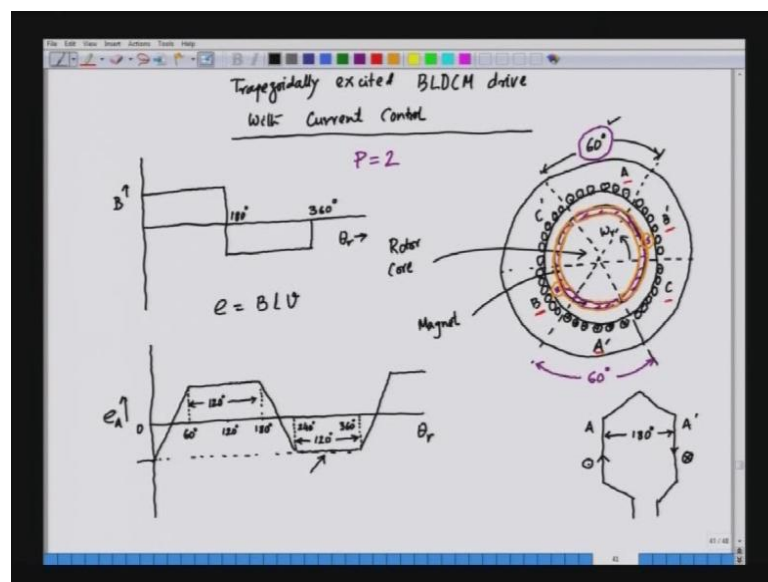


Advanced Electric Drives
Prof. S. P. Das
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture – 24

Hello and welcome to this lecturer on Advanced Electric Drive, in the last lecture we are discussing about the trapezoidal excited a permanent magnet d c motor drive. In trapezoidal excited motor the back EMF is a trapezoidal excited form, let see an example of a trapezoidal motor.

(Refer Slide Time: 00:41)



So, this is the view of the trapezoidal motor, and here we have 3 phase distributed winding and the windings are phase a, phase b, and phase c, the windings phase a, phase b, and phase c are position in the phase 120 degree apart. Phase b shifted from phase a by 120, and phase c is shifted from b by 120 and these are distributed by windings, it means the windings circulate in the slots.

So, we have the slot and each winding if we see the phase a, the phase a spade over 60 degree, here and this is a prime the return parts of phase a. So, if a conductor carriage dot current, dot current means it is coming out of the plane and a prime will carry the cross current, something like this we have a winding, we have a term of winding and this 2 side of the term are place 180 degree apart.

So, if it is one term of winding a, now this will be lying under a and this will be lying under a prime, so if a is carrying a current like this, which is upward coming out of the plane a prime will be carrying current, which is downward. So, we can say this is dot current, and this one the cross current, that is what is shown in a and a prime, similarly we have b and b prime, this is b, and then we have b prime.

And b and b prime are the 2 sides of the winding, one side is a, other side is b prime, they are shifted by 1 pole pitch that is 180 degree electrical angle. And if one of them is carrying dot current, the other 1 would carry the return current that is the cross current. Similarly we have phase c, phase c is shifted from phase b by again by 120, and phase c has got a c and c prime, c is for the one side of the term, and c prime for the other side.

So, this is how the windings are distributed, they are not concentrated that, in fact they are distributed, in phase the distributed harmonics or distributed windings have one advantage that space harmonics are reduced. When we distributed the winding in the space the harmonic the space harmonic of the MMF or very much reduced, because they can be approximated to a trapezoidal wave form, and the harmonics are laid compact to a concentric winding.

Now, here the windings are distributed, so if we see that phase a, and a prime that each one is occupying 60 degree, so here this also 60 degree here, mechanical as well as electrical, because here we have assumed that the number of poles in this case is equal to 2. It is a 2 pole structure, we have one north pole, and one south pole, what is the north pole here. We have the north pole of the rotor, the rotor is the permanent magnet, and has the 2 poles one is the north pole, other is the south pole, and the south pole is here.

And these permanent magnets are placed in the rotor it looks like it is a surface mount permanent magnet rotor, the permanent magnet is housed in the surface. So, this is the permanent magnets are the north pole, and similarly this is the permanent magnet for the south pole, so these magnets are placed in such a way that this side is north, and this side is south. And the flux is coming out of this the permanent magnet is almost constant, so in fact if we plot the flux lines, the flux line almost like a square wave.

So, if we plot the flux lines of this magnet against theta r, here we have in the theta r in the x axis and flux densities are b is in the y axis. So, we will see the flux density here is nearly constant, this for the north pole and for the south pole something like this, so this

is 180 degree, and this one is 360 degree. So, this is the flux density, where form of this kind of motor, that the rotor is the permanent magnet having 2 pole structure, now if we see here in the stator phase a, the phase a speed over 60 degree here.

So, when the permanent magnet is north pole is entering the stator phase a, gradually the south pole is going away north pole is entering, so that is a gradual rise of the induce emf, because induce emf. We know by definition is $B l v$ flux density, l the length of the conductor in the linear velocity, so when it is changing from north pole to south pole, gradually it take some time to become fully south pole, or from north pole to south pole this will takes some time and this overlap time is 60 degree.

It means for 60 degree interval of phase will have both north pole and south pole, so during that time the induce time will be gradually rising after the end of the 60 degree the induce emf will be constant. So, here if we plot the induce emf of a phase, let us say the this is phase a, e_a and we are plotting this induce emf in this case against θ_r , so this induce emf will be function of speed.

So, this will be gradually raising in this case, and at the end of 60 degree intervals it will be fully north pole to fully south pole, at the present situation is the induce emf in the phase a, the phase a in this case is seeing full south pole, a prime is seeing full north pole. So, the south polar fully enter under phase a, so it means the induce emf is maximum there, and this induce emf is going to stay there for how much duration, this will be staying for another 120 degree.

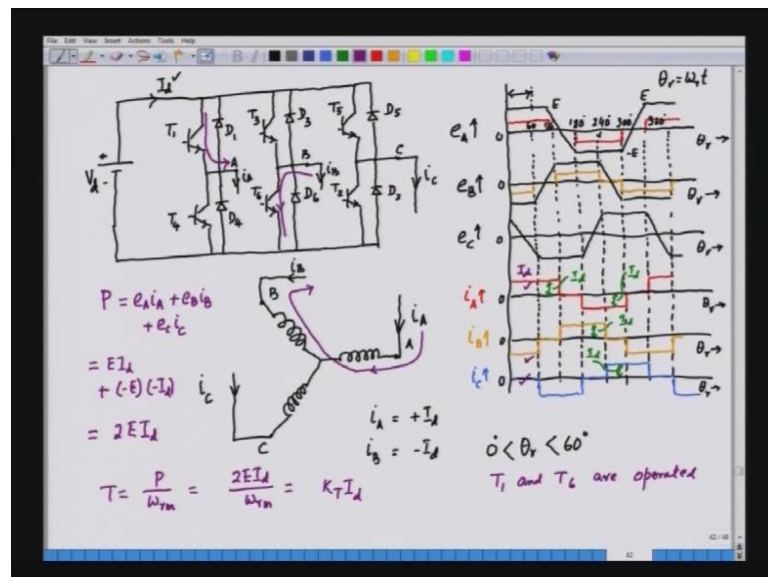
So, in that case what we see here, the up to this time it will stay for 120 degree here, so this is 60 degree, and this is 120, and this is 180 degree and after the end of a span of 120 the flux will again, the winding will again see the north pole. So, the south pole will going away, and the north pole will be entering and again that the duration of 60 degree, so here it will again come back here, and again this is 60 degree intervals. So, this is 240 here 180 plus 60 is 240, and this is going to stay for again in another 120 and 240 plus 120 is 360 degree, and 360 is the complete of 1 cycle.

So, we see that the induce emf have has competent 1 cycle, at the end of a the same thing will repeat the induce emf will again rise and will the maximum here and the same period will repeat. So, this is basically 1 cycle of induce emf a raises from negative maximum to passively maximum for 60 degree interval, it is stays are the maximum

value for a duration of 120 degree. Again, a reverses from passively maximum to negative maximum within a duration of 60 degree is stayed are the negative maximum, here again for 120 degree.

And this is the nature of the induce emf in a single phase, that is in phase a, so if it is an induce emf in phase a, phase a, phase b, phase c are symmetrical. So, whatever is happening to the phase a, the same thing will be happening to the phase b, phase c respectively, but after delay of 120 degree. It means phase b induce emf will be shifted from phase a by 120, so we can plot the complete induce emf in the following fashion.

(Refer Slide Time: 12:13)



So, we have a structure like this, so we are plotting the induce emf of if is individual phases, phase a, phase b, and phase c, these are the 60 degree intervals, so we will start with phase a for phase a the induce emf will be maximum here. Let say we are talking about phase a, and then it will take about 60 degree for reversing it voltage amplitude, then this will be stew the negative maximum, again it will reverse the voltage to the positively value, then it will stay here.

So, this is of phase a, induce emf in phase a and this is the origin, and what we are plotting, we are plotting against theta r, and theta r is nothing but omega r into t, omega r is a rotor electrical speed, theta r is the rotor electrical angle. So, we are plotting the induce emf against the rotor electrical angle for a 2 pole structure, the electrical angle is same as the mechanical angle, so theta r is the same mechanical angel of the rotor. So, in

this case this is the induced emf seen in phase a, and for phase b, the phase b is shifted from the phase by 120, so this is 60 degree intervals.

So, we can say 60, 120, and this is 180, and this is 240, 300 and this 360 completion of 1 cycle, so phase b will have this positive voltage after 120 degree of phase a, and this will display the same v form like this, this is for phase b. So, this is e_B here the induced emf of phase b of the trapezoidal excited motor, and what about phase c, phase c will be shifted from phase b by again 120. So, phase c will have the maximum positive voltage here, and then this will have the negative voltage here, for 120 degree, and again the maximum positive voltage here, so this is the wave from phase c.

So, we are plotting everything against θ the rotor angle, so and what we do here, how to control this kind of motor, we have being a trapezoidal excited permanent magnet motor, and we would like to control it, like a brushless dc motor. Now, here what we do since the wave from the back emf of the induced emf of phase a, phase b and phase c are not sinusoidal are, in fact trapezoidal here. We excite the respective phases by quasi rectangular current wave form, so the current of phase a looks like this, this is for phase a will say I_a , similarly will have I_b , and similarly we have I_c .

So, let say we can start with I_a will be a quasi rectangular wave, if we explain this will mean to draw the current of phase a, so this is what we want that phase a current should be quasi rectangular, it should be something like this. If you have a value I_d for sometime, then 0 this minus I_d then 0, then again plus I_d , because what we want in this case that E_a and I_a should be in phase.

It means whenever the back emf in this case, in fact what we doing here, we are have this current block like this, when the induced emf is positive, the current is positive, when the induced emf is negative the current is also becoming the negative. And as a result of the product of induced emf on current is always positive, and hence we have the positive power, and thus we have a positive torque.

Similarly for phase b, we can draw the current for phase b and phase c respectively, so these are the 60 degree intervals we have and for phase b we can have this I_b , I_b we have this nature positive here than 0 here and then negative here. Similarly, we have here 0, and we have negative, so that in this case the current poles essentially exist here, this is I_b when the back emf is positive, we have the current positive, when the back emf is

negative, the current is becoming negative. Similarly, here also we have the negative current, and the result in phase b also E_a , E_b and I_b are in phase and the product of E_b and I_b will be positive, and hence we have positive power.

Similarly, for phase c we can have the current in the following fashion, this is the current again a quasi rectangular current here, and here also we have 0 current, negative current and this is I_c . And we have the origin here, and these are all against θ_r the rotor angle, so as a rotor rotates the back emf are trapezoidal nature, and the current which has to be injected in to the various phases will be a quasi rectangular in nature.

How do we inject the currents, the currents are injected using a voltage source in water, with the 3 voltage source in water which is use to inject the current into the 3 phases of the motor. So, we will see how the currents are injected by using a voltage source in water, so we have a voltage source in water, a 3 phase voltage source in water having the transistors features. These are the diode, anti parallel diode for the feedback diode, for feeding back inductive energy that to the source, we have the voltage here v_d , and this is the input current that is I_d .

This for phase a, similarly for phase b, we have another lage for phase b, this for phase b of the in water feeding the phase b of the motor, and then we have the c phase, so these are the various transistors we have. Now, we can call this to be transistors t_1 and the corresponding with the diode the feedback that is d_1 , and this is t_4 , and the corresponding diode is d_4 , this is t_3 and the diode is d_3 . And the transistors here, is t_6 and the diodes here is d_6 , here the transistors is t_5 the diode is t_5 , the transistors is t_2 and the diode is d_2 .

So, we have 6 features here, 6 transistors features and 6 diodes and the diodes are require to feedback the inductive power back to the source, and we have 3 phases of the motor 3 phases are like this, we have phase a here, phase b, phase c may be star connected. So, this is a, this is b, and this is c, and these are connected to the respective output of the in water, and the current which is injected in to the phase a is coming from phase a of this.

So, this current is flowing here this is called I_a or the phase a current that we have plotted in this case, and similarly the current which is coming to phase b is actually coming from the in water, this also I_b , for phase a it is I_a same current of this. Similarly, for phase c is here is connected to this phase of in water, and this current is I_c

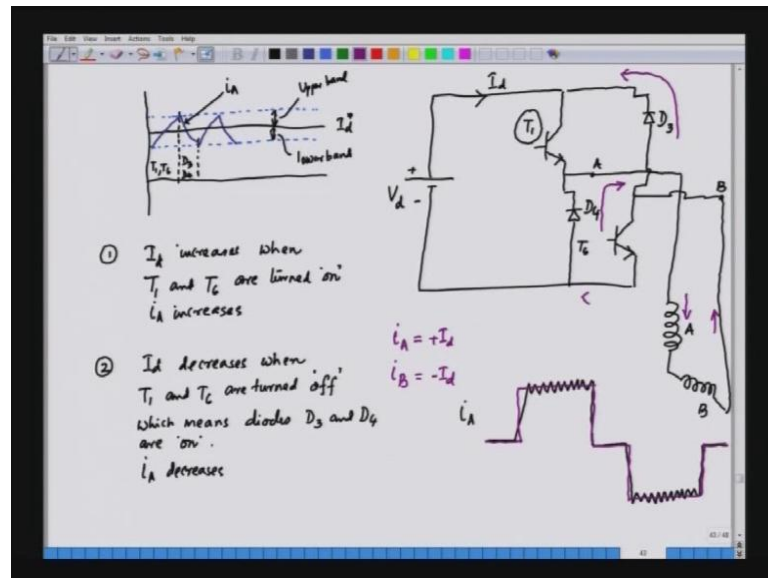
same as this current here is I_c , these as per the convention, now let us say for the first 60 degree intervals.

So, then θ_r in this case is greater than 0, and lays then 60 degree, we are in the first sector, the first 60 degree, in the first 60 degree the current I_a is positive and current I_b is negative, and the amplitude of this current are all I_d , I_d is the dissolving current, which assume to be approximately constant. So, in this case what we have here is this, that this current amplitude is I_d , I_d is constant, this also I_d in the dissolving current, this is I_d , and this is also I_d .

So, this currents I_d is basically coming from the dissolving that is this current, and here for the first 60 degree, we can see that I_a is positive, I_a is plus I_d , and what about I_b , I_b is minus I_d . So, in the first 60 degree, we see that I_a is positive, but I_b is negative, it means the first 60 degree the transistors t_1 is conducting. So, the current is flowing like this to I_a to this particular phase, and it is turning to phase b like this, and going to phase b and to the transistors t_6 .

So, for this particular region 0 to 60 degree, we can say that t_1 and t_6 are operated, so that I_a is positive and I_b is negative. Now, the objective is that the current has confine to a band, so that the current becomes a rectangular nature. It is basically quasi rectangular current wave form, but we cannot keep the dissolving current absolutely constant, so we have to control the current how to control the current, the current can be control using hysteresis current control method.

(Refer Slide Time: 28:06)



We can see how the current is control the following wave, we can control the current like this, this is the reference current, we can say it is I_d^* , and the actual current will be controlled within 2 bands here. Actual current will vary like this, going up and going down, going up and again going down, so this actual current is essentially the current i_a , so we are controlling current i_a by controlling the switches, on the switches are t_1 and t_6 .

If you see the equivalent circuit, in the equivalent circuit we will see that we have 2 switches here, one switch is here, and the other switch is here, and we have a DC source which is present here that is V_d . And we have t_1 , and we have t_6 and this is connected to phase a, and we have phase b, which is connected to this particular transistors. So, in this case when we turn on t_1 and t_6 the current flows in the following fashion the current flows like this, it goes to phase a and returns back to phase b like this, that to the source here.

So, this is the direction the pass of the current in phase b and phase c, so we can say here that i_a is positive, i_a is plus I_d this is our I_d , and i_b as per the convention is negative, it is flowing against the source. So, we can say that i_b is minus I_d , so that I_d will increase, if we turn on t_1 and t_6 , so we can say that I_d increases, when t_1 and t_6 are turned on. And what about decreasing, this is the increasing current, so we will say that for this reason, we are basically turning t_1 and t_6 , what about decreasing current, the current can be reduced, when we turn off the switches.

Now, when we turn off the t_1 and t_6 , what happens, when t_1 and t_6 are turned off the load is basically an inductive load, in the inductive load current will try to maintain its path. So, when the transistors are turned off, the current looks for an alternative path, and the alternative is provided by the feedback diode, the feedback diodes are here. So, we have the diode in this case the diode are present here also, so we have the diode here and this is the feedback diode for the other transistors, which is not control here, so this is d_4 and we have another feedback diode which is present here, which is d_3 .

So, when the transistors are turned off, the current directions would be maintain, so we will maintain the current direction, so this path is not there. So, the transistors are an are the off, so the current has to be maintain, so the current which was flowing in this direction positive here, and negative here will look for an alternative path, and the alternative path if is provided by the diode.

The diode are in this case is d_4 to the current will come from d_4 , d_4 terms on, and then this current will flow back to d_3 , so in fact when we turn off t_1 and t_6 , automatically d_3 and d_4 will be turned on, because of the nature of the current. And when these 2 diode are turned on, and negative v_{dc} is applied across this 2 windings, so we can see here that earlier this point was positive, now we see that when d_4 is on this terminal, the terminal of phase a.

This is phase a terminal, and this is phase b terminal, phase b was earlier positive, because t_1 was conducting, when d_4 is conducting a becomes negative, so it mean that is a tendency to reduce the current. Similarly, when t_6 was conducting the b terminal in connected to the negative of the dc voltage, when d_3 is on now, the diode is on b is connected to the positive of the dc voltage.

So, in fact when we switch off t_1 and t_6 d_4 and d_3 are automatically turned on, and there an to reduce the current I_a , so I_d increases when t_1 and t_6 are turned on, it means I_a also increases. This is number 1, and we will say that I_d decreases, when t_1 and t_6 are turned off, which means it there off which means diode d_3 and d_4 are on, and this also means I_a decreases.

So, for this reason we have the conduction of d_4 and d_3 , d_3 and d_4 , so we are able to control the current. The current can go high or increased and current can also decrease, and this is also done with 2 band, and this band is called a hysteresis band, so in fact we

have one band called a upper hysteresis band. And the other band are the low hysteresis band, so this is the upper band, and this is the lower band, so if we control this current within 2 band we can keep the current approximately quasi rectangular in nature.

So, the current as a result would be something like this is being control, and then after 120 duration comes here and becomes 0, and this is again control the negative direction like this, and then it again goes to 0. So, this is the nature of I_a which can be approximately a quasi rectangular wave form like this all though we have reference current, which is a quasi rectangular the actual current follow the reference within an hysteresis band.

Now, let us try to find out the power in a brushless d c motor drive, in this case we have a trapezoidal excited motor, when the wave forms are trapezoidal, and the current in this case are quasi rectangular in nature. And the current exists when the voltage is positive, now let as try to calculate the power in such a drive. Now, here if we see the power in this case p is, because of the currents in all 3 phases, so in fact we can see that p is equal to $E_a I_a$ plus $E_b I_b$ plus $E_c I_c$.

Now, if we take the 60 degree intervals, the first 60 degree interval that is d 60 degree interval, now we see that I_a is positive, and I_b is negative and $I_c = 0$, so at any giving time only 2 phases are conducting. So, for the first 60 degree we will see that I_a is positive, and what about E_a E_a is also positive, let as assume that this E_a value is capital E , the peak value is capital E here.

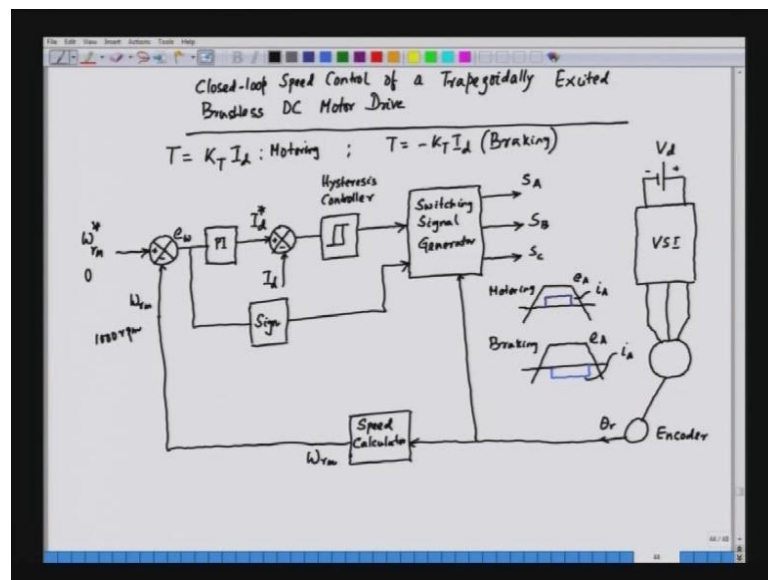
Similarly, this is minus capital E this is plus E and this is symmetrical for all 3 phase, now the power is phase a is given as E into the current is I_d the current here is I_d here, this current is I_d . So, the power is in phase a is E into I_d , what about the power in phase b the power in phase b, this is phase b the voltage has reversed this is minus E here, the current is also minus I_d . So, we can say that the power here is minus E into minus I_d , and what about phase c, in phase c the current is 0, so we have the current phase a is positive phase b is negative and phase c 0.

So, phase c does not contribute to power for the first 60 degree, so if we calculate the power for the first 60 degree, it is $E I_d$ plus minus E into minus I_d , and that is equal to $2 E I_d$. So, the power here is $2 E$ into I_d , so what about the torque, the torque is

power by the mechanical speed that is $\omega_r m$, and that is equal to $2 E I_d$ by $\omega_r m$.

And we can say that is equal to we have a torque constant that is k_t into I_d , so we can show that the torque is proportional to the dissolving current, so in fact we can increase the torque by increasing the current, and decrease the torque by decreasing the dissolving current. So, the torque wave from will be a smooth wave form, that is a function of I_d and torque is equal to k_t into I_d just like a d c machine.

(Refer Slide Time: 41:40)



Now, let us see how we can control this motor with a close loop control, so we will see a close loop control here, closed loop speed control of a trapezoids excited brushless d c motor drive. So, we have the expression here, T is equal to torque is equal to k_t into I_d , so in fact we can control the torque by controlling current, so if we want to have a close loop speed control, we should be having a closed loop speed feedback.

So, we have a reference feed, and then we compare the speed with the actual speed, so we have the reference speed here, that is ω_m^* or $\omega_r m^*$ the rotor mechanical speed. And that we compare with the actual speed of the rotor that is $\omega_r m$, and we feed that to a p i controller, and then the output of this will give as the reference current or the reference torque I_d^* .

And this is subtracted, we can subtract this I_d from this and we have a hysteresis control here as we have discussed, so this is hysteresis controller, and then the hysteresis controller will be giving the signal to a switching signal generator. So, we have a switching signal generator, which will be generating the gate drives of the inverter that discussed, so this is the switching signal generator, and this will be generating the voltages.

The signals s_a for the phase a, the switching signals, s_b for phase b, and switching signal, s_c for the phase c, and then we have the inverter voltage source inverter. So, this is the v_{si} and here we have the inverter in this case v_d , and this is the v_{ldc} motor, and then we have an encoder here, or the encoder gives the information about the rotor angle, the rotor angle is very important for synchronizing the power signal with the motor.

So, we have the rotor angle, which is generated or obtained from the encoder θ_r and this rotor angle information is sent to the switching signal generator, and then we have a speed calculator here. The information about θ_r is sent to the speed calculator, which calculates the speed and we are closing the loop, so this is the output of the speed calculator is a speed the rotor mechanical speed, which is used as a feedback for the closed loop speed control.

Now, we know that we can also have braking operation, for the braking we want to reduce the speed, so the braking operation is identified by negative speed error, suppose the speed is now 1000 rpm. So, if you want to brake the machine to 0 speed, this error will be negative, and accordingly we have to apply the signal in such a way the torque will be negative. The torque is given by $k_t I_d$ or if the phase sequence is reverse or the current signals are phase shifted by 180 degrees, we can say this is for motoring reason, and for braking t is equal to minus k_t into I_d .

The same inverter current, but we are going for braking, how do we go for braking for the braking instead of applying positive current to the positive voltage, will apply the negative current to a positive voltage. So, when the voltage is positive, we will make the corresponding phase current negative that is possible, so if we can do that instead of having positive torque, we have a negative torque, and torque will be given by minus of k_t into I_d .

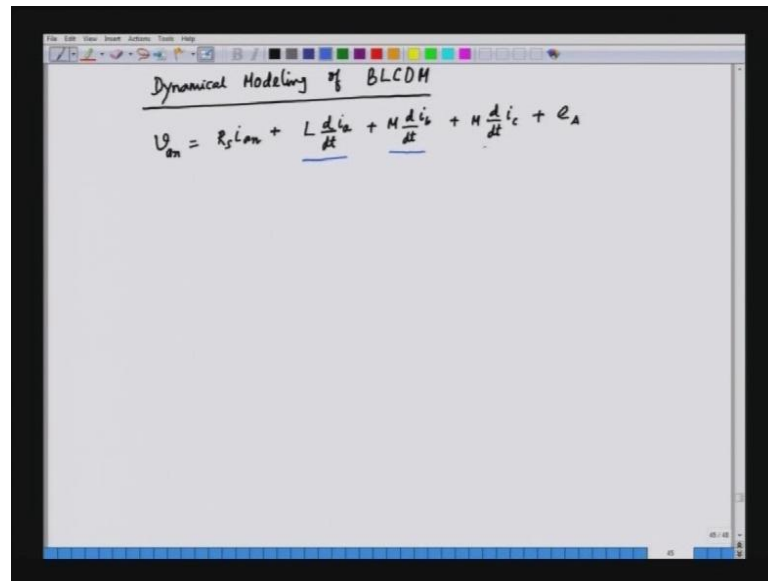
So, in fact to have the braking operation, we will sense the sign of this error, so this is the speed error, and we will be sensing the sign of this error, so this is basically a sign block and this is sent to the switching signal generator. So, this sign information will tell us whether the motor is motoring or braking, so if the error is positive, it is motoring if the error is negative it is a braking operation.

For braking operation that the torque is negative, so instead of applying current, which is in phase with the voltage like this. If this is the trapezoidal for motoring, what we do we apply positive phase current like this is for any given phase, so I_a and E_a , suppose this is our voltage E_a or induced emf, these are current that is I_a . For braking we will have a different kind of form this is the voltage waveform here, and the current waveform is 180 degree phase shifted, so this is the current for the braking operation.

So, in this case this could be E_a , and this is I_a , so this is for motoring, and this is the figure below this and for braking operation, we need to know the sign of this error, and this error is phase here to have a braking operation. So, this is this is a basically closed loop speed control brushless dc motor drive, where the trapezoidal excited motor and this can be applied for applications.

Say for example, automotive applications like electric scooters, electric vehicles also, and here we can have both forward motoring also forward braking, now let see how to model this brushless dc motor drive. Now, for the modeling we would like to have an idea of back emf, so we will see the modeling aspect of brushless dc motor drive, and this modeling, we will do dynamically.

(Refer Slide Time: 51:13)



Dynamical Modeling of BLCDM

$$V_{an} = R_s i_a + L \frac{d i_a}{dt} + M \frac{d i_b}{dt} + M \frac{d i_c}{dt} + e_a$$

So, we will say the this the dynamical modeling of BLDC motor, now we now that we have 3 phases, so the basic principle is that for any phase, we have the applied voltage, and then we have the resistive drop, the rate of change flux linkage, and the induce emf. So, we can write down the equation for each phase, so we will say that for phase a with start this that the applied voltage a n is equal to R s i a n plus the induce emf, because of the inductions that is l d by d t of i a.

And we have the mutual between phase b and c d by d t I b plus m d by d t I c plus the back emf, that is E a, similarly we can write down the equation for phase b and phase c. So, this equation for phase a voltage phase b voltage, and phase c voltage equations and dynamical in nature, because they involve the derivative term, and the derivative terms are l d i a by d t m d i b by d t and m b i c d t.

So, we can write down the equation for phase b and phase c, and then for that equation we can solve this equation to get an idea about the various current, the currents are I a, I b, and I c. When we know the back emf and currents, we can find out the torque, and hence the speed, the detail modeling of this brushless d c motor drive will be discussed in the next lecture.