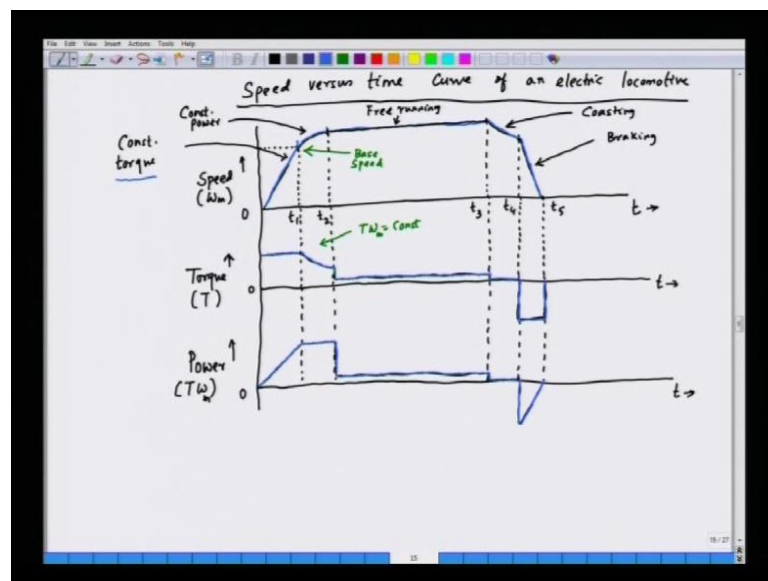


Advanced Electric Drives
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Lecture – 38

Hello, and welcome to this lecture on advanced electric drives. In the last lecture, we were discussing about the traction drives, and more specifically the speed time curve of a traction drive. Now we have seen that when a traction drive or a locomotive start from rest, it goes to the various modes of operation. It starts and then goes to the various modes and then come to a halt or a stop, and the various modes are as follows. We were seeing this diagram in the last lecture, we will just see it once again.

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So, we have the constant power region constant torque region, that is from this point to this point which can also be known as constant acceleration, because during the constant torque region the acceleration remains constant, and then we have the constant power region. This is the constant power region from this point to this point, and in this region the power remains constant, and then what we have is the free running region. So, this is the free running region where the motor speed is constant and the vehicle speed is constant that the free running region. The vehicle is running at constant speed or the locomotive is running at constant speed that the free running period. And then before we

stop the power is switched off to the motor and that is called coasting region, the motor speed is reduced.

So, this is basically the coasting region, the speed reduces. And then before we come to stop we apply brake, and this could be electrical braking or mechanical braking; especially in electrical drives we prefer to apply electrical braking, so that there will be minimal wear and tear of the mechanical part. So, we brake the motor electrically and the kinetic energy of the whole locomotives is converted into electrical energy which could be fed back to the supply in case of a regenerative braking mechanism. So, this is basically the speed variation. We have constant torque or constant acceleration, then we have constant power free running, coasting and braking. What about the torque variation? The torque variation is as follows.

During the constant torque region the torque remains constant, and then when we go for constant power region the torque decreases because here what we have in this case during this region, the torque into omega is constant as the power. So, we have constant power $T \text{ into } \omega$ is constant. And since the speed is increasing or this ω is increasing torque has to reduce to keep this $T \text{ into } \omega$ constant; ω is the mechanical speed. So, $T \text{ into } \omega$ is kept to constant, and when the speed increases the ω increases, torque has to reduce. Then we come to the free running region. This is the free running region, and here the torque requirement is minimal, because the locomotive is running at constant speed; there is no need of acceleration.

So, it has to only cater for the frictional losses, the internal friction and also the external friction. So, the torque requirement is minimal there. So, the torque is minimum but constant. So, that is basically the constant torque and during coasting what we do, we switch off the power. So, the torque becomes equal to 0, and the motor runs because of the stored energy in the kinetic energy in the mechanical system. So, the vehicle has got a large inertia, it stores energy and during the coasting region this kinetic energy is stored in the inertia of the vehicle caters to the motion. So, we have zero internal torque; the motor torque is zero, the vehicle speed decreases. And then we go for braking because we have to stop in a scheduled period.

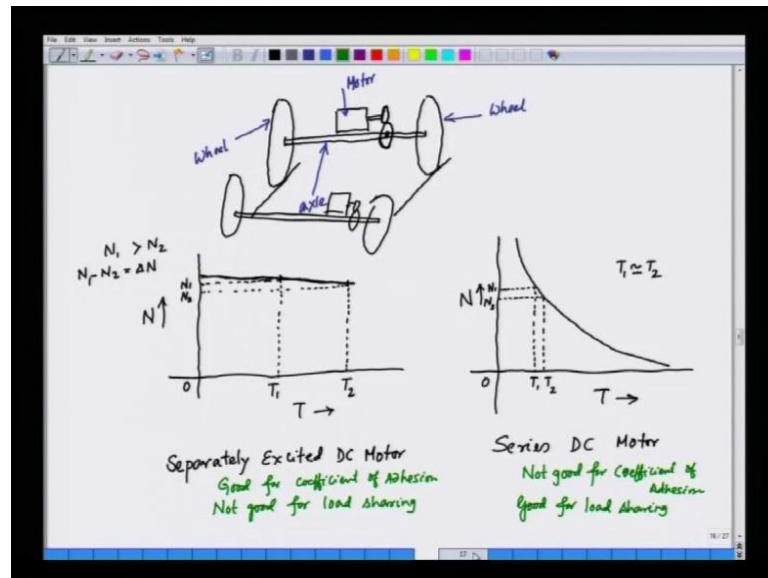
So, what we do here we apply mechanical brake in this case, and it means the torque becomes negative. So, we apply a negative torque here, and hence that is deceleration

that is braking we can say it is constant deceleration, and hence the motor come to a stop at $t = 5$. Now if we see the power the power diagram here, here we have constant torque and speed is increasing. So, we have power increasing here, and at this point we reduce the base speed and maximum possible power is raised; we cannot go beyond the rating of the motor. So, the power remains constant. So, the power here remains constant and then at the maximum possible value that the rated power. And then when we go for the free running the power requirement becomes really low, because the torque is also low.

So, we have reduces power here, and then when the coasting starts the power becomes equal to 0, because the motor is not giving any torque. It is basically coasting; the electric supplied the motor is switched off. So, the machine is running or the motor is running, the vehicle is running, because of the inertia and hence the power requirement is 0 here. And then when we start the breaking process we apply electrical break, and here initially the speed is quite significant, and we have large initial power. This is the electrical power, and then gradually the power decreases, and when the motor comes to a stop the power becomes equal to zero.

So, these are the basically the variation of the speed, torque and the power of a motor when the vehicle is moving from one place to the other place. Now we would like to see how does the low torque gets shared between the various motors; ultimately, when we have a locomotive it is basically powered by electric motors. And we cannot have a single motor, because the power requirement is quite large something like 6000 h p or close to 4.5 megawatt. So, what we do? We have multiple motors to supply the various axles of the locomotives. We could have six motors, ten motors, and all this motors are attached to the axle, and the axle is connected to the wheel and these motors drive the vehicle.

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We can see a diagram which will be clear; say for example, if we have these wheels these are wheels here. This is one pair of wheel, and we have the axle which connects to this wheel. And the wheels are running on the track, so we have the track here. And where does the motor sit? The motor actually sits on the axle; the driving motor sits on the axle. So, we have the motor here which basically it sits on the axle, mounting is on the axle. And then we have the soft output of the motor, and then we have the axle, and these are the two gears. We have the motor gears, and we have the axle gears, and this two gears help transmit the torque from the motor to the axle.

So, this is the electrical motor that we have, and this is the axle. And these are the wheels, another wheel. Now we can have multiple axles; we can have one more axle. So, if suppose we have a choice of motors. Say for example, let us talk about d c motors. We have the choice of using either a separately excited motor or a series motor. Now if we use a separately excited motor, how is the low torque shared? Now let us see the torque speed characteristic of a separately excited motor. So, we have the speed in the y axis and torque in the x axis. Now this is for separately excited dc motor. So, the torque speed characteristic is doping like this. So, it is a straight line, but it is doping in this following passion.

Now on the other hand we have a choice of using a series motor. Now if we use a series dc motor the characteristic is something like a hyperbola. At very low torque we have

very high speed, and at full torque we have low speed as follows. So, if we plot the speed versus torque for a series dc motor. This is series dc motor, the origin here, and we have the speed in the y axis and torque in the x axis, and this is the nature of the torque speed characteristic. So, at low speed we have a high torque. So, it means the starting torque is very high. For a series motor the starting torque is very high, and when we have full speed the torque is very low as we see here.

When we increase the speed here the torque developed by the motor decreases. Now if we see which one shall we choose we have this wheels and we have multiple wheels; we have another wheel here or similarly we will have an axle here connecting this 2 wheels, and we have the corresponding motor. Now the wheels are supposed to run at constant speed. They run at constant that provided the wheel diameter is the same, but unfortunately the diameter of the wheels are not same because of wear and tear; one wheel may run at a different speed, very minor different speed that the other wheel. So, in that situation the two motors will not be running at the same speed. Suppose let us take the first situation separately excited dc motor, and we are supplying this from the same converter. So, say for example, say, the motor number one is running at n_1 speed.

This is N_1 , and motor two is running at a little low speed which is N_2 . So, this speed variation is very minor but no doubt there is a variation on the speed. So, N_1 and N_2 . So, N_1 is higher than N_2 , and $N_1 - N_2$ is equal to ΔN is the change in speed. So, if they run at two different speeds how are the low torque share? Now we see that the first motor N_1 having a higher speed is having a torque corresponding to T_1 , and then the lower speed N_2 will have a torque and that torque is T_2 . So, N_1 torque is T_1 and N_2 torque is T_2 , and there is a large difference between T_1 and T_2 . So, it means the load is shared unequally. It means the motor which is having higher speed is sharing a lower torque as the motor which is having a lower speed is sharing a higher torque; that is not desirable.

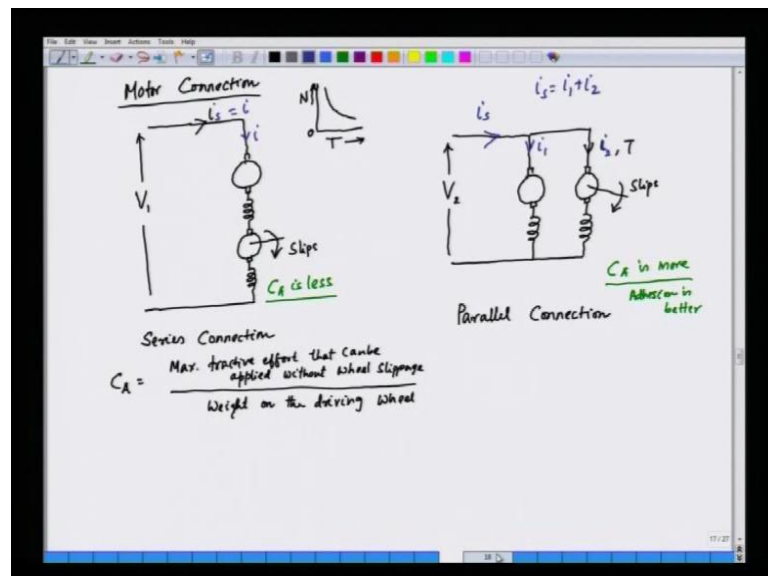
The two motors are having the same rating they let us sat at the same lower torque. Now on the other hand if you go for a series dc motor, suppose we have the same situation there we have N_1 here and then we have N_2 . N_2 is less than N_1 and the corresponding torque will be T_1 , and the corresponding torque for N_2 will be T_2 . And here we see that T_1 is the approximately equal T_2 , very close to each other. So, it means although there is a minor speed variation between the two motors for a series dc motor that torque

sharing are almost equal, and that is why the series motors are sometimes used for traction applications.

It means if we have multiple series motors the sharing of the torque will be almost equal, but if we have already understood the coefficient of adhesion but the coefficient of adhesion it is better to have a motor with is steep motor speed characteristic; it means the speed regulation has to be low. So, the separately excited dc motors are good for coefficient of adhesion but not good for loads sharing. On the other hand the series dc motors are good for load sharing, but not good for coefficient of adhesion. So, we can conclude here in this case good for coefficient of adhesion, because adhesion will be good in this case, the possibility of wheel slip will be less but not good for load sharing.

On the other hand the series dc motor we can say that these are not good for coefficient of adhesion but good for load sharing. So, we have a choice. So, each one is having its pros and cons. So, sometimes depending upon the application we select separately excited dc motor, sometimes we go for series dc motor. Now let us see the type of connection. If you see the type of connection if you have multiple motors the motors can be connected in series or can be connected in parallel; which one is better?

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Now say for example here we have two motors connected in series, motor connection, say, we take let us say series motor. So, we have two motor which are connected in series; this is one motor which is series filled. Then we have other motor which is series

filled, and they are connected in series. And we have applied voltage here that is v . This is series connection, and then we can have another situation where the two motors are connected in parallel. So, these are the traction motors. They could be connected in parallel, because we have multiple motors. So, we apply some voltage here. This is v_1 and this is v_2 ; of course the voltage will be different here, because the series voltage will be higher than the parallel voltage, and here we have the parallel connection.

Now if we connect two motors in series, suppose due to the bad patch of the track, one patch of the track is bad and one motor one axle slips one wheel slips. So, if that wheel slips because of the bad patch of the track, the speed increases. Wheel slipping means the wheel is running at a higher speed not able to have the grip on the track. So, it means the speed increases. Suppose this motor speed will increase this slips. Now if this motor speed increases torque will decrease, because we know that the torque speed characteristic is like this for series motor, this is speed, and this is torque. So, if speed increases the torque is going to reduce; there is the torque we have in this axis. So, if the torque increases I mean the speed increases torque will reduce, and if the torque reduces the current through this series combination will also reduce.

So, when the motor slips the speed increases; the current will come down, and the current will come down. The torque will reduce, and the torque is going to reduce in both the motors, because both the motors are series connected. And hence when the wheel slips there will be net reduction in the torque and what about the coefficient of adhesion? Now the coefficient of adhesion if we have seen the definition, the definition of C_A is maximum tractive effort that can be applied without wheels slippage divided by weight on the driving wheel. C_A as per the definition is basically a ratio; the maximum tractive effort that can be applied without the wheels slippage divided by the weight on the driving wheel.

Now if the wheels slips both the torque are going to decrease and hence we have net reduction in the torque. And if we have a net reduction in the torque we have a lower value of coefficient of adhesion for this connection. So, we can say that for series connection C_A is less. Now on the other hand if we go for the parallel connection we have two motors in parallel. Suppose this wheel slips, so if this slips, only this current is going to decrease, and this torque is going to decrease. So, this current and this torque of

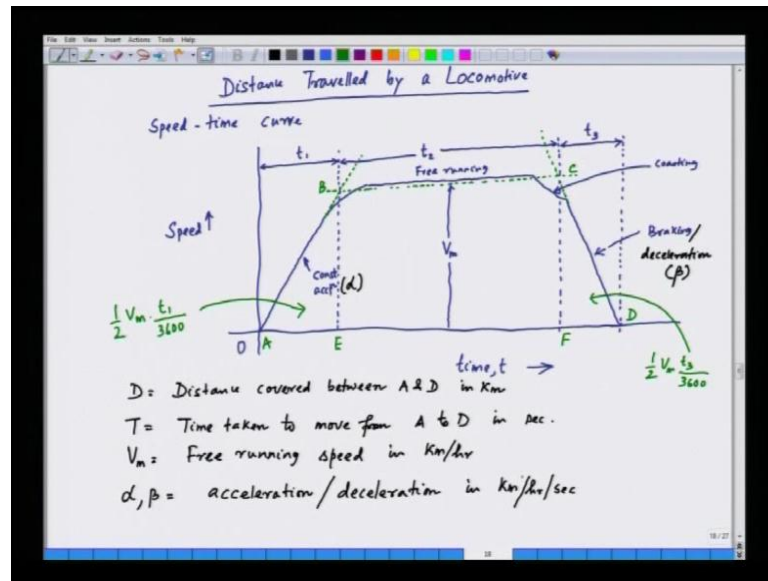
this motor which is slipping that torque is going to reduce, and the other motor which is connected in parallel does not see any change.

So, here effectively we have a higher torque. So, as per the definition the tractive effort which can be applied without wheel slippage divided by weight on the driving wheel here the coefficient of adhesion will be more. So, here the adhesion is better so we can say $C A$ is more here. So, for this combination the adhesion is better. We can say that here the adhesion is better, better compared to a series connection. So, we have the choice; usually when you have the motors we can connect them in series or in parallel. So, we have a choice whether to go for series connection or parallel connection. Now what is the drawback of parallel connection?

Now if we see the parallel connection this current is more; the current which is drawn from the supply is more. Suppose this is i source, and this is i_1 , and this is i_2 ; let us say. So, i_s is going to be equal to i_1 plus i_2 . So, the current requirement from the convertor increases. On the other hand in this case the source current is same as i , and i is the current flowing in the motor. So, we have the same current.

So, if the motor is rated for 100 amperes the 100 ampere will be drawn from the source. Here we have a reduced current requirement from the source. So, we will have a choice between the two, or we can go for a tradeoff. So, we can connect some motors in series and then the combination of the series in parallel to have an optimized connection. So, with this background let us try to see how can we analyze the speed time curve in a better way, and how to calculate the distance covered by the train and also the drive rating.

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So, we will again look at the speed time curve of a locomotive, distance travelled by a locomotive. So, we will again see the speed time curve. So, here we have the speed in the y axis and time t in the x axis. So, we go through this region constant acceleration or constant torque, then constant power, then we have the free running period, then we have the costing and then braking. So, this is a curve which is very well known, but to analyze this we will make some approximation. The approximation is as follows. This can be approximated to be a trapezoid. So, if we approximate this to be a trapezoid the calculation will be simple. So, what we do here is the following. We extend this and then we take an equivalent speed in this case and again we extend this.

So, we will consider this to a trapezoid. So, the trapezoid is a, b is this point, now c is this point, and d is here. And we have the various times here; the acceleration is constant. So, we call that to be t 1. So, what we have here is the following. Now this time is the time for acceleration approximately. So, we call that to be t 1 as the duration of for which it accelerates, and then the free running time and we call that to be T 2. And then we have the braking time, and we define the acceleration as alpha and deceleration as beta. So, this is our T 1. So, we call this to be constant acceleration, and then we have braking in this case. This is the braking region; this is the free running period, and this is costing.

So, here what we do this? The area under this curve will give us the distance traveled, and we define the following variables and the variables as follows. This is the maximum velocity that we have here v_m and we have so drawn this curve so that here we have a little bit increase of area in this case. And then here we have some reduction in the area, an increase of area here. So, we are increase of area here and increase of area here that is compensated by a reduced v_m ; v_m is not the actual v_m . V_m is little less than the actual v_m , because we are trying to approximate the curve to be a trapezoid. And hence we are equalizing the area of the actual curve and that of a trapezoid. So, we have the various variables in this particular graph as follows.

So, D is the distance covered in kilometer, distance covered between A and D in kilometer, and capital T is the time taken to move from A to D in second. Here we have A , B , C and D . So, we are starting from A and we are ending at D . So, this is basically the capital T is the time taken to move from A to D in second, and then we have A_m is a free running speed in kilometer per hour. And α and β are acceleration or deceleration in kilometer per hour per second. So, acceleration is constant acceleration. So, we can say that acceleration is α here and braking or deceleration and the rate of deceleration is β . So, if we have seen this graph, now the objective is to calculate the distance. So, the distance is found out as follows.

So, the distance covered is the area under this curve. Now we can find out this area as follows. We will give this name again as E and F . So, the area under this curve will be the area of the triangles ABE plus the area $BCFE$ plus the area FCD . So we will calculate this as follows. So, this area the area ABE , this area will be half, the maximum speed is V_m into T_1 and T_1 are in second. So, that is T_1 by if you convert this second into hour. So, we have 3600; what about area of this triangle? We have the area is half; V_m is the speed, and this is t_3 , and we have to convert this into hour 3600, and then we have T_2 here. And since we know the acceleration we can find out what is the distance covered. So, we will calculate as follows.

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The image shows a whiteboard with handwritten mathematical derivations. On the left side, the area of a trapezoidal curve is calculated in three steps:

$$D = \text{Area of trapezoidal Curve}$$

$$= \frac{V_m}{3600} \left[\frac{1}{2} t_1 + t_2 + \frac{1}{2} t_3 \right]$$

$$= \frac{V_m}{7200} [2T - (t_1 + t_3)]$$

$$= \frac{V_m}{7200} \left[2T - \left(\frac{V_m}{\alpha} + \frac{V_m}{\beta} \right) \right] \text{ Km}$$
 On the right side, the total time and velocity relationships are defined:

$$T = t_1 + t_2 + t_3$$

$$t_2 = T - (t_1 + t_3)$$

$$V_m = \alpha \cdot t_1$$

$$V_m = \beta \cdot t_3$$

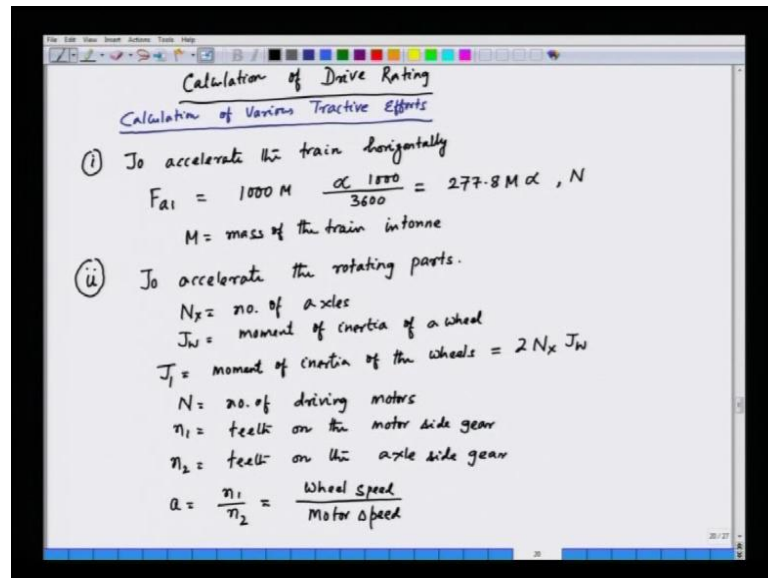
So, we can do this calculation D is equal to area of the trapezoidal curve that is equal to we will take this V_m outside, and this is half t_1 plus t_2 plus half t_3 . So, if we see here this area the area of this rectangle is basically V_m into t_3 . So, we have V_m is the maximum speed here into t_3 . So, that is how we have shown here this is t_2 . So, we have t_2 here. So, V_m into t_2 . So, we have shown that here half t_1 plus t_2 plus half t_3 . Now this we can simplify as follows. If we simplify this we will have V_m by 7200 we will take this two out, and then we will have this $2T$ minus t_1 plus t_3 , because we know that the total time is equal to t_1 plus t_2 plus t_3 . So, what we will do from this we will basically eliminate t_2 .

Now if we eliminate this t_2 , t_2 is given as the capital T minus t_1 plus t_3 . So, we will eliminate this t_2 time. So, we can substitute that here. So, the V_m by 7200 into $2T$ minus t_1 plus t_3 , and that can be simplified as follows, V_m by 7200 $2T$ minus V_m by α plus V_m by β . So, α and β are the acceleration. So, if we know the acceleration you can find out what is t_1 and what is t_3 , because we know to reach that particular speed we have to accelerate that, and that is a constant acceleration. So in fact, we can say that V_m is equal to α into t_1 ; t_1 is in second, and also V_m is equal to β into t_3 .

That we can substitute here, and we have the expression for the distance travelled from A to D. So, we have calculated the distance travelled by the locomotive, and this D the unit

is in kilometer. So, we can write that this is in kilometer because t 1 is in second. So, we will get the expression of the D in kilometer. Now let us see how we can calculate the rating of the motor or rating of the drive. So, if we see the locomotive. There are so many forces coming on the locomotive. We will take each force one by one and try to find out the tractive effort which will be required to overcome that particular force.

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Now let us see calculation of drive rating, and before we do that we have to find out the tractive effort calculation of various tractive efforts. Now the train is having a mass and it is a huge mass. So, the train has to be accelerated linearly. We have equivalent mass of the train that is usually in tones and it as to be accelerated linearly, and we need to have some tractive effort for linear acceleration. So, the first one is to accelerate the train horizontally and that is equal to F_{a1} ; how is it calculated? That is calculated as follows; m is the mass in tone. So, we 1000 m, and we have the acceleration. Acceleration is alpha, and if you convert that into meter per Second Square into 1000 by 3600, and that is equal to 277.8 m into alpha, and the unit is Newton.

So, here m is the mass of the train in tone. So, one tone is 1000 kg. So, we convert that into kg and then the acceleration into meter per Second Square, and we find out the effective tractive effort in Newton. Now this is basically to accelerate the train linearly, and if you see the train has got so many rotating parts; we have the wheels; we have the axels; we have the motors. So, all these rotating parts will also have to accelerate in an

angular fashion. So, we also have angular acceleration. So, we need to also find out the equivalent tractive effort or equivalent torque which is required to accelerate these rotating parts in an angular way.

So, the second part of the tractive effort is to accelerate rotating parts. Let us effectively find out the moment of inertia; what is the moment of inertia of the rotating components? We have two rotating components; one is the wheel, other is a motor. Now suppose we have number of axels equal to N_x . So, if N_x is the number of axels and j_w is the movement of inertia of a wheel, so how many wheels we have? Each axel will have two wheels. So, if we have N_x axels we have $2 N_x$ numbers of wheels. So, the effective inertia for all the wheels will be $2 N_x j_w$. So, we call that to be j_1 . So, j_1 is the moment of inertia of the wheels that is equal to $2 N_x j_w$. And then let us try to find out the moment of inertia of the motors.

So, for example, we have N number of motor, and each motor is connected to the wheel by means of a gear, and the torque is transmitted from the motor to the wheel. So, there is a transmission mechanism there, and we need to find out the effective inertia or the equivalent inertia under this geared condition. So, we have the following parameters with us. N is the number of driving motors, and then N_1 is a teeth on the motor side gear; N_2 is a teeth on the axel side gear. So, we have the ratio that is equal to N_1 by N_2 is the gear ratio that is equal to the wheel speed by the axel speed by the motor speed.

So, this is what we have. So, when we refer this total moment of inertia to the wheel side, because the motor is not on the wheel; motor is actually connected to the axel, and it is having its own speed. So, this has to be converted; this inertia has to be referred to the wheel side. Now since we have the gear ratio that is equal to A , the wheel side equivalent moment of the motors is $N j_m$; j_m is the inertia of each motor divided by A square. So, we can calculate that.

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Moment of inertia of the motors referred to the wheels = $J_2 = \frac{N J_m}{a^2}$

$J =$ Total moment of inertia = $J_1 + J_2$

Linear acceleration = $\frac{\alpha \times 1000}{3600} \text{ m/s}^2$

Angular acceleration = $\frac{\alpha \times 1000}{3600 \times R} \text{ rad/s}^2$

$R =$ Radius of the wheel in m

$T_{a2} = (J_1 + J_2) \frac{\alpha \times 1000}{3600 \times R} = \left(2N_s J_m + \frac{N J_m}{a^2} \right) \frac{\alpha}{3.6 R} \text{ Nm}$

$F_{a2} = \frac{T_{a2}}{R} =$ Tractive effort for angular acceleration of the rotating parts

$F_a =$ Total tractive effort to accelerate the train horizontally including the rotating parts = $F_{a1} + F_{a2}$

So, moment of inertia of the motors referred to the wheels is equal to J_2 . We have this J_2 that is equal to N into J_m is the inertia of each motor divided by a square. And hence we have the total moment of inertia, J is equal to total moment of inertia; that is equal to J_1 plus J_2 , and this total moment of inertia has to be accelerated angularly. And we know the linear acceleration that is α . α is the acceleration of the train. Now this α which is actually the distance traveled per hour per second or kilometer per hour per second has to be converted to radian per second square or angular acceleration. And to do that we have to have the radius of the wheel, because the wheel is rotating and the train is moving forward. So, we have linear acceleration and that has to be converted into angular acceleration.

So, if we have the diameter of the wheel are meters, we can convert α into angular acceleration as follows. So, what is the linear acceleration? The linear acceleration is equal to α is in kilometer per hour per second. So, we have to convert into meter per Second Square into 1000 divided by 3600 that is meter per Second Square. And then we have the angular acceleration that is equal to α into 1000 divided by 3600 into R that is radian per Second Square. Now what is R here? R is the radius of the wheel in meter. So, when we have the angular acceleration we can find out the torque that is necessary for accelerating the rotating parts, and the torque is the moment of inertia into the angular acceleration. So, we can find out that.

So, T_{a2} is equal to $J_1 + J_2$. We have the moment of inertia of the wheel and also of the motor referred to the wheel side into α into 1000 divided by 3600 into r , and that is equal to we can calculate this $2 N \times J_w$ plus $N_j m$ by a square. We are replacing this J_1 and J_2 into α by $3.6 R$. So, this is basically the effective torque, and if we want to find out what is the effective tractive effort F_{a2} , and this is basically in Newton meter. F_{a2} is the tractive effort which has be applied due to I mean to accelerate the rotating parts that is basically the torque by the radius; that is equal to T_{a2} by R . So, we can calculate what is the total tractive effort? F_a is the total tractive effort to accelerate the train horizontally including the rotating parts is F_{a1} which we already valuated plus F_{a2} .

And F_{a2} is obviously the tractive effort for angular acceleration of the rotating parts. So, F_{a2} is important, because F_{a2} is a component of the tractive effort which has to be spent to accelerate the rotating parts like wheels, the motors and so on. And then we have a linear acceleration also. So, the total tractive effort is the summation of the linear acceleration, the force because of the linear acceleration, and the force required for the angular acceleration of the wheel and the motors. Now if we add these two tractive efforts that is F_{a1} plus F_{a2} we will obtain the final one.

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The image shows a whiteboard with handwritten mathematical derivations. The first part calculates the total tractive effort F_a as the sum of F_{a1} and F_{a2} . The second part, labeled (ii), calculates the force F_g due to gravity on a gradient, showing it is equal to $Mg \sin \theta$ in Newtons and $M \sin \theta$ in kilograms. A diagram on the right shows a block on an inclined plane with a 1000m horizontal base and a 1000m vertical height, with a weight Mg acting vertically downwards.

$$F_a = F_{a1} + F_{a2} = 277.8 M \alpha + \left[2N \times J_w + \frac{N J_m}{R^2} \right] \frac{\alpha}{3.6 R}$$

$$= 277.8 M_e \alpha$$

(ii) Gravity / Gradient

$$F_g = 1000 M g \frac{1}{1000}$$

$$= M g \sin \theta, N$$

$$= M \sin \theta, Kg$$

So, what about the total tractor effort? So, F_a is equal to we have this expression F_{a1} plus F_{a2} and that is equal to we have the expression $277.8 M$ into α that is in

Newton plus we have the angular acceleration $2 N \times J w$ is the moment of inertia of the wheel plus $N J m$ by a square, the gear ratio square into α by $3.6 R$. Again we have 1 by R in this case, and that will be 2.77 into 8 into M_e is the effective mass into α . So, we can simplify that the actual mass of the train is not M ; M is just the weight of the train as the mass of the train in kg or in tone.

Now the effective mass includes the rotating part also. So, this effective mass M_e is ten percent higher than the actual mass. So, this is basically for the acceleration which is the horizontal plus angular, and then we have other tractive efforts also. So, let see the other tractive efforts. We have the gravity. Now sometimes the track is actually not linear, not horizontal; that is having an inclination. So, if you see the track this is basically nature of the track here and here we have the G of the gradient, and then this is 1000 meters. So, what we have here because of this we have a tractive effort which is required. This is basically the mass of the train, the weight; the weight is $1000 m$ in to G . So, we have a component of the force which is opposing the motion.

It is trying to go forward here, and that is basically the tractive effort which has to be supplied by the locomotive. So, that is evaluated as follows. So, we have F_g is the tractive effort because of the gravity that is equal to $1000 M$; M is the mass in tone, then 1000 is basically to convert that into kg into G is the gravitational acceleration 9.812 meter per second square into G is the gradient by 1000 . G is defined as the change in the vertical inclination in 1000 meters. So, effectively what we have here? This is basically the sign component if this angle is θ ; this is $1000 mg$ in $\sin \theta$ that is approximated as $1000 mg$ into g by 1000 . And that is equal to M small g capital G Newton or that is equal to $m G$ in kg . So, this is the gravity or sometimes we call this to be gradient.

So, in this lecture we have discussed the speed time curve of a locomotive. We have seen how we can calculate the overall distance travelled, and we got an expression for the distance traveled by the locomotive from one place to the another place when it goes through constant torque mode, constant power, free running, costing and braking. And then we tried step by step to evaluate the various tractive effort primarily to include the train, to accelerate the train horizontally, to accelerate the angular or the rotating parts of the train, and also when the train is overcoming a gradient the track is not a horizontal track.

It is having an up gradient or a down gradient. So, we found out the tractive effort required to overcome a gradient. So, in the next lecture we will be discussing the other type of tractive effort which is required to overcome the air resistance. And we will see how the total rating of the electric motor which is used for driving this train can be calculated. So, these things we will be discussing in the coming lecture.