Advanced Electric Drives
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Lecture – 3

Hello. In the last lecture, we were discussing about the equation of Kron's primitive machine model. And we have derived the equation for the voltage in terms of the resistive drop, the inductive drop and the speed emf.

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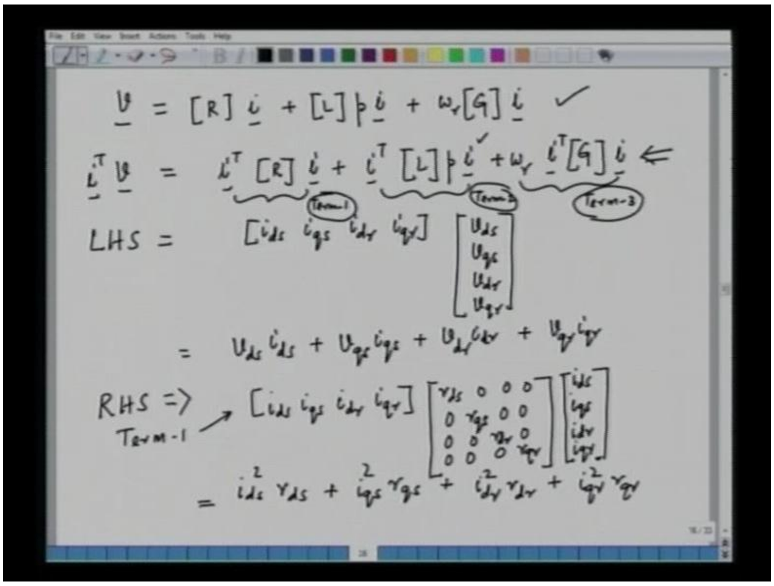

Now, this is the equation that, we were discussing in the last class that, v is equal to R i plus L p i plus omega r G i. Now, if we pre-multiply this equation with i transpose, what you obtain is i transpose R i plus i transpose L p i plus omega r i transpose G i, this equation. Now, we have already seen that, the first term – term 1 represents the losses of the system. This is basically the first term – term 1 – R transpose R i; term 1 represents the system loss – i square R loss. And the term 2 represents the power associated with the magnetic field or the power stored in the magnetic field. And the third term – term 3, which is omega r i transpose G i is the mechanical power output. This is what we were discussing in the last lecture that the second term is the power associated with the magnetic field, the third term is the mechanical output of the system.

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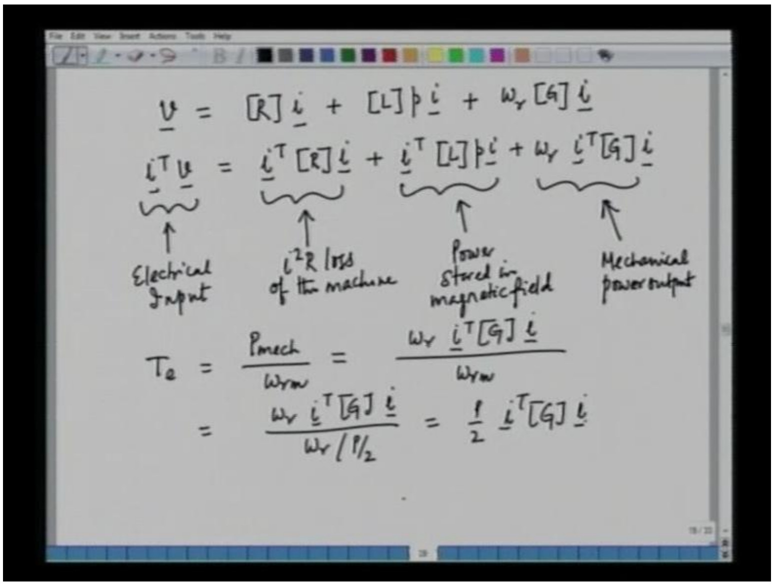

Now to just to recapitulate, we have already seen that v is equal to R i plus L p i plus omega r G i. Now, this equation is the voltage equation. If we pre-multiply this with i transpose, we get the following equation: i transpose R i plus i transpose L p i plus omega r i transpose G i. Now, this equation is interesting, because the first part is the electrical input. The second part, which is i transpose R i represents the system loss. This is basically i square R loss of the system. We know that, the Kron primitive machine model has four windings: one winding in the d-axis stator, one winding in the d-axis rooter, one winding in the q-axis stator, and one winding in the q-axis rooter. And i transpose R i – this term represents the system losses – i square R loss of the machine. And this term is associated with the inductance L. And we know this L matrix; L is a matrix we have seen in the last lecture, which has got the self inductance and also the mutual inductance.

And, this equation or this particular term, that is, i transpose L p i is the power stored in the magnetic field. So, we can say here this is the power stored in magnetic field. This is not converted to the mechanical power; this is basically the power stored in the magnetic field, which is the self inductance and the mutual inductance. And this power is not converted to the mechanical output. What we have here – this is the third term, is the mechanical output So, we can say that this is the mechanical power output. And if we find out the torque, the torque is p mech – the mechanical output by the mechanical speed – omega r m. And that is equal to… This is the p mechanical.

So, I can say here omega r i transpose G i by omega r m. And that is equal to omega r i transpose G i by omega r by p by 2; and that is equal to p by 2 i transpose G i. This is the expression for the torque. This i is a vector; i has more than one element here. So, this is the expression for the torque of a Kron primitive machine. This is what we have discussed in the last lecture.

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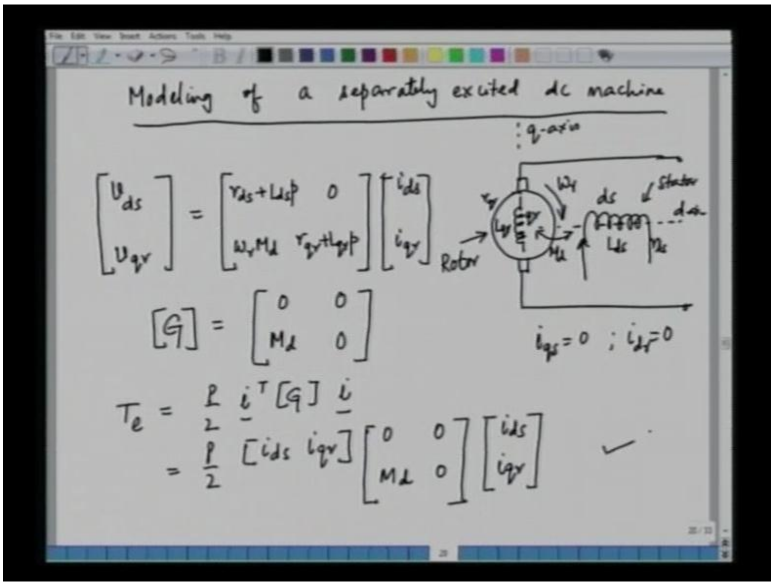

Now, in this lecture, we will try to discuss how this Kron primitive machine model can be used to simulate a DC machine. First of all, we will simulate a separately excited DC machine. Let us see modeling of a separately excited DC machine. Now, what we have here; if you see a separately excited DC machine, we have the armature and we have the field winding. And these are the armature terminals. We can apply some armature voltage here; and this is the field winding.

And, this armature winding can be replaced by a pseudo stationary winding as you have already seen that, this is the two brushes and this is the pseudo stationary winding. So, here what we have; we have only two windings: one in the d-axis; this is the d-axis; and this is the q-axis. So, we have one winding in the d-axis stator and one winding in the q-axis rotor. This is basically the rotor here. And the field is on the stator. So, we have the stator in this case.

Now, this we can call to be the ds winding; and this we can call to be the qr winding. So, we can write down the equation here. We have just two windings here. So, we do not have to write a 4 by 4 matrix equation. We have to just write here a 2 by 2 matrix equation, because we have only two windings in this case. Other windings are nonexistent. So, we can write down the equation like this – v d s and v d r. This is the rotor and this is the stator. That is equal to… We have to write down the impedance matrix. And the currents are i d s and i d r. So, we can write down the equation here that, v d s equal to r d s plus L d s p.

And, this is 0. And then we have r d r plus L d r p. And the rotor as we have already seen that, in rotor, we have statically induced emf as well as the rotationally induced emf, because this is rotating in the clockwise direction at a speed of omega r. So, the rotor will have rotationally induced emf as well. So, r d r is the resistive drop; L d r p is the inductive drop or statically induced emf. And we have omega r M d here. This is the rotationally induced emf. There is a small change here; this is the q-axis. So, we have in this case, v q r. And this is i q r; and this is r q r plus L q r p. So, this is the resistive drop; the statically induced emf and the rotationally induced emf in the q-axis rotor.

Now, of course, we have already said here that, we do not have any q-axis stator current and we do not have any d-axis rotor current. So, these two variables are 0. So, we have a 2 by 2 matrix here; and. we have to model the DC motor using this 2 by 2 matrix. So, in this case, we can find out what is this G matrix. G matrix is the matrix associated with the speed, that is, omega r. So, we can write down this G matrix. This G matrix would be 0 and 0 here and this is M d and 0. This is the G matrix. It is a 2 by 2 matrix associated with the speed, that is, omega r.

And, we can find out the torque equation in this case that, as per the Kron's primitive machine equation, we can say that, T e is equal to p by 2 i transpose G and i. Now, here we can say here that, the torque equation is as follows: p by 2… What is i transpose? i transpose is i d s i q r. This will be a row matrix. i is a column matrix. And if you transpose it, it will be a row matrix.

And then what is the G? This is the G matrix. And then we will post-multiply with i. This is what we have. And if we do this; this M d is a mutual inductance between the stator d-axis and the rotor d-axis. This is actually M d. And L d s is the self inductance of the stator. L d r, L q r is the self inductance of the rotor. So, these are the various inductances here. And similarly, the resistance of the stator winding is r d s. And the resistance of the rotor q-axis winding is r q r. So, these are the various parameters of the machine, which are given here.

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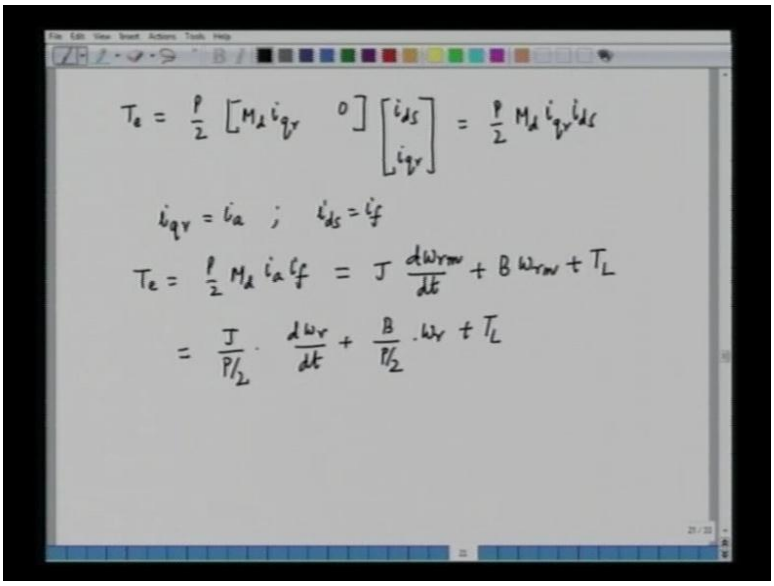

And then if you simplify this torque equation, we will get the following expression. Now, T e is equal to p by 2 M d i q r, 0; i d s and i q r. Now, this we can simplify this as p by 2 M d i q r into i d s. So, this is the expression of the torque of a separately exited DC motor. The field is separately excited. And in this case, you can see that, i q r. What is i q r? i q r is the current in the rotor winding.

So, i q r is same as armature current. So, if you have… This is the armature current; I can say this is i a; i a is same as i q r, because this is going through this winding. And this is the i d s and this is same as the field current. So, i d s is same as the field current and i q r is same as the armature current. So, we can replace this i q r by i a, is the armature current; and i d s by i f is the field current. So, we can say that, T e is equal to p by 2 into M d into i a into i f. So, this is the torque equation of a separately exited DC motor.

So, we have the voltage equation. In this case, this is the voltage equation. And we have the torque equation and we can find out the speed from the torque. How do we find the speed from the torque? This is the toque equation. So, we can write down the electromechanical equation to find out the speed. Now, that is equal to J d omega r by d t. Now, omega r is the electrical speed. So, if you want to find out the torque, this has to be the mechanical speed – omega r m. And this J d omega r m by d t is the inertial torque. J is the moment of inertia; and d omega r m by d t is the angular acceleration of the rotor – the mechanical angular acceleration of the rotor. We can also have the viscous friction, that is, B omega r m. Again, omega r m is the mechanical speed and B is the coefficient of viscous friction. Of course, you can also have the load toque – T L. So, this equation… If we solve this equation maybe numerically, we can get the expression for omega r m; omega r m is the mechanical speed.

Now, we can also simplify that, if you express this omega r m in terms of omega r, we can divide by p by 2. So, J by p by 2 into d omega r by d t plus B by p by 2 into omega r plus T L. p by 2 is the pole-pair; p is the number of pole of the machine; and p by 2 is the pole-pair. Now, this is a first order differential equation; we can solve for omega r. So, this actually… We have the model of the DC machine, which has been obtained from the Kron primitive machine theory. And we can find out the current, find out the torque, and then find out the speed.

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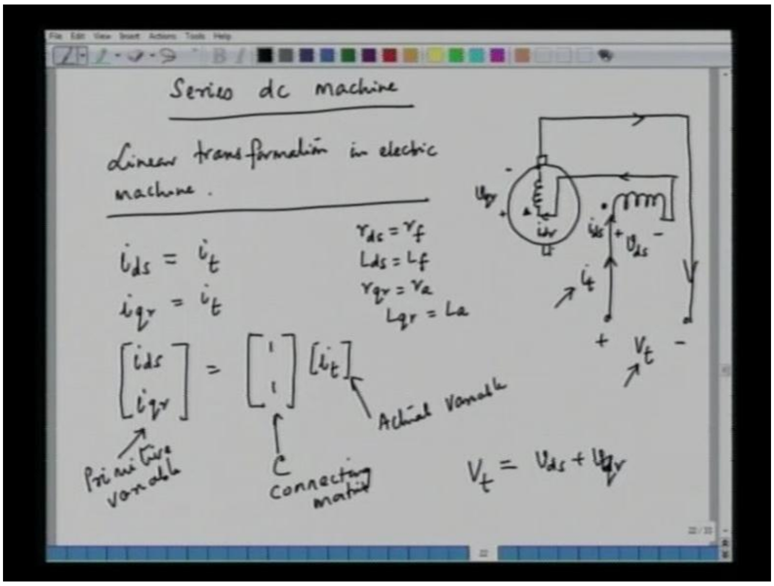

Now, let us take a little complex problem. Say for example, if you simulate a separately excited DC machine; this is… We have already done a separately excited DC machine; we can now go for little complex problem, that is, series DC machine. In series DC machine, what we have; we have the rotor; the rotor is the armature; and then we have the field winding; and they are series connected. Let me just make the connection here. Now, this is the stator – d-axis stator. We have this voltage as v d s. This is the rotor winding; we only have q-axis rotor winding. And this voltage will have v q r. And the current is flowing in this particular direction, is returning back to the supply in this particular fashion. So, when we go for series DC machine, it is little complex problem. So, we cannot solve it directly. So, we have to take the help of what is called the linear transformation. So, we will be discussing about a linear transformation in electric machine. So, we will be discussing about linear transformation in electric machine.

Now, what is the meaning of linear transformation, Linear transformation is a transformation, which transforms the actual variable to the primitive variable without any nonlinear term. Nonlinear term we mean there should not be a product of two variables; there should not be a square of a variable. Say how we can tackle this particular problem? Now, this series DC machine have been connected like this. The armature is in series with the field winding. And we can write down this equation that, i d s here… Now, we have one stator and one rotor. This is the i d s – d-axis stator winding. And this one is i d r.

So, we can say that, i d s equal to i t. Now, i t is the terminal current or the machine current. And similarly, I can say that, i q r is also equal to i t, because the field current is same as the armature current. So, i d s is the field current and i q r is the armature current; and both of them are same. So, we can say that, i d s equal to i t and i q r also equal to i t; i t is the machine current that is coming from the supply. So, we can write down this in a matrix form.

So, we can say that, i d s and i q r is a matrix. That is equal to… I will have another matrix; it is a column matrix – 1, 1 into i t. i t is just a single variable. So, this is an interesting equation, which relates the actual variable with the primitive variable. Now, this variable is the actual variable. I can say that, this is the actual variable. Now, this variable is the primitive machine variable. So, I can say that, this is the primitive variable. And this matrix – 1, 1 – this is called the connecting matrix. So, I call this to be matrix C. And C is called the connecting matrix. So, you know that, what we are trying to do; we are relating the actual variable with the primitive variable by a linear transformation. That is the meaning of linear transformation. And this 1, 1 is not a nonlinear transformation; it is a linear transformation, which relates the actual variable with the primitive variable. Now, this is about the current. What about the voltage?

Now, let us see the voltage here. Now, if you see in this case; the terminal variable is v t; this is v t. If I write down the Kirchhoff’s voltage law, I can say that V t is equal to v d s plus v d r. This is a simple equation, which can be written in a straightforward fashion from the circuit equation. So, I can rewrite this in a matrix form.

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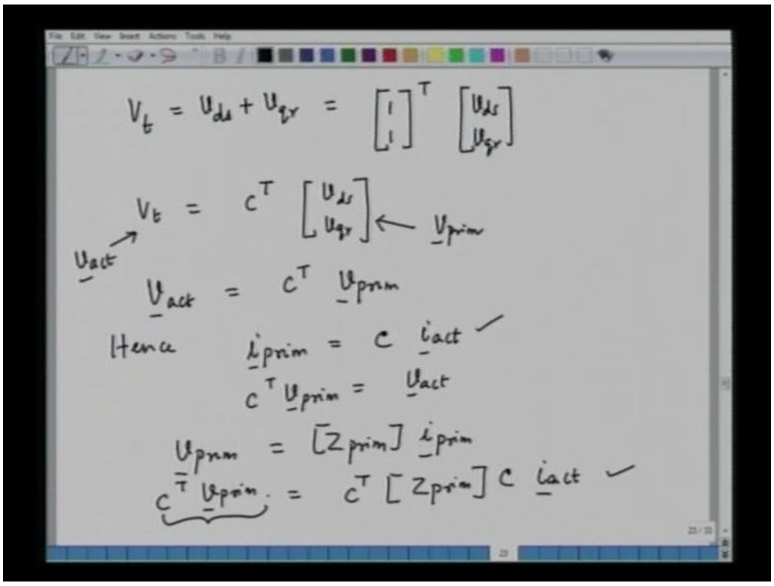

Now, if I rewrite this in a matrix form, I will have the following expression: V t is the actual variable; that is equal to v d s plus v q r. That is what we have written in the previous thing. And that is equal to the transpose of this matrix. So, this is the equation once again. And that is equal to… I can say here the transpose of the C matrix into v d s and v q r. So, I can say here that, v actual is equal to C transpose into v primitive. This is the v primitive; and V t is the V actual. So, I have two sets of equation; that we have i primitive is equal to C i actual; and I have C transpose v primitive is equal to v actual. So, I have two equations. And the actual variables are related to the primitive variables by a linear transformation, that is, C.

Now, you know that, the objective of linear transformation is to take help of Kron primitive machine model and get back the variables in the actual variable form, because the primitive variables like i d s, i q s, i d r, i q r or v d s, v q s, v d r and v q r – these are the variables of the primitive machine; they are hypothetical variable; they are not the real variable. The real variables could be something different from the hypothetical variables. So, if you have the transformation matrix, that is, C, you can write down the equation of a primitive machine, which is from the Kron primitive machine model. And using the linear transformation, you can get back the variables in the actual machine variables.

Now, in this case, it is a series motor and the machine variables are i t and V t. These are the actual variables: V t and i t. So, we have to find out the torque and the voltage equation in terms of the actual variables. So, this is what we have here. And we know that, v primitive is equal to Z primitive into i primitive. So, this is basically the voltage equation; and Z primitive is the impedance matrix of the primitive machine, that is, the Kron primitive machine; and i is the primitive current of the primitive machine. Now, I can pre-multiply this equation by C transpose. So, I have the C transpose here. And i primitive can be written like this – that is, C into i actual. So, this equation will lead to a different result like C transpose into v primitive. This is same as v actual. So, I can write down this in the following fashion.

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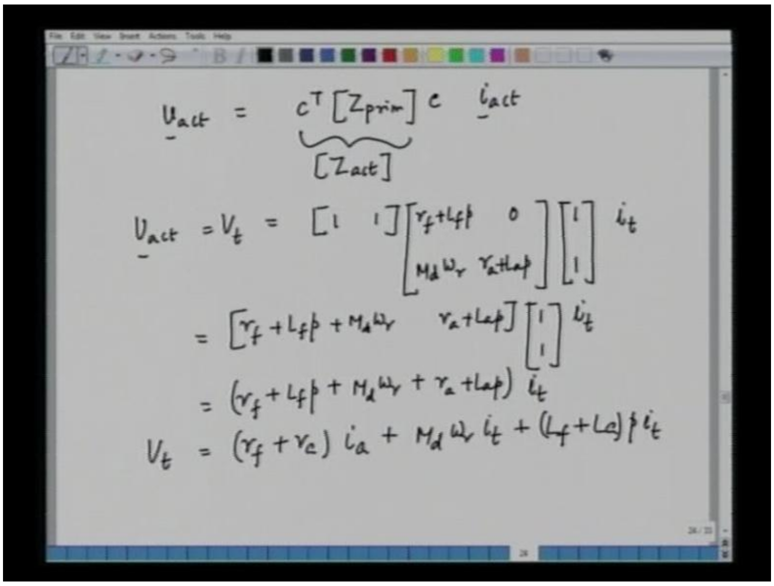

v actual is equal to C transpose Z primitive C into i actual. Ultimately, I have to get the equation in the actual variable. So, the impedance matrix Z primitive can be transformed with C transpose pre-multiplied and C post-multiplied. So, this is the Z actual. So, this is the advantage of linear transformation. It means using the linear transformation, I can get back the impedance matrix in the actual variable – in the actual of the actual machine. So, for this equation, what is Z primitive here?

So, we can say that, v actual in this case is V t; and C transpose is 1, 1; and Z primitive is given by r f plus L f p, 0; M d omega r, r a plus L a p. And it is post multiplied by C; and i actual is i t. Now, in this case, if you go back to the previous slide, you see here that, in the stator, we can say that, i d s r d s equal to r f. This is the field winding. L d s is equal to L f – the field inductance is same as the L d s. And similarly, r q r is equal to r a – the armature resistance; and L q r is equal to L a, is the armature induction. So, these are the various values of the parameters of the machine.

Now, we can replace this r d s by r f, L d s by L f, r q r by r a, L q r by L a. So, this equation is the equation of the DC series motor in actual variables. Now, we can simplify this, because this is a matrix equation. So, if we simplify this, we can pre-multiply and see what happens here. So, it is r f plus L f p plus M d omega r. And then here r a plus L a p. And then we can post-multiply this and see r f plus L f p plus M d omega r plus r a plus L a p. So, we can also simplify this.

We will have r f plus r a into i a plus M d omega r i t plus L f plus L a into p of i t. So, this equation that is equal to V t, is the equation of a DC series motor. We have seen that, in a series motor, the field and the armature are in series. So, the resistive drop is r a plus r a. And then the inductances are also in series. So, the self inductance is L f for the field winding; the self inductance is L a for the armature winding. So, it is L f plus L a into p of i t; i t is the current. And the rotationally induced emf is M d omega r i t. That is also appearing in this equation.

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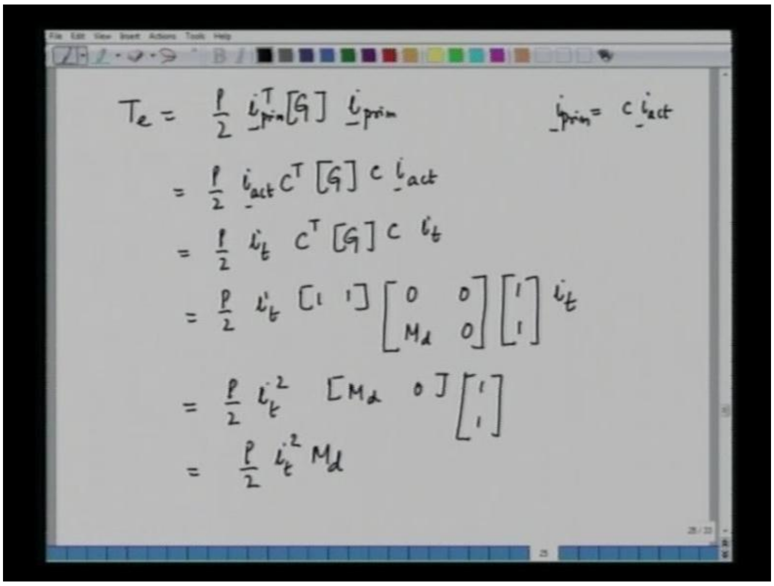

Now, we can directly find out the torque equation taking the help of the primitive equation. So, let us see how we can find out the torque. Now, we know that, the torque is given by p by 2 i transpose G i. Now, these are the primitive currents. I can say primitive. So, I know that, the primitive current – i primitive is equal to c into i actual. This i; we know this. So, I can replace this primitive by C i actual. So, that will be C transpose i actual G C… This is i actual C transpose G C i actual. And i actual is i t; this is C transpose G C i t.

And, I can write down what is this G matrix. C transpose is 1, 1; and the G matrix is 0, 0, M d and 0. So, further simplification will lead to i t square; i t is here and i t is also here. So, then I have M d and 0 and post multiplied by 1 and 1. And then what I finally obtain is p by 2 i t square M d. So, this is basically the expression for the torque of a DC series motor. So, this we have obtained from the Kron primitive machine model using the principle of linear transformation in electric machine.

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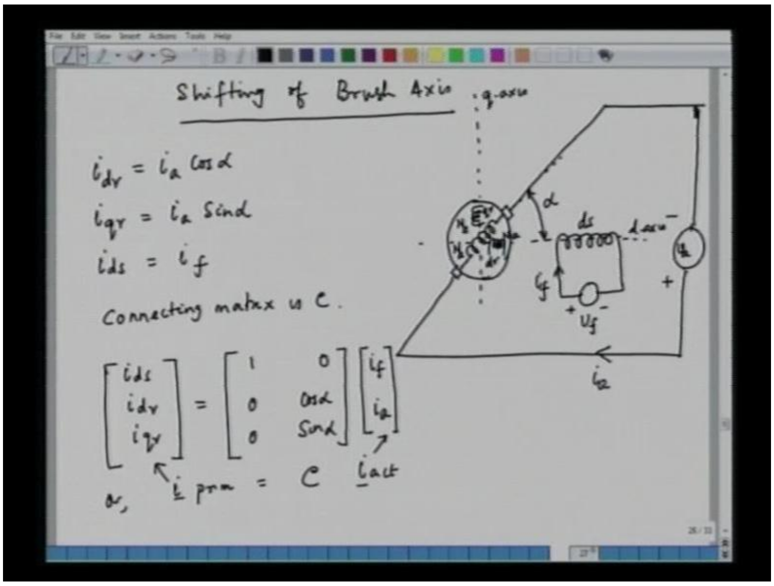

Now, let us take a different problem. We will take a separately excited DC machine; but in this case, the brush is little shifted from the magnetic neutral axis. So, what we have here is this; that we have a DC machine here; this is the field winding. We apply the voltage v f here; this current is i f – the field current. But, the brushes are not in the magnetic neutral axis. That is not in the q-axis. This is the d-axis and this is the q-axis. The brushes are shifted and they are placed in this direction. And this angle is alpha. So, this is the situation that we have; I meant we have little different type of DC machine here, where the brush axis is shifted from the magnetic neutral position. This may be true when we have the shifting of the brush from the magnetic neutral position. We will have the situation like this. And this is the armature; and armature is applied with a voltage. So, this voltage is v a. And this is passing a current here, that is, i a.

And, we have a pseudo stationary winding as we have already seen that, the armature can be replaced by a pseudo stationary winding; the commutator and the brushes can be replaced by a pseudo stationary winding; the axis of the pseudo stationary winding is the brush axis. So, this is a pseudo stationary winding. And what we can do here; we can have a component of the winding; we can have a d-axis rotor and a q-axis rotor. And this is dr and this one would be qr.

So, if the original armature winding was in a, this winding will also have the number of turns that is equal to N a. The q-axis winding will also have the number of turns that is equal to N a. So, what we want to do here is the following; that since we have the armature winding in some arbitrary axis inclined with the d-axis at an angle alpha, we can simulate that by having a d-axis rotor winding, that is, dr and q-axis rotor winding, that is, qr. And the number of turns of this dr and qr will be the same as that of the original armature winding, that is, N a. So, the number of turns of this winding are same as the armature winding number of turns, that is, N a.

And, we can say that, i d r – if you equate the mmf in this case, we can say i d r is equal to i a cos alpha; very naturally, we can say that, this is the projection of this along the d-axis. So, this is i a cos alpha. And i q r is equal to i a sine alpha. So, we can… So, we are talking about the shifting of the brush axis and i d r equal to i a cos alpha and i q r equal to i a sine alpha. And this is the d-axis stator winding d s. So, we can also say that, i d s is equal to i f. So, we are relating the primitive variables with the actual variables. Actual variables are here – the armature current and the field current.

And, the primitive variables are i d r, i q r and i d s. So, we can write down the connecting matrix in the following way. So, the connecting matrix is C. And we can say the i d s, i d r, and i q r; that is equal to… This is i f – the field current and the armature current. And this matrices is 1, 0, because i d s equal to i f, and then i d r equal to i a cos alpha. So, this is 0 cos alpha. And i q r equal to i a sine alpha. So, this is 0 and sine alpha. Or, we can say that… It means we can write here that, i primitive is equal to C – the connecting matrix C into i actual. So, this is the i actual and this is the i primitive. So, this C has been determined. C is equal to 1, 0; 0, cos alpha and 0, sine alpha. So, in a similar way, we can write down the expression for the voltage. So, the voltage and currents are related.

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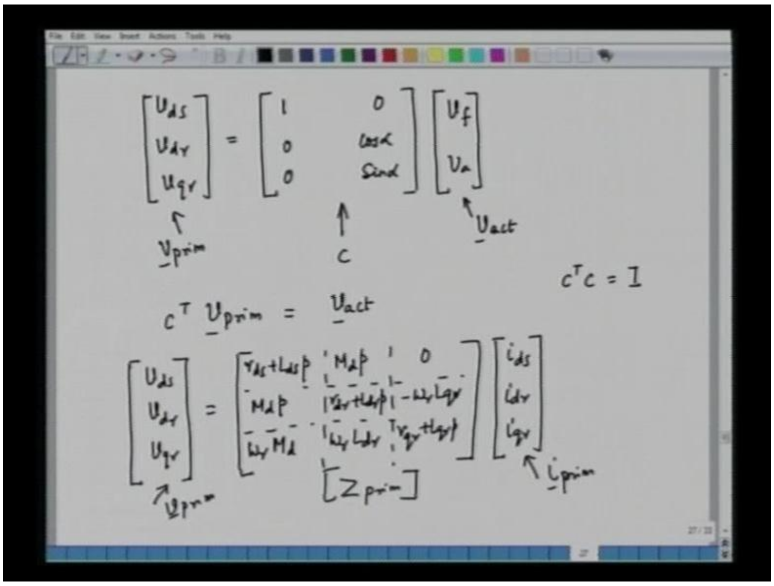

So, we can write down in this case, v d s and v d r and v q r are the voltages of the primitive variables in terms of the actual variable voltage, that is, the v f and v a. Now, what is v f? v f is the field voltage here and v a is the armature voltage, that is, the v a applied here. So, we can rewrite like this. And v d s equal to v f. We can see here that, v d s and v f are the same. This is v d s. And this terminal is a positive terminal closer to the center as per the convention. So, v f is equal to v d s. And what about v d r? v d r would be v a cos alpha just like the currents. See if the current is multiplied by cos alpha, voltage will also be multiplied by cos alpha, because only when the voltage is reduced, current is also reduced.

Similarly, we can say that, v q r is equal to v a into sine of alpha. So, will have here that, 1 and 0; v d s equal to v f; and v d r is v a cos alpha; v q r is v a sine alpha. So, this is the matrix, which relates the primitive variables. This is… The primitive is here. And this is the v actual. So, we will pre-multiply with C transpose. This is the C. Now, if you pre-multiply with C transpose, C transpose v primitive. Now, if you C transpose into C here, that will be i matrix. So, we can that, C transpose v primitive is v actual. So, if you pre-multiply this with C transpose; in this case, C transpose into C is an identity matrix. That can be verified here. So, we can say here that, C transpose into v primitive is equal to v actual. So, this is what we have for this situation.

Now, if you see that, the primitive equations are as follows; we have three windings here. And the windings are ds, dr and qr. And similarly, we will have three different currents. And the currents will be i d s, i d r and i q r. So, we have to fill up this 3 by 3 matrix. You can see here that, we have three windings; that is, ds, dr and qr. And correspondingly, we have v d s, v d r and v q r; i d s, i d r and i q r. So, that is what we have written here. And this would be the resistive drop plus L d s p. We can fill up this matrix by inspection; we did not remember this.

The stator does not have any rotationally induced emf; the rotor will have the rotationally induced emf, because this is rotating in the clockwise direction at a speed of omega r. The rotor is in rotation. So, there is a mutual coupling between the d-axis rotor and the d-axis stator, that is, M d p and 0. Similarly, for the second row, we have r d r L d r p. Now, this is M d p. And we have the rotationally induced emf, that is, omega r L q r. The third row would be r q r plus L q r p. And then we do not have any coupling with the d-axis stator, but that will be rotationally induced emf. So, we have omega r into M d. And here we have omega r into L d r. So, this is the v primitive and this is i primitive and this is the z primitive.

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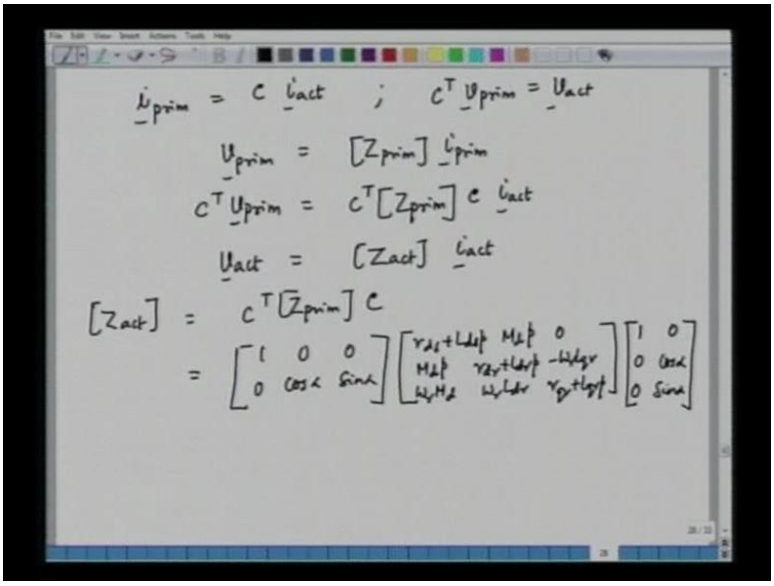

Now, we want to have the equation in the actual variable. So, we have to use the transformation here. So, using the transformation, we can transform this primitive equation into the equations of the actual variables. So, we have here that, i primitive is equal to C into i actual and C transpose v primitive is equal to v actual. And we also have v primitive is equal to Z primitive into i primitive. This we have already derived. v primitive is equal to Z primitive into i primitive. And here if you pre-multiply this by C transpose, C transpose v primitive is equal to C transpose Z primitive. We can replace i primitive by C into i actual. So, this equation will give us the equation with actual variables, because C transpose v primitive is v actual; v actual is equal to Z actual into i actual. So, Z actual in this case is C transpose Z primitive into C.

So, we can evaluate what is Z actual. So, Z actual has given us C transpose Z primitive into C. And that is equal to C transpose is 1, 0, 0; 0, cos alpha, sine alpha; and then Z primitive is r d s plus L d s p, M d p and 0; M d p, r d r plus L d r p, minus omega r L q r; omega r M d, omega r L d r r q r plus L q r p. And then we can post-multiply with C, that is, 1, 0; 0, cos alpha; 0 sine alpha. So, this is what is Z actual. So, we can simplify this; I will just write down the simplified equation; we can pre-multiply this and post-multiply this and write down the simplified equation.

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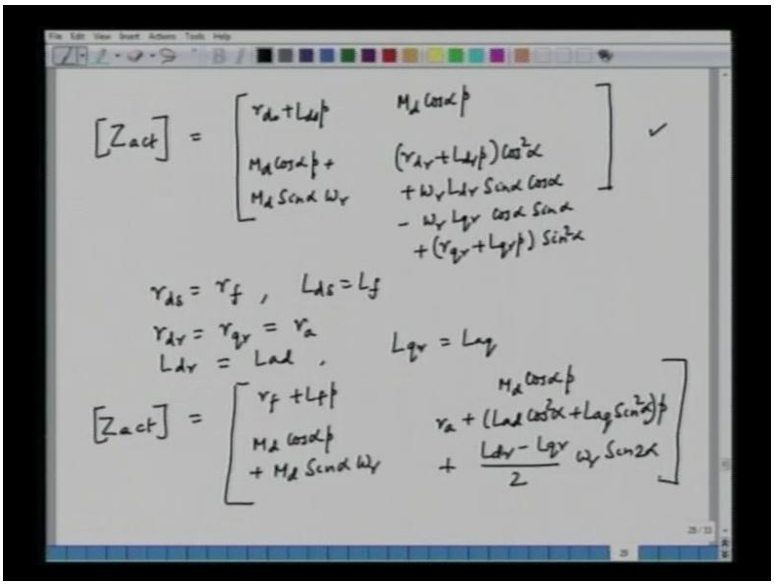

So, let me just we write down the simplified equation of Z actual. Z actual after simplifying, is obtained in the following fashion, which is r d s plus L d s p, M d cos alpha p; p is the derivative operator – d by dt; small p is d by dt; and M d cos alpha p plus M d sine alpha into omega r. Then here we have r d r plus L d r p cos square alpha plus omega r L d r sine alpha cos alpha minus omega r L q r cos alpha sine alpha plus r q r plus L q r p into sine square alpha. So, this is Z actual. Now, we can replace r d s by r f – the field resistance; r d s is the field winding resistance; the d-axis stator is basically the field winding. So, r d s is equal to r f. And L d s is equal to L f; r d r equal to r q r. That is equal to r a. The two windings in the d-axis and q-axis rotor have same number of turns; each one is having the same number of turns. And that is the resistance of armature; that is equal to r a.

And then L d r is equal to L a d – the armature inductance in the d-axis. And L q r is equal to L a q – the armature inductance in the q-axis. So, when we substitute this r d s, r d r, r q r, L d s, L d r, L q r in the equation for the Z actual, what we obtain is the following. So, we can say that, Z actual after simplifying will be of this sort – r f plus L f p. Now, this is M d cos alpha p. And here we have M d cos alpha p plus M d sine alpha omega r, r a plus L a d cos square alpha plus L a q sine square alpha p plus L d r minus L q r by 2 into omega r sine of 2 alpha. So, this is the equation of the Z actual. It means the actual impedance matrix of this machine has been found out by using the linear transformation. So, the last thing what remains here is that, we have been able to find out the voltage equation, because once we know Z actual, we can write down v actual is Z actual into i actual. The voltage equation has been written; we have to find out the torque equation.

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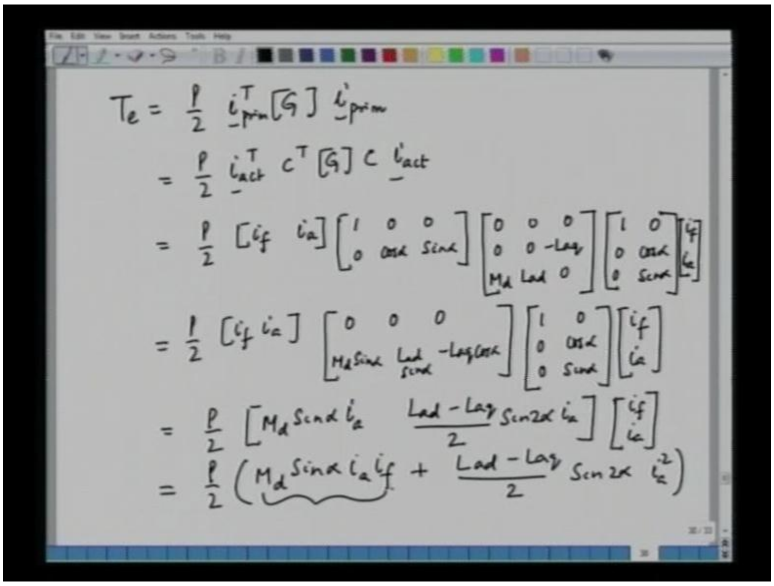

Now, the torque equation can also be found out from the Kron primitive machine equation. We know that T e is equal to p by 2 i transpose G i. So, this we have seen from the Kron's primitive machine equation. Now, this i is i primitive. And here is i is i primitive. So, again, we replace i primitive by C into i actual. So, that is we could do, What we have here is i actual transpose into C transpose G C i actual. Now, that is equal p by 2. What is i actual? i actual is i f i a. And the transpose of i actual is a column will be a row matrix. That will be i f i a row matrix. What is C transpose? C transpose is given by 1, 0, 0; 0, cos alpha, sine alpha. And the G matrix is the matrix associated with speed torque. So, we can find out this G matrix from the primitive machine equation that is 0, 0, 0; 0, 0, minus L a q; M d, L a d, 0. And then we will post-multiply with C; the C is 1, 0; 0, cos alpha; 0, sine alpha. And then we have i actual; i actual is i f, i a. So, this…

If we simplify this equation, we will get the equation for the torque in the actual variable, because actual variables are i f and i a. The field current and the armature current respectively. So, we can simplify this. So, if we pre-multiply this, what we have here is i f, i a and this would be 0, 0 and 0; M d sine alpha, and L a d sine alpha, and then minus L a q cos alpha. So, this is basically a 2 by 3 matrix: 2 rows and 3 columns. And then we post-multiply this with if C; C is 1, 0; 0, cos alpha; 0, sine alpha. And then we have the current, that is, i f, i a. We can further simplify this and get the final equation. So, further simplification will give us p by 2; we can post-multiply this, and then we can pre-multiply with i a into i f. So, what we obtain here is that, M d sine alpha i a, L a d minus L a q by 2 into sine of 2 alpha into i a; and post multiply with i f, i a.

Now, further if you simplify this, we get p by 2 M d sine alpha i a i f plus L a d minus L a q by 2 into sine 2 alpha i a square. Now, this equation is an interesting equation in the sense that, there are two components of the torque: one component is coming because of the first term. Now, this is i f into i a. This is something like a normal torque of a DC machine: the product of i a and i f. But, the second component is basically proportional to i a square. So, this is not the convention; I mean in case of a DC machine, the torque is produced by interaction of field current with armature current. But, the second term shows that, this torque is proportional do i a square. So, you can see here that if you take the original machine in this case, the alpha; for a normal machine, alpha is pi by 2; alpha is 90.

Now, if you substitute alpha is 90 in this equation; you see that, sine of 90 is 1. So, this is M d into i a into i f, which was the equation for a separately excited DC machine. And here if you put alpha is 90, it will be sine of 180; that is equal to 0. So, this torque will be 0 if alpha is 90. So, this is actually for this lecture. So, in this lecture, what will have studied is the following. To summarize, we have seen how the generalized theory of electric machine can be used to simulate a DC machine. We have taken the example of DC separately excited machine – DC series motor and also a DC machine, where the brush is shifted from the magnetic neutral axis.

And, we have seen the equation for the torque. We have also seen how linear transformation in electric machine will be helpful in transforming the variables from the Kron primitive machine model to the actual machine model. So, this is a linear transformation, because this transformation does not involve any product of variables or any higher square or cube of the variable. So, in the next lecture, we will be discussing the modeling of induction machine, which is a three-phase machine, which is an AC machine, which is more complex than a DC machine. And we will see how the generalized ((Refer Time: 01:00:49)) machine can be applied to simplify the model of a three-phase induction machine.