

Advanced Electric Drives
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Lecture - 17

Welcome to this lecture on advanced electric drives. In the last lecture, we were discussing about the control of synchronous motor drive, and we have seen that the synchronous motors are actually applied for very high power application especially the wound-field synchronous motor where the field winding is mounted on the rotor. And these kinds of synchronous motors are applied for very hyper application beyond one megawatt rating.

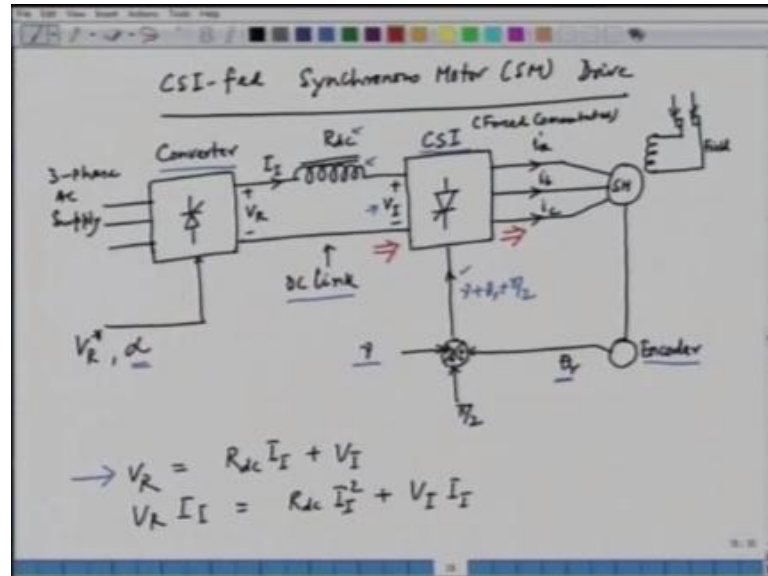
And we have also seen that when we control the synchronous motor, there could be two modes of operation; one mode is called a true synchronous mode in which the synchronous motor is applied from a voltage source of independent frequency. Say for example, it could be a synchronous motor connected to an infinite bus. So, when a synchronous motor is connected to an infinite bus, the infinite bus frequency is fixed that, and that is independent of the rotor speed. And hence, sometimes the synchronous motor undergoes hunting and instability. When the rotor speed is different from the synchronous speed, the synchronous motor exhibits hunting, and sometimes if the load is very high and if it is suddenly applied it may also lose synchronism which is instability.

So, when we talk about a synchronous motor under true synchronous mode as we have seen in case of an infinite bus feeding a synchronous motor, the applications are limited not mainly for the variable speed application. But when we talk about the variable speed application, we want the speed to be continuously variable from rest to the full speed, and hence a closed loop becomes necessary, and that is called a self-controlled synchronous motor where the rotor position is used to determine the voltage applied to the stator.

Hence, the rotor frequency or the rotor speed is tied with the stator frequency, and hence, there is no question of falling out of step; there is no question of hunting, and hence, the motor becomes the stable one at any given speed. And one of the examples that we are discussing in the last lecture was that of a CSI phase synchronous motor. The motor is being fed from the CSI a force-commutated CSI, and the application could be for fan

application, could be for pump application, but especially for high power application. Let us see the circuit diagram of a CSI fed self controlled synchronous motor drive.

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Now this is what we were discussing in the last lecture that we have a converter, and this converter is a AC to DC converter which could be a control rectifier which is changing the 3 phase AC supply to a DC supply; that is V_R . V_R is the output of the converter which is the DC supply. Then this is fed to the DC link, and this is the DC link, and the DC link has a very high inductor. This is the inductor in this case, and the purpose of this inductor is to keep the current approximately constant. And then we have the CSI, and the CSI is a force commutated CSI, and the outputs are the three current I_a , I_b , I_c , which are feeding the synchronous machine.

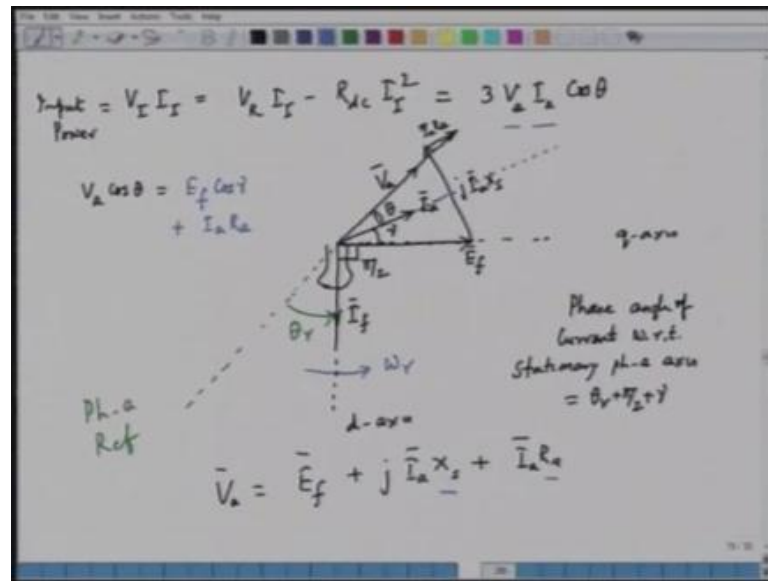
The field may be separately excited as we will have a field winding here. This could be the field in this case, and field in this case can be excited separately; it is a wound-field synchronous machine. So, we can excite the field separately by DC source. We have slip rings and gausses, and this is the field. The synchronous machine when it is self controlled requires the position sensor. The rotor position sensor is essential to determine the phase angle of the applied voltage so that the rotor and the stator have always synchronized. So, we have an encoder here, and this encoder will be used for determining or finding out θ_r . θ_r is the angle of the field or the rotor with respect to the stationary phase a axis.

And this rotor angle can be used to determine the phase of the applied currents that is i_a , i_b and i_c . So, we defined an angle γ ; little later we will see what is the significance of this γ , and we have a space shift of π by 2 and we determined this angle that we get here is $\gamma + \theta_r + \pi/2$. That is fed to the CSI to determine the phase of the current that is injected into the phase a, phase b, phase c of the stator of the synchronous motor. And to vary the speed we need to control the current, and hence, we control the voltage that is V_R of the converter. Voltage V_R is controlled by controlling the triggering angle or the firing angle of the 3 phase convertor bridge. This is basically consisting of SCR. So, we can control the triggering angle of the SCR to be able to control the DC link current and which in turn which control the torque and the speed of the machine. Now this is the complete power circuit; to be able to understand the operation in detail we will be only talking about the steady state operation.

We will ignore the transient for the time being. We will only concentrate of the steady state operation of the drive and try to draw an equivalent circuit of this CSI fed synchronous motor drive system. Now let us see that in this DC link we can apply the Kirchhoff's voltage law; that is V_R is equal to $R_{dc} I_i$ plus V_i ; V_i is the inverter input voltage. And if we multiply both the sides by I_i we can say that $V_R I_i$ is equal to $R_{dc} I_i^2$; that represent the loss in the DC link. The inductor that we are using can have a resistance that is R_{dc} . R_{dc} is the resistance of the DC link inductor and $R_{dc} I_i^2$ represent the loss in the inductor and plus $V_i I_i$ that is the input to the CSI.

So, what we see here is that the power is entering the CSI, and then it is going out of the CSI. So, in this case we can assume that the inverter is having 100 percent efficiency; there is no loss in the inverter in the ideal case. So, if we assume that the inverter does not have any loss whatever is the input to the inverter will be the output of the inverter. The input is the DC power, and the output is the AC power. So, this $V_i I_i$ represents the input power to the CSI.

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Now we can see here that this $V I I I$ is the input power that is equal to this equation we can manipulate little bit. We can take out this $V I I I$, and then if we simplify we will have $V R$ into $I I$ minus $R d i$ square. Now this $V I$ into $I i$ is the input of the CSI, and we have assumed that the inverter is lossless, and hence the input should be equal to the output, and that is equal to the input of the motor. The motor power factor is $\cos \theta$, and the power phase voltage is V_a , and power phase current is I_a . So, we can say that the output of the CSI is 3 into V_a into I_a into $\cos \theta$. Now this is the equation; this is the power balance equation.

Whatever is the input the same is coming as the output, and the output of the converter minus the losses in the DC link is the input of the CSI as a current source inverter, and the current source inverter is lossless does not have any loss. So, the input is the same as the output, and what we have here is that the output of the CSI is the input of the motor which is 3 into $V_a I_a \cos \theta$. Now let us draw the phasor diagram. So, if we draw the phasor diagram here we can start with let us say the field axis. So, this is our field axis. So, we can say this as the d axis. So, we can show the field pole like this, just show the field of the synchronous machine, and then the q axis is right angle to the d axis.

So, this is our q axis, and this angle is 90 degree. And of course, the field current is in the direction of the field axis. So, we can say this is I_f ; this vector is I_f . I_f is in the d axis, and the field induced EMF; this I_f is induced some open circuit voltage; that voltage is

in the q axis. So, this is our E_f . We know that the induced TMF and the flux or the MMF are right angle to each other. So, this E_f is along the q axis, and I_f which is inducing E_f is around the d axis. There is quadrature relationship between the flux and the induced TMF which is very well known; from the DC machine we know that the field is in the d axis and the induced TMF in the armature is in the q axis.

Similarly, in case of synchronous machine if we take the rotor reference frame, the field MMF is in the d axis, and the field induced EMF is in the q axis. Then we have the current, and the current can be in any arbitrary direction based on the power factor. So, let us say that this is the current, armature current I_a . So, if this is the armature current we know that in case of a motor, we can write down this equation that the applied voltage V is equal to E_f plus $j I_a X_s$ plus $I_a R_a$. This is the phasor equation; this is the steady-state phasor equation.

We have ignored saliency for simplicity, and synchronous reactance is X_s , the armature resistance is R_a , and I_a is the current phasor is the phasor quantity in the steady state condition. So, this is the equation of a motor. So, we can translate this equation into a phasor diagram. So, in this case what we have here is that we have E_f and we have I_a and we can of course draw $j I_a X_s$. So, what is $j I_a X_s$? So, we can draw this. This is the current, and the current and $j I_a X_s$ will be right angle to each other. And then we have $I_a R_a$ and $I_a R_a$ will be in this direction. So, this is they are all phasor quantity. So, we have to show them appropriately by phase angle and amplitude.

This is $I_a R_a$, and the resultant of this would be the terminal voltage that is V or V_a ; we can call this to be V_a , and this is to be the power factor angle. So, we can say that the angle between V_a and I_a is θ . θ is a power factor angle, and this angle the angle between phase a current or the current phasor on the q axis is γ . And this actually is the rotating phasor; each one is the rotating phasor, and these are DC variable when we talk about the rotor reference frame. Rotor is rotating at a speed of ω_r ; this is moving at a speed of ω_r , what about the stationary phase? Now if it is the stationary phase, we can have a reference for the stationary phase that is the phase a. The phase a axis is the reference of the stationary phase. So, we can say this is the phase a axis.

So, if you take the phase a reference in this case this angle that the rotor sustains with phase a is the rotor angle that is θ_r ; rotor is moving away at a speed of ω_r . So, θ_r is the angle between the d axis of the rotor and the reference phase a axis which is stationary. Now if we see the phase angle of the current that is I_a , what is the phase angle of the current? This phase angle of the current is with respect to stationary phase a axis that is equal to we can say this is equal to $\theta_r + \pi/2$; this angle is right angle. So, we can add a $\pi/2$ here plus γ ; γ is angle between the q axis and phase a.

So, this phasor diagram is also a space phasor diagram because these are also there is a one-to-one correspondence between time phase and space phase. So, if we take a snapshot in the machine, and if we visualize all this induced TMF, the MMFs, the flux linkages and the voltage drops in the space; this would be the representation in the space. So, we can call this to be a space phasor diagram as well. So, if we want to find out the phase angle of the current phasor with respect to stationary phase a axis; that is that will be $\theta_r + \pi/2 + \gamma$; that is what we were talking about little earlier that if we want to find out the phase angle of phase a let us say that is $\gamma + \theta_r + \pi/2$.

Of course, the actual currents are I_a , I_b , I_c , and once we find out the phase angle with respect to I_a , we can find out the phase angle with respect to I_b and I_c also, because the phase b and phase c are shifted from phase a by 120 and 240 respectively. So, this is what we have here that we have the phase angle of the current phasor with respect to the stationary phase axis is $\theta_r + \pi/2 + \gamma$. So, this is the phasor diagram that we have, and from the phasor diagram we have to find out what is $V_a \cos \theta$. Now we can write down the expression for $V_a \cos \theta$. Now if we find out the expression for $V_a \cos \theta$ what we need to do is the following.

This is axis of V_a , I mean I_a . So, if we project the V_a along I_a we get $V_a \cos \theta$. Now what are the other voltage drops on the same axis? So, what we do, we project the other voltage drops also on the same axis; the other voltage drops are E_a , E_f into $\cos \gamma$ plus the resistance drop that is $I_a R_a$. So, what we do here we refresh this $V_a \cos \theta$ by $E_f \cos \gamma$ plus $I_a R_a$. So, if we substitute that and simplify what we get is the following.

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$$V_T I_T = V_R I_T - R_{dc} I_T^2 = 3 V_a I_a \cos \theta$$

$$\text{or, } V_R I_T - R_{dc} I_T^2 = 3 I_a [E_f \cos \gamma + R_a I_a]$$

$$I_a = \frac{1}{\sqrt{2}} \cdot \frac{4 I_T}{\pi} \cdot \cos 30^\circ = \frac{2\sqrt{2}}{\pi} I_T \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{6}}{\pi} I_T \quad \text{or, } I_T = \frac{\pi}{\sqrt{6}} I_a$$

So, we have the original equation here that $V I I_i$; that is equal to $V R I_i$ minus $R d c I_i$ square; that is equal to $3 V a I_a \cos \theta$. So, we can replace that in the following fashion $V R I_i$ minus $R d c I_i$ square; that is equal to $3 I_a$ into $V a \cos \theta$ is $E_f \cos \gamma$ plus $R a I_a$. Now here in this equation we have two currents; one is I_i . This is I_T , and then we also have I_a ; I_a is the motor currents. This is the motor fundamental of current, and I_i is the DC link current, and there is the relationship between the DC link current and the motor fundamental component of current, and the relationship can be derived as follows.

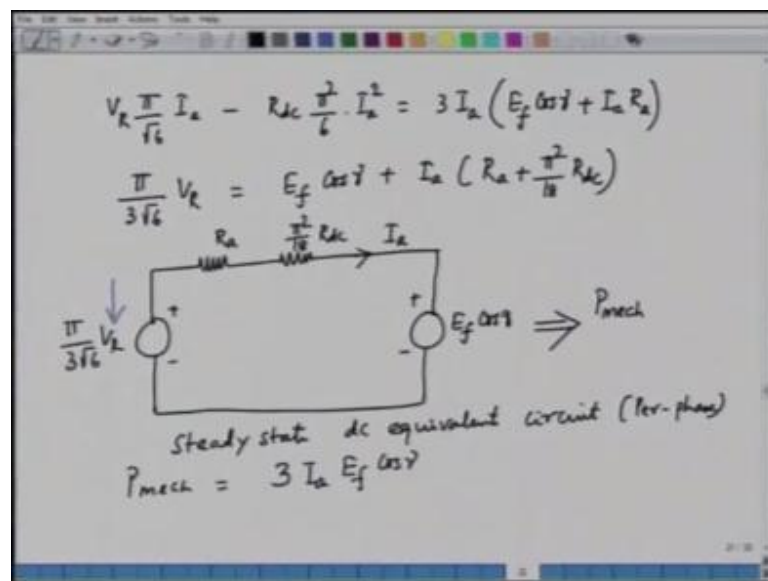
We know that in a CSI, if we see the output current profile of a CSI it is basically like a quasi square wave current pattern. This is our time axis or ωT axis like this, and then this is I_a . Say for example, we are talking about the phase a currents, and the peak value of the current is I_i . It is the current of the DC link; the DC link current is reflected in the AC side, and the current in the AC side is not a sinusoid. It is basically a quasi square wave when the inverter is the current source inverter. Now this quasi square of current will have a fundamental components and without so many harmonics. So, we ignore the harmonics, because harmonic do not contribute to the torque production I mean the fundamental torque production.

When we are talking about the average torque production, we are only concentrating on the fundamental component of current, and that has to be derived. So, this can be broken

down into a fundamental component of current which is sinusoid, and this is having this width of 120 degree. Similarly, in the negative half cycle also this is symmetrical 120 degree, and this duration between the two pulses here is 60 degree. So, we can find out what is I_a ? I_a is the fundamental component RMS value of the fundamental component of motor current; that is equal to $\frac{1}{\sqrt{2}}$ the peak value of this fundamental component is $\frac{4}{\pi} I_i$ by π into \cos of 30 degree. This can be easily derived by finding of the Fourier series of the current.

So, we keep the fundamental component and the expression is given as follows. So, we can simplify that. So, we have $\frac{2}{\pi} I_i$ by π I_i , $\cos 30$ is $\frac{\sqrt{3}}{2}$. So, what we have here is this that this is $\frac{\sqrt{6}}{\pi} I_i$. So, what we do here is that we replace this I_i by I_a . So, we can say that this implies I_i is equal to $\frac{\pi}{\sqrt{6}} I_a$. So, we can substitute this in this equation. So, in this equation what we have we can take the values of I_i and replace this I_i by $\frac{\pi}{\sqrt{6}} I_a$ and find out the final equation. So, we can find out the final equation.

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So, we have V_R into $\frac{\pi}{\sqrt{6}}$ into I_a . We have replace I_i by $\frac{\pi}{\sqrt{6}} I_a$ here minus $R_{dc} I_i^2$. So, I_i is $\frac{\pi}{\sqrt{6}} I_a$. So, we have here $\frac{\pi^2}{6} I_a^2$ into I_a . So, we have here $\frac{\pi^2}{6} I_a^2$; that is equal to $3 I_a$. Here we do not have to replace; it is already in I_a . E_f into $\cos \gamma$ plus I_a into the resistance, and the resistance here is R_a . So, this is what we have here, and we can divide the left hand side and the right hand side by 3.

And cloak these current parts, and there will be the cancellation. We have I_a here in the left hand side and the right hand side. So, I_a will be cancelled.

So, what we have finally is the following, $\frac{\pi}{3} \sqrt{6} V_r$; that is equal to $E_f \cos \gamma + I_a R_a + \frac{\pi}{6} R_d c$. So, this equation is the steady-state equation. We have ignored all the transient conditions, and of course, this is basically something like a DC equation. There is no induction term here; there is no $L \frac{di}{dt}$ term here. So, this is something like a DC equation. So, we can write a steady state equivalent circuit based on this equation, and the steady state equivalent circuit is shown as follows.

So, what we have here is the applied voltage $\frac{\pi}{3} \sqrt{6} V_r$, and then we have the resistance here R_a , then we have the DC link resistance $\frac{\pi}{6} R_d c$; this is $\frac{\pi}{6} R_d c$. And then we have the back EMF which appears as $E_f \cos \gamma$. So, we can call this to be a steady state DC equivalent circuit. Now this DC equivalent circuit will be sufficient to find out the steady state torque and steady state speed. So, if we are only interested in the steady state response in steady state speed and steady state torque, we did not consider the transient conditions. And this equivalent circuit is going to give us the steady state response of the CSI phase synchronous motor drive.

Now this resembles that of a DC machine. We have seen that in case of a DC machine we have a resistance and we have a back EMF in the armature, and the field is separately excited. We can keep in the special equation the field current constant. So, in a synchronous machine like this, we can keep field current constant, and if we want to change the speed we change the converter voltage, and the converter voltage is V_r . So, if we change this V_r , we will have an increase current, and the current in the circuit here is I_a .

So, if we increase V_a , I_a will be increased, and if I_a is increased there will be more and more torque, and the speed will be increased. Now let us find out the expression for the mechanical output power and the speed. So, here in this case what we have the power here P_{mech} is power coming out of this, the gross mechanical power, and this mechanical power is the product of current into the induced TMF. So, the current is I_a is a DC circuit. There is no phase angle, and this induced TMF is $E_f \cos \gamma$. Now

this is the power phase equivalent circuit because we divided this equation by 3. So, we can say it is a power phase circuit.

So, the synchronous machine has three phases. So, if we want to find out the total mechanical output power, we have to multiply this with 3. So, we can multiply this with 3 here, and then this is the total mechanical power. And if we interested to find out the torque, the mechanical power has to be divided by the mechanical speed. So, this is what we have.

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The image shows a handwritten derivation for torque T_e . It starts with the definition of torque as mechanical power divided by mechanical speed: $T_e = \frac{P_{mech}}{\omega_{m}} = \frac{P}{2} \frac{P_{mech}}{\omega_{re}}$. This is then substituted with the expression for total mechanical power: $= \frac{P}{2} \frac{3 I_a E_f \cos \gamma}{\omega_{re}} = \frac{3P}{2} \frac{I_a (\lambda_{af} \omega_{re}) \cos \gamma}{\omega_{re}}$. A note indicates $E_f = \lambda_{af} \omega_{re}$. The equation simplifies to $T_e = \frac{3P}{2} I_a \lambda_{af} \cos \gamma$. For a synchronous machine, $\gamma = 0$, so $T_e = \frac{3P}{2} I_a \lambda_{af}$. A final note states: "(Similar to that of a separately excited dc machine)".

And then if we want to find out the torque in this case T_e , T_e is P_{mech} by $\omega_{r m}$, and $\omega_{r m}$ is the mechanical speed. We can replace that by the electrical speed divided by pole pair. So, we can multiply that by pole pair; this equation by p by 2 and p by 2 is pole pair, because p is the number of poles. And we can substitute the value of P_{mech} ; the P_{mech} is $3 I_a E_f \cos \gamma$. So, $3 I_a E_f \cos \gamma$ by $\omega_{r e}$, and we multiply that by p by 2. Then we can say that this is $3 p$ by 2 into I_a , and the induced EMF we can replace by $\lambda_{a f}$ is the field flux linkage into $\omega_{r e}$ into $\cos \gamma$ divided by $\omega_{r e}$.

Here we have assumed that the field induced TMF E_f we have assumed to be the field induced TMF, and that is the induced TMF in the armature due to the field current, and we defined a flux linkage called $\lambda_{a f}$ which is the flux linkage in the armature due to the field current. So, this E_f is equal to $\lambda_{a f}$ into $\omega_{r e}$. If we assume that e

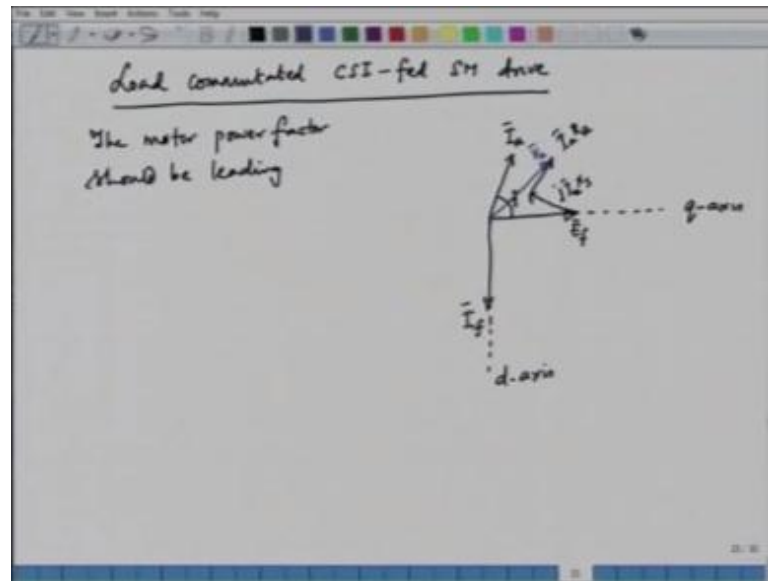
f equal to $\lambda_a f$ into $\omega r e$, we can write that here. And then finally, the expression for the torque is $3 p \text{ by } 2 I_a \lambda_a f \cos \gamma$. Now this equation is a very simple equation which resembles that of a separately excited DC machine. Of course γ in this case can be kept constant.

Now if we keep γ constant and field flux constant, it means this is constant, and this is also constant. We are not interested to change the field current; we keep the field current constant. The torque can be controlled by controlling the armature current, and that is something like a DC machine. And if we want to maximize this torque, we make γ equal to 0. We know that we have a freedom here; because we have a CSI and in the CSI we can have any phase angle and γ in a special case can be equal to 0. So, if we keep γ equal to 0 then the torque is maximized, and that is something like a torque of a DC machine, because the current is only in the q axis, and the field is in the d axis, because γ is equal to 0 and the torque is maximized.

So, we can say here that if γ equal to 0 we can say that T_e is equal to $3 p \text{ by } 2$ is a constant into I_a into $\lambda_a f$. So, that is similar to that of a separately excited DC machine. So, it means whenever we have a close loop position feedback we can control a synchronous machine like a separately excited DC machine, and in this case the torque is also maximized. So, this equation shows that the torque in this case is controlled like that of a DC machine. We can keep the flux constant; that is $\lambda_a f$ constant, and we can change I_a to control the torque.

Now in the very beginning we assumed that the CSI is force commutated. Now in many situations the CSI need not be force committed. Say for example, the force commutation requires separate force commutating components like inductance and capacitance. So, if the CSI is to be load commutated, it means the CSI would be commutated by the back EMF of the synchronous machine; the power factor has to be ready. So, we will just briefly discuss that how the situation changes when we talk about a load commutated synchronous machine.

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So, we are talking about load commutated CSI-fed synchronous motor drive. We will just talk very briefly about this. Now for the load commutation, the power factor has to be leading. So, that is very important that the motor power factor should be leading. So, if the motor power factor has to be leading that has to be obtained by a proper control of the synchronous machine. So, what we do here we do not make γ equal to 0, because our objective is to achieve leading power factor; we make γ very large in fact, So, if γ is very large the power factor automatically becomes leading. Say for example, if you take the phasor diagram, we have the field current here and we have the field induced TMF in this case that is E_f .

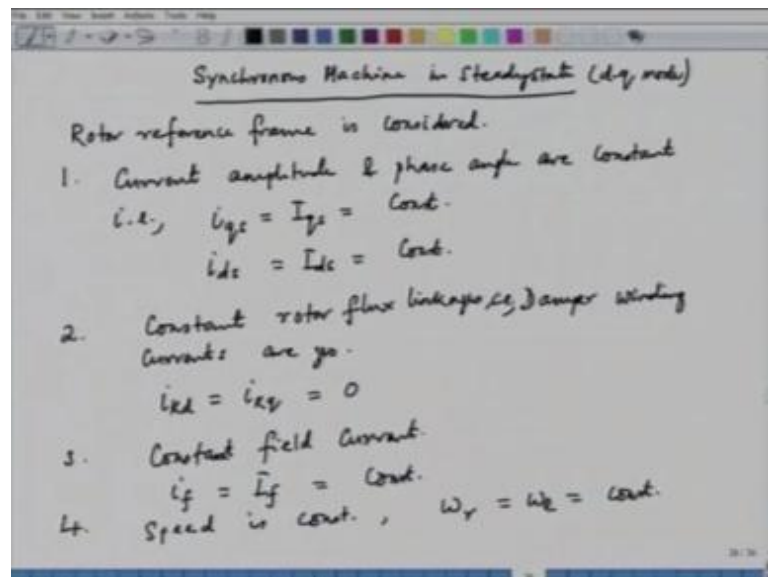
This is the q axis, and this is the d axis, and here we deliberately make I_a large. So, this is our γ ; γ is the angle between E_f and I_a . These are all phasor quantities. So, if this is our γ what about $j I_a X_s$. So, this is $j I_a X_s$ the synchronous reactance drop, and then we add this to the resistance drop, and the resistance drop is somewhere here $I_a R_a$. So, we have the synchronous reaction drop; that is $j I_a X_s$ and we have the resistance drop that is $I_a R_a$.

Now the resultant will be giving us the applied voltage. So, the applied voltage here is this vector; this is V_a . Now if this is V_a , what about the power factor? The power factor is the angle between V_a and I_a . So, we see here that the power factor in this case is leading; this angle is now θ . So, for load commutation when the CSI has to be commutated by

the back EMF of the machine, we have to make gamma large. And when we make gamma very large, the current leads the voltage and the power factor becomes unity. So, this is a discussion on the CSI-fed synchronous motor drive which is used for very high power application.

So, let us now see how we can describe or how we can study the steady state response of the synchronous machine. Let us now see the behavior of the synchronous machine in the steady state condition. The steady state we mean the speed is constant, the phase angle of the applied current is constant; the flux linkages are constant.

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So, we will try to derive the equations of the synchronous machine in rotor reference frame or steady state condition. So, we will try to write an equation for the d-q model, and we will consider the rotor reference frame. So, what is the meaning of steady state? By steady state we mean the current amplitude is constant. It means I_a , I_b , I_c are constant, and the current phase angle is also constant. So, we can say that the current amplitude and phase angle are constant which means I_{qs} ; in the rotor reference frame we can define the currents in terms of d and q axis currents. And the component of the current along the q axis is called i_{qs} . i_{qs} is equal to capital I_{qs} , because this is a constant quantity. So, we can say this is constant.

Similarly, the d axis current I_{ds} is equal to capital I_{ds} that is constant. And then we can also consider the rotor flux linkages to be constant, constant rotor flux linkages. So,

this implies the damper winding currents to be equal to 0, because if the flux linkages in the rotor are constant, the damper windings do not have any current. So, i_{kd} is equal to I_{kq} equal to zero. So, we can say here that damper winding currents are zero. This implies damper winding currents are zero which means i_{kd} is equal to i_{kq} is equal to 0. And we can also assume that the field current is also constant; the field is separately excited.

So, we can assume that the field current is also constant, constant field current which means i_f is equal to capital I_f is equal to constant. Of course, the speed is constant; it means ω_r is same as ω_e is constant. The electrical speed of the rotor is same as the electrical synchronous speed that is ω_e . So, these are the conditions for the steady state. Now if we consider the steady state condition, the equation for the stator voltages can be written as follows. So, we can write down the stator voltage equation, but before that we can derive the torque equation.

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The torque expression is given by

$$T_e = \frac{3}{2} \frac{p}{2} (\Psi_{ds} i_{qs} - \Psi_{qs} i_{ds})$$

$$= \frac{3}{2} \frac{p}{2} \left[(L_{ds} i_{ds} + L_{md} (i_{ds} + i_f)) i_{qs} - (L_{qs} i_{qs} + L_{mq} i_{ds}) i_{ds} \right]$$

$$= \frac{3}{2} \frac{p}{2} \left[\underbrace{L_{md} i_f i_{qs}}_{\substack{\text{Reaction Torque} \\ \text{Field torque}}} + \underbrace{(L_{md} - L_{mq}) i_{ds} i_{qs}}_{\text{Reluctance Torque}} \right]$$

$$= \frac{3}{2} \frac{p}{2} \frac{1}{\omega_s} \left[X_{md} i_f i_{qs} + (X_{md} - X_{mq}) i_{ds} i_{qs} \right]$$

So, the expression for the torque is given by, we now that the torque is equal to 3 by 2 into p by 2 into ψ_{ds} into i_{qs} minus ψ_{qs} into i_{ds} . So, here the flux linkages ψ_{ds} and ψ_{qs} can be expressed in terms of the inductances and the respective currents. So, we can replace this ψ_{ds} by L_{ds} into i_{ds} plus L_{md} into i_{ds} plus I_f , because the damper winding currents are zero. So, we need not write the expression for the damper winding currents here, this into I_{qs} . Similarly, for ψ_{qs} we can write down as L_{qs} into i_{qs}

this is the leakage component. And then we also we have the magnetizing component I_n into I_q into I_d s. So, this is the expression for the torque, and this can also be simplified. If we simplified that the leakage components will be cancelled in this case.

So, what we have finally is the following $\frac{3}{2}$ and $\frac{p}{2}$. We have here $L_{m d}$ I_f I_q s plus we have $L_{m d}$ minus $L_{m q}$ into I_d s I_q s. So, this is in terms of inductances $L_{m d}$ and $L_{m q}$ are the d axis magnetizing inductance and the q axis magnetizing inductance respectively, and the first component here is the synchronous machine torque. So, we can say here this is the reaction torque or the synchronous torque or the field torque. This is better termed as the field torque. So, we can say this as field torque. This is because of the field winding current. If the field winding current is equal to zero, this component of torque will vanish, and the second term is called the reluctance torque.

This torque comes into picture when there is a difference of inductance between the d and q axis. So, this can also be expressed in the terms of the reactants. We are talking about a variable speed drive and the reactance of the machine is defined as the best frequency. And the best frequency in radian per second we can call that to be ω_b . So, we can also express that as $\frac{3}{2}$ $\frac{p}{2}$ ω_b into $X_{m d}$ I_f into I_q s plus $X_{m d}$ minus $X_{m q}$ into I_d s into I_q s. So, this is the expression for the torque in terms of the reactants, and this is the steady state torque.

So, in the steady state we have two components of the torque; one component is called the reaction torque or field torque, and the other component is called the reluctance torque. And the reluctance torque basically comes up due to difference of inductance between the q axis and d axis. If $X_{d s}$ or $X_{m d}$ is not same as $X_{m q}$, there will be a reactions torque. This torque of course is not a very large amount; this is about 20 to 30 percent of the rated torque, but never the less this torque is present in the steady state condition when we talk about the salient pole synchronous motor. So, this is the expression for the torque. What about the voltage equation? The voltage expressions can be derived as follows.

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Steady state voltage equations

p-terms = 0 , $\omega_e = \omega_r = \text{const.}$

$$\begin{cases} V_{ds} = r_s I_{ds} - \frac{\omega_e}{\omega_b} X_{qs} I_{qs} \\ V_{qs} = r_s I_{qs} + \frac{\omega_e}{\omega_b} [X_{ds} I_{ds} + X_{md} I_f] \end{cases}$$

$$\begin{cases} V_{ds} = r_s I_{ds} - \omega_e L_{qs} I_{qs} \\ V_{qs} = r_s I_{qs} + \omega_e [L_{ds} I_{ds} + L_{md} I_f] \end{cases}$$

So, we are talking about the steady state voltage equations which is very simple to write, and the idea of writing the steady state equation is that I mean the operating principle of the motor will be very clear, because in the transient condition we do not have much understanding of the machine that only the thing is that we have some damper currents. So, the steady state equations are very good for understanding the working principle of the machine, the steady state torque production, the steady state flux, the speed, etcetera can be obtained from the steady state voltage equations. Of course, in the transient condition in addition to the field or the armature, we also have the damper currents which will have some dynamics or which will delay the torque response.

So, in the steady state condition damper currents do not exist. So, what we have here is the following. So, we do not have any p-terms here; the flux linkages are constant. So, we do not have any p-terms, and all the quantities are the DC quantities, because we are talking about the rotor reference frame. So, we can write down the d axis voltage; that is equal to $r_s I_{ds}$ minus we have the rotationally induced TMF, and the rotationally induced TMF is ω_e by ω_b into $X_{qs} I_{qs}$. So, these are the equation; this is the equation in the rotor reference frame.

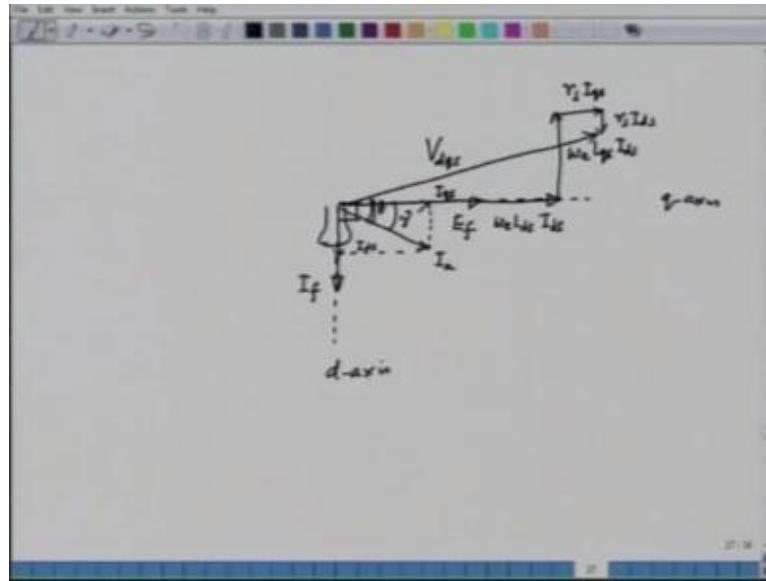
We do not have any p term, because the flux linkages are constant, and we have ω_e . And please remember that ω_e is same as ω_r , and that is also constant. We are talking about the constant feed operation, and at constant speed the electrical speed is

same as the electrical frequency of the current. So, this is what we have here, and then what about the q axis? V_q is equal to $r_s I_q$, and here also we do not have any p-term, because the flux linkages are constant. We have a positive rotationally induced TMF. ω_e is same as ω_r by b, and q axis will have the rotationally induced TMF due to the d axis flux. And in the d axis, in addition to the armature winding or the stator winding we also have a field winding. So, here what we have in this case is $X_{ds} I_{ds}$ plus X_{md} into I_f .

So, this is basically the equation in terms of the reactances, because reactances are given at base frequency, and hence we divide that by ω to obtain the inductance. So, can also be written in the following fashion that V_{ds} is equal to $r_s I_{ds}$ minus ω_e ; instead of reactance we can also have the inductance. The inductance will be $L_q I_q$. Similarly, we can have V_{qs} equal to $r_s I_{qs}$ plus ω_e into $L_{ds} I_{ds}$ plus L_{md} into I_f . Now we have two equations; one in the d axis, other in the q axis, and these equations are valid for any given speed. ω_r can be anything; it can be 2 radian per second; I mean 15 radian per second; it can also be 300 radian per second. It is varied for any speed, because we are talking about a variable speed drive; it is frequency independent.

So, from this we can of course write down a phasor diagram. We can draw a phasor diagram based on the two equations, and the phasor diagram will be in two axis mode. We have voltage in the q axis, and accordingly we can show the voltage as vectors in the two independent axis d and q respectively.

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So, we can draw the phasor diagram here. We have the q axis in this case, and this is the d axis. And we have the field in the d axis field pole, and the field current I_f is in the d axis. We can show that to be I_f here; this is right angle. And then what we will do here we will first write down the q axis induced TMFs. We have this term $L_m d I_f$ into ωe ; $L_m d I_f$ into ωe is same as the field induced TMF. So, we can say it is same as E_f . So, we can write down that this is E_f the field induced TMF as you have seen little earlier, and then what we have in this case? We have another induced TMF; that is $L_d s$ into $I_d s$. That is basically a reactance drop.

We know that we have two different reactances, and the MMF can be split into two axes and the corresponding reactances will give the voltage drop. So, here this $L_d s$ into $I_d s$ will be this reactance. So, this is $X_d s$ into $I_d s$ and then we add here the drop in the d axis. So, this is basically the d axis drop. If you see that this is our d axis equation. In the d axis the induced TMF is minus ωe into $L_q s$ into $I_q s$. So, this component of the induce TMF will be in the negative d axis. So, this will be in the negative d axis. So, we can call this to be $X_q s$ into $I_d s$, or we can write this in terms of the inductances. So, we have ωe $L_q s$ into $I_d s$; this also we write in terms of the inductances.

So, we have ωe into $L_d s$, and here we can add the resistance drop. The resistance drop in the q axis is $r s$ into $I_q s$. So, we can have $r s$ $I_q s$, and then we have the resistance drop in the d axis is $r s$ into $I_d s$. So, this is $r s$ into $I_d s$, and then we can

complete the complete phasor diagram by drawing the voltage, and the voltage is applied voltage that we can call V_d , and what about the current? The current is here I_a and the current will have two components as we have already discussed. This is I_q the q axis components, and this is I_d the d axis component. And this angle is the angle that is γ we have seen previously.

So, this is the phasor diagram of a synchronous machine in the steady state condition, and here the power factor angle is θ . And this power factor angle is the angle between V and I ; we can call that to be θ . So, this shows the various components of the voltage drop in the d axis and the q axis respectively. And we have also seen the torque equation, and from this we can find out the conditions for the maximum torque. We also can find out how we can change the current to control the torque. This will be discussed in the next lecture.