Advanced Electrical Drives
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Lecture - 37

Hello, and welcome to this lecture on advanced electrical drives. In the last lecture, we were discussing about the selection of motor ratings, and we know that a motor can we operated continuously or it could be subjective to an intermittent load. When it is operated continuously the determination of rating is straightforward, but if it is operated intermittently, say for example, if you have a pulsating load or if you have a motor having a short time rating, the motor rating has to be chosen carefully. So, we will again discuss how to select motors if a motor is continuously rated or if a motor is rated intermittently. Let us have a look at these ratings.

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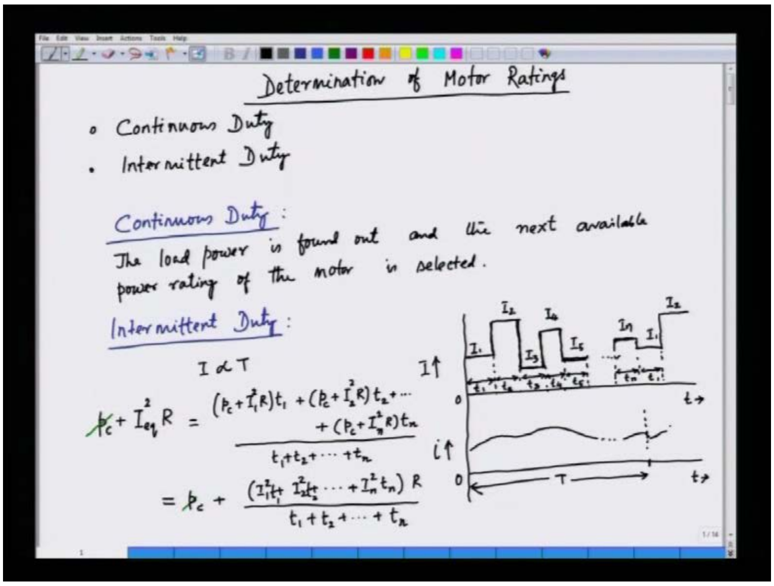

So, we will be discussing about determination of motor rating, and we have two types of motor ratings like continuous duty and intermittent duty. Now let us have a look at the continuous duty motors. When a motor is continuously rated is running continuously, so we know the load power. We know the power of the load, and we select the next available rating of the motors. Motors are not available for all ratings; they are available for selective ratings. So, when we know the load power we select the next higher power of the motor, and that will be the motor rating. So, what we do here in the first case continuous duty, the load power is found out and the next available power rating of the motor is selected.

Now what if if the load is fluctuating in nature, if suppose load is continuously changing? So, if the load is continuously changing there is a need to find out an equivalent torque. So, if you can find out the equivalent torque we can find out the rated torque of the motor based on the equivalent torque. Now let us see if the load is continuously changing; how we can find out this equivalent torque or in that sense equivalent current of the motor. So, we have intermittent duty. So, let us assume that the load is changing as follows. So, we have let us, say the time in the x axis, and lets we have the current of the motor; the motor is drawing the current in the following fashion. It is not constant but it is changing, and the corresponding durations are known.

So, this is basically for t 1, this for t 2, this for t 3, for t 4, then t 5. So, we have fluctuating load and the load is taking varied current, and the currents are I 1, I 2, I 3 and so on. Now the load is taking the fluctuating current, and let us assume that load is also periodic. So, we have n intervals in which currents are defined as fallows. So, we have I 1 here for t 1 interval, and I 2 for interval t 2, and this is I 3 for interval t 3, and I 4 for interval t 4, I 5 for interval t 5. And similarly we have I n for interval t n, and then the load repeats; the same current profile will repeat here. I 1 for again for interval t 1 and so on, and similarly again I 2 the same thing repeats here. So, what we can say is that the currents are defined here, I 1 to I n and the respective interval are also known.

Now why we say current? Sometimes the torque and the currents are related. So, we assume here that the current is proportional to the torque. So, if you know the torque variation we can easily find out the current variation. So, here our assumption is this; the torque and the currents are proportional. In fact, I is proportional to the torque.

So, if the torque variation is I 1, t 1, t 2, t 3, t 4, up to t n, the corresponding current variations are also I 1, I 2, I 3, I 4, up to I n. Now the question here is this that how to find out an equivalent current. Now this is going to flow in the machine; the current is going to flow in the motor, and the motor will have the losses because of this current. So, usually a motor has got a core loss and the copper loss. And the copper loss let us say we have an equivalent current that is I e q, and the effective resistance of the motor is R, very simplified expression

The core loss plus the copper loss, and that is equal to this basically is the total loss, and this loss is equal to the loss occurring in the motor in the respective intervals. The intervals are I 1 to I n. So, when the motor is operating we have got distinct intervals like I 1 I 2, t 1 t 2, t 3 t 4 up to t n, and the respective current are I 1, I 2, I 3, I 4 up to I n. So, we have to find out the loss in each and every interval.

Now the core loss is independent of the load. We know that when a motor operates the core loss does not change; only depending upon the load the copper loss changes. So, we can write down the individual losses as follows. So, for the interval number 1 we can say that it is the core loss plus we have the current I square R and this is for interval t 1. Now similarly for the interval t 2 we have the same core loss plus we have a new current that is I 2.

The resistance is the same in the motor R and it is for t 2. And similarly we can go on writing these losses up to the n th interval; for n th interval what we have here? We have the same core loss, and we have the copper loss that is I n square r into t n. Now this has to be divided by the effective time, and the time is t 1 plus t 2 up to T n. Now in the left hand side we have the loss which is expressed in terms of an equivalent current p c plus I equivalent square into R. In the right hand side we have found out the average loss of the system. Now if we simplify this we get the following expression. So, what we can say here is that we have in fact, p c here can be taken common, and p c can be taken out. So, what we are here is that p c we can take out plus I 1 square plus I 2 square up to I n square.

Of course, what we are here I 1 square into t 1, I 2 square into t 2, and I n square into t n into r here divided by t 1 plus t 2 plus T n. So, in the right hand side we have been able to remove or separate p c, the core loss, and this p c being common, this p c will be cancelled from the right hand side and the left hand side. And what remains is the individual currents multiplied current square multiplied by the individual intervals like I 1 square into t 1, I 2 square into t 2 and so on. Let us see what expression we will get in this case. Now for sure we can cancel this left side and the right side; this can be cancelled.

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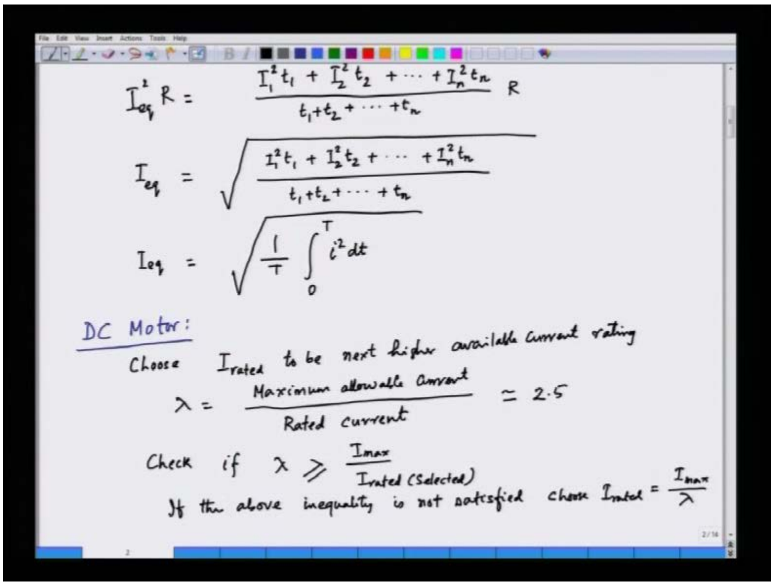

And what remains here is the following I equivalent square into R that is equal to I 1 square t 1 plus I 2 square t 2 plus I n square into t n divided by t 1 plus t 2 plus t n into R here; R is the resistance of the machine or the motor. Now this R will be also be cancelled. So, we can say that here from this equation we will say that I equivalent is equal to under root I 1 square t 1 plus I 2 square t 2 plus up to I n square t n divided by t 1 plus t 2 up to t n. So, we have been able to find out an equivalent current. Now in this case the current variation is not continuous but discrete. Now let us see the nature of the current variation. Now if you see the nature of the current variation we see that the current variation is discreet. Here we have I 1, then we have a state change; these currents are discontinuous, and then we have I 3, I 4, I 5 and similarly up to I n.

So, these variations are not continuous. Now sometimes if you have a continuous current variation how do we find out this? For the discreet current variation we can find out the sums I 1 square t 1 plus I 2 square t 2 and so on. Now if the current variation is continuous instead of a summation what we will require is an integral. So, if the current variation is continuous the current is not constant but fluctuating; we have the variation like this. And here we have the current which is not constant, but it is basically changing like this; it is a variation. So, this is the nature of the current variation. Let us say this is I, small I; it is continuous. So, it is something like this, but it is periodic. After sometime let us say after this interval up to this we have a time period in this case and the same thing repeats.

So, if the current variation is continuous what we will do here is the following. Now what is I equivalent there? I equivalent in that case would be 1 by T integral o to capital T is the time period i square d t under root. Now what is the time period here? Let us say that the current is having the time period that is capital T, and after this capital T the same thing repeats. So, if this is the nature of the current and the current variation is continuous we can integrate the current over this whole time period. And the integration of that will give us the equivalent current as we have seen in the equation. So, this is the equivalent current that we have. Now that we have find out the equivalent current we do not have any more the district nature. The current is although varying, we have been able to find out an equivalent current. So, the next able would be find out the rating of the motor.

Now what we will do the motors are not the same; d c motor is different from induction motor; induction motor is differ from synchronous motor. So, definitely we would like to choose the next higher current rating or the next higher torque rating. So, let us take one by one; in d c motor if we know the equivalent current I e q, how shall we find out the rating of the motor? So, first let us take the example of a d c motor. In d c motor we have the rating of the motor; let us say which would usually what we do here. The first step is to choose I rated to be next higher available current rating. So, what we do here which choose I rated based on the next higher available current rating. In d c motor when we operate the motor, and in this case of course we do not have continuous load; we have a fluctuating load. The load can go high and low.

We can allow a maximum current, and this maximum current should not be 2 to 2.5 times rated current. Say for example, if the rated current is 10 ampere the maximum allowable current should not be more than 25 ampere, and this is called lambda. Lambda is a constant which is defined in a d c motor as fallows. Lambda is equal to maximum allowable current divided by the rated current, and this is approximately equal to 2.5 let us say, why? Because if the current the maximum current goes beyond that value there will be excessive sparking at the commutators and brushes. Now this is to limit the sparking, we can allow the maximum current 2.5 times higher than the rated current. Say, if the rated current is s10 ampere the maximum allowable current should not be 25 ampere even for a short time more than 25 ampere.

So, we have already selected this I rated. So, we will substitute our condition. So, each check in that case if lambda is higher than I maximum by I rated selected. So, lambda is 2.5, and I maximum in this case should be such that the actual values would be less than lambda. So, if this condition is not met; it means say for example, rated current is 5 ampere and the maximum current that is taken by the motor is 20 ampere. So, in that case I maximum by I rated is 4 and 4 is definitely higher than 2.five. So, this inequality is cannot satisfied. So, if it is not satisfied, if the above inequality is not satisfied, choose I rated as I maximum by lambda. So, if it is not satisfied what we do there that we go for a higher rated motor, we select I rated as I maximum by lambda.

So, this is the process because it may so happen that the load can have a very high peak. If the load is having a very high peak and if you choose a rating of the motor which is very low, for that instant when the load is having high peak there will be excessive sparking at the brasses and that should not be allowed. And to avoid that what we do we choose the rating of the motor or I rated is equal to I maximum by lambda. So, this is actually for the d c motor; what about induction motor? In induction motor what we have we are not bothered about the commutation, because commutation does not happen in induction motor, but we are bothered about the break down torque or the peak torque. So, let us see how we select the motor rating for induction motor.

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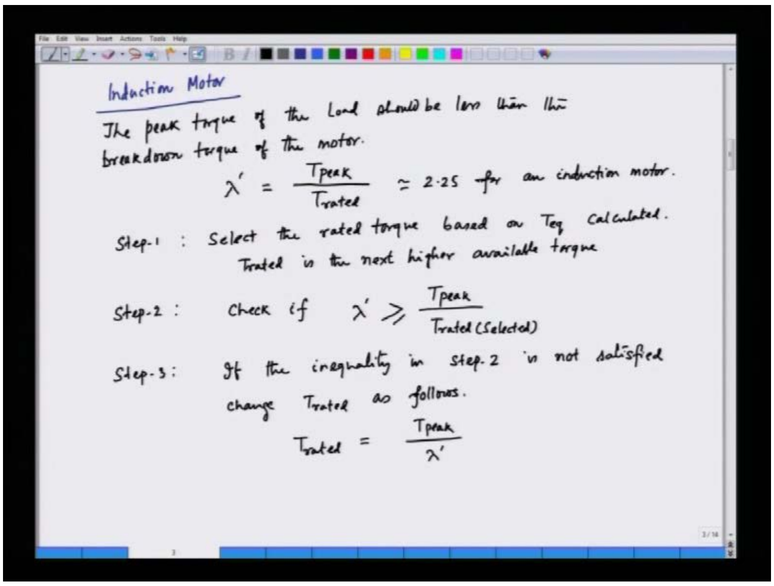

So, right now we will be discussing about induction motor; the peak torque of the load should be less than the break down torque of the motor, the peak torque of the load should be less than the break down torque of the motor. Now usually what we have we defined lambda prime which is equal to T peak by T rated, and this is approximately 2.5 times for an induction motor. So, we know that we have already found out the equivalent current I equivalent, and we know that approximately we can say that the current and torque are proportional. So, we can also find out T equivalent. So, what we do usually based on T equivalent that we have calculated earlier we choose the rated torque. So, the rated torque is next available torque higher than the equivalent torque that we have obtained.

So, after we have obtained this rated torque we apply this condition, and the condition is that the peak torque by the rated torque should be less than lambda prime. So, we can note down the various steps. Step one here; select the rated torque based on T equivalent calculated. In fact, what we do here is that T rated is the next higher available torque, and then after we have obtained this T rated we apply this inequality. Step two; check if lambda prime is greater than T peak by T rated selected. So, we know that the torque is fluctuating in nature, because the current is fluctuating in nature. So, we can always find out the actual maximum torque of the load. So, we know the maximum torque; we know the rated torque that selected, we check this inequality. If the inequality is satisfied then this rating is okay or the rating is proper. So, we can go ahead.

If the inequality is not satisfied we change the rating as follows. Step three; if inequality in three in step two is not satisfied, change T rated as follows. So, what should be T rated now? T rated should be equal to T peak by lambda prime. So, we know the peak value of that torque. We divide that by lambda prime we get the T rated. Now this is how we do the selection in case of an induction motor. Now if we see in case of a synchronous motor the process is almost similar. In synchronous motor lambda prime is having a higher value; lambda prime is something around 3 to 3.5, and the process is exactly similar.

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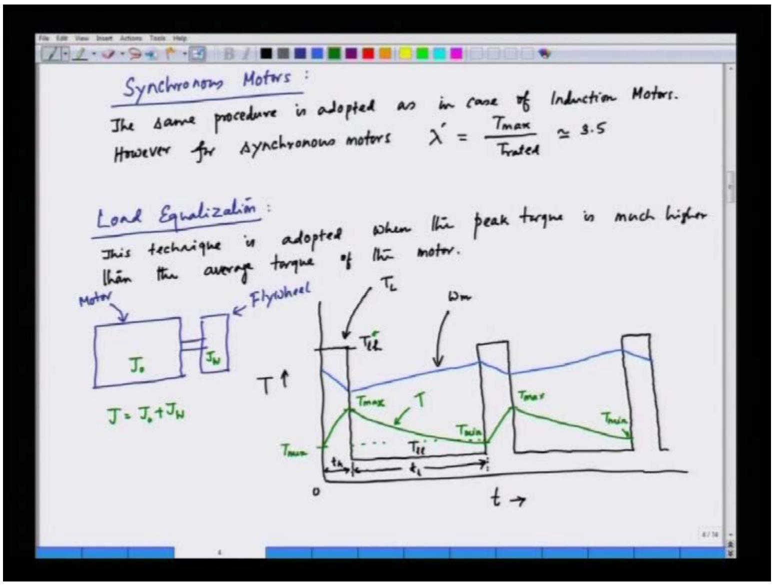

So, what we can do here is the following that in case of synchronous motors the same procedure is adopted as in case of induction motors; however, for synchronous motors lambda prime which is equal to the T maximum by T rated is about 3.5 higher than that of a synchronous motor. So, we adopt the same procedure that we have followed in case of an induction motor for synchronous motor, but in this case lambda prime is little bit higher. It is approximately 3.5 which is the ratio of the peak torque to the rated torque.

We now come to an understanding how to select the rating of the motor when the load is fluctuating. The load is fluctuating we can find out an equivalent current or equivalent torque, and then go ahead with the selection of the rating based on the equivalent current or equivalent torque, because in most of the cases the nature of the load variation is known either in the form of a current or in the form of a torque, and in fact the torque and the current are proportional.

So, if we know one the other can be calculated or other can be evaluated; now in certain situation we have very high peak torque. Now if the peak torque is extremely high we will get by this procedure a very high rating of the motor which is not sometimes economical. So, if the peak torque is very high we go for what is called load equalization. So, let us see what is the meaning of load equalization and how it is achieved? So, we will be discussing now load equalization, and this is employed or this is adopted. This technique is adopted when the peak torque is much higher than the average torque of the motor.

Say for example, we can take an example that a load is periodic, but it is having a high peak torque. We have a high value and we have a low value, but the load is periodic. Say for example, we can have the torque peak characteristic like this. So, we have torque here and we are changing with time. The load is changing as follows. We have a high value here, then we have a low value, then it is remaining low for quite a significant amount of time. Then it is becoming high once again, and then it is again becoming low and remaining low for significant amount of time, and then becoming high once again and it is periodic. So, it changes like this. So, we see the nature of the load torque variation. Now this is the load torque variation T L.

So, T L is fluctuating in nature. It is periodic definitely, but it is having a very high peak. Now if we try to find out the equivalent torque employing the equivalent current method or equivalent torque method. Now we will get a torque which is much higher than what is required. So, unnecessarily we have a bigger motor. So, now that is not very much economical. So, now what we do here is that in the motor we attach a flywheel; the flywheel has a large inertia. So, when we attach a flywheel the flywheel absorb the energy when the motor is running under a load torque condition, and it gives some amount of the load torque when the torque becomes high. So, what we do here is the following.

We have the motor here, and with the motor we attach a flywheel. This is a flywheel, and this is the original motor. And when we attach a flywheel the flywheel takes some part of the kinetic energy, and the kinetic energy stored in the load torque region, and this kinetic energy given in high torque region. In fact, what we see here if we plot the speed against time we see that when the torque is applied high torque is applied the speed decreases, and when the load torque is applied the speed increases as follows. So, here we can also plot in this case the speed; the speed will fall down here. And it will slowly increase in the load torque region; again it little fall down and it is also periodic variation like this. So, this one is the speed.

Now what we have here this is called the load high T L h the high load and this one is called T l l the low load, and these loads are applied for various intervals; the high load is applied for T h. So, this is T h the time for application of the high load torque, and T L is this duration. This is T L the time for application of the low load. So, we have a high load torque and we have a low load torque. Now under this situation the speed is also changing, and we can imply this flywheel technique to even out the torque burden on the motor, and ultimately the motor will be a small motor be sided by the average torque of T L h and T L l.

So, we are talking about the load equalization. In this case the load torque is having peaks, and we have a high peak, and we have a low peak, but the load is periodic as you have seen. And the load is varying between T L h and T L l as we have seen in the diagram. So, we have T L l is basically the low part of the load, and this is applied for T L duration. And T L h is the high part of the load, and this applied for T h duration. Now what we do in this case is that the motor is fixed with or we fix a flywheel with the motor. So, when we fix the flywheel with the motor, the effective moment of inertia increases.

So, we assume that effective movement of inertia is j and j is equal to j naught plus j flywheel. So, here what we have in this case is the following. Now this is having a movement of inertia that is equal to j naught. We know that this is the moment of inertia of the motor, and the flywheel is having a movement of inertia that is j w. And we can say that the total movement of inertia of the combination is equal to j naught plus j w and we choose a motor which is having a drooping speed torque characteristics as follows. So, the motor is selected as follows.

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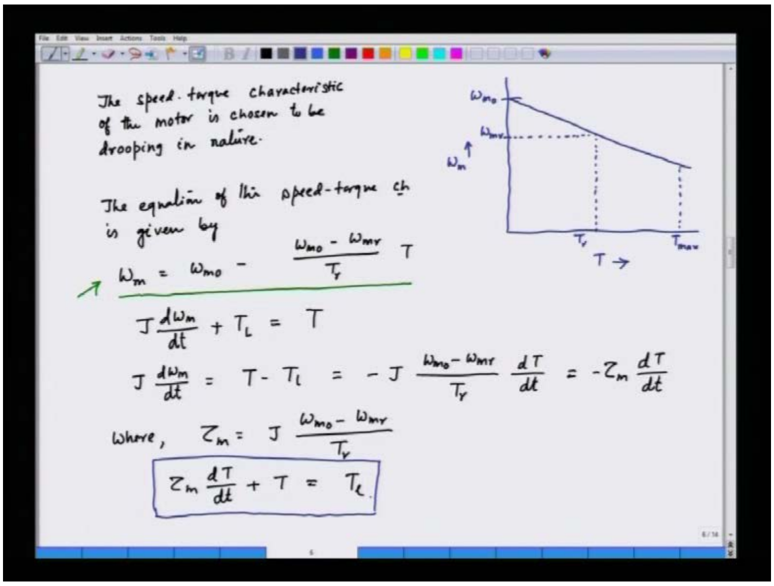

This is the speed torque characteristics of the motor. The speed is in the y axis and torque in the x axis. And this is the speed torque characteristics of the motor which is linear, but which is drooping. The speed torque characteristic of the motor is chosen to be drooping in nature. Now we define this as the no load speed that is omega m naught and this is our rated load T r. And let us say this is maximum torque T max, and corresponding to the rated torque we have the speed that the rated speed that is omega m r. So, in this case we can find out the equation of this speed torque characteristics, and the equation of the speed torque characteristics is as follows.

The equation of the speed torque characteristics is given by omega m is equal to omega m naught minus omega m naught minus omega m r divided by T r into T. So, it is the torque. So, this equation shows the variation of omega m with capital T that is the torque. So, we have the no load speed, and we have the slope of the line which is given by omega m naught minus omega m r by T r, and it is a negative slope. So, we can very easily determine the nature of the torque speed characteristics; for a given torque we can also find out the corresponding speed. So, this is the torque speed characteristics of the motor. Now what we have we can apply the dynamic condition here. We know that whenever a motor runs it basically obeys a dynamic equation, and the equation is as follows.

So, we know that j d omega m by d t is the inertial torque. We have here the moment of inertia is j and d omega m by d t is the angular acceleration plus the load torque T L is equal to capital T is the torque which is given by the motor. So, the motor has to support the inertial torque which is basically the acceleration or deceleration torque plus the load torque. Now we know what is omega m? Omega m is given by this acceleration. So, we can substitute this omega m here and simplify. So, after substituting omega m we can say here that d omega m by d t is equal to T minus T L, and that is equal to what is j d omega m by d t; we will basically differential this equation. So, if we differential this with respect to capital T, the time small t, the time here; so this is basically constant which will give 0, and we get the following equation.

So, that is equal to minus j, and then we have omega m naught minus omega m r by T r. This is basically a constant d T by d t small t, or we can say that is equal to minus tau m d T by d t. So, this is what we obtain here if we differential this omega m with respect to T we get minus tau m d m by d t. So, in this case we have first order differential equation. The equation which we have got out of this is the first order linear differential equation whose solution is quite straightforward. So, we can write down the linear differential equation as follows, but before that tau m is a constant is a time constant is defined as fallows.

Tau m is given as follows. Tau m is basically in this case is j omega m naught minus omega m r by T r. So, this is what is tau m and from this above equation we can say that tau m into d T by d t plus T; that is equal to T L. So, this is the first order differential equation. So, this equation has to be solved, and if we solve this equation we get the torque variation with time. How does the torque vary with time? Now we go back to our previous diagram that we have drawn with the torque was changing from a low value to a high value. Now when the torque is changing from a low value to a high value that is basically the load torque, how does the machine torque changes? Now in this equation has that two variables. One variable is the load torque, and the other variable is the machine torque. So, this is the load torque and the machine torque is T.

So, the load torque is changing and the machine torque or the motor torque has to fallow the load torque, and this variation is exponential because solution of this differential equation will give us an exponential salvation. So, we go back to our previous diagram which we have drawn showing the torque variation, and in the same diagram we will try to draw the variation of the motor torque that is capital T. So, this is what we have here and here if we draw the motor torque, how does it vary? The motor torque will vary like this; exponentially we have applied a high load torque, the motor will accelerate exponentially from let us say T minimum to a T maximum, and what you are drawing is a periodic waveform. So, then again when a load torque is applied T L l the motor torque will again come down exponentially.

So, raising exponentially and coming down exponentially, and this will exponentially decrease to a T minimum value, same as the previous one, and this is periodic in nature. So, we have again T maximum exponentially, and then again we have T minimum here; the decrease is also exponentially. So, this is the variation of capital T the motor torque. So, the motor torque as we can understand here the motor torque is not same as the load torque; it is much less than the load torque. In the high load region it is increasing but not exactly reaching the peak load torque that is T L h. In the low load region it is not going as low as T L l, and the differential torque is basically absorbed or catered by the flywheel. The inertial torque that is available in the flywheel.

The flywheel gives the energy when it is decelerating; the speed is decreasing here. And when it is accelerating it is giving the energy. So, this is how the variation looks like. The load torque is increasing; the actual torque is also increasing, and the load torque is decreasing; the actual torque is decreasing. Now this can be explained by means of a differential equation which you have already shown here; so this is the equation that we have. And what is the solution of this equation? The solution of this equation will exactly give us the same thing that we have seen. Now if we solve this equation, this equation will have the following solution.

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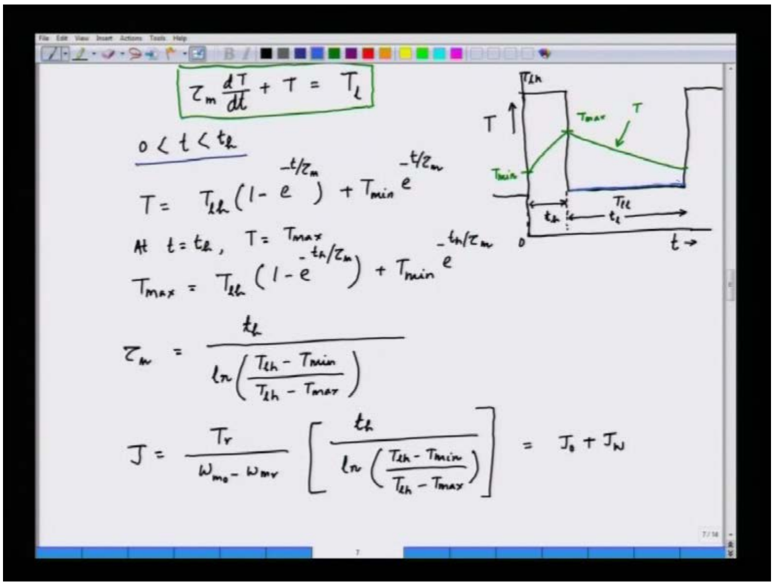

Now what we have here is this T m d T by d t plus T is equal to T l. So, we have a high load region, and we have a low load region, and this basically repeats. So, if this is T L h and this is T L l, and this time is T h, and this time is T l. We will start our coordinates from this. Let us say we have the time axis here, and this is the torque axis. And the load torque is changing in this case, and the motor torque is increasing from a T minimum to T maximum and again it is coming back to a T minimum here. Now we need to find out the equation of these curves. So, this equation can be obtained by solving the above differential equation. So, let us take the time interval from 0 to t h when we apply the high load T L h. Now in this case the load torque is T L h.

So, what is the solution of this equation? The solution of this equation T is equal to, the final value will be T L h into 1 minus exponential minus T by tau m plus the initial value is T minimum, exponential minus T by tau m. So, this is the solution of the above differential equation between 0 to T h. So, if we substitute here T is equal to T h, what we obtain is T L or T maximum. So, in this case we can say that at T equal to T h, T is equal to T maximum. So, we can say that T maximum is equal to T L h into 1 minus exponential minus T h by tau m plus T minimum into exponential minus by T h tau minimum.

Now from this we can also find out what is the flywheel inertia or tau minimum. So, tau minimum can be very easily evaluated from this as fallows. So, if we simplify this and find out what is tau minimum here. So, taw m is equal to T h divided by the natural log of T L h minus T minimum divided by T L h minus T maximum. So, tau minimum or tau m is the time constant is given as fallows; that is basically given by an expression. So, if we know tau m which is the time constant we can evaluate the inertia, and the inertia is important to us because if we know what is inertia we can find out the flywheel inertia of the overall system. So, from this equation we can find out the inertia as follows. The movement of inertia is j that is equal to T r by omega m naught minus omega m r into above expression T h divided by the natural log of T L h minus T minimum by T L h minus T maximum, and that is equal to j naught plus j w.

So, if we know the inertia of the motor that is j naught we can find out the inertia of the flywheel that is j w. Now this is a way to find out the moment of inertia of the motor and also the flywheel. The flywheel inertia has to be calculated, the moment of inertia of the motor is known, and j is the total moment of inertia of the overall system. We can also have the similar expression if we consider the other part of the graph. The other part is when the low load is applied. Now if we apply this low load we have this part of the graph which is so near for the low load. Now for this part we can have a similar expression.

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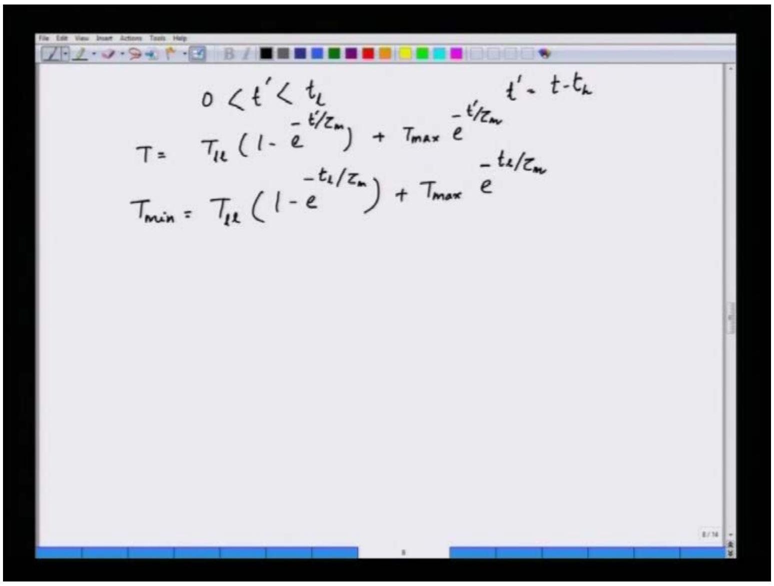

We have the equation for t prime is greater than 0 is less than t L. Now what is t prime? T prime is t minus t h. So, if we have this part if you concentrate on this part, what is the initial condition here? The initial condition is T maximum, and what is the final here? Final is t minimum. So, if we apply our equation here, and we find out the solution which is T is equal to T L l into 1 minus exponential minus t prime by tau m plus T maximum into exponential minus t prime by tau m. From this also we can calculate the movement of inertia, because we know that T minimum happens when T prime is equal to T l. So, we can apply that condition here. So, we can say that T minimum is equal to T L l into 1 minus exponential minus t L by tau m plus T maximum exponential minus t L by tau m.

And from this we can find out what is tau m, and when we know tau m we can very easily calculate the moment of inertia that is z. So, in this equation which is very simple we will get the same result that we have obtained from the other equation. So, this lecture we have already seen how to calculate the rating of the motors based on the equivalent current and equivalent torque. We have also seen how to choose the rating of the d c motor, induction motor and synchronous motors. And we have also discussed if the load has a very high peak torque, how we can equalize the load by having a flywheel attached to the rotor. So, in the next lecture we will continue with this, and we will see other type of drives those have got the practical applications; specially, the traction drives we will be discussing in the next lecture.