Advanced Electric Drives

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Lecture - 25

Hello, and welcome to this lecture advanced electric drives. In the last lecture, we have just started the dynamic modeling of brushless DC drive, we will start form that. So, let us take a look at the dynamic modeling of brushless DC drive. For simplicity, we will consider a cylindrical rotor and the stators have 3 phase winding a, b, and c. The rotor is the permanent magnet rotor, but have been a cylindrical structure, and hence the air gap is uniform. So, let us took a look at the dynamic modeling of dusted decimated drive.

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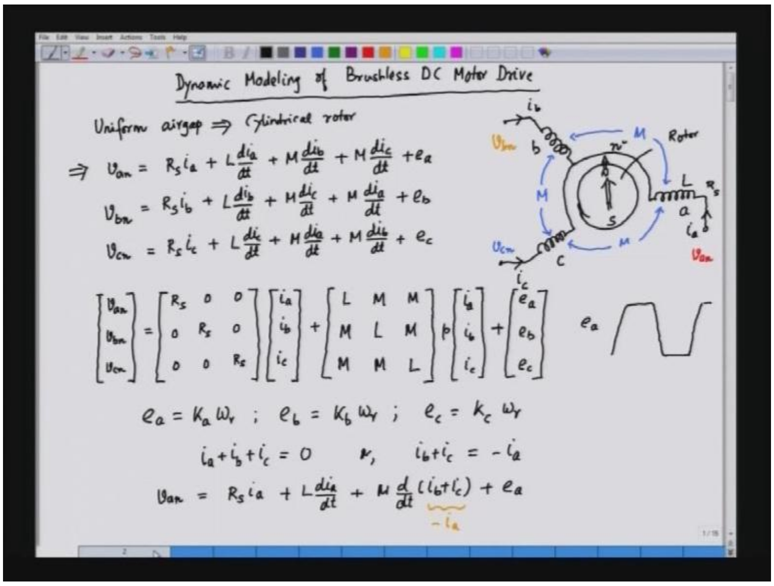

So, we are talking about the dynamic modeling of brushless DC drive for simplicity we will be taking a cylindrical structure in which the air gap is uniform. So, we have a motor in which the air gap is uniformed. So, we will take uniform air gap which means cylindrical rotor, the stator have 3 phase winding which could be distributed. And the coupling has the 2 phase we have coupling rotor is the cylindrical structure. So, we have we have a rotor and then we have the stator phase a phase b and phase c and this is the rotor. So, the stator phases are ABC and this could be star connected. So, in that case we can connect them in star, and this a neutral point that is n.

So, we have applied the voltage to the stator phase a phase b and phase c in the stator currents are i a i b and I c. Now, we will assume that e stator winding has a inductance of L, the self inductance is L and the mutual inductance between 2 phases will be M. So, the capital L is the self inductance and the mutual inductance between 2 phases would be M. And this inductance is between say phase a and phase b and similarly between phase a and phase c is also M. Now, if we write down the dynamic equation of phase a, we will say the following that the voltage v a n is equal to each phases having a resistance that is R s.

So, we can say R s into i a the self inductance is L d i a by d t and M is the mutual inductance we can say M d i b by d t that the mutual between a and b phase a and phase b and then we will have the mutual between phase a and phase c M d i c by d t and each phase will have a back k M f because the rotor is the permanent magnet rotor this could be a north pole here we have south pole here and this is the permanent magnet structure. So, when the rotor is rotating the flux linkage will be is the direction of the flux we just coming out of the rotor like this. And this is linking stator and as a result each phase will have a induced d M f. And we know that induce d M f in a brushless DC motor is trapezoidal in nature

So, in fact, the induce d M f nature we have seen in the last lecture is trapezoidal. So, it looks like this, in every phase we have an induced d M f which is trapezoidal in nature. So, this could be the induce d M f is phase phase a. So, we will put here plus e s similarly, we can write down the equations for phase b and phase c, for phase b will have v b n. The voltage of phase b with respect to the neutral point and the neutral here is n that is equal to R s into i b plus L d i b by d t. This is the self inductance the self inductance drop of phase b plus M d i c by d t plus M d i a by d t and this is phase b. And in phase b also, we have some induce d M f and the induce d M f in phase b e b plus e b e b is also a trapezoidal way form which is phase shifted from e a by 120.

Similarly, we can have the voltage equation for phase c v c n is equal to R s i c plus L d i c by d t plus M d i a by d t plus M d i b by d t plus e c and e c is a induce d M f in phase c. Now, we have 3 equation, the rotor is a permanent magnet; the rotor does not have any winding. So, we do not have to write a equation for the rotor. So, this 3 equation can be represented in the form of a matrix equation. So, we can write down this in the form of a matrix and matrix equation. So, v a n v b n and v c n is a vector is a column vector that is equal to we have the resistance matrix that is R s 0 and 0 it is a diagonal matrix 0 R s and 0 0 0 and R s into i a i b and i c. And the 3 currents of 3 phases plus we have the inductance drop and the inductance we have the self inductance of a phase and the mutual inductance between 2 phases. So, that inductance drop can be written again in the form of a matrix.

So, we have the self inductance L and the mutual between a and b is M and the mutual between a and c also M is the inductance matrix. And this is multiplied by the derivative operator is p and the currents are i a i b and I c. So, L M M is for the first row, the second row we can write down the equation for phase b. So, this is the self inductance of phase b and then the mutual between b and a is M and the mutual between b and c is M. Similarly, we can write down the equation for the phase c the self inductance and the mutual inductance drops the self inductance is l. So, we can have here L p i c and the mutual between c and b is M and the mutual between c n a is M.

In fact, if we see the 3 phases are symmetrical. So, the mutual between phase a and phase b is M phase b and phase c we can say that this mutual inductance is also M. Because of the symmetry they are pre sifted from each other by 120 degree and the of the mutual inductance between phase a and phase b phase b and phase c phase c and phase a are all equal and the mutual inductance here is M. So, this will be added with the back e M f and the back e M f of the respect, we know that is e a for phase a e b for the phase b e c for the phase c. So, we have the voltage equation of a 3 phase machine where the rotor is a permanent magnet and the rotor rotation is reflected as a back e M f in the 3 phases.

Now, in fact, we can also say that the back e M f's e a e b and e c are all proportional to omega R. So, we can say that e a is equal to there is a function which is k a; k a is not a constant which is a function and this is function of theta r into omega r. Similarly, we can say that e b is again a function k b into omega r and e c is again a function that is k c into omega r.

So, the induced d M f here are the functions of the speed rotor speed and omega r in this case it is electrical speed. Because we are assuming a 2 pole structure in general rotor could be a p 4 structure. So, we have to appropriately take into account the pole pear. Now, these are the induce d M f, now can we simplify this set of equations we have 3 equation. But there is a possibility of simplification and out of this 3 equation we need the currents. Currents are not known we have apply the voltage here this voltage is v a n and this voltage is v b n phase b and the other voltage is v c n.

So, the voltages are given they are independent variables we can apply the voltage to the machine by using a inverter. Now, when we apply the voltage the currents will be flowing into the machine now this we can simulate. So, the currents up to be calculated I have to be found out by numerical technique. So, we can simplify this set of equation in the following fashion. Now, we know that we have a star connected machine the machine is star connected. So, we can say that i a plus i b plus i c is equal to 0 there is no neutral connection the neutral the forth wire is not there.

So, we have a 3 phase 3 wire system. So, we can say that i a plus i b plus i c is equal to 0 or we can say in this case i b plus i c is equal to minus i a. So, we take the first equation; the first equation is this voltage equation is to take v a n is equal to R s i a, we will have here in this case plus L d i a by d t. We can have here plus M d by d t of i b plus i c plus e a now here we have a term which is d by d t of i b plus i c and i b plus i c can be replace by minus i a. So, we will have this and this we will be replacing by minus i a. So, if we do that and derive this equation will have the following equation. So, for the phase a will have this following equation.

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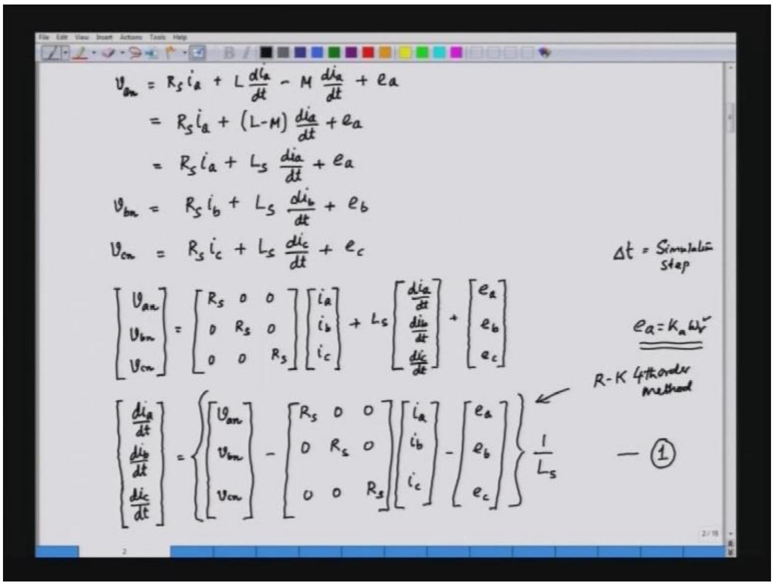

After simplification v a n is equal to R s i a plus L d i a by d t minus M d i a by d t plus e a or we can also write down this equation in the following fashion R s i a plus L minus L

M d i a by d t plus e a. Now, this equation is much more simplified then the previous equation for phase a. In the phase a we had expression for i b and i c also, but this particular equation only involve i a. So, it is a simplified equation compare to the previous equation. The previous equation was this equation where we had both i a i b and i c now we have simplified this equation. And the equation that we have is only having d i a by d t and also i a. So, we can further rewrite this equation as R s i a plus L s d i a by d t plus e a. We can say L s a something like the inductance of phase a, because as it appears from this equation that phase a can be treated independently. Because we have a star connected machine i a plus i b plus i c equal to 0.

So, by virtue of that equation i a plus i b plus i c equal to 0, we can independently consider each page. So, L s we can say the inductance of each page. Similarly, for phase b we can have R s i b in a same way L s d i b by d t plus e b and also for phase c. We can say v c n is equal to R s i c plus L s d i c by d t plus induce d M f that is e c. So, we have now 3 equations and 3 unknowns and unknowns are i a i b and i c.

This 3 equation can also be written in the form of a matrix equation as follows we can write down this as a matrix equation v a n v b n and v c n. That is equal to R s 0 and 0 0 R s and 0 0 0 R s i a i b and i c, the resistance drop plus, we have the inductance matrix inductance matrix is L s here L s in this case. And we will see in this case d i a by d t d i b by d t d i c by d t inductance L s is the same in all 3 phases. So, we have the derivative of the current plus the back e M f e a e b and e c.

Now, we need to find out the values of d i a by d t d i b by d t and d i c by d t. So, we can we can take this out in the left hand side we can write down d i a by d t d I by d t d i c by d t. And the right hand side what we have here? We have the voltages the voltages are v a n v b n and v c n and then minus the resistance drop R s 0 0 0 R s 0 0 0 and R s. Then we have the currents here i a i b and i c minus the back e M f e a e b and e c. Of course, we have an L s here L s can be taken to the right hand side we can multiply here 1 by L s. So, this equation we can say equation number 1.

So, this equation gives us d i a by d t d i b by d t and d i c by d t. So, we have 3 simultaneous differential equations which can be solve by any numerical technique. For example, we can solve this equation using Runge Kutta fourth order method to solve the 3 equations and get the values of i a i b and i c. What about the speed? Unless we know the speed we cannot calculate the back e M f, because we have seen that e a here is a function k a into omega r. So, we need to know the value of the speed, the speed can be obtain from torque equation, torque can be obtain as follows. So, we can we can find out the value of the torque as follows.

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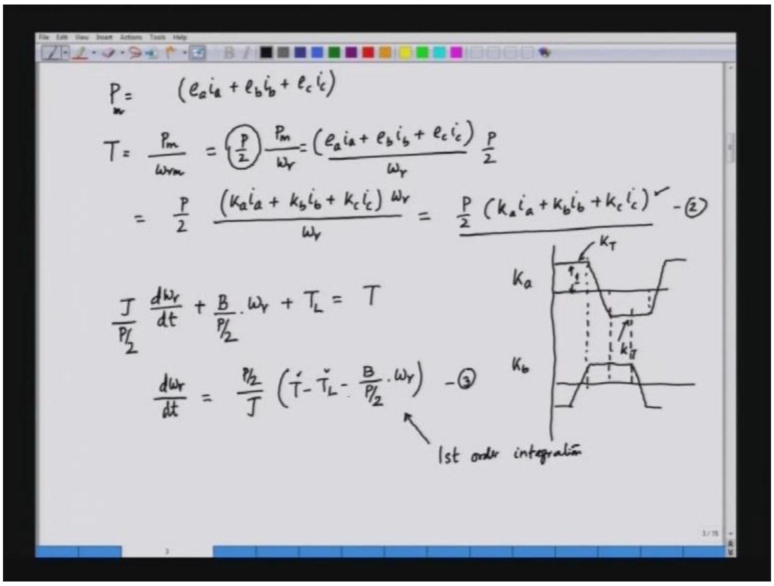

So, T e or the power in this case before that we can we can see the power expression the power in this case is P by 2 times we have mechanical power out. So, P n here this is the mechanical power that is equal to you do not have any P by 2 term here. Because we are talking about the mechanical power that is e a i a plus e b i b plus e c i c this basically the respective back e M f into the respective current. So, if we add the back e M f and the current product in all 3 phases, we get the total mechanical power and the torque can be found out by dividing this by the mechanical speed.

So, the torque here T is equal to P m by the mechanical speed omega r M that is equal to p by 2 into p M by omega r because we know that the mechanical speed is the electrical speed by pole pair. So, omega r by pi by 2 is omega r m. So, we have p by 2 coming here into this product we have the sum of the product of the back e M f and the currents e a i a i b i b plus e c i c and that can be written as P by 2. This is what we have here that is equal to this divided by omega r and we have to multiply here P by 2 in this case. So, we can now replace e a by k a i a and e b by k b i b plus k c i c. And omega r is in the numerator and the omega r is also in the denominator they would get canceled.

So, finally, the expression for the torque is P by 2 k a i a plus k b i b plus k c I c. So, this is the expression for the torque of a brushless DC motor in which k a k b and k c are function of theta r. Now, how does this k a look like? k a as we have seen, it is basically a trapezoidal function and the k a for phase a if we see the function k a k a may look like this 120 here this is 60 degree in this case and this is 120 again and so on. So, this is of unity magnitude. So, this magnitude is 1. So, this is k a now, when we multiply this by omega r we get the corresponding back e M f.

So, this is the nature of k a this magnitude can be suitably skilled say for example, this depends upon the strength of the permanent magnet this could be in general; this could be some K T. So, we can say that the magnitude of this is K T, the torque constant K T is the peak magnitude.

Similarly, in this case also this magnitude is minus K T and the constant for the phase b that is k b will be shifted from k a by 120. So, this is k b k b will have a pattern which is shifted from k a by 120. So, this is the nature of k b and so on. And similarly, we can have k c k c will be again shifted from k b by 120. So, we have the functions for phase a phase b and phase c when the respective functions are multiplied with the corresponding current. We get the torque and the torque in this case is k a i a plus k b i b plus k c i c into P by 2 and omega r get cancel. So, this is the expression for the torque.

Now, when we get the torque we can get back the speed by having electromechanical equation. So, the electromechanical equation is. So, inertial torque d omega r by d t into J by P by 2 we neglect for simplicity the viscous friction. But the viscous friction will also come here B by P by 2 into omega r plus the low torque is equal to the motor torque the motor torque is T. So, from this we can we can get the expression for B omega R by d t. So, d omega r d t can be obtain in the following fashion is equal to T minus T L minus B by P by 2 into omega r. This B is the coefficient of viscous friction any machine will have some viscous friction and that is represented as B.

So, B into omega r M or omega r by P by 2 this is multiplied by P by 2 and divided by J. So, we have the expression for the speed and this speed equation can be solved again by a numerical integration technique to give us a value of the speed. So, once we know the speed we can find out the back e M f and the back e M f's are e a e b and e c. The multiplication of corresponding constant k a k b and k c with the rotor electrical speed omega r. And then this back e M f’s can be substituted in this equation to give us the derivative of the currents and the derivative of the current can be solved using any numerical integration technique.

Now, here we can understand that the electrical variables faster rate speed is a mechanical variable. So, we can offered to solve the electrical variable with a better integration method. So, these 3 equations can be solve by Runge Kutta fourth order technique. If we are trying to solve these 3 equations, we can use Runge Kutta fourth order integration technique to solve this 3 equations. And then the speed equation after we have obtain i a i b and i c. This i a i b and i c can be substituted here to get the value of T and T is known here T L is the low torque omega r is the previous value.

So, this equation can be solved by a first order integration method, first order integration to give us the value of omega r. In fact, when we are simulating the first order equation that is equation number 1; this is equation number 2 and this is equation number 3. So, when we are solving the first equation, we are assuming that e a e b and e c at constant for the time delta t delta t is the simulation step. So, we are simulating this in step of delta t this is the simulation step and for that small delta t we can assume the back e M f to be constant and that is only true with the speed is constant.

So, while solving equation 1, we are assuming that the speed is constant. We after solving this equation 1, we get i a i b i c and this 3 currents can be use to get the speed omega r. So, this is the dynamic modeling of a DC motor now, as we have already seen that we can have a trapezoidally exited permanent magnet motor. Also we can have a sinusoidally excited permanent magnet motor and the sinusoidally excited permanent magnet motors are call permanent magnet synchronous motor. So, we will be now discussing about the sinusoidally excited permanent magnet motor or permanent magnet synchronous motor.

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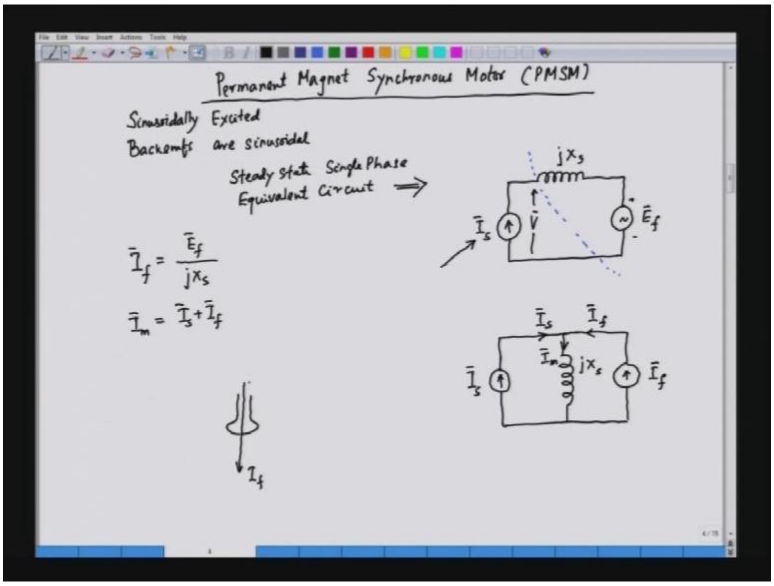

So, our next discussion is permanent magnet synchronous motor and we call this to be PMSM. And these are sinusoidally excited which means the back e M f in this case is sinusoid the back e M f e a e b and e c are not trapezoidal they are sinusoidal. So, we will say that here in this case the back e M f’s are sinusoidal now, we will be right. Now, we will be only confiding to the steady state analysis of this motor.

Then this will be followed by the dynamical analysis in steady state we assuming that the speed is constant and we can represent the motor by its equivalence circuit. So, we have the synchronous motor where the field is a permanent magnet, we can replace this by single phase steady state equivalence circuit. So, we can have an equivalence circuit like this here we have let us say we excite this; this; this from a current source. And then we have the synchronous reactant that is j X s and the field induce d M f is e f it is a c quantity. And we excite this from let us say a current source, we have a current source inverter or an inverter which can be current control.

So, we are injecting the current into the stator winding. So, in that case we can assume that the current in the stator is constant is AC current. But it is having fixed amplitude and fixed frequency let us say in the steady state condition. So, in the steady state condition we can say it is I s. So, this is the steady state single phase equivalence circuit. So, we are discussing about the steady state operation, steady state singe phase equivalence circuit. In this case since we have a sinusoidally excited motor the voltage which is applied and the currents which are applied will also be sinusoidal. So, in fact in this case i a s i a s will be sinusoidal. So, we have i a s in phase a phase b and phase c. So, they all are be sinusoidal. So, this is a fager and we represent the fager by bar on the top I s bar. And this voltage is the voltage here that is v is the terminal voltage and e f is the field induce d M f.

Now, we can rewrite this or this equivalence circuit in a different way, what we will do here? We will be repressing this part by Norton equivalence circuit. So, what we will do? We will take this branch and we will be repressing that branch by again a current source. This is I f; this also a Fraser here and then we have a parallel branch in this case and the source here is I s this is the voltage; this is a current source. So, what we have here is a current I s and this reactance is x s, the synchronous reactance. Now, in this case, we have 2 currents one is coming this way, other is coming from the rotor and this current which is flowing through this is call the mutation current. So, we have I m.

So, we can have the expression for I f I M also. So, what is I f here? i a f is the Norton's equivalent current that is equal to e f by j X s. And this I M is the sum of the other 2 current the stator current is I s and this is the field current which is coming from the field side this is I s. So, we can say I M is the sum of I s and I f. So, we can now draw the equivalent circuit, after drawing the equivalent circuit, we can draw the diagram Fraser diagram. In the Fraser diagram, what we have? We have the field here, this is the field is in the d axis and this is the current i a f. And the induced d M f will be right angle to the d axis; this is e f and the stator current is somewhere along this axis I s they are all Frasers and the resultant of this 2.