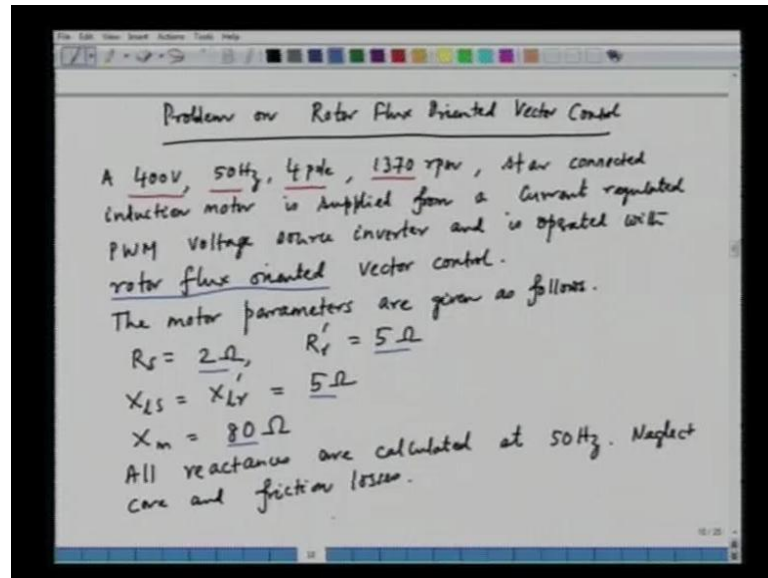


**Advanced Electric Drives**  
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**Lecture - 14**

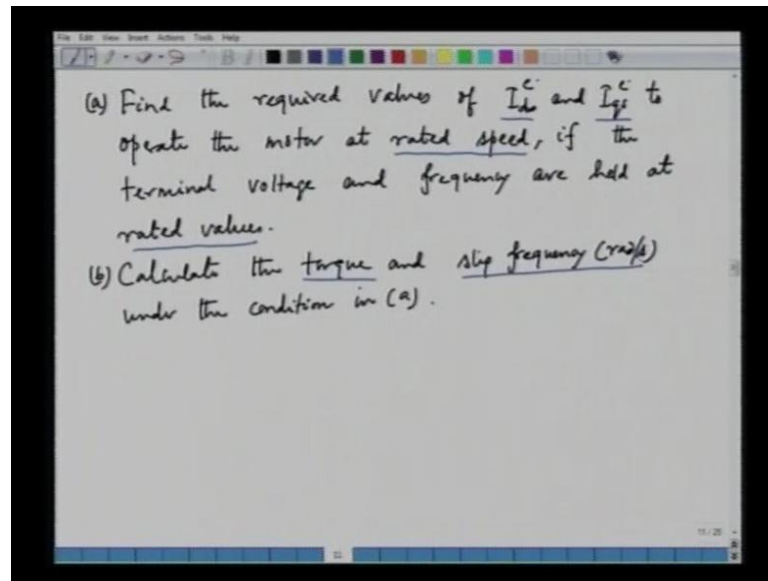
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Hello, and welcome to this lecture on advanced electric drive. In the last class we discussing about a numerical problem on vector control of induction motor drive. We start from that the problem statement as follows; a 400 volt, 50 Hertz, 4 pole, 1370 rpm, star connected induction motor is supplied from a current regulated falls with modulated voltage source in inverter and is operated with rotor flows oriented vector control.

The parameters of the machine are as follows; the stator resistance is given as 2 ohms, the rotor resistance refer from the primary side is 5 ohms, the stator leakage reactance and the rotor leakage reactance are 5 ohm respectively, the magnetizing reactance of the machine is 80 ohms. All reactances are calculated at 50 Hertz. The rated seed, the rated frequency of the machine is 50 Hertz and hence all the reactants of the machine are given with 50 Hertz frequency. The core loss and the friction loss are neglected. Therefore, we are neglect core loss and friction loss.

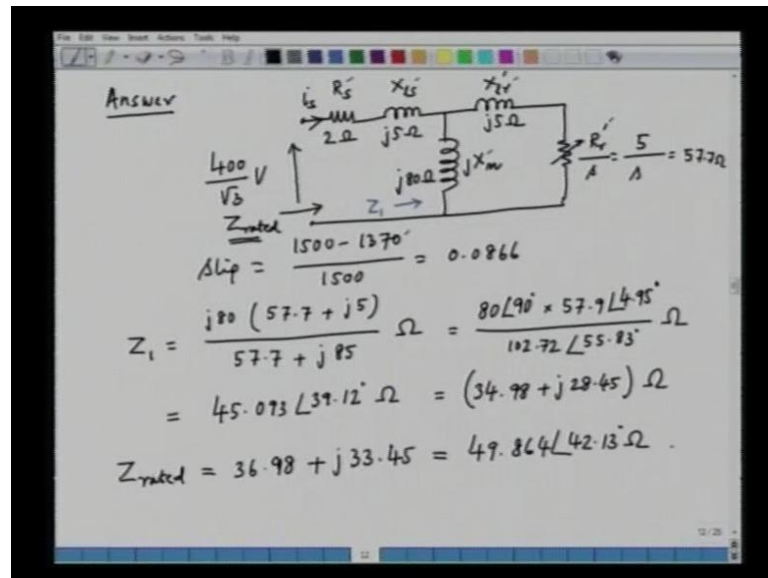
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So, what you have to find out is as follows; find the required values of  $I_{ds}$  and  $I_{qs}$ . This is  $I_{ds}^e$  and  $I_{qs}^e$ , because; we are talking about a rotor flux oriented vector control. So, this  $I_{ds}$  and  $I_{qs}$  are in a rotating reference frame, rotating with the rotor flux vector. And, hence we have the super script e here,  $I_{ds}^e$  and  $I_{qs}^e$ . In fact, this  $I_{ds}^e$  and  $I_{qs}^e$  in the steady state this variable are d c variables. Find out the required value of  $I_{ds}^e$  and  $I_{qs}^e$  to operate the motor at rated speed, if the terminal voltage and frequency are held at rated values.

So, we have to find out  $I_{ds}^e$  and  $I_{qs}^e$ ; there the flux component of current, and the torque components of current respectively to operate the motor at rated speed with the terminal voltage and frequency at held at the rated values. Also the part b is calculated the torque and slip frequency is radian for second under the condition in a. So, this is the problem statement and this is the problem of vector control induction motor drive and we will see how to solve this problem.

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So, actually if we see this particular problem, this problem talks about the steady state condition only. So, we do not have bother about the tangent condition and hence a steady state power phase equivalent circuit will be sufficient to analyze the various current and voltages. So, we know that we have very well known power phase equivalent circuit of an induction machine in steady state condition. So, this is given as follows. So, we have the stator resistance, the stator leakage reactance, the rotor leakage reactance in the further stator side, the resistance of the rotor of the magnetizing reactance.

And, of course, the motor is star connected and hence, the power phase voltage, the power phase rated voltage is 400 by root 3, root 3 is coming the picture, because; we are talking about the star connected machine. And, 400 volt is line to line voltage and power phase voltage is 400 by root 3. So, and the motor is operating at rated speed, the rated speed is 1370 rpm. And, hence we can find out the slip of the machine is 1500 is the synchronous speed minus 1370 by 1500 and that is equal to 0.0866 is the rated slip of the machine. And, then we have to find out the current  $I_s$  here, and always we find out the current  $I_a$ , we can find out  $I_d$  and  $I_q$ .

So, we know the slip of the machine, the operating slip of the machine, we know the motor parameter, we have the equivalent circuit, it is a macro of simplifying the impedance and finding out the current. So, we will find out the impedance looking from this side. So, if I look from this side this impedance is placed it is  $Z_1$ , it is a

compress quality. So, it is  $Z_1$ . So, I will just find out what is the value of  $Z_1$ . So, if we find out the value of  $Z_1$ ,  $Z_1$  is given as follows; that is equal to this basically the parallel combination of  $j 80$  ohms with the leakage reactance of the rotor and the resistance of the rotor by the slip.

So, we have in this case  $j 80$  into  $57.7$  plus  $j 5$  divided by  $57.7$  plus  $j 85$ . The slip resistance in this case is  $57.7$  ohms. So, if we divide the  $5$  by  $0.0866$  which is the slip of the machine, we get here  $57.7$  with the slip resistance in the rotor. So, this is what we have here, and the units are ohms. And, we can further simplify this as  $80 \angle 90$  into  $57.9 \angle 4.95$  degrees divided by  $102.72 \angle 55.83$  degree. So, this also a ohms. So, this is the impedance  $Z_1$ . We can further simplify this  $Z_1$ , we can write this as  $45.093 \angle 39.12$  degrees, this is the polar forms. The rectangular form would be;  $34.98$  plus  $j 28.45$  ohms. So, this is the value of the impedance that is  $Z_1$ .

Now, we can we can find out the impedance seen from the source side. This is the  $j$  rated this is what we have to find out. So, in order to find out this resistance  $j$  rated we have to add the impedance of the stator. The stator impedance consists of the resistance of the stator  $R_2$  there is the ohms and the leakage reactance of the stator is  $j 5$  ohms. So, we can calculate what is  $Z$  rated. And, the rated slip  $Z$  rated means; the impedance of the machine from the source side at rated slip. So, that is equal to  $36.98$  plus  $j 33.45$  that is equal to  $49.864 \angle 42.13$  degree and the unit is ohm. So, this is the basically the impedance of the machine are rated condition. And, if you want to find out the current we have to divide the voltage for the impedance.

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The image shows a series of handwritten equations and a phasor diagram on a whiteboard background. The equations are as follows:

$$I_{s(\text{rated})} = \frac{400/\sqrt{3}}{49.864} \text{ A} = 4.63 \text{ A}$$

$$I_{s(\text{peak})} = I_{s(\text{rated})} \sqrt{2} = 4.63 \sqrt{2} = 6.549 \text{ A}$$

$$\hat{I}_s^2 = I_{ds}^2 + I_{qs}^2$$

$$I_{ds}^2 + I_{qs}^2 = 6.549^2 = 42.89 \text{ A}$$

$$\omega_{sl} = A \cdot \omega_e = 0.0866 \times 2\pi \times 50 = 27.226 \text{ rad/s}$$

$$\omega_{sl} = \frac{L_m}{Z_r} \cdot \frac{I_{qs}^c}{N_{dr}^c} = \frac{1}{Z_r} \frac{I_{qs}^c}{I_{ds}^c} \quad (\because \mu_{dr}^c = L_m \frac{I_{ds}^c}{N_{dr}^c})$$

$$Z_r = \frac{L_r}{R_r} = \frac{L_m + L_{lr}}{R_r} = \frac{X_m + X_{lr}}{(2\pi \times 50)}$$

To the right of the equations is a phasor diagram showing a vector  $\hat{I}_s$  in the first quadrant of a coordinate system. The horizontal axis is labeled  $I_{ds}^c$  and the vertical axis is labeled  $I_{qs}^c$ . The vector  $\hat{I}_s$  is the hypotenuse of a right-angled triangle formed by  $I_{ds}^c$  and  $I_{qs}^c$ .

So, we can we can do that to find out the current which is I rated, I s rated, the rated stator current that is equal to 400 by root 3 is the voltage divided by the impedance value is 49.864. So, we do not have to bother about the angle here, we are only interested in the amplitude. So, what we do here is that the power phase voltage, here we can say this is V phase, the power phase is 400 by root 3 divided by 49.864 that will give us the stator current that is I s in r m s value. So, this current is the r m s value. So, we can say this is the root mean square value of the current of the stator for the rated condition and the unit here is ampere. So, that is equal to 4.63 amperes of course, this is the side way of current.

Because we are talking about the sinusoidal quantity we are talking about the impedances, we are talking about the voltage which is a sin wave. So, this current will have a peak value that is the r m s value into root 2. So, we can find out the peak value of the current, I s peak and that is equal to I s rated into root 2. So, rated we know here 4.63 that into root 2. So, we can find out that is equal to 6.549 ampere. So, this is the peak value of the stator current.

And, if we want to find out the torque and flux components of current, their basically in 2 axis so what we need to do is this that we have this current and the current has 2 components; one is the d axis components other is the q axis of the component. So, this is the d axis and this is the q axis of course, this axis is rotating. This is rotating at a speed of omega e and this is our I s. and we can have 2 components; one is I d s e and

other components is  $I_{qs}$ , and this is the peak value of the current. So, we have a situation, where we can apply the Pythagoras theorem and find out the peak square, this is peak square is equal to  $I_{ds}^2$  plus  $I_{qs}^2$ . This is same as the rated  $I_s$  hash is the same rated value of the current, the peak value of the current.

So, we can say here that  $I_{ds}^2$  plus  $I_{qs}^2$  is equal to  $6.549$  whole square and that is equal to  $42.89$  ampere. So, this is one of the very useful equations that we have. So, this relate  $I_{ds}$  and  $I_{qs}$ . Now, we have to the slip speed. Now, we know the slip speed is very well known quantity, is a very important quantity in rotor flux oriented in vector control. And, that is basically; the products of the slip and the supply frequency in radian for second. So, we can find out for the slip speed.

Slip speed here is given as  $s$  into  $\omega_e$ . Now, what is  $s$ ?  $s$  we know, it is  $0.08$  into  $\omega_e$  is  $2\pi$  into  $50$ ,  $50$  Hertz. So, we have  $2\pi$  so that is we have  $2\pi$  into  $50$ . And, that is approximately equal to  $27.226$  radian per second. This is one of the answers. The slip speed is  $27.226$  radian per second. So, the slip speed in a rotor flux oriented vector control drive; it is  $L_m$  by  $\tau_r$  into  $I_{qs}$  by  $\psi_{dr}$  and that is equal to, if we keep the flux constant, we know that that is equal to  $1$  by  $\tau_r$  into  $I_{qs}$  by  $I_{ds}$ .

Because we know that  $\psi_{dr}$  is equal to element to  $I_{ds}$ . So, what we have here, we can write down  $\psi_{dr}$  is equal to  $L_m$  into  $I_{ds}$ . So, if we substitute for the rotor flux in the  $d$  axis, the decider  $L_m$  and  $L_m$  get cancel. So, we get here  $1$  by  $\tau_r$  into  $I_{qs}$  by  $I_{ds}$ . So, this is the slip frequency or slip speed. So, from this we can have another equation involving  $I_{ds}$ ,  $I_{qs}$  and slip speed. Slip speed it is known. Now, what is  $\tau_r$ ?  $\tau_r$  is the rotor time constant and that is equal to the rotor inductance by the rotor resistance.

What is the rotor inductance? The rotor inductance with the total inductance of the rotor it is not, the leakage inductance. It is basically the some of the magnetizing inductance and the rotor leakage. So, we can express for  $L_r$ ,  $L_r$  is equal to  $L_m$  plus  $L_{Lr}$  prime. And, this we can find out, because; we know the reactance value. So, we can say that we have got  $L_m$  is corresponding to  $X_m$  and  $L_{Lr}$  is corresponding to  $X_{Lr}$  prime. What we can see here? We have the magnetizing reactance, we have the leakage reactance.

So, we can sum this up and this reactance's are calculated at  $50$  Hertz. And, we can divide by  $2\pi$  into  $50$  to find out the inductance. So, we can divide this by  $2\pi$  into  $50$ .

And  $R_r$  prime is already given here;  $R_r$  is specified in the question. So, we know what is the value of  $R_r$  is 5 ohms. So, we can find out  $R_r$ ,  $R_r$  is 5 ohms here. So, if we divide this, what we get  $\tau_r$  as 0.0541 and the unit is second. So, this is the value of the rotor time constant. Now, from the rotor time constant an  $I_{ds}^e$  and  $I_{qs}^e$  we can get another equation. And, the equation is as follows; we have got one equations here, this is equations number 1 and we can get the second equations by substituting the values here. We know the slip speed we know  $\tau_r$ , we know I mean, the expression for  $I_{ds}^e$  and  $I_{qs}^e$  is here. So, we can get the second equation.

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The image shows a whiteboard with handwritten mathematical derivations. The equations are as follows:

$$\frac{1}{0.0541} \frac{I_{qs}^e}{\sqrt{42.89 - I_{qs}^{e2}}} = 27.226 \quad \text{--- (2)}$$

$$0.6787 I_{qs}^e = \sqrt{42.89 - I_{qs}^{e2}}$$

$$r, \quad 0.46096 I_{qs}^{e2} = 42.89 - I_{qs}^{e2}$$

$$r, \quad I_{qs}^e = \underline{\underline{5.4182 \text{ A}}}$$

$$I_{ds}^e = \sqrt{42.89 - I_{qs}^{e2}} = \sqrt{13.532} = \underline{\underline{3.6784}}$$

$$T_e = \frac{3}{2} \frac{P}{2} \frac{L_m}{L_r} \gamma_{dr}^c I_{qs}^e = \frac{3}{2} \frac{P}{2} \frac{L_m}{L_r} I_{ds}^e I_{qs}^e$$

$$= \underline{\underline{14.3264 \text{ Nm}}} \quad \text{--- Ans}$$

So, we can obtain the second equations in the following fashion; 1 by 0.051, 0541 is the time constant of the rotor and then we have  $I_{qs}^e$ . and what is  $I_{ds}^e$ ?  $I_{ds}^e$  is 42.89 minus  $I_{qs}^e$  square under root. And, that is equal to slip speed 27.226. So, this is our second equations. So, from this equation, if we simplify this equation we get the expression for  $I_{qs}^e$  or the torque components of current. So, this basically will be a quadratic equation.

So, we can simplify this. So, we can get it in the following fashion; 0.6789  $I_{qs}^e$  and that is equal to 42.89 minus  $I_{qs}^e$  square or we can simplify this 42.89 minus  $I_{qs}^e$  square. So, we can take this  $I_{qs}^e$  square at one side. So, we can get the value of  $I_{qs}^e$  is equal to 5.4182. So, this is one of answer. So, we have in able to find out the value of  $I_{qs}^e$  and  $I_{qs}^e$  is equal to 5.4182 ampere. So, we can evaluate the value of  $I_{ds}^e$  also.

From this we can find out, what is the value of  $I_{ds}$ ?  $I_{ds}$  is equal to  $\sqrt{42.89 - I_{qs}^2}$  and that is equal to we have the value of  $I_{qs}$  and we get it a 13.532 under root and that is equal to 3.678 ampere. So, we have been able to find out the torque component of current and we have been also able to find out the flux component of current. And, the flux component of current is  $I_{ds}$ . So, that is equal to 3.678 ampere. So, these 2 currents are known.

And, what is next? The next ask in the question in part b is to find out the torque. So, the b part says that we have to find out the torque. And, what is torque? The expression for the torque is given by  $\frac{3}{2} \frac{p}{2} L_m L_r \psi_{dr} I_{qs}$  that is sampling like the torque equation that of a d c machine. The product of flux and current and the flux can be express in terms of current  $\psi_{dr}$  here is constant. We are talking about the steady state condition. So,  $\psi_{dr}$  is constant and we can replace  $\psi_{dr}$  by  $L_m I_{ds}$ . So, we can do that. So, what we have here is  $\frac{3}{2} \frac{p}{2} L_m^2 I_{ds} I_{qs}$ .

So, we know the value of  $L_m$ , we know the value of  $L_r$ , we also know the value of  $I_{ds}$  and  $I_{qs}$ . We can find out the torque, it is the matter of simple substituting the value of various parameters and variables to find out the torque. So, we can find out the torque. So, if you find out the torque here, the torque comes out to be 14.3364 Newton meter. So, this is the answer. So, in this case we have been able to find out for the vector control drive, the torque component of current  $I_{qs}$ , the flux component of current  $I_{ds}$ , the slip speed and the torque for the rated condition. So, this source how we can tackle the problem. and if a problem is given specially for the steady state condition it is not very difficult and most of the numerical we can do in the steady state condition.

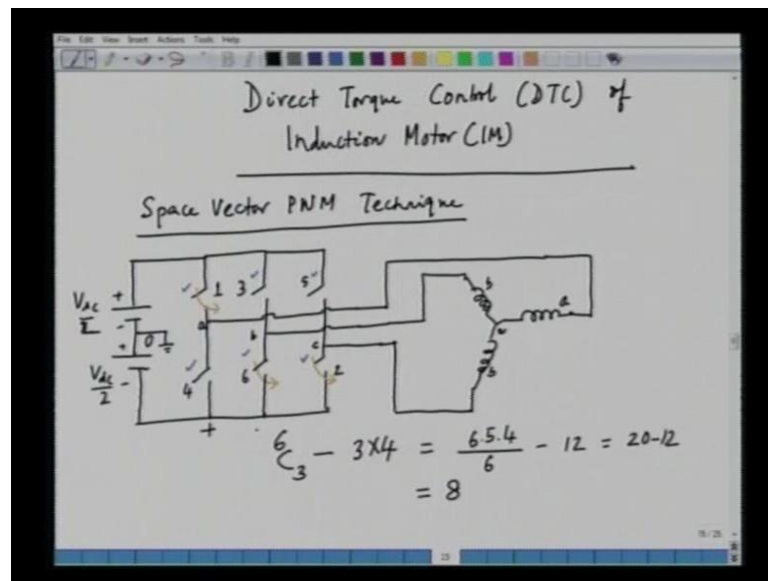
The steady state condition is easier to solve then the tangent condition. In the tangent condition can be simulated numerically using a computer. So, if you have understood procedure of this vector control drive and the principle of rotor flux oriented vector control drive. We can simulate this vector control drive with a computer numerically. So, we can starts from 0 speed, we can we can go to the rated flux and then we can apply a step change of speed to achieve the speed change, the speed response. So, the tangent condition as well as the steady state condition can be simulated with the digital computer.



So, this finishes our discussion on vector control of induction motor drive which is one of the important control of induction motor. And, which is basically used in industry also, because; it gives a very fast torque response. There is another type of control of induction motor that is called direct torque control of induction motor. Now, the direct torque control of induction motor was initially invented by A B B sampling times in early 1990 s, this is basically invented by A B B.

And, the advantage of having a direct torque control is that it is easier to implement then vector control. So, if we are talking about each of implementations the direct torque control are in brief D T C has an upper hand. It is not easier to implement a D T C based induction motor drive compare to a vector control based induction motor drive. So, we will be discussing about the direct torque control of induction motor.

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This called D T C. As I already said that this is basically a technique of control which is straitened by A B B. So, this is straitened of A B B. But this has been implemented by many industrial houses are before we talk about the direct torque control of induction motor we will be discussing about, what is called the space vector of P W M of induction motor. This technique which is called space vector P W M technique are called S V P W M technique will help us understand the direct torque control in a better way.

Now, we know that we have a 3 phase in inverter. We have a 3 phase voltage source in inverter. So, we can show this by this diagram. So, we have a voltage source inverter

which looks like this. This may be feeding to the 3 different phases. So, we call this to be phase a, phase b, and phase c. So, this is phase a, and this one phase b and we have phase c. And, this I will connect to a phase a, and this I will connected to phase b, this is to phase c. And, of course, this is the neutral of the machine that is n, small n. And, often you know that the dieseling is basically for convenience what we can do here; we can split this into 2 half's. So, we can split into upper half and split this into lower half. We have a battery which is split into 2 parts for convenient analysis.

So, this is  $V_{dc} / 2$  and also this is  $V_{dc} / 2$ . And, this point is the center point, we can say this is our local ground and this point we can call to be O, point O. So, this is what we have a 3 phase in inverter feeding a 3 phase induction machine we have initially having 2 supplied  $V_{dc} / 2$  and  $V_{dc} / 2$ . Now, here if we analyze this is the phase a, I can also write a here, b here, and c here, these are the 6 switches. So, we have the switch number 1, switch 2, switch 3, switch 4, switch 5, and switch 6. And, we will see that where; you want operant the particular in inverter the 3 switches will be on at the same time.

It can be 1 2 3, it can be 2 3 4 and so on, but; with a condition that is no 2 switches on the same lane should be turn on. And, for example; we cannot turn on this switch and this switch at the same time. 1 and 4 cannot be turn on at the same time. Let there should result in a shortly circuit. Similarly, we cannot turn on 2 and 6 at the same time, their result in a short circuit. Similarly, we cannot turn on 5 and 6 at the same time. So, these are actually called complementary switches; 1 and 4, 3 and 6, and 5 and 3 are the complementary switches, they cannot torque at the same time. So, we have 6 different switches and we would like to turn on any 3 of the switches.

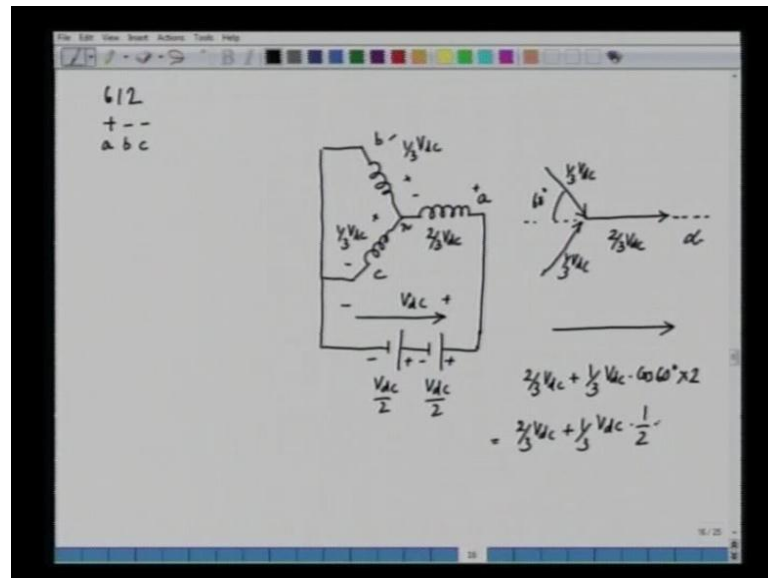
How many combinations do we have, how many legal combinations, how many possible combination we have? So, we can find it out by this particular formula, that we have 6 switches we would like to take 3 out of 6. So, it is the selection of 3 out of 6, but; we have to prevent some illegal combinations. The illegal combinations are those combinations in which 2 switches on the same lane are on. So, you know that if I suppose; I turn on 1 and 4. So, if I turn on 1 and 4, I even, if I turn on 3 2 6 and 5 that is should be a illegal combination. So, we can subtract this 3 into 4, if we turn on 2 switches on the same lane there will be 4 illegal combinations and we have 3 different lanes. So, for every lane there will be 4 illegal combinations.

And, I have 3 lanes; phase a lane, phase b lane, and phase c lane. For every lane we have 4 illegal combinations and since; we have 3 lanes, the numbers of illegal combinations are  $3 \times 4$ , that is 12. So, that is we subtract, what we obtain here is this the  $6 \times 5 \times 4$  divided by 6 minus 12. So, so that is equal to  $20 - 12$  we have 8 possible combinations which are legal. So, this 8 possible combination will leads to what is called 8 switching's, 8 possible switching. And, what are the switching's? We will see right now.

So, what we have here is this that we have phase a, and phase b, and phase c. This is a, this is b, and this is c, suppose; what I do I have a switching which is called 6 1 2 say for example, I will close the switch 1 is closed 6 is closed and 2 is closed. So, this is one of the combinations. So, I can call this as 6 1 2, this combination also be named as plus minus minus. And, this plus corresponding to phase a, minus corresponding to phase b, minus corresponding to phase c. When I have plus or upper switch and upper switch in the same lane is turned on, when we have minus a lower switch of the same lane is turned on.

So, we just go back and see that is phase a on upper switch is turned on. So, I can say it is plus for phase a. And, similarly; for phase b a low switch is turned on I can say it is minus for phase b, similarly; for phase c a low switch is turned on I can say it is minus for phase c. So, sequence is a b c. So, plus minus minus means upper switch of phase a, lower switch of phase b, and lower switch of phase c.

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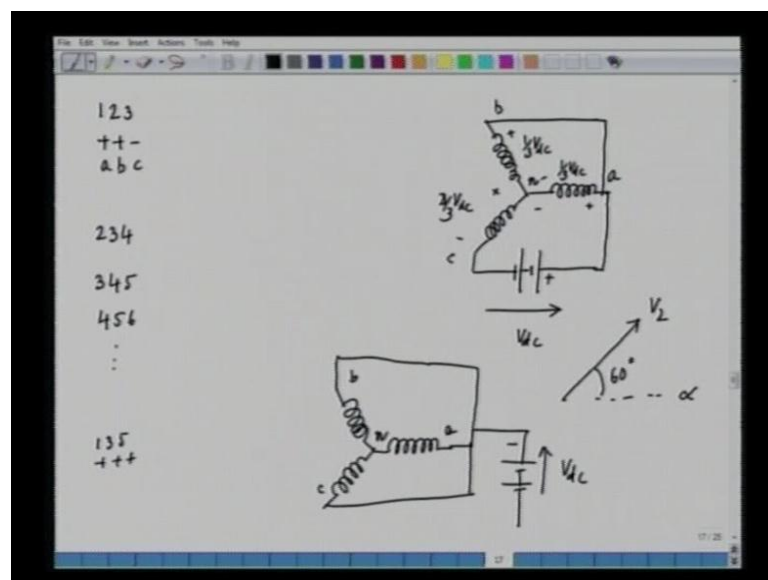
So, this is what I have store here as plus minus minus. So, this will be resulting into equivalent circuit of the machine like this. So, the upper switch of phase a, is connected or is turned on. So, that will be connecting phase a, to the positive bar. So, this is my positive bar and similarly, the low switch of phase b, and phase c are connected. So, phase b, and c are tied together and connected to the load negative ((Refer Time: 32:54)). So, this is  $V_{dc}$  by 2 and  $V_{dc}$  by 2. So, in total what we have here, we have a total voltage of the  $V_{dc}$ , this voltage is  $V_{dc}$  this positive and this negative.

And, we know that when we have a symmetrical machine; we can assume that the impedance of each phase is the same. So, we can say that the drop up in phase a across phase a will be  $\frac{2}{3} V_{dc}$  with this positive and this negative. And, drop up in phase b and c will be  $\frac{1}{3} V_{dc}$  right. Similarly, here also we can have this plus and this minus this is  $\frac{1}{3} V_{dc}$ . So, we have a situation where we have 3 windings and the 3 windings we have 3 different drops. So, if we show this voltage drop as vector, because; each windings is having a special orientation; phase a having a special orientation, phase b is having a special orientation, and phase c is also having a special orientation. So, when I am applying a voltage to a particular phase I can consider that particular voltage should be a vector.

So, I can show that phase a is having a vector like this. And, amplitude of the vector is  $\frac{2}{3} V_{dc}$  and the direction is as per the phase action with this positive and this negative.

So, this is shown in the direction as shown here. And, similarly phase b the voltage is like this, and phase c the voltage is like this. And, what about the magnitude, the magnitude here is  $\frac{1}{3} V_{dc}$  and magnitude here also is  $\frac{1}{3} V_{dc}$ . So, if we find out the resultant of the 3 vector, the resultant of the 3 vector will be in the direction and the magnitude here will be  $V_{dc}$ . We can show that it is  $\frac{1}{3}, \frac{2}{3} V_{dc}$  in the x axis this is my x axis let us say or alpha axis, I can say this is alpha axis. So, a  $\frac{2}{3} V_{dc}$  plus  $\frac{1}{3} V_{dc}$  into  $\cos$  of 60 degree, this angle is 60. And, we have 2 vectors b and c that is equal to  $\frac{2}{3} V_{dc}$  plus  $\frac{1}{3} V_{dc} \cos$  of 60 is half into 2 that is equal to  $V_{DC}$ . So, this is basically the vector  $V_1$ .

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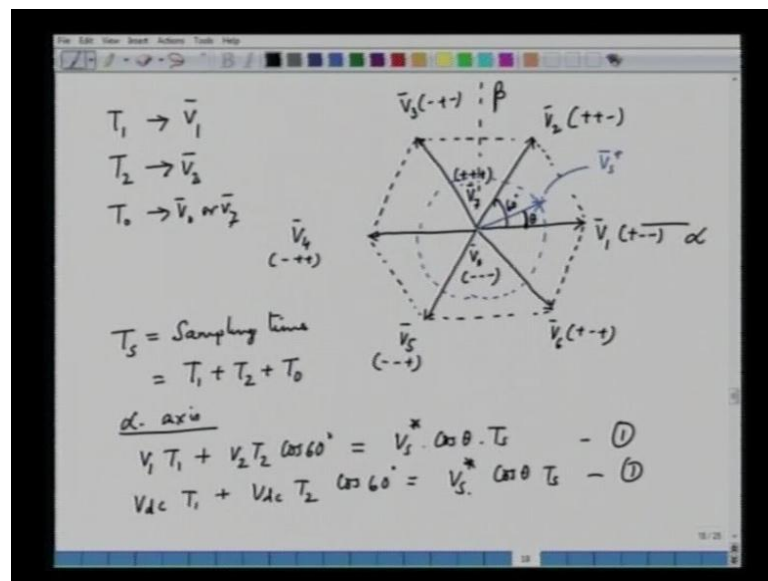


Similarly, I can also switch the switching, the switching can be from 6 1 2 to let as say I will go to 1 2 3 and if it is 1 2 3, I can say here. So, if go back in this case 1 means a is plus, 2 means c is minus, 3 means b is plus. So, I can say here that 1 2 3 same as plus plus minus. So, plus plus minus means; phase a, and phase b are connected to the positive bar and phase c is connected to the negative bar. So, in a similar way if you draw this various binding of the induction machine, this is the neutral in this case, and this is phase a, this phase b, and this phase c. So, what we do here is that phase a, and b here tie together and this connected to the supply in this case that is the positive of the supply and the total in this case is  $V_{dc}$ .

And, here we have plus minus, here also we have plus minus and here we have plus minus drop and this drop is 1 third, 2 third of V dc. And, this one is 1 third V dc; this is 1 third V dc. So, in similar fashion we will see that resulted vector here will be in this direction, which angle the 60 degree with the phase alpha. So, we have already seen about the vector V 1 and V 2. So, this we call as V 2. And, this is the vector which we call as V 1. So, it means due to the switching of the inverter we are able to generate some voltage vector. Similarly, we can have switching like 2 3 4 and 3 4 5, 4 5 6 and so on.

So, we have seen that we can have 8 possible switching's. And, out of 8 possible switching's, the 6 switching's will give as 6 non-zero vector and this vector will be shifted by 60 degree. And, hence we will have 6 vector shifted by 60 degree and 2 vector will be corresponding to all upper switches on and all lower switches on. So, we can now draw various vectors here. So, we can drop the vector in the following fashion.

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This is V 1 is the vector which is corresponding to the switching of plus minus minus. And, then we have vector V 2 with the corresponding to the switching plus plus minus and then we have V 3 which is corresponding to minus plus minus. And, we have this vector V 4 this as the switch which is corresponding to the switching minus plus plus and then we have vector as V 5 which is corresponding to the switching minus minus plus then we have the vector this is V 6 this corresponds to the switching plus minus plus.

So, we have 6 non-zero voltage vectors and this angle is 60 degree, we can say that this angle here is 60 degree. Similarly, the angle between 2 adjacent voltage vectors is 60 degree. Of course, we have 2 other vectors, and the 2 other vectors are  $V_7$ ;  $V_7$  is the special voltage vector in which all upper switch will on. So, we call this to be plus plus and plus. And, this will corresponds to the situation if we see their, suppose; we have a situation like 1 3 5 which is plus plus plus; if we take this combination, this combination will lead to the situation that we have phase a here, phase b here, and phase c here and this is our neutral.

So, the 3 pages are tied together to the positive of the dc bar. So, we can say here that this all are connected together phase a, phase b, and phase c are all tied together to the positive of the d c bar. And, the negative is floating; the negative is not connected in anywhere so this is the d c. So, this total is  $V_{dc}$  here, and the negative will floating, but; this will be leads to a zero voltage vector so for in the machine is concern. So, this phase b, and phase c are sorted together. So, there is no voltage vector, there is made from voltage vector. So, we can call this to be a zero voltage vector. The same thing we happen if we have other condition like 2 4 and 6. So, if we have condition like 4 6 and 2 this will be known as minus minus minus.

So, again phase a, phase b, and phase c are tied together, but; connected to negative of the d c bar that will also lead to a zero voltage vector. So, we have 2 zero voltage vector. And, those 2 voltage vector can be shown as  $V_7$  have plus plus plus, and  $V_0$  as minus minus minus. So, we have 6 non-zero voltage vectors and 2 zero voltage vectors in a 3 voltage source inverter feeding a 3 phase machine. So, we can say that the of the voltage vector if we join this step, this will constitute a regular hexagon. So, this is, these are the voltage vectors for a  $V_{si}$ , 2 level  $V_{si}$  feeding a 3 phase machine. And, what we are trying to do here, we are generating a reference voltage vector from this 6 voltage vector.

For example,  $V_1$  a voltage vector to rotate on the voltage vector having on amplitude that is equal to  $V_s$ . So, we have that is a voltage vector which is  $V_s$  this is my  $V_s$ , I can call this to be  $V_s$  star this is what I want to generate and the  $V_s$  star is the rotating voltage vector and this will be moving as a circle. The tip of this vector will make a circle in the space. So, what a given instant this is here, and this angle that is that this make with alpha axis, I can call this axis as alpha axis and this axis which is right angle to this is beta axis. So, the angle that makes with alpha axis ((Refer Time: 44:26)). So,

we have to produce this voltage vector by the combination non-zero voltage vector and zero voltage vectors.

So, at a given instant, I can say that the voltage vector is making an angle  $\beta$  with the alpha axis. And, if we taking some time of  $T_s$  living the sampling time, the average of the voltage produced by the reference voltage vector should be same as the average of the voltage produce by in the inverter. So, we can equate the voltages in the alpha axis and the beta axis respectively.

And, we can say that we have 3 distinct time  $T_1$  for which  $V_1$  is turned on or  $V_1$  is applied  $T_2$  for which  $V_2$  is applied and  $T_0$  for which voltage  $V_0$  or  $V_7$  is applied depending upon the situation. So, we have 3 different time  $T_1$   $T_2$  and  $T_0$  and  $T_1$  plus  $T_2$  plus  $T_0$  is called a sampling time. So, we can say here the  $T_s$  is sampling time is a small time that is equal to  $T_1$  the time for voltage vector  $V_1$ ,  $T_2$  time for voltage vector  $V_2$ , and  $T_0$  the time for voltage vector  $V_0$  are  $V_7$ .

So, we can equate the whole second in the alpha axis beta axis respectively. So, if we do that. So, we can say that in the alpha axis; we can say here that  $V_1 T_1$  plus  $V_2$  into  $T_2$  into  $\cos$  of 60 degree this angle between  $V_1$  and  $V_2$  60 degree. So, that is why we have  $\cos$  of 60 degree that is equal to  $V_s$  in to  $\cos$  theta into  $T_s$ .  $T_s$  is sampling time. So, we are multiplying in this case the sampling time. So, this is 1 equation. So, we can define which is  $V_s$  star; which we have here. And, we can define  $V_{dc} T_1$  plus  $V_{dc} T_2$  into  $\cos$  of 60 that is equal to  $V_s$  star  $\cos$  theta into  $T_s$  same is the equation 1.



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$$a = \frac{V_s^*}{V_{dc}} = \text{amplitude ratio}$$

$$T_1 + T_2 \cos 60^\circ = a T_s \cos \theta \quad - (1)$$

$$\frac{\beta\text{-axis}}{T_2 \sin 60^\circ} = a T_s \sin \theta \quad - (2)$$

$$\text{or } T_2 = a T_s \frac{\sin \theta}{\sin 60^\circ} \quad - (3)$$

$$T_1 = a T_s \cos \theta - \frac{a T_s \sin \theta}{\sin 60^\circ} \cdot \cos 60^\circ$$

$$= \frac{a T_s \sin (60^\circ - \theta)}{\sin 60^\circ} \quad - (4)$$

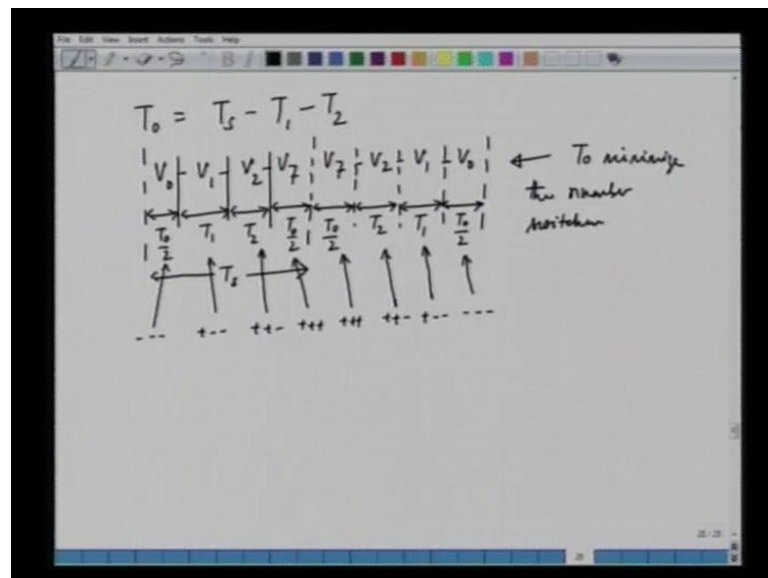
So, we can define a, a is a ratio of  $V_s^*$  by  $V_{dc}$  and this is called the amplitude ratio. So, we can simplify this equation using a. So, we can write down that  $T_1 + T_2 \cos 60^\circ$  that is equal to  $a T_s \cos \theta$ , we have to know that although we have 6 non-zero voltage vectors  $V_1, V_2, \dots, V_6$ . Each voltage vector having an amplitude of  $V_{dc}$  each one is equal having amplitude of  $V_{dc}$  and hence we have replaced this  $V_1$  by  $V_{dc}$ ,  $V_2$  by  $V_{dc}$  also.

So, this  $V_{dc}$  will be taken to the right hand side and we can get a here; a is amplitude ratio. And, we define equation number 1 as this and similarly, in the beta axis; we can say here that  $T_2 \sin 60^\circ$  that is equal to  $a T_s \sin \theta$ . So, this is what we have here, and from the equation 2 we can directly get expression for  $T_2$ . So,  $T_2$  we can find out as;  $a T_s \sin \theta$  by  $\sin 60^\circ$ .

And, we can substitute for  $T_2$  in equation number 1; find out the value of  $T_1$ . So, if we substitute this in this equation and evaluate, but; it is  $T_1$  we can say that  $T_1$  is equal to  $a T_s \cos \theta$  minus  $a T_s \sin \theta$  by  $\sin 60^\circ$  into  $\cos 60^\circ$ . So, we can further simplify this, we can say here that is equal to  $a T_s$ , if we simplify this; we get the following equation  $a T_s \sin (60^\circ - \theta)$  divided by  $\sin 60^\circ$ . So, we have been able to evaluate  $T_2$  and we have been able to  $T_1$ . Now, what is  $T_0$ ?  $T_0$  is  $T_s$  minus  $T_1$  minus  $T_2$ . So, this is our  $T_0$ . So, we can call it as equation number 3.

And, this is our  $T_1$  which is given in the equation number 4. So, we can find out what is  $T_{naught}$ ?  $T_{naught}$  is given as  $T_s$  minus  $T_1$  minus  $T_2$ . So, for a given sector is angle is  $\theta$  between the alpha axis or direct axis and the vector  $V_s^*$  we can find out the values of numerical values of  $T_1$ ,  $T_3$  and  $T_0$ . Similarly, this can repeat in sector 2, sector 3 and so on. So, we have 6 different sector we have this is 1 sector and similarly, we have this second sector here and third sector here so on. So, we can do the calculation in sector 2, sector 3 and sector 4 respectively. And, we can evaluate the values of  $T_1$ ,  $T_3$  and  $T_0$ . And, we can go for false modulation. So, in this case to have minimum number of switching, what we do in this case that we split this 0 vector duration into 2 half's  $T_0$  by 2 and  $T_0$  by 2.

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So, what we do here, instead of going for single  $T_0$ . What we do here is that we switch like this  $V_0$ ,  $V_1$ ,  $V_2$ ,  $V_7$  this is 1. And, then we start with  $V_7$ ,  $V_2$ ,  $V_1$  and  $V_0$ . So, you know that we have to repeatedly have this  $V_1$ ,  $V_2$  and  $V_0$ . So, what we do here is this; we divide this into 2 equal half's. This is  $T_0$  by 2 and this is  $T_0$  by 2 and this is  $T_1$  and this  $T_2$  and this is whole is  $T_s$ . Similarly, in the next sampling time what we do here we divide this into again, 2  $T_0$  by 2 and  $T_0$  by 2. So, we divide the zero voltage vector duration into 2 half and then this is our  $T_2$  and this is  $T_1$ . And, this will help as to minimize the number of switching's of the inverter.

Say for example, we can justify that in the following way. So, in the  $V_0$  we have the switching states are minus minus minus. So, we can say that  $V_0$  means minus minus and minus. What about  $V_1$ ? If we talk about  $V_1$  it is plus minus minus. We have a just change of 1 switching in a lane, the state of the other 2 lanes are remaining the same will be 1 switch in a lane turned off and other switch is turned on.

So, only 1 lane is affected. Similarly, in  $V_2$  we have the switching state is plus plus minus. So, phase a, and phase c are remaining the same only phase b is changing from minus to plus. And, here we can see here again, we have plus plus plus. So, phase a, and b are remaining the same only phase c changing from minus to plus. And, in this sector we are retaining that so that is plus plus plus. So, we have no change to the  $V_7$  again we have  $V_7$  here.

And, then  $T_2$ ,  $T_2$  is plus plus minus, the phase c is changing from plus to minus. Similarly, in  $T_1$  what we have here is plus minus minus. So, phase b is changing from plus to minus. And, then we have  $V_0$  and the  $V_0$  is minus minus minus. So, we have minimum number of switching. So, this particular sequence is adopted to minimize the number of switching in the inverter. So, this is how we go for what is called as space vector P W F. The voltage in this case is represented by space vector, and we are trying to approximate the giving voltage vector by taking  $V_1$  for some time  $V_2$  for some time and the zero vector for some time. So, in the next lecture we will see how we can go for the direct torque control in which we essentially try to approximate the flux vector by the voltage vector. So, that be in the topic in the next lecture.