maximum possible reactive power. This is Q_{max} ; this is maximum.

This is in fact 1 here. And then when the still increase this output voltage again at 1, this is corresponding to alpha equal to 0, and this is 0 output voltage; we have alpha is 180 here. Alpha is 180 at 0 output voltage and alpha is 0 at 1 power unit output voltage.

So, when we increase this output voltage the reactive power increases reaches maximum for 0.5 power unit of output voltage, the reactive power is maximum that is 1 power unit. And again when we increase this output voltage further the reactive power decreases. This reactive power in fact is a big burden for the traction drive. When you have the traction drive, when the motors are being controlled we have to vary the output voltage. And hence this reactive power requirement is also excessively high.

So, in this lecture we have discussed about the locomotive, the drive rating, the various tractive efforts. And then we have seen how we can control the dc traction motor using tap changing transformer and also the semi controlled converter bridge. In the next lecture we will see how we can have a two stage converter to minimize or to reduce this reactive power burden on the supply side. The source will have less reactive power where we instead of a single stage converter we go for a two stage converter. This we will be discussing in the next lecture.