

**Advanced Electric Drives**  
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**Lecture - 11**

Welcome to this lecture on advanced electric drives. In the last lecture, we were discussing about the flux estimation scheme the rotor flux estimation scheme, and we have seen actually there are two methods two ways to estimate the rotor flux. The first method is the voltage based estimation, and the second method is the current based estimation. So, in this lecture we will be discussing about the current based estimation of rotor flux.

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Current based estimation of rotor flux

Rotor equations in stationary reference frame

d-axis:

$$p \psi_{dr} + i_{dr} r_r + \omega_s \psi_{qr} = 0$$

q-axis:

$$p \psi_{qr} + i_{qr} r_r - \omega_s \psi_{dr} = 0$$

$$i_{dr} r_r + p \psi_{dr} + \omega_s \psi_{qr} + \frac{\omega_s L_m i_{ds}}{L_r} = \frac{r_r L_m i_{ds}}{L_r}$$

$$p \psi_{dr} + \frac{r_r}{L_r} (L_r i_{dr} + L_m i_{ds}) + \omega_s \psi_{qr} = \frac{r_r L_m i_{ds}}{L_r}$$

$$p \psi_{dr} + \frac{1}{\tau_r} \psi_{dr} + \omega_s \psi_{qr} = \frac{L_m}{\tau_r} i_{ds}$$

Now primarily in this method we will be estimating the rotor flux from the currents; current means the currents of the stator and the stator currents when we are transforming into the d q they are  $i_{ds}$  and  $i_{qs}$  respectively. So, before we start discussing about the estimation let us derive the next three equations which will be helpful in estimating the rotor flux. So, we can write down the rotor equations in stationary reference frame and the d-axis equation is as follows,  $p \psi_{dr} + i_{dr} r_r + \omega_s \psi_{qr}$  that is equal to 0.

So, this equation is basically the d-axis equation in the stationary reference frame. So,  $\omega_s - \omega_r$  is the slip speed. Slip speed is  $\omega_e - \omega_r$ , the slip speed of the rotor, and similarly we can have the equation in the q-axis. We can write down  $p\psi_q + i_q r - (\omega_s - \omega_r)\psi_d = 0$ . So, these are the two equations which will help us in estimating the rotor flux, and the rotor fluxes are  $\psi_d$  and  $\psi_q$ . So, let us do some mathematical rearrangement to these two equations. We have two equations with us. We will first take up the d-axis equations and do some mathematical readjustments, and then we similarly do the same thing for the q-axis equations.

So, we take these equations and what we do here is the following. We can just rewrite this equation  $i_d r + p\psi_d + (\omega_s - \omega_r)\psi_q$ . We will be adding what we will add here is the following. We will add here  $r$  by  $l_r$  into  $i_d$  in the right hand side,  $r$  by  $l_r$  into  $l_m i_d$  in the left hand side. So, the value will not be changed, and then we will do some adjustment. And what we will do here we will club these two, and if we club these two we get the following. What we obtain here is the following,  $p\psi_d$  plus we can take here  $r$  by  $l_r$  common.

And if we do that what we obtain here  $l_r i_d + l_m i_d + (\omega_s - \omega_r)\psi_q$ ; that is equal to  $r$  by  $l_r$  into  $l_m i_d$ . And this equation we know that the second term, the part of the second term is the rotor flux. So, this term will give us  $\psi_d$ . So, we can again rewrite this equation in the following fashion that  $p\psi_d$  plus we can say here  $l_r$  by  $r$  is  $\tau_r$ . So, this is  $1/\tau_r$ ,  $\tau_r$  is the rotor time constant, into  $\psi_d + (\omega_s - \omega_r)\psi_q$ ; that is equal to  $l_m/\tau_r$  into  $i_d$ .

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$$p \psi_{dr} + \frac{1}{\tau} \psi_{dr} + \omega_r \psi_{qr} - \frac{L_m}{\tau} i_{ds} = 0$$

Similarly in the q-axis

$$p \psi_{qr} + \frac{1}{\tau} \psi_{qr} - \omega_r \psi_{dr} - \frac{L_m}{\tau} i_{qs} = 0$$

$$p \psi_{dr} = \frac{L_m}{\tau} i_{ds} - \frac{1}{\tau} \psi_{dr} - \omega_r \psi_{qr}$$

$$p \psi_{qr} = \frac{L_m}{\tau} i_{qs} - \frac{1}{\tau} \psi_{qr} + \omega_r \psi_{dr}$$

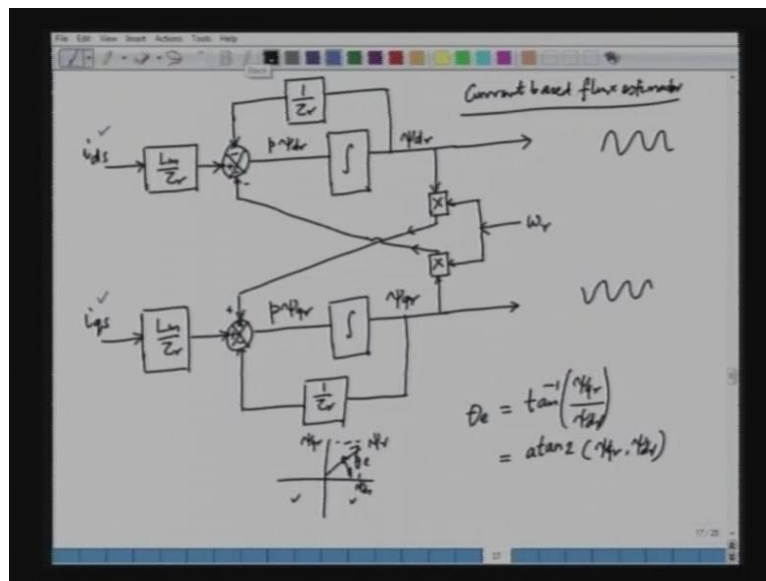
So, we can further simplify this, and after simplifying what we obtain here is the following.  $p \psi_{dr} + \frac{1}{\tau} \psi_{dr} + \omega_r \psi_{qr}$  that is equal to minus of  $\frac{L_m}{\tau} i_{ds}$  that is equal to zero. Now in this previous thing this will be since we are taking a stationary reference frame this  $\omega_s$  because  $\omega_e$  is equal to 0. When we take the stationary reference frame we can say that the reference frame speed is equal to 0. So, since the reference frame speed is equal to 0 the  $\omega_s$  will be replaced by  $\omega_r$ . So, we have here that is  $\omega_r$  into  $\psi_{qr}$  and  $\omega_r$  into  $\psi_{dr}$ ; this will also be  $\omega_r$  here. So, we have  $\omega_r$  in this case, and here also we have this  $\omega_r$ , and here also we have this  $\omega_r$ . So, we have  $\omega_r$  here, and we have  $\omega_r$  here, and the rotor speed is a known quantity.

So, we can always sense the rotor speed by using a sensor; the rotor speed can be measured by using a speed sensor. So, this is what we have. And similarly in the q-axis we can rewrite the equation in the following fashion.  $p \psi_{qr} + \frac{1}{\tau} \psi_{qr} - \omega_r \psi_{dr} - \frac{L_m}{\tau} i_{qs}$  that is equal to 0. So, these two equations will help us estimate the rotor fluxes that is  $\psi_{dr}$  and  $\psi_{qr}$  in the two axis respectively. And this actually requires little bit of readjustment. So, if we do a small readjustment here we will get this equation in the following fashion.

This equation will be  $p \psi_{dr}$  if you want to evaluate but  $p \psi_{dr}$  that is equal to  $\frac{L_m}{\tau} i_{ds} - \frac{1}{\tau} \psi_{dr} - \omega_r \psi_{qr}$ . And this equation will

give us  $p \psi_{qr}$ ; that is equal to  $l_m$  by  $\tau_r$  into  $\psi_{qs}$  minus  $1$  by  $\tau_r$  into  $\psi_{qr}$  plus  $\omega_r$  into  $\psi_{dr}$ . So, these two equations if they are simplified if they are solved we will get respectively  $\psi_{dr}$  and  $\psi_{qr}$ . Now we can show this by means of a simple block diagram. Now what we will do, we will take these two equations and we will evaluate what is  $p \psi_{dr}$  and  $p \psi_{qr}$ , and the integration of  $p \psi_{dr}$  and  $p \psi_{qr}$  will be  $\psi_{dr}$  and  $\psi_{qr}$  respectively. So that we can show by means of a block diagram.

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So, we can show that in form of a block diagram in the following fashion. So, we have the currents and the current is  $i_{ds}$  here. We multiply here  $l_m$  by  $\tau_r$ , and then output is  $p \psi_{dr}$ , and we can use integrator here. And what we obtain is  $\psi_{dr}$ . And similarly for the  $q$  axis we have  $l_m$  by  $\tau_r$  and then we have  $i_{qs}$  plus here, and we integrate this. This is  $p \psi_{qr}$ ; what we obtain here is  $\psi_{qr}$ . So, these are the two fluxes that we are talking about, but we have some other terms here. As per the equation we have to complete the equation in the block diagram.

So, here we have a feedback, and this is  $1$  by  $\tau_r$ , and we feedback this flux. This is negative here, and similarly in this case we also have a feedback, and we have  $1$  by  $\tau_r$ . We feedback this flux, and this is negative. The feedback is again negative here, and then we have another term that is  $\omega_r$ , and the  $\omega_r$  term can be obtained in the following fashion. So, we have two multipliers. These are the outputs of the estimator, and the two multipliers are supplied from the scheme. The speed can be measured. We

can, obviously, have an encoder to measure the speed; we can have a tachogenerator to measure the speed.

There are so many speed measuring devices. Encoders it is quite common for measuring speed for a vector control drive. So, we can use an encoder to find out the speed, and this speed can be fed back here. So, we can use speeds in this case, and then this is multiplied by  $\psi_{dr}$ , and this is fed here, right. And similarly we can multiply here  $\psi_{qr}$ , and we can feed this here. This is the output in this case, this is also the output, and this feedback is negative here, and this feedback is positive. So, we have a negative feedback in this case, we have a positive feedback. So, this is as per the equation. So, this is the current based estimator of the flux.

So, what we are evaluating here is  $\psi_{dr}$  and  $\psi_{qr}$ , and from  $\psi_{dr}$  and  $\psi_{qr}$  we can get the information about the flux angle that is  $\theta_e$ . So, how do you obtain  $\theta_e$ ?  $\theta_e$  can be obtained by taking  $\tan^{-1}$  of  $\psi_{qr}$  by  $\psi_{dr}$ , and please remember these are basically fluxes in the stationary reference frame. These fluxes will be alternating in nature. They will be a function of time. If you see this flux is also varying with time; this flux is also varying with time. So, we know that these two fluxes are function of time, and if you take  $\tan^{-1}$  of this you will get the information about the flux angle, the angle of the flux vector.

Now when we simulate it that is a function called  $\tan^{-1}$ ; when we are simulating may be in MATLAB or in C this particular find out this angle.  $\tan^{-1}$  is sometimes it is not preferred. So, what we do here is this. We use a  $\tan^{-1}$ ; a  $\tan^{-1}(\psi_{qr}, \psi_{dr})$ . Now why do we use a  $\tan^{-1}$ , because the flux can be in any of the four quadrants? We have four quadrants here, and we have a flux which is rotating that is  $\psi_r$ , and this is our angle that is  $\theta_e$ . This is angle of the flux vector. The flux can be in any of the four quadrants, and the component of this flux is like  $\psi_{dr}$ , and  $\psi_{qr}$  can be positive can be negative. So, to find out the angle in any four quadrants we have to take the help of a  $\tan^{-1}$ .

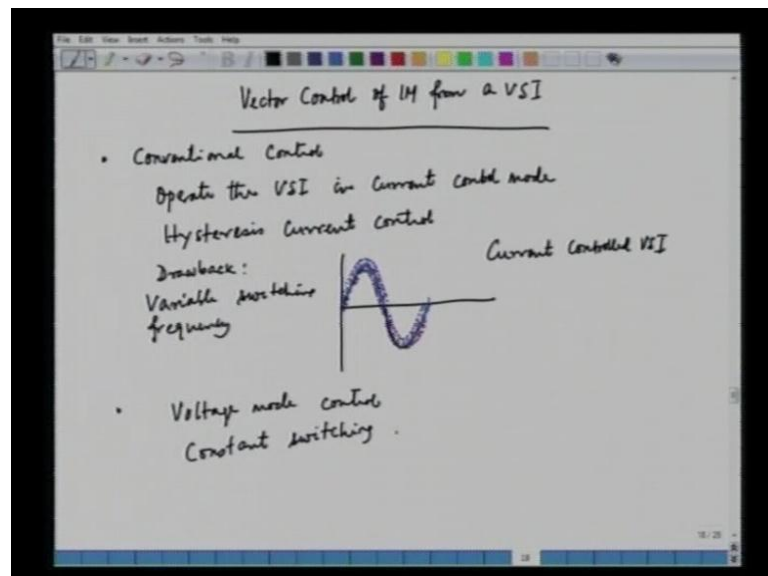
If both are negative they are in the third quadrant. If suppose  $\psi_{dr}$  is positive, but  $\psi_{qr}$  is negative it is in the fourth quadrant and so on. So, this is the current based estimator, and this depends upon  $i_{ds}$  and  $i_{qs}$ . And in the previous lecture we have already explained how we can find out  $i_{ds}$  and  $i_{qs}$  from the currents  $i_a$ ,  $i_b$ ,  $i_c$ . So, we have

the actual currents of the machine. We can use some sensor, and the sensors are commonly Hall Effect sensors. We can use the Hall Effect sensors to measure  $i_a$ ,  $i_b$ ,  $i_c$ , and this  $i_a$ ,  $i_b$ ,  $i_c$  can be transformed into  $i_d$  and  $i_q$ . And  $i_d$  and  $i_q$  are the stationary reference currents, and we can use those  $i_d$  and  $i_q$  here.

So, we do not have to integrate the voltage. Since, we do not have to integrate the voltage; this estimator was well also at low speed condition. The integration does not depend upon the voltage. So, if in at zero speed this estimator is going to give us current result, okay. So, in many situations we prefer current based estimation to voltage based estimation for this reason that we avoid the integration of the voltage which becomes very low at low speed current. So, we will discuss about the vector control fed from a voltage source inverter.

We have already seen that we have to use an inverter to inject the currents, and the currents are the physical currents of the 3 phases, and the currents are  $i_a$ ,  $i_b$ ,  $i_c$  of the 3 phase machine. And to do that commonly we use voltage source inverter, because voltage source inverter can be easily fabricated. But we need to inject current  $i_a$ ,  $i_b$ ,  $i_c$ , and hence, we have to operate the voltage source inverter in current control mode.

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Vector control of induction motor from a VSI, okay; so what is the conventional practice? Conventional control is operating the voltage source inverter in current control mode, and one of the ways to have current control is the hysteresis current control that

we have already seen that what we have; we have a reference current, and we can track the reference current within a band. So, one of the ways is to go for hysteresis current control. And in this case what we do here is that suppose this is our reference current which are speed tracked; this is for phase a.

Similarly we can have for phase b and c. We control the actual currents within a band. So, this is how the tracking takes place. So, the actual current tracks a reference current within hysteresis band. So, we can define a hysteresis band here. We can have an upper band, and we can also have a lower band. And the actual current is confined within these two bands, and this method is quite popular also conventionally used. But one of the drawbacks of this method is that the switching frequency is variable. When we go for hysteresis current control the switching frequency of the inverter does not remain constant. And we do not know what is the switching frequency. It actually depends upon various parameters like the dc-link voltage, the parameters of the machine and so on.

So, it can be as low as a few hundreds of hertz; it can be as high as few kilohertz. So, the switching frequency continuously varies as the actual current tracks the reference current. Now this is one of the drawbacks of this method that the switching frequency is not known. Now if you want to operate the inverter under constant switching frequency mode, what we do we operate the inverter not as a current control VSI but as a VSI. So, this is called a current control VSI, alright. So, we can go for voltage mode control. In the previous one the drawback is variable switching frequency.

Now in the voltage mode control the switching frequency can be maintained constant. So, we can operate the inverter like a through VSI and we can go for constant switching frequency operation. Now to be able to do that we have to convert the currents into the voltage, because you know that the vector controller will be giving us  $i_d$  and  $i_q$ ;  $i_d$  is the flux component of current,  $i_q$  is the torque component of current. So, we have to convert these two currents into the voltages  $v_a$ ,  $v_b$  and  $v_c$ , okay. So, we will see that how this is torque.

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Rotor flux oriented vector control from a  
VSI with Voltage Mode Control

Sync. Rotating Ref frame (Rotor flux orientation)

$$U_{ds}^e = r_s i_{ds}^e + p \psi_{ds}^e - \omega_e \psi_{qs}^e$$

$$= r_s i_{ds}^e + p (L_s i_{ds}^e + L_m i_{dr}^e) - \omega_e (L_s i_{qs}^e + L_m i_{qr}^e)$$

$$i_{dr}^e = \frac{\psi_{ds}^e - L_m i_{ds}^e}{L_r} ; \quad i_{qr}^e = \frac{\psi_{qs}^e - L_m i_{qs}^e}{L_r}$$

$$= -\frac{L_m}{L_r} i_{qs}^e$$

$$U_{ds}^e = r_s i_{ds}^e + p \left[ L_s - \frac{L_m^2}{L_r} \right] i_{ds}^e + \frac{L_m}{L_r} p \psi_{dr}^e - \omega_e \left[ L_s i_{qs}^e - \frac{L_m}{L_r} i_{qr}^e \right]$$

So, what we have we are talking about a rotor flux orientation vector control from a VSI with voltage mode control. So, we are operating this with voltage mode control. Please remember that it is not only true for VSI, it can also refer a cycloconverter; say for example, if we want to feed an induction motor from a cycloconverter and the cycloconverter is essentially a voltage fed system. So, we can operate the cycloconverter in voltage control mode and use this technique to our vector control. So, let us see how this is implemented. So, we derive some equations here. So, we take synchronously rotating reference frame, and here what we do we we take the rotor flux orientation and the equations are pretty straightforward.

So, we can write down the equations for the voltage  $v_{ds}$  is equal to  $r_s i_{ds}$  plus  $p \psi_{ds}$  minus  $\omega_e \psi_{qs}$ . And we can simplify this; that is equal to  $r_s i_{ds}$  plus we can replace this  $\psi_{ds}$  by the currents  $i_{ds}$  plus  $i_{dr}$  minus of  $\omega_e$  into; again we can replace this  $\psi_{qs}$  by this current  $i_{qs}$  plus  $i_{qr}$  minus of  $\omega_e$  into. And we know that  $i_{dr}$  equal to  $\psi_{ds}$  minus  $i_{ds}$  by  $L_r$ , and similarly we can say that  $i_{qr}$  is equal to  $\psi_{qs}$  minus  $i_{qs}$  by  $L_r$ . So, these are the known equations which we can obtain from the flux linkages. And we know that in a vector control drive which is rotor flux orientation  $\psi_{qr}$  is equal to 0.

So, we can say that this quantity is equal to 0, because this does not exist in a rotor flux oriented vector control drive. Now, if that is equal to 0 we can say that  $i_{qr}$  equal to



minus of  $l_m$  by  $l_r$  into  $i_q$  s e. So, what we will do now we will substitute for  $i_d$  r and  $i_q$  r in the previous equation and simplify. And if you substitute that what we obtain is the following that  $v_d$  s e is equal to  $r_s i_d$  s e plus  $p$ . This is  $l_s$  minus  $l_m$  square by  $l_r$  into  $i_d$  s e plus  $l_m$  by  $l_r$  p psi d r e minus  $\omega_e$  into  $l_s$  i q s e minus  $l_m$  square by  $l_r$  into  $i_q$  s e.

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$$v_{dc}^e = (r_s + \sigma L_s p) i_{dc}^e + \frac{L_m}{L} p \psi_{dr}^e - \omega_e \sigma L_s i_q^e$$

$$\sigma = \text{Leakage factor} = 1 - \frac{L_m^2}{L_s L_r}$$

If there is no leakage  $\sigma = 0$

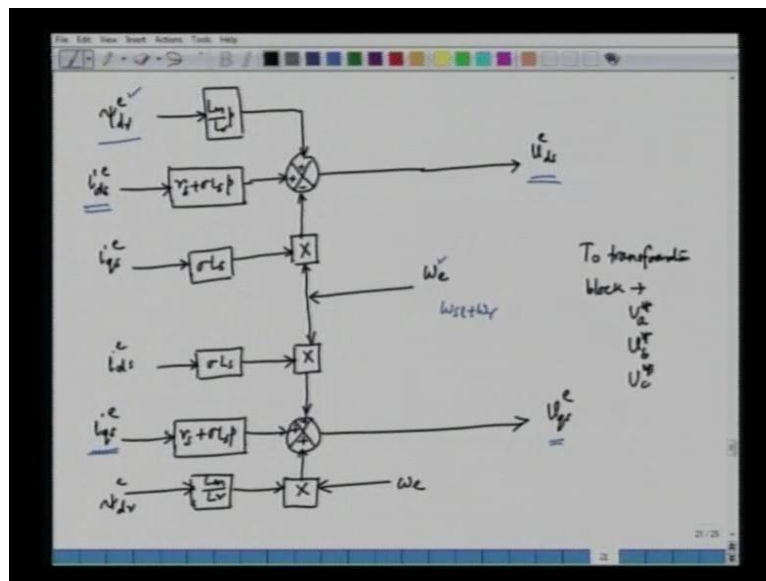
$$v_{qs}^e = (r_s + \sigma L_s p) i_{qs}^e + \omega_e \sigma L_s i_{dc}^e + \omega_e \frac{L_m}{L} \psi_{dr}^e$$

And this can further be simplified in the following fashion that  $v_d$  s e is equal to  $r_s$  plus  $\sigma l_s p$  into  $i_d$  s e plus  $l_m$  by  $l_r$  p psi d r e minus  $\omega_e$  sigma  $l_s$  i q s e. Now this equation is basically the equation for the voltage. So, we are able to find out  $v_d$  s from  $i_d$  s and  $i_q$  s. Now  $\sigma$  is called the leakage factor;  $\sigma$  is called the leakage factor and that is equal to  $1$  minus  $l_m$  square by  $l_s$  into  $l_r$ .  $L_m$  is the magnetizing inductance, and  $l_s$  and  $l_r$  are the self inductances of the stator and rotor respectively. Now if the coupling is 100 percent, suppose we have a very tight coupling between the stator and the rotor. So, we can say that  $l_m$  square is equal to  $l_s$  into  $l_r$ . So, if there is tied coupling the leakage factor will be equal to 0.

So, it means if there is no leakage  $\sigma$  is equal to 0. So, basically this is the measure of the leakage inductance of an inductance machine, and hence it is called the leakage factor. Now similarly in the  $q$  axis we can derive this equation  $v_q$  s e is equal to  $r_s$  plus  $\sigma l_s p$  into  $i_q$  s e plus we can have here  $\omega_e$  sigma  $l_s$  into  $i_d$  s e plus  $\omega_e$  into  $l_m$  by  $l_r$  into psi d r e. So, this is the equation for the voltage in the  $q$  axis. So, we

have the equation for the d axis and equation for the q axis voltage. So, it means if we know  $i_d$  and  $i_q$  or  $i_s$  and  $i_r$  we can evaluate what it is  $v_d$  and  $v_q$  and which you can show in the form of block diagram. So, this basically will translate the currents into the voltages which will be useful for controlling a voltage source inverter in the voltage control mode. So, we will see how we can have that in the form of a block diagram. So, this is shown in the following fashion.

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So, we have two summers here. This is  $r_s$  plus  $\sigma l_s p$ , and we have  $i_d$  here. And then what we have in this case  $i_m$  by  $i_r$  this goes here, and this is  $\psi_d$ , and what we obtain here is  $v_d$ . And we have some multipliers also here. We have another multiplier here, and what we are feeding in the multiplier one of the variables is  $\omega_e$ .  $\omega_e$  can be measured, because  $\omega_e$  is the synchronous speed which can be obtained by adding the rotor speed and the slip speed, okay. So, this speed here and we have some gain block here  $\sigma$  into  $l_s$  as per the previous equation. Then what we have here is  $i_q$  as we have in the d axis.

Similarly we can have in the q axis. Here this is for the q axis,  $r_s$  plus  $\sigma l_s p$  and this is our  $i_q$ . It is positive here, this is also positive. And here we have another multiplier and as we have  $\sigma$  into  $l_s$  here. Similarly we have here  $\sigma$  into  $l_s$ . It goes here, and this quantity is  $i_d$ . And here also we have another multiplier, and one

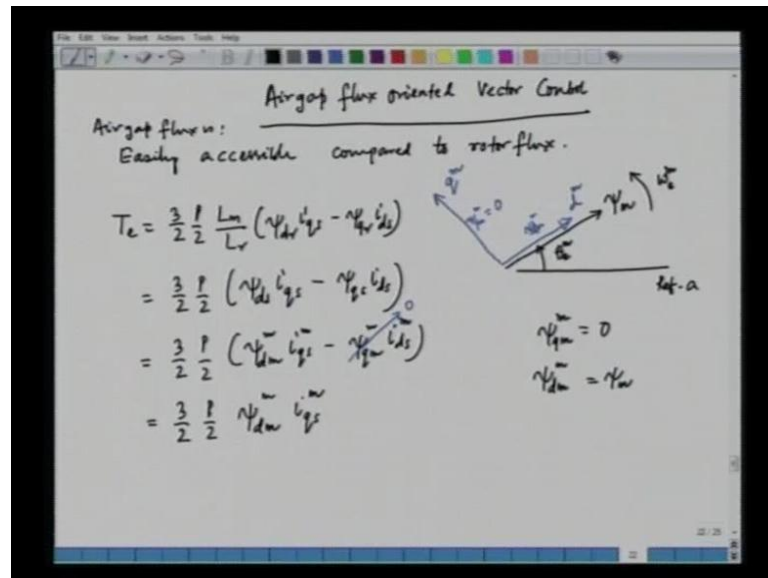
of these inputs is  $\omega_e$ . And here we have  $i_m$  by  $i_r$  feeding here and here this is  $\psi_{dr}$ , and this output is  $v_{qs}$ , and this is positive.

So, this is actually the complete block diagram which shows how we can evaluate the voltages from the currents. We know that these are available to us. We know  $i_{ds}$ , the flux component of current; we know  $i_{qs}$  the torque component of current. We of course know  $\psi_{dr}$  is the rotor flux in the d axis which is basically which is the total rotor flux that we know because  $\psi_{qr}$  equal to zero that we know. So, this flux is also one of the inputs here, and of course this synchronous speed can be evaluated by adding the rotor speed with the slip speed  $\omega_{sl}$  plus  $\omega_r$ . So, in a way we know  $\omega_e$ , and then when we use this we obtain  $v_{ds}$  and  $v_{qs}$  which will be used for a VSI.

So, what we can say here is that to the transformation block which will be giving us  $v_a$  star,  $v_b$  star and  $v_c$  star. And once we obtain  $v_a$  star,  $v_b$  star and  $v_c$  star which are the reference voltages of the 3 phases a, b and c respectively we can use an inverter to control the various voltages of the 3 phases, and we can also operate the inverter at a constant switching frequency. Now we will be discussing about the air gap flux oriented vector control. We have already seen that vector control can be done with respect to the rotor flux vector. It means the d axis of the reference frame is oriented along the rotor flux vector.

Now we have so many flux vectors in an induction machine. We can have rotor flux vector, we can have air gap flux vector, we can also have stator flux vector. So, it is not mandatory that always we have to go for speed orientation with respect to the rotor flux vector. It can always be done with respect to other other flux vector; say for example, it can also be done with respect to air gap vector. Now what is the motivation? The motivation is this that when we talk about air gap flux vector, air gap flux is easily available compared to the rotor flux.

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So, in this lecture we will also discuss about the air gap flux oriented vector control. So, one of the motivation that I have already said that air gap flux is easily measurable compared to the rotor flux. The rotor flux inside the machine inside the rotor we cannot have any sensor to measure the rotor flux directly, but however, for the air gap flux we can have some sensors in the air gap to measure the air gap flux. And hence the air gap flux vector is more easily available than rotor flux. So, the air gap flux is easily available compared to rotor flux. So, we can find out the corresponding air gap flux angle to have the transformation. When we are doing the transformation we have to use angle of the transformation. Here what we do we track the air gap flux vector.

Suppose this is the air gap flux vector, and we can call this to be  $\psi_m$ . And what we do here we take a rotating reference frame, and the d-axis of the rotating reference frame is oriented along the air gap flux vector. So, we can have the d-axis aligned along this vector; this is my d axis, and the q-axis is oriented right angle to this, this is the q-axis. So, the air gap flux vector is rotating at a speed of  $\omega_e$  superscript m because we use this  $\omega_e$  to show the speed of the rotor flux vector. And in this case we are talking about the air gap flux vector. So, we use a superscript  $\omega_e$  m, and this we can call as  $d_m$  and  $q_m$ . So, symbolize that this reference frame is a special reference frame whether d-axis of the reference frame is aligned along the air gap flux vector.

So, we have the stationary phases. The stationary phase is here. This is a reference phase, and this angle that we have here is  $\theta_{em}$ . So, what we do here is that we have to find out  $\theta_{em}$ , and if we know  $\theta_{em}$  we can transform the variables in the air gap flux reference frame to the stationary a b c. We have phase a here, maybe phase b 120 degree phase away from this, phase c 240 degree away from phase a. And hence if we know the angle  $\theta_{em}$  we can evaluate  $i_a, i_b, i_c$  in the actual phase variables of the induction machine. So, let us see the torque equation. Now we know that the torque of an induction machine is given as  $\frac{3}{2} p \frac{1}{2}$ . If you see the original equation what we used for the rotor flux orientation was  $\psi_{dr}$  into there was a constant here. And the constant is  $\frac{1}{2} l_r$  into  $\psi_{dr} i_{qs}$  minus  $\psi_{qr} i_{ds}$ .

Now this was the equation that we use for rotor flux oriented vector control. Now if we simplify this equation we can also derive this equation in the following fashion. Simplify this we can have  $\frac{3}{2} p \frac{1}{2} \psi_{ds}$  into  $i_{qs}$ , this is max of rearrangement  $\psi_{ds}$  into  $i_{qs}$  minus  $\psi_{qs}$  into  $i_{ds}$ . So, what we do we replaced  $\psi_{dr}$  and  $\psi_{qr}$  by  $\psi_{ds}$  and  $\psi_{qs}$  respectively. So, this torque equation is in terms of the stator flux and the stator currents. Further simplification can give us the following equation. We can say that this is  $\frac{3}{2} p \frac{1}{2} \psi_{dm}$  into  $i_{qs}$  minus  $\psi_{qm}$  into  $i_{ds}$ , and  $\psi_{dm}$  and  $\psi_{qm}$  are the air gap fluxes in the d-axis and the q-axis respectively. So, what we mean here is that that this flux in this axis is called  $\psi_{dm}$ , and this is called  $\psi_{qm}$ . And furthermore you know that this reference frame is rotating synchronously with the air gap flux vector.

So, we add a superscript here. We can say that that is equal to  $\psi_{dm}^m$  and  $\psi_{qm}^m$ , and the currents will be  $i_{qs}^m$  and  $i_{ds}^m$ . This superscript m shows there is a special variable. These variables are all defined in a hypothetical reference frame rotating with the air gap flux vector; of course, when we are talking about the d q axis remember that the d q axis is in line. It is basically the hypothetical situation that we are talking about a d-axis and q axis which is actually non-existent, but these are useful for understanding the control.

So, this reference frame d and q is the reference frame which is attached to the air gap flux vector that is  $\psi_m$ . So, we symbolize this by using a superscript that is m. So, when we orient the d-axis along the air gap flux vector we can very well say that  $\psi_{qm}^m$  is

equal to 0 because of the special choice of the reference frame; we do not have any q-axis flux.

So, it means this flux is equal to 0. So, we can say this flux does not exist, and what about  $\psi_{dm}$ ?  $\psi_{dm}$  is equal to  $\psi_m$ ; the total air gap flux is oriented along the d-axis. So, we can say that  $\psi_{dm}^m$  is equal to  $\psi_m$ . So, in this case this equation; this will be equal to 0. So, what we can say here that is equal to  $\frac{3}{2} p \psi_{dm}^m$ . So, this equation is also a very simple equation; in the sense that if you remember the equation of d c machine that the torque is a product of flux and current or the torque is a product of q currents.

This is a simpler product of a flux and a current. So, it means an induction machine torque equation is similar to that of a d c machine torque equation even when we are talking about air gap flux orientation, but is not as simple as rotor flux orientation. You will see little later that in air gap flux orientation we do not have inherent d coupling; that we will see when we derive the equations.

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Rotor equations (Air gap flux reference frame)  
 d-axis:  

$$0 = r_r i_{dr}^m + p \psi_{dr}^m - \omega_{sl} \psi_{qr}^m$$

$$= r_r i_{qr}^m + p (\psi_{dm}^m + L_r i_{dr}^m) - \omega_{sl} (\psi_{qm}^m + L_r i_{qr}^m)$$

$$\psi_{dm}^m = L_m (i_{ds}^m + i_{dr}^m) \quad \text{or} \quad i_{dr}^m = \frac{\psi_{dm}^m}{L_m} - i_{ds}^m$$

$$\psi_{qm}^m = L_m (i_{qs}^m + i_{qr}^m) = 0 \quad \text{or} \quad i_{qr}^m = -i_{qs}^m$$

$$p \psi_{dm}^m = -\frac{r_r}{L_r} \psi_{dm}^m + \frac{L_m}{L_r} (r_r + L_r p) i_{ds}^m - \omega_{sl} \frac{L_m}{L_r} i_{qs}^m$$

$$i_{qs}^m \text{ is coupled to } \psi_{dm}^m$$

So, what we have right now is the following that we can write down the rotor equations, and this we are writing all in the air gap flux reference frame. So, we can write down in the d-axis, the equation is as follows. We have  $v_{dr}$  equal to 0. So, we can say 0 equal to  $r_r i_{dr}^m + p \psi_{dr}^m - \omega_{sl} \psi_{qr}^m$ . This  $\omega_{sl}$  is again the slip speed with respect to the air gap flux vector. So, this is the equation that is coming

straight from the rotor d-axis, but this needs some simplification. Now we can simplify as follows. What we can do here is the following that we can take this equation and instead of writing in terms of the rotor flux write in terms of the air gap flux.

So, we have  $\psi_{dm} + l_{lr} i_{dr} - \omega_s l_m$ , and  $\psi_{qr}$  we can replace by  $\psi_{qm} + l_{lr} i_{qr}$ . So, this is actually directly coming from the previous equation, and here we know that this flux is 0, and this flux does not exist. So, in this flux is 0 we can further rewrite this equation in the following fashion, and before that what we will do here is the following.

We will try to replace for  $\psi_{dr}$  and  $\psi_{qr}$ . So, what we will do here we can find out the expression for  $i_{dr}$  and  $i_{qr}$ . So,  $\psi_{dm} = l_m i_{ds} + \psi_{dr}$ , or we can say that  $i_{dr} = \psi_{dm} / l_m - i_{ds}$ . In a similar way we can say that  $\psi_{qm} = l_m i_{qs} + \psi_{qr}$  that is equal to 0 because there is no q-axis rotor flux.

So, we can say here  $i_{qr}$  is equal to minus of  $i_{qs}$ . So, we have been able to find out the expression for  $i_{dr}$  and  $i_{qr}$  respectively. So, this is our  $i_{dr}$ , and this is our  $i_{qr}$ . So, we can eliminate these two variables from the equation from the d-axis equation. So, if we eliminate these variables we have got the following result.

So, after we simplify this what we obtain here is the following.  $P \psi_{dm} = -r_r l_r i_{dr} + l_m l_r i_{ds} - \omega_s l_m$  into  $l_{lr} l_m i_{qs}$ . Now here one interesting thing has been observed. This equation shows that there is coupling between  $i_{ds}$  and of course  $i_{qs}$  and the flux linkage. Say for example, in this case you know that we want to keep the flux constant. This is the flux, and to keep the flux constant we can keep  $i_{ds}$  constant.

So, this shows a coupling between the d and q-axis. So, if we change the torque that is  $i_{qs}$ ,  $i_{qs}$  is the torque component of current, suppose, I would like to change the torque component of current. Now if I change this automatically even if I keep  $i_{ds}$  constant  $\psi_{dm}$  will be affected. It means what we can say here that  $i_{qs}$  is coupled to  $\psi_{dm}$ . So, this is a very interesting situation where you know that  $i_{qs}$  is coupled with  $\psi_{dm}$ . It means I have one current in the q-axis, and I have one flux in the d-axis; they are orthogonal to each other. Although, they are orthogonal to each other they are coupled. We normally understand that suppose we have two vectors, means I have one vector in

the x-axis and other vector in the y-axis; these two vectors should be normally non-interfering to each other.

They should be decoupled from each other. But in this case although they are two different components in the d and q-axis respectively variation of  $\psi_q$  in the q-axis affects  $\psi_d$  in the d-axis, and hence this were coupled. It means I cannot independently control the torque and the flux. When I want to control the torque the flux will be affected, and if the flux is affected the flux will be delayed; the torque response is also delayed. So, this is basically one of the drawbacks of having air gap flux orientations, but however, this coupling has to be cancelled. If you have a decoupling network this so call coupling that is present between the torque and the flux can be eliminated, okay.

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The image shows a whiteboard with handwritten mathematical equations. The top equation is:

$$i_{ds}^m = \frac{1}{1 + \tau_{lr} p} \left[ (1 + \tau_{lr} p) \frac{\psi_{dm}^m}{L_m} + \omega_{sl} \tau_{lr} i_{qs}^m \right]$$

Below this, two definitions for  $\tau_{lr}$  are given:

$$\tau_{lr} = \frac{L_{lr}}{r_r}$$

$$\tau_r = \frac{L_r}{r_r}$$

Then, the text "Similarly in q-axis" is written. Below it, the following equation is derived:

$$0 = r_r i_{qr}^m + p i_{qr}^m L_{lr} + \omega_{sl} (\psi_{dm}^m + L_{lr} i_{ds}^m)$$

From this,  $\omega_{sl}$  is solved as:

$$\omega_{sl} = \frac{\frac{1}{\tau_{lr}} (1 + \tau_{lr} p) i_{qs}^m}{\left( \frac{\psi_{dm}^m}{L_m} - \frac{\tau_{lr}}{\tau_r} i_{ds}^m \right)}$$

Finally, the total angular speed  $\omega_e^m$  is given as:

$$\omega_e^m = \omega_{sl} + \omega_r ; \quad \theta_e^m = \int \omega_e^m dt$$

So, if you simplify it further what we have here is the following; say we can have  $i_{ds}$  can expressed in the following fashion,  $1 + \tau_{lr} p$  into  $1 + \tau_{lr} p \psi_{dm}$  by  $L_m$  plus  $\omega_{sl} \tau_{lr}$  into  $i_{qs}$ , okay. And here  $\tau_{lr}$  is the leakage time constant of the rotor; that is equal to  $L_{lr}$  by  $r_r$ , and  $\tau_r$  is the time constant of the rotor that is  $L_r$  by  $r_r$ . It means if I want to keep my flux constant I have to change  $i_{ds}$  as per the equation, and if I change  $i_{ds}$  as per this equation  $i_{ds}$  can no longer be constant when  $i_{qs}$  is changing. If I want to keep the flux constant, say for example, if I want to keep this



constant let us say I have to change  $i_d$  with  $i_q$ ; this is reflected from this equation.

And, similarly what we can do we can have the equation in the q-axis. We can write down the following equation  $0 = r i_q + p \lambda_m + \omega_s \lambda_m + l_r \frac{di_q}{dt}$ . And from this we can obtain expression for the slip speed that is equal to  $1 - \frac{l_r p}{\lambda_m} \frac{di_q}{i_q}$  divided by  $\lambda_m$  minus  $\frac{l_r}{\lambda_m} \frac{di_q}{i_q}$ . So, this is expression for the slip speed, and the expression for the slip speed is little more complicated than compared to rotor flux oriented induction motor drive.

Now here the slip speed contains the derivative terms like  $\frac{di_q}{dt}$ , and this also has got  $\frac{di_q}{i_q}$ ; of course, this will again depend upon the parameters like  $\lambda_m$  and  $\tau_r$ , okay. And this slip speed has to be evaluated, because when we evaluate slip speed we can find out the speed of the air gap flux vector. So, when we obtain slip speed we can find out  $\omega_e$ . Now here  $\omega_e$  that is equal to the slip speed plus the rotor speed, and once we know  $\omega_e$  then  $\theta_e$ , the angle of the air gap flux vector can be evaluated by integrating  $\omega_e$ . So, in the next lecture we will try to see that how we can have this decoupling network to cancel the decoupling which is existing in case of an air gap flux oriented vector control.