Advanced Electric Drives
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Lecture - 2

Hello. This is the second lecture of the course on advanced electric drives. In the last lecture, we were discussing about the generalized theory of electric machine in which we have a common framework for analyzing all sort of rotating electrical machines. Now, we also discussed about the Kron's primitive machine model to just repeat that, I can show that once again.

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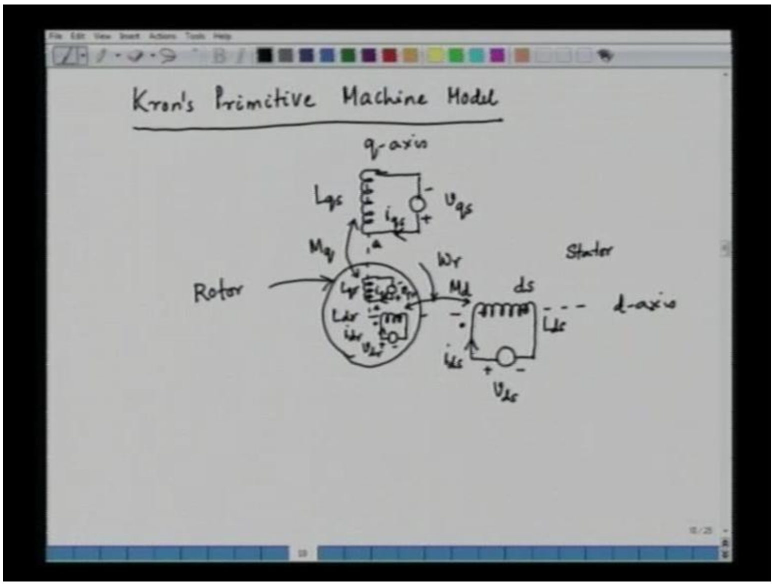

We have discussed already Kron's primitive machine model, which was given by a scientist called Kron in early a twentieth century. Now, if you see this particular model – the Kron primitive machine model, what we have here, we have rotor structure and we have the stator. And in the stator, we have two different axes. One is the d-axis, this is called the d-axis. And this is known as the quadrature axis or q-axis. And we have two different windings. One set of windings in the d-axis and the other set of windings in the q-axis. For example, in the d-axis, we have a winding in the stator. This is basically the stator; the stator winding, which is stationary, not rotating. And this is the rotor, which is rotating, this is the rotor. And the rotor is rotating in the clockwise direction. So, the direction of rotation is clockwise at the speed of omega r, this is the speed of the rotor.

Similarly, in the rotor winding, we have a winding in the rotor in the d-axis. And in the q-axis, we have a stator winding. And in the q-axis rotor, we have rotor winding. So, we have shown the actually machine as a 2-axes machine, where we have two well-defined axes: one in the d-axis and one in the q-axis. The objective is this that, it may be an AC machine; it may be a DC machine; the effect of an AC machine or a DC machine can be well understood or can be simulated by a 2-axes model. It is sufficient to have two orthogonal axes; two orthogonal axes means two axes should be perpendicular to each other: one in the d-axis and other one in the q-axis.

Now, please remember that, this machine that we are talking about – the Kron's primitive machine is essentially hypothetical machine; it does not exist in reality. It is per our own understanding, per our own convenience; we have taken help of this machine to simulate an actual machine. So, in this case, we have two different axes. In the d-axis, we have the stator winding. And this we call to be the ds winding. And we can have applied voltage here. And the applied voltage is v ds; v stands for the direct axis and s stands for the stator. And this is the terminal marking. We can show this as dot. And this is the current that is entering into the winding. And this current is i d s.

Similarly, in the rotor, I can have applied voltage. And the applied voltage here is v dr. And this the dot here. These two windings are coupled. The d-axis stator is coupled with the d-axis rotor. These two are the coupled windings. The d-axis stator is coupled with the d-axis rotor. And this dot shows the positive terminal. Here the current in the rotor is entering the rotor and this current is i d r.

Similarly, in the q-axis, I have the terminal marking here; I can show this by a triangle – a small triangle here. Similarly, I can show this by a small triangle indicating that, these two terminals are similar. In the q-axis, similarly, I can have applied voltage; I can have applied voltage here in the q-axis. And this voltage terminal is positive; this is negative. And I can call this voltage as v q s. And the current in the q-axis winding is entering this terminal and this current can be i q s.

Similarly, in the q-axis rotor, I can have applied voltage; this voltage is plus here and minus here and this is v q r. And this current in the rotor… q s is winding, is entering the triangle terminal. And this is i q r. So, I have these two windings – these two sets of windings and they are orthogonal to each other. It means the d-axis stator is not coupled with the q-axis stator. Similarly, the q-axis rotor is not coupled with the d-axis stator; they are orthogonal windings. Now, in this case, what I can do here; I can write down the voltage equations.

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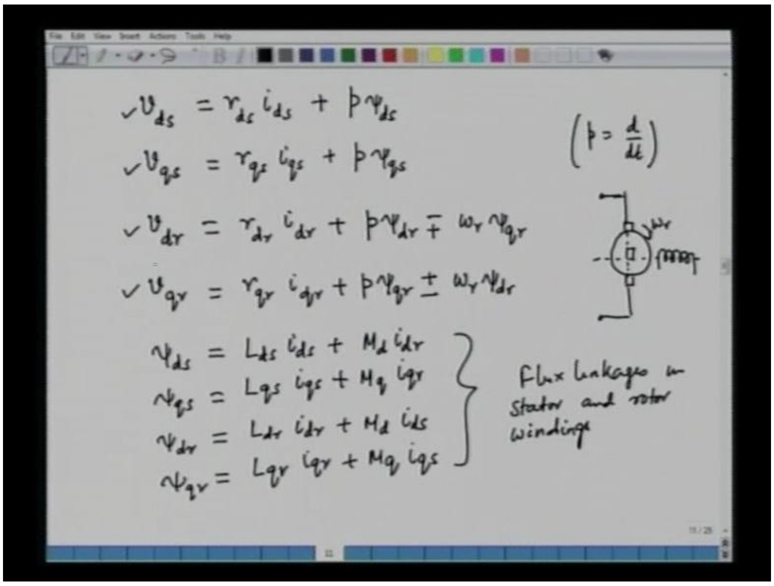

Now, when I write down the voltage equations, I can start with the d-axis stator winding – v d s. This winding is a stationary winding. So, I can just write down the voltage as r d s, is the resistance of the winding – into i d s, is the current flowing through the winding plus the statically induced emf, because the stationary winding can have a statically induced emf. And v d s is a stationary winding in the stator. Unless the winding is rotating, there is no rotationally induced emf.

So, I can just say that, v d s is r d s or i d s – the resistance drop plus p psi d s – the statically induced emf in the d-axis stator. In the similarly, I can write down the q-axis voltage equation in the stator. That is equal to r q s i q s plus p psi q s. And this winding is also stationary. And hence, there is no rotationally induced emf. I just have a statically induced emf in the q-axis, that is, p psi q s; and the psi at the flux linkages. Psi d s is the flux linkage in the d-axis stator; psi q s is the flux linkage in the q-axis stator.

Coming to the rotor winding, when I write down the equations for v d r – the d-axis rotor winding; I will have both statically induced emf and rotationally induced emf, because the rotor is rotating. So, I can have a component of emf, which is coming due to the rotation of the rotor. And that emf is called rotationally induced emf. So, I can just write down the equation for the d-axis rotor; that is equal to r d r i d r, is the resistance drop in the d-axis rotor plus p psi d r. p psi d r – p is the derivative term, that is, d by dt. This is the differentiation of the flux with the respect to time. p is equal to d by dt; it is a derivative operator.

So, I can just write down v d r equal to r d r i q r plus p psi q r. This is a statically induced emf. And here I will have a rotationally induced emf, but I do not know its sign. But, I know that, the rotationally induced emf as we have already seen in the last lecture, it appears in the quadrature axis, in the orthogonal axis. So, I can just write down it is omega r into psi q r. It means the rotationally induced emf in the d-axis is due to the q-axis flux; something similar to the DC machine.

In the DC machine, what we have here; we have the armature; this is the armature axis. And we have the speed winding here. This is the schematic diagram of a DC machine; and the voltage that is appearing in the armature of the DC machine due to its rotation. Omega r is because of the flux in the d-axis produced by the field winding. The field winding and the armature winding are orthogonal to each other in the sense that, the field axis here and the armature brush axis are orthogonal to each other. And hence, the induced emf in the q-axis rotor is due to the d-axis stator.

Same thing is here appearing here that, the voltage equation in the d-axis rotor will have a rotationally induced emf due to the q-axis flux. However, we will find out the polarity of the rotationally induced emf little later. Similarly, the voltage induced in the… voltage in the q-axis rotor is given by r q r i q r. This has to be i d r; r q r i q r plus p psi q r. Again, I do not know the polarity of the rotationally induced emf. So, we will have plus minus omega r i d r. Here again, the rotationally induced emf in the q-axis is appearing because of the clock in the d-axis; that is, omega r into psi d r.

Now, what are the various fluxes? We have to define the various fluxes. What are these psi d s, psi q s, psi d r and psi q r? They are the flux linkages in the stator and the rotors. Now, psi d s is equal to L d s i d s plus M d into i d r. L d s is the self inductance of the d-axis stator. M d is the mutual between the d-axis stator and d-axis rotor. Let us go back to the previous slide. Now, the self inductance of the d-axis stator is L d s. And the mutual between the d-axis stator and d-axis rotor is M d. Similarly, the self inductance of the d-axis rotor is L d r; and the self inductance of the q-axis stator is L q s; self inductance of the q-axis rotor is L q r. And the mutual between the q-axis stator and q-axis rotor is M q.

So, psi q s produced due to the current in the d-axis stator and the current in the d-axis rotor. And hence, we can write down psi d s is equal to L d s i d s plus M d into i d r. In a similar way, I can write down the flux linkage in the q-axis stator; that is, L q s i q s plus – in the q-axis stator, due to its own current and the flux in the q-axis stator due to the current in the q-axis rotor – M q into i q r. In the similar fashion, I can write down psi d r; psi d r is equal to L d r i d r plus M d i d s. And psi q r is equal to L q r i q r plus M q into i q s. So, these are the flux linkages in stator and rotor windings.

So, let us try to see actually; we have the voltage equations: v d s, v q s, v d r and v q r. And what we have to find out right now is the direction of the rotationally induced emf. Now, we have already seen that, the rotationally induced emf appear due to the rotation of the rotor. And the rotationally induced emf also appear in the quadrature axis. It means the rotationally induced emf in the d-axis rotor is produced due to the flux in the q-axis rotor. Similarly, the rotationally induced emf in the q-axis rotor is produced due to the flux in the d-axis rotor. So, let us try to see how the rotationally induced emf is produced and what is the direction of the rotationally induced emf?

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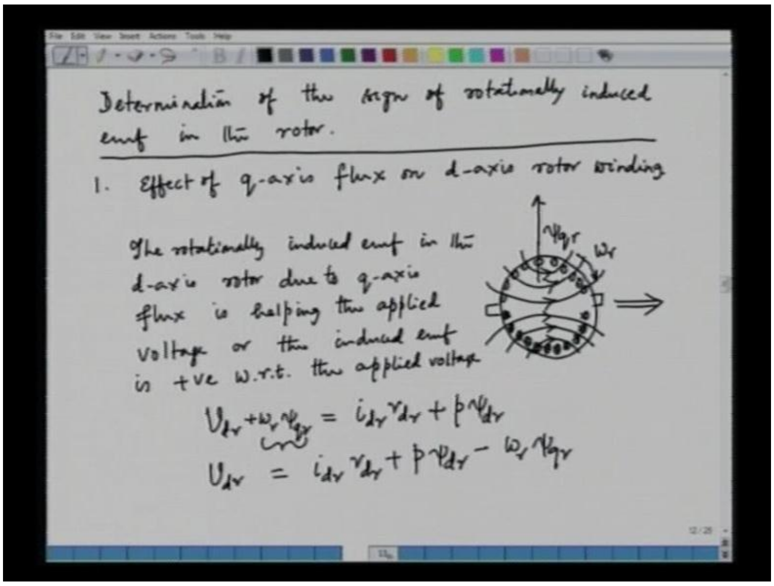

Now, we will take the first instance – the determination of the sign of rotationally induced emf in the rotor. So, if you see the rotor; let us say that, initially, we will take the effect of q-axis flux on d-axis rotor winding. So, we have the rotor winding here. And what we are trying to find out – the effect of q-axis flux; the q-axis flux is here. This is the q-axis. And this is linking the rotor. So, I will call this to be psi q r. And I am trying to find out the effect of the q-axis flux on the d-axis rotor winding. Now, the rotor winding as we have already seen; that although we have the rotor windings dr and qr, they are pseudo stationary winding. They are something similar to the windings of a DC machine.

So, I can replace the rotor winding by an armature winding having two brushes in the same axis. So, this is the d-axis winding. And this is something like a DC machine winding, which are called pseudo stationary winding. So, we can have the conductors like this like DC machine rotor. So, these are the conductors – cross-section of the conductors. So, we have the cross-section of the conductors like this here. And the rotation as per the convention is in the clockwise direction. So, this is the direction of rotation. So, we have omega r.

Now, we know that, whenever a conductor is rotating; if you want to find out the direction of induced emf, you have to take the help of the Fleming's right-hand principle. The Fleming's right-hand principle says that, if the thumb shows the direction of motion of the conductor; if the index finger shows the direction of the flux; then the middle finger will show the direction of the induced emf. The same principle we can apply here; we have the cross-section of the machine in which we have the cross-section of the conductors, are shown here.

And, in this case, we are interested to find out what is the direction of the emf. We can apply the Fleming's right-hand principle. If you see, this is the direction of flux here. The flux is radially outward. As per the convention, the flux is radially outwards. And the thumb shows the direction of motion of the conductors. And hence, the middle finger is coming out. That is the direction of the induced emf. So, I can show that, the induced emf in the upper half of the conductors will be dot. So, I can show them as dot. This is the induced emf that I am showing – coming out of the plane of the slide.

Now, similarly, in the lower half or the bottom half, I can find out the direction of induced emf, that is, cross. So, one half of the conductor are carrying dot current that is coming out of the plane of the slide. And the other half of the conductors are carrying cross current that, the induced emf is going into the plane of the slide. Now, let us assume that, the current and the induced emf are in phase. If you assume that, the current and the induced emf are in phase; it means the currents have been produced by the induced emf. So, if you assume that, we can assume that, the currents are in phase with the induced emf. So, if you assume that the currents are also in the same phase of that of the induced emf, we can find out the flux produced by the current carrying conductors in this particular structure.

We can apply Ampere thumb rule. You can see that, the Ampere thumb rule says that, if the thumb shows the direction of the current, the finger encircling the thumb would shows the direction of flux linkage. So, we can have the same principle here in the upper half of the conductors; we can have the flux linkage like this. It is encircling the conductors according to Ampere thumb rule. Similarly, in the bottom half; if you see in the bottom half, the thumb will be the direction of the induced emf and the current; and the finger would show the flux linkage – the pattern of the flux linkage. So, we can show that, this is the direction of the flux linkages. And the net flux linkage we can see is coming in this direction due to the flux in the q-axis; we have the currents producing fluxes in the d-axis in this particular fashion.

Now, this is a positive flux as per the convention; the flux, which is radially outward; it is a positive flux. So, it means the rotationally induced emf in the d-axis due to the q-axis flux is producing a positive flux and a positive current. It means the rotationally induced emf is helping the applied voltage. So, we can say here that, the rotationally induced emf in the d-axis rotor due to q-axis flux is helping the applied voltage or the induced emf is positive with respect to the applied voltage. So, we can say that, if you write down the equation for the rotor, we can say that, v d r is the rotor applied voltage in the d-axis.

And, we wrote the equation that, it is equal to i d r r d r; the resistance drop plus the p psi d r, is the statically induced emf and we had a confusion about the sign; we did not know whether the sign of the rotationally induced emf would be positive or negative. As per this analysis, we have shown that, the rotationally induced emf will be helping the applied voltage; helping means it is plus omega r psi q r. It means the sign of the applied voltage and the rotationally induced emf in the d-axis are additive are in the same sign. And if you take it to the right-hand side, the induced emf if you take it to the right-hand side, what you have here is v d r is equal to i d r r d r plus p psi d r minus of omega r psi q r. So, this actually clarifies the sign that, here it has to be negative. In the right-hand side, if you take a positive thing from the left-hand side to right-hand side, the sign changes. And hence, we have minus omega r into psi q r.

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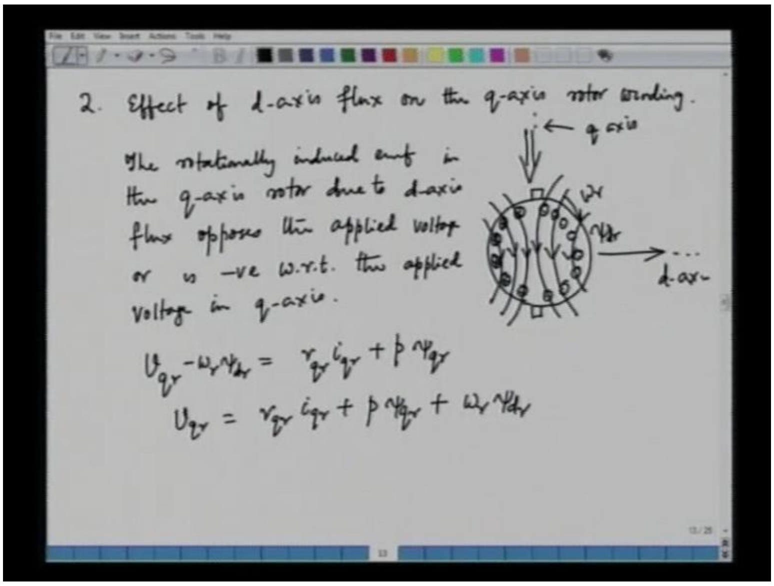

Similarly, we can find out the effect of the d-axis flux on the q-axis rotor winding, effect of d-axis flux on the q-axis rotor winding. We can again draw the picture of the rotor. So, we can show this as a rotor. And here we are interested to find out the effect of the d-axis flux in the q-axis rotor winding. The flux is in the d-axis. And we can call this to be this psi d r linking the rotor d-axis. And we have a winding in the q-axis. So, the winding in the q-axis can be represented by a pseudo stationary winding; which means I can have something similar to a DC machine armature winding having the two brushes in the q-axis. And if you take a cross-section; we are showing it in terms of a cross-section. In the cross-section, we can see that, the conductor cross-sections will be visible. These are the conductors and we are seeing the cross-section of the conductors across the periphery of the armature. And the router is rotating in the clockwise direction as per the convention. So, we have the rotation is omega r; omega r is the speed of the rotor.

Now, again we can apply the Fleming's right-hand principle. Applying the Fleming right-hand principle, we can determine the direction of the induced emf. And the positive induced emf will inject a positive current. So, we can apply the same principle here that, if this is the direction of the field and this is the direction of the motion, the middle figure would be the direction of the induced emf. So, we can do that. And after this, we can find the direction of the induced emf will be dot in the right-half from the brushes. In the opposite half, we will see that, the induced emf direction will be cross; that will be entering into the plane of the slide.

And, we can again assume that, the induced emf will be circulating a current. And the current and the induced emf will be in phase. If they are in phase, the induced emf will be circulating a current. And the current will give rise to a flux linkage. And the current and the flux linkage relationship can be given by Ampere thumb rule, where the thumb shows the direction of the current and fingers will show the direction of the flux linkages. So, in a similar fashion, we can say that, the flux linkages would be in the following fashion. So, these are the flux linkage due to the conductors in the right-hand side; and this would be the flux linkage due to the current carrying conductor in the left-hand side. And the resultant flux linkage will be in this direction. So, if you say that, the resultant direction of the flux linkage; it will be downwards; it will be towards the center of the circle, because this is the d-axis.

The d-axis is in this direction and the q-axis in this direction. This is the q axis. So, if I see in terms of the q-axis, the flux is entering the circle – entering the center of the circle. It is not outward, but it is inward. So, it is a negative flux linkage. So, we can conclude here that, due to the rotation of the rotor, the rotationally induced emf in the q-axis will be producing current and flux linkage, which is negative. It means that, the q-axis rotationally induced emf due to the d-axis flux is in opposition to the applied voltage. So, we can say here that, the rotationally induced emf in the q-axis router due to d-axis flux opposes the applied voltage or is negative with respect to the applied voltage in q-axis. So, we can write down the rotor equation in the q-axis once again as we have done for the d-axis. We can do that; v q r – that is equal to r q r i q r is the resistive drop plus p psi q r. And we did not know the sign of the rotationally induced emf.

And, as per this analysis, we have seen that, the rotationally induced emf is in opposition with the applied voltage. So, we can say this is minus of omega r psi d r. So, if you simplify this equation, we can say that, v q r is equal to r q r i q r plus p psi q r plus omega r psi d r. So, we have no ambiguity right now; we have been able to find out as per the convention; we applied the convention that, we have said it at the very beginning.

And as per the convention, in the q-axis, it is plus omega r psi q r in the d-axis; it is minus omega r psi d r. So, we will just write down the equation.

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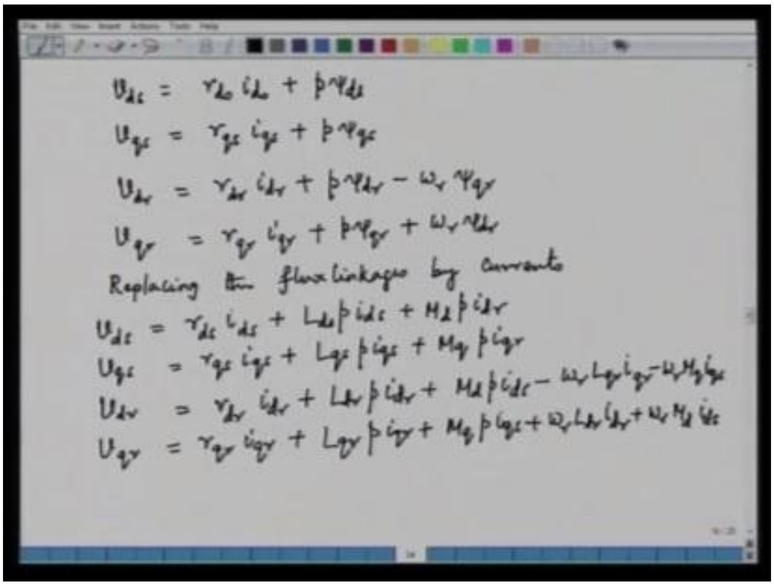

Once again, we will say that, v d s is equal to r d s i d s plus p psi d s; v q s is equal to r q s i q s plus p psi q s; v d r is equal to r d r i d r plus p psi d r minus omega r psi q r; v q r is equal to r q r i q r plus this p psi q r plus omega r psi d r. So, these are the four equations that will be essential to simulate the generalized electric machine, which is the Kron's primitive machine. But, these are in the flux linkages. We have seen that, the voltage is expressed as a function of current and flux linkages. So, if you replace the flux linkage by the current, we can rewrite this equation as follows.

Replacing the flux linkages by currents, we can say v d s equal to r d s i d s plus L d s p i d s plus M d p i d r; v q s is equal to r q s i q s plus L q s p i q s M q p i q r. And v d r is equal to r d r i d r plus L d r p i d r plus M d p i d s. This would be minus omega r L q r i q r minus omega r M q i q s; and v q r is equal to r q r i q r plus L q r p i q r plus M q p i q s; and then we have the rotationally induced emf plus omega r L d r i d r plus omega r M d i d s. So, these four equations are quite important and interesting. These relate the currents with the voltages; v d s, v q s, v d r and v q r are the voltages; and the currents are i d s, i q s, i d r and i q r are the currents. So, from this, we can write down this equation in a matrix form, which will be more interesting. And the equation in the matrix form, you do not have to remember; you can just write down by inspection.

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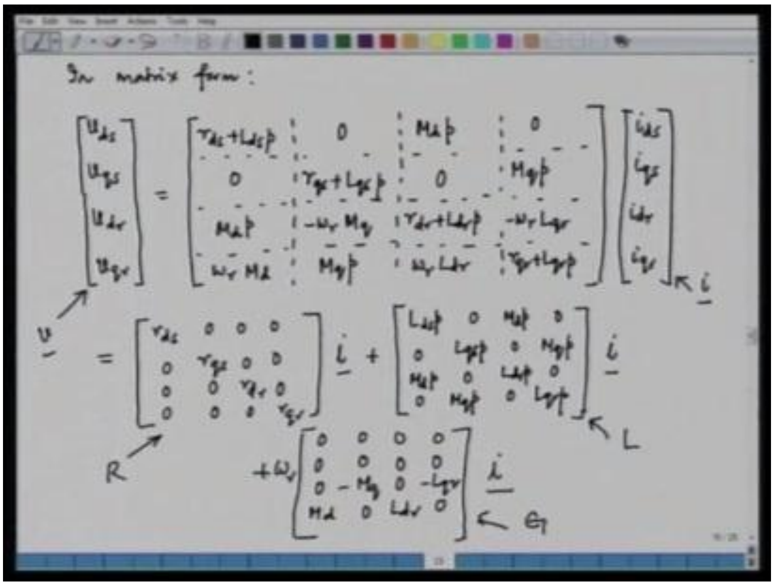

Let us see how we write this equation in a matrix form. In matrix form, we can write down this equation as v d s, v q s, v d r and v q r. This is a vector. And then we have a 4 by 4 matrix and we have to fill up the elements of this matrix. And this is i d s, i q s, i d r and i q r. Now, we have a 4 by 4 matrix. And we have to find out the elements of this matrix. And this matrix can just be written by inspection without remembering the detailed element that we wrote little before. So, we can say here that, this is the self-inductance ((Refer Time: 35:29)). So, we can have the resistance drop here – r d s plus L d s p. And in the stator, we do not have any rotationally induced emf. And we just have a statically induced emf. And the coupling between the d-axis stator and the d-axis rotor will give us M d p; p is a derivative operator – d by dt. And this term – you can put comfortably 0 without any second thought.

Similarly, in the second row, we can have here r q s plus L q s p. And then here we have the coupling between the stator and the rotor in the q-axis. So, we have M q p here. And these elements will be 0 without any second thought. Similarly, in the rotor equation, this is the d-axis rotor – the third row. So, we have r d r plus L d r p. And then we have the coupling here – M d p. This is the statically induced emf – L d r p M d p. Then we have the rotationally induced emf in the rotor; the rotor is rotating. And due to rotation, we have rotationally induced emf. And those will be coming in the cross axis; it means the q-axis flux will produce rotationally induced emf in the d-axis. So, here we have minus omega r; also, minus omega r here. And this is the rotor. The rotor here we will have L q

r. And here we have M q. Similarly, in the q-axis rotor, we have r q r plus L q r p. The coupling term here is M q p from the stator. And then we have the rotationally induced emf; but it will be positive in this case. This is omega r L d r. And here it is omega r M d. So, we have a 4 by 4 matrix relating the current and voltages that we can just write down by inspection by proper understanding without having to remember each and every element.

Now, if you see this matrix, this matrix has got the resistance term; has got the inductance term; has also got… has the speed term. So, we can break up this matrix into three different parts. One part containing only the resistance term; the other part containing only the induction term with p; p is the derivative operator; and the third component consisting of the speed; the speed is omega r here. So, we can split this into three different matrices. That is equal to… We can have the resistance term here – r d s, 0, 0, 0; 0, r q s, 0 and 0; 0, 0, r d r, 0; 0, 0, 0, r q r. This is the resistance term. So, this matrix as I was talking, that this can be broken down into a resistance matrix and an inductance matrix having p term; and a matrix associated with omega r or the speed term. So, this is the first component; that is, r d s, r q s, r d r, r q r; it is a diagonal matrix.

And, I will multiply this vector, that is, i. And what is i vector? i vector is this vector. And then I can have the inductance matrix having p terms. And this matrix is L d s p, 0, M d p, 0; 0, L q s p, 0, M q p; then we have M d p, 0, L d r p, 0; 0, M q p, 0, L q r p. This is again multiplied with the current vector, that is, i; i is a vector, which has got the elements i d s, i q s, i d r and i q r. And then what is remaining is the matrix with the speed term. That we can write down in the following fashion. We can… This also a 4 by 4 matrix.

And, what we have; this matrix has got the speed term. So, I can take this speed term out. And the first row does not have any speed term; we can put all these equal to 0 same as the second row. The speed terms are only appearing in the rotor. So, I have got minus M q here; this is 0; this is minus L q r. And then we have M d, 0 L d r, 0. Then we multiply with the current vector, that is, i. So, this is an interesting equation. And this equation has got the resistance term, the inductance term having the derivative of the current, and the matrix having the omega r or the speed term.

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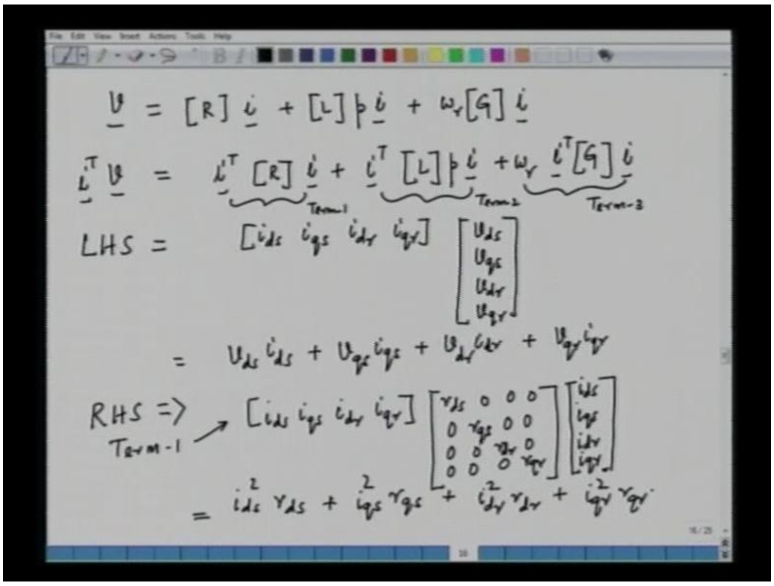

Now, if we… See if we try to see this equation, once again we can rewrite this equation as follows. The voltage equation that we are writing; the voltage vector v, which has got the components; what is this v? This one is the vector v. So, this is the v vector. And this matrix I can call to be the resistive matrix or the resistance matrix, that is, R. And this matrix we can call to be this inductance matrix of L. I can take this p term out of this. And the third matrix is the matrix, which we can call as G. So, we can rewrite the equation once again; v is equal to r into I; this is capital R I can write here – plus L p i plus omega r G i. So, this is again a representation of the same equation in a different form having the resistance drop. And then we have the current derivative and the p d m f or omega r.

Now, what I will do here; I will premultiply this equation by i transpose; i is a column vector. So, I can transpose it and premultiply. So, I can have this matrix here – i transpose R i plus i transpose L p i plus omega r is a ((Refer Time: 43:21)) quantity – i transpose G and i. Now, if you see in this equation, what is the left-hand side? The left-hand side of this equation is i transpose into v. Now, i transpose into v is the power input to the system – the electrical power input to the system. Now, what is i transpose? i transpose is the left-hand side of the equation is as follows; I transpose is i d s i q s i d r and i q r. And what is v? v is v d s, v q s, v d r and v q r. Now, if you simplify this, you will have v d s i d s; this i d s is multiplied with v d s – plus v q s i q s plus v d r i d r plus v q r i q r. As we know that, in the Kron's primitive machine model, we have four different windings: d s, q s in the stator; d r, q r in the rotor. So, the electrical power input to the whole system is v d s i d s, v q s i q s, v d r i d r plus v q r i q r. So, that is basically the left-hand side of the equation.

What about the right-hand side? Now, if you see the right-hand side, the right-hand side of the equation has got three different terms. So, if you see the right-hand side of this equation; I can call this to be term 1. And this is term 2. And this one is term 3. Now, what about the term 1? The term 1 is i transpose R i. Now, let us see what is i transpose R i; i transpose is i d s, i q s, i d r and i q r. What is this R? What is the diagonal matrix? r d s, all elements as 0; 0, r q s, 0, 0; 0, 0, r d r, 0; and then 0, 0, 0, r q r – post multiplied by i d s, i q s, i d r and i q r.

Now, if we simplify this equation, what we… We can pre multiply and post multiply these currents with the matrix and then simplify. The results I will just write down here. The result of this term 1 – this is the term 1, is i d s square into r d s plus i q s square r q s plus i d r square r d r plus i q r square r q r. Now, what is this? This equation represents the loss of the system – the electrical loss of the system. i square R loss of the system. So, the term 1 represents the i square R loss of the system. We have assumed that, there is no core loss, there is only copper loss. So, this four components: i d s square r d s plus i q s square r q s plus i d r square r d r plus i q r square r q r represent the loss of the entire system.

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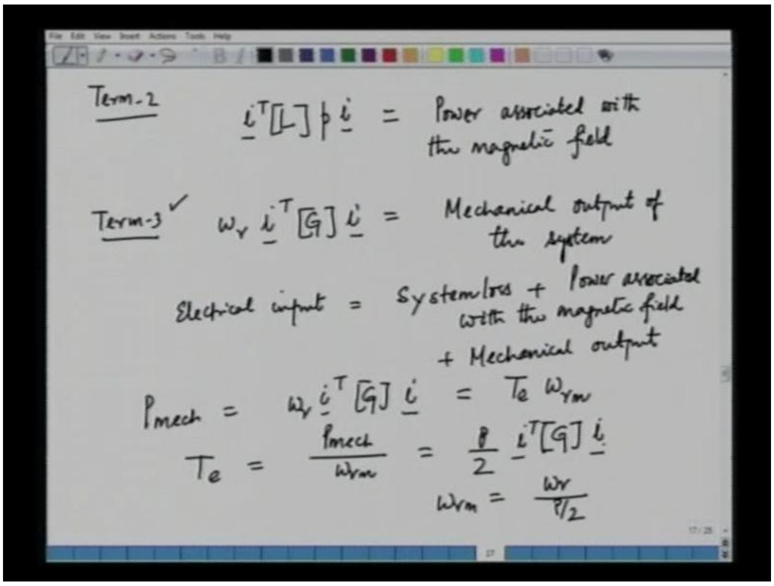

Now, what about the second term? The second term is this term. Now, if you see this particular term; we can just see what is this term 2? The term 2 is i transpose L matrix and p i; i is a vector. Now, if you see; this would represent the power associated with the magnetic field. If you expand this, you will see that, this term is the power associated with the magnetic field. So, this is also the power, and then what about the third term? The third term is i transpose G i into omega r. This is the mechanical output of the system. It is quite natural, because if you see the entire equation, what we can say here is the following that, the electrical input is equal to the system loss plus the power associated with the magnetic field plus the mechanical output. So, this actually is the energy balance of the system; that we are giving some electrical input to the system. And some component as wasted as losses – i square R loss.

Then, some component is stored in the magnetic field that is associated with the self and the mutual inductance. And then the remaining part, which is not stored, is coming out of the system as the mechanical output. So, then if you want to find out the mechanical output and the torque, we have to concentrate on the third term. And the third term is this term. So, we can say here that, the p mechanical is equal to omega r i transpose G i. And then that is equal to the torque into the speed. So, we can say that, that is equal to the electrical – electromagnetic torque coming out of the shaft of the machine – T e into omega r m; omega r m is the mechanical speed.

Now, if you want to find out the torque, the torque is equal to p mech by omega r m. That is equal to p by 2 into i transpose G and i. So, this is basically the torque that is coming out of the system. And then we have p by 2 torque here, because the mechanical speed is given as the electrical speed divided by the pole-pair, that is, p by 2; p is the number of poles. And hence, the electrical speed divided by the pole-pair will give us the mechanical speed. And if you divide the mechanical speed here – p mechanical by omega r m, what you obtain here is the torque output, that is, T e; that is equal to p by 2 into i transpose G i. So, this is the expression for the torque. And the expression for the torque will be beneficial for us when we go for the stimulation of electric machines. The mechanical output is the torque; and the torque will be leading to the speed. So, we have to know the torque output to simulate the entire machine.

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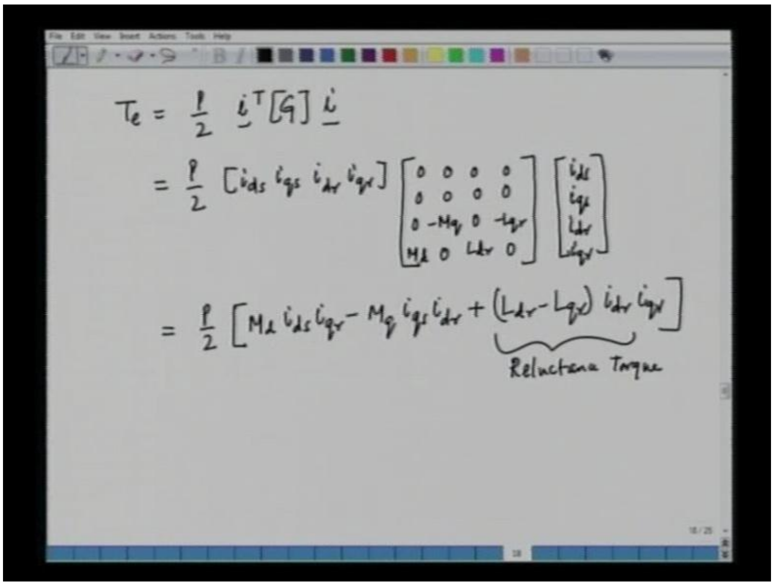

So, here we have already seen that, T e is equal to p by 2 i transpose G and i. So, this is the expression for the torque. And if you want to find out the torque expression, we can simplify this equation. And this equation has got the product of the current, the matrix G and again the current. So, we can simplify this equation p by 2. And what about i transpose? i transpose is i d s, i q s, i d r and i q r, Now, what is the G matrix? G matrix we can write here. And then we have i d s, i q s, i d r and i q r. Now, we can fill up this matrix G. And the matrix G as we have already seen has got these elements. These are the elements of matrix G – minus M q, 0, minus L q r, M d, 0, L d r, 0. So, if we simplify this, we will have the expression for the torque. So, we can simplify this. And what we obtain here is the final expression for the torque; that is equal to p by 2 into M d i d s i q r minus M q i q s i d r plus L d r minus L q r into i d r and i q r. So, if we simplify this equation, we get expression for the torque. This is the expression for the torque that we have seen.

Now, this torque is a very interesting component. This torque is called the reluctance torque, because this component of the torque is coming out due to the variation of inductance between the d-axis and q-axis; L d r minus L q r. You can see this particular torque. So, this is the total expression for the torque for a Kron's primitive machine model. Now, in these two lectures that we have already seen that, we have introduced the generalized theory of electric machine; also, we have introduced the concept of the Kron primitive machine model and we have derived the equation for the voltage and current, and also the equation for the torque. Now, as we have already said, the generalized theory will help us to analyze all machines in a common framework. So, we need some examples. So, we will see that, how the generalized theory can be used to simulate a simple DC machine in the next lecture. In the next lecture, we will try to take a DC machine model using the generalized theory, we will try to simulate the DC machine.