

Computer Vision and Image Processing – Fundamentals and Applications
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Lecture – 09
Image Reconstruction from a Series of Projections

Welcome to NPTEL MOOCS course on Computer Vision and Image Processing, Fundamentals and Applications. In my last class, I discussed about the concept of stereo vision that image formation in a stereo vision setup. In that discussion, I discussed about two camera configurations. One is general camera configuration and another one is canonical camera configurations.

In general camera configuration, I have nonparallel epipolar lines, but in a canonical configuration epipolar lines are parallel. So, I can convert the general camera configuration into canonical camera configuration by using some transformation and that is called image rectification. So, after image rectification 2D search problem is converted into 1D search problem and that is the advantage of image rectification.

After this, I discussed about the concept of disparity map so we can find disparity map by finding the horizontal displacement of the pixels of the left image and the right image and if I compute the disparity values for each and every pixel of the images then I can determine the disparity map and from the disparity map I can determine the depth map. So, also I discussed some concepts like the constraints, some assumptions in finding the disparity maps.

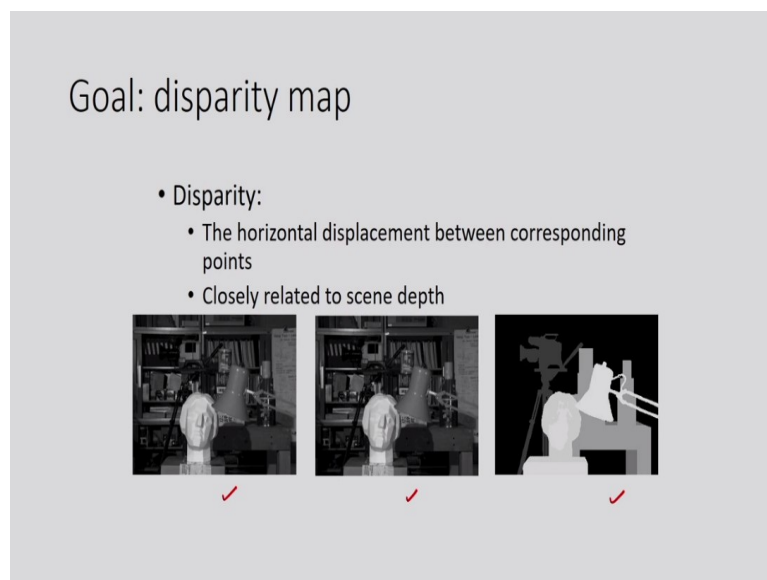
And also, some problems in finding the matching the problems like noise, the perspective distortions, foreshortening factors, specular surface. So, all these cases I have discussed in my last class. So, today I am considering the second part of this the stereo vision setup that is how to find the matching between the left image and the right image that is called the stereo correspondence.

After finding the stereo correspondence, I have to determine the disparity map and from the disparity map I can determine the depth map. Now for the matching of the left image and the right image there are two approaches mainly one is the pixel-based method and another one is feature based method. In pixel-based method, I have to compare the pixel intensity value of the left image and the right image.

And based on this comparison, I can find the matching pixels of the left image and the right image and in this comparison actually we are getting the dense disparity map. In a second approach, we can consider features I can select some of the image features and based on this feature I can find the correspondence between two images, the left image and the right image. This method will give the sparse disparity map.

In pixel based method, I am getting the dense disparity map, in case of the feature based approach I am getting the sparse disparity map and in case of the pixel based method I can consider a particular window that is called the search window and in this window I can find the corresponding pixels. So, let us discuss these two approaches one is the pixel-based approach and another one is the feature-based approach.

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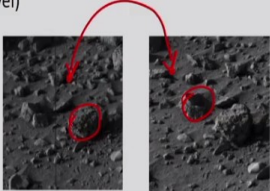


So last class I have shown that is the disparity map here in this case you can see the disparity is nothing, but the horizontal displacement between corresponding points. So, here I have shown the left image this is the first one is the left image, the next one is the right image and I have shown the disparity map. So, this disparity map you can determine based on the horizontal displacement between corresponding points.

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The matching problem

- Which image entities should be matched?
 - Two main approaches
 - Pixel/area-based (lower-level)
 - Feature-based (higher-level)



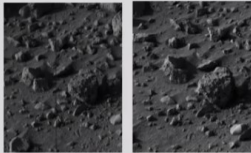
And already I told you that there are two approaches for stereo matching. One is the pixel or maybe the area-based approach and another one is the feature-based approach. In pixel based approach, I can compare the pixel intensity values of the left image and the right image and I can find the corresponding pixels in the right image and the left image and in the feature based technique I can extract some features and based on this features I can do the matching.

So, first let us consider the pixel based that is the lower level method so I am discussing the pixel-based method for matching the images the right image and the left image.

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Matching challenges

- Scene elements do not always look the same in the two images
 - Camera-related problems
 - Image noise, differing gain, contrast, etc..
 - Viewpoint-related problems:
 - Perspective distortions
 - Occlusions
 - Specular reflections



Also I discussed about the matching challenges the problems like the perspective distortions, image noise, the gains of the cameras may be different, specular reflections, occlusion. So these cases I have already discussed about the stereo matching.

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More matching heuristics

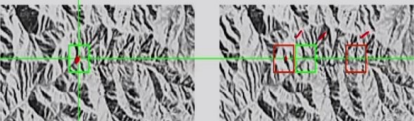
- Always valid:
 - (Epipolar line)
 - Uniqueness
 - Minimum/maximum disparity
- Sometimes valid:
 - Ordering
 - Local continuity (smoothness)

And also I have considered some assumptions, some constraints like epipolar constraints, uniqueness constraints, the maximum and the minimum disparity values, the ordering constraints, the local continuity assumptions. So, this assumption I have considered for stereo matching the matching between the left image and the right image.

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Area-based matching

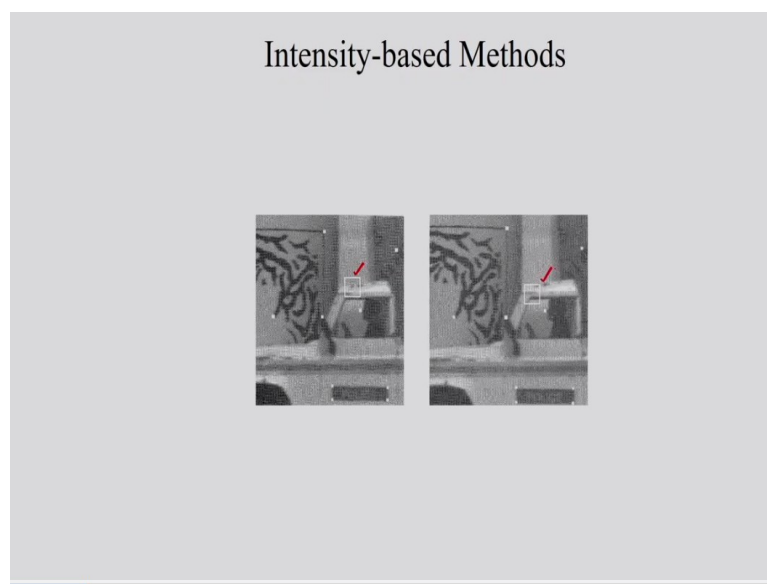
- Finding pixel-to-pixel correspondences
 - For each pixel in the left image, search for the most similar pixel in the right image
 - Using neighbourhood windows



So, in the pixel based correspondence I am searching the corresponding pixels. Suppose, I have a pixel in the left image and I want to find the corresponding pixel in the right image for this I have to do the matching and in this case I can consider one window that is the search window. So in this search window I can find the corresponding pixels in the right image.

So, corresponding to this pixel in the left image I can find the corresponding pixel in the right image by searching in this particular window. The window is shifted you can see the window is shifted and shifted like this and I want to find the corresponding pixels in the right image.

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This is the same thing I am showing here that is the pixel correspondence one is the left image and another one is the right image and I am considering one window and in this window I am doing the searching. So, that is the intensity based method.

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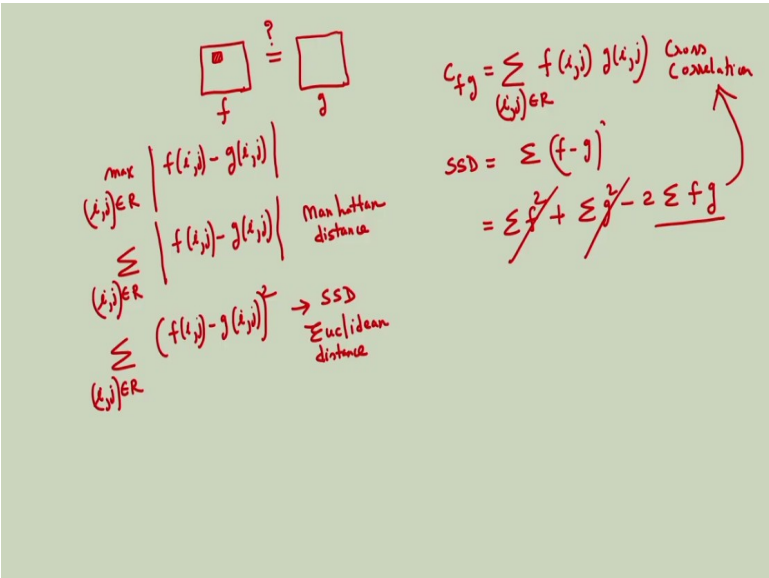
Area-based matching

- Similarity measures for two windows
 - SAD (sum of absolute differences)
 - SSD (sum of squared differences)
 - CC (cross-correlation)
 - ...



In intensity based method, we can consider this similarity measures like SAD sum of absolute difference or we can consider the SSD sum of squared differences or maybe the measures something like the cross correlation factor. So, based on this we can do the matching the matching between the left image and the right image. So, how to do the matching I can show you.

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Handwritten mathematical formulas for similarity measures:

- Manhattan distance: $\max_{(i,j) \in R} |f(i,j) - g(i,j)|$
- Sum of absolute differences: $\sum_{(i,j) \in R} |f(i,j) - g(i,j)|$
- Sum of squared differences: $\sum_{(i,j) \in R} (f(i,j) - g(i,j))^2 \rightarrow$ SSD Euclidean distance
- Cross Correlation: $C_{fg} = \sum_{(i,j) \in R} f(i,j) g(i,j)$
- Sum of squared differences (expanded): $SSD = \sum (f-g)^2 = \sum f^2 + \sum g^2 - 2 \sum fg$

So, suppose I have the image one image is f and one another image is suppose g I want to find the correspondence between this two images. So, for this I am considering the window based method so I am considering one window I am considering this window I am

considering. So, this measures I can consider the maximum and in this case i, j the pixel i, j is within this particular window the window means the region particular window that is considered.

So, this measure I can use or maybe something the measure something like this I can use this one i, j in this particular window and $f_{i,j}$ minus $g_{i,j}$ this measure also I can use. So, this is the nothing but the first one is the chessboard distance the second one is this is the Manhattan distance, so already you know about the distances. One is the chessboard distance, the second one is the Manhattan distance or maybe I can also consider this distance.

This is called SSD sum square difference so sum square difference that is also the Euclidean distance which is very popular Euclidean distance the Euclidean distance is very popular. So, by using this we can also find the corresponding pixels that means this measurement I can do. This cross correlation factor between these two images I can write like this, this is the cross correlation.

And in this case also I am considering window this is called the cross correlation. So one is the chessboard distance, one is the Manhattan distance, one is the Euclidian distance and the cross correlation also I have defined. So, suppose what is SSD? SSD is nothing, but the summation of f minus g whole square so I can write like this simple expression $f^2 + g^2 - 2fg$.

So, in this case this is constant if you see this is constant and this is also constant. So this is nothing, but you can see this is nothing but the cross correlation that means the SSD actually also give the value of the cross correlation in this case. So, by using this measure we can find the similarity between the pixels and based on this I can do the matching.

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Feature-based matching

- Matching features:
 - Edge points
 - lines
 - corners
 - ...
- Sparse reconstruction sets
- Best if scene type is known *a priori*

The next point I want to show you that feature based matching. In feature based matching, we can consider features like edge points, lines in the image, the corner points in the image or maybe the boundary pixels by using this features we can compare the left image and the right image and in this case I am getting the sparse disparity map.

In case of the pixel based method, we are getting the dense disparity map. But in this case I am getting the sparse disparity map. So, in this case in feature based technique I can give one example.

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Left Right

Edge points
line segments
corner points.

$$\text{Similarity measure} = S = \frac{1}{\omega_1 (\theta_1 - \theta_2)^2 + \omega_2 (x_1 - x_2)^2 + \omega_3 (y_1 - y_2)^2 + \dots}$$

Image Fusion → Multi-focus image
Multi-exposure image

Image Mosaicing → Constructing multiple images of a scene into a large image.

The example suppose I am considering one image. The left image is something like this there is a image one suppose object is something like this one object is there and I have another objects that is in the left image. One is the left image and one is the right image. So, from these images I can determine I can find the features the features maybe something like the edge point.

Line segments or maybe the corner points I can consider. This features I can consider and based on this I can determine the similarity measure S I can determine θ_l minus θ_r l means the left image and r means the right image. So, θ_l I am considering some orientation of a particular line. Suppose l means the orientation in the left image and θ_r means the orientation in the right image.

And w_1 I am considering the weight some weight I am considering and suppose another one is w_2 l_l minus l_r depth also I can consider. This will be square or w_3 maybe something like this l_l minus l_r whole square so like this I can find the similarity measures. So, first one is I am considering orientation of a particular line. So θ_l means the orientation in the left image θ_r means orientation in the right image.

Similarly I can consider the length of a line particular line in the left image and the length of a line in the right image and in this case the average intensity along a particular line in the left image, average intensity along a particular line in the right image. So like this I can consider the features based on this similarity measures. So, this image matching problem is quite important some applications like this, the applications like image fusion.

In my first class, I explained the concept of the image fusion, the image matching problem is quite important for the applications like image fusion. In image fusion, suppose we have the images like the multi focus image or maybe the multi exposure image. I will explain this one image fusion. Another application is image mosaicking that means I can consider the construction of multiple images.

I can write like this constructing multiple images of the scene into a larger image that means the output of the image mosaic will be the union of two input image. Image fusion concept I have explained. Suppose, I have one image that is the multi focus image another one is suppose multi exposure image or maybe suppose I have the image corresponding to infrared imaging and another one is the visible vision of the spectrum that is the visible image.

Then in this case I can fuse the image both the image I can fuse and in this case I am considering the important information from both the images I can consider and I can neglect the redundant information that is the concept of the image fusion and imaging mosaicking mainly the output of the image mosaic will be the union of two input images. So, for this application we have to do image matching.

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Intensity-based vs feature-based approaches

- Intensity-based methods
 - Provide a **dense** disparity map.
 - Need textured images to work well.
 - Sensitive to illumination changes.
- Feature-based methods:
 - Faster than correlation-based methods.
 - Provide **sparse** disparity maps. ✓
 - Relatively insensitive to illumination changes.

Now in case of the intensity based method already I have explained that we have to compare a pixel values. So, in this case I will be getting the dense disparity map and for this we need the textured image and in this case it will be sensitive to illumination changes, but in case of the feature based method it is relatively insensitive to illumination changes because we are considering the features only not the pixel values.

And in this case we will be getting the sparse disparity map in case of the feature based method because we are comparing the features so this method will be faster than the correlation based method. You can see the comparison between the intensity based method and the feature based method.

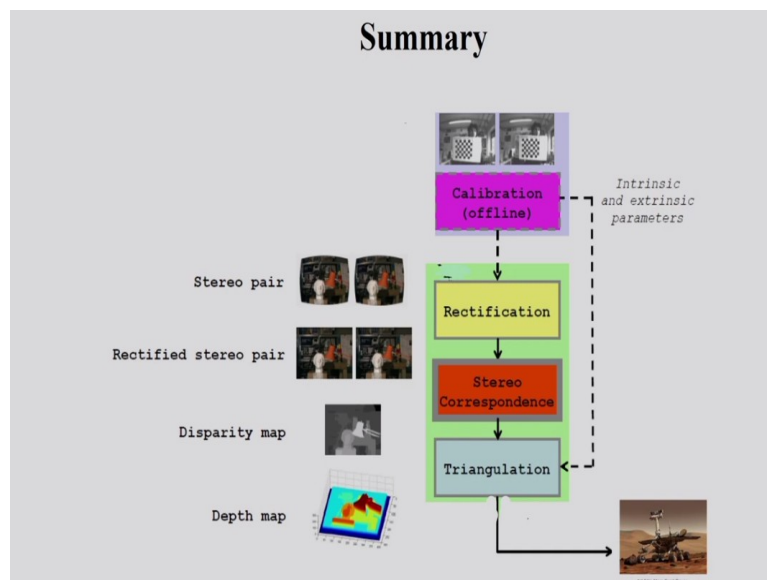
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Three or more viewpoints

- More matching information
 - Additional epipolar constraints
 - More confident matches

And if I consider three or more viewpoints then in this case I will be getting more information for the matching, but in this case we have to consider additional epipolar constraints. If I consider more viewpoints if I use more cameras then the common field of view will also increase.

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So, in summary I have shown here first one is the left image and the right image obtained from the stereo cameras one is the left camera another one is the right camera and after this we have to do camera calibration. Camera calibration means the estimation of the intrinsic

and the extrinsic parameters of the camera that is camera calibration. After doing the camera calibrations, we are getting stereo image pairs.

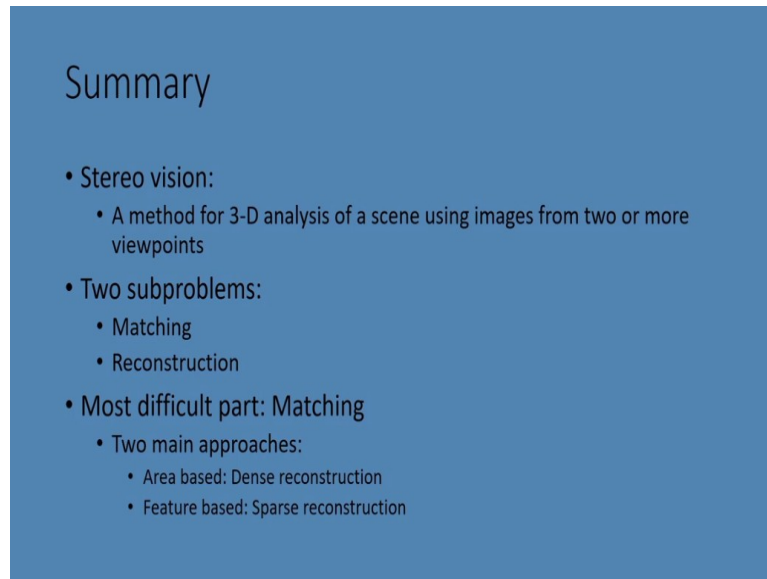
One is the left image another one is the right image and that corresponds to general camera configurations. After this, I can do image rectification. So, if I do the image rectification the 2D search problem is converted into 1D search problem. So, after this we have done the image rectification. After doing the image rectification we do the stereo correspondence. We can do the stereo matching.

So already I have explained two method one is the pixel based method another one is the feature based method. So by this we can find the correspondence between the left image and the right image. After determining the disparity map the next step is we can determine the depth map from the disparity map we can determine the depth map and finally this depth map can be applied for some applications.

In this case I have shown one application that is for the robotic vision and for robotic vision this depth map can be used. So, this is the summary of this discussion. First, we have to do the camera calibration the estimation of the intrinsic parameters and the extrinsic parameters and after this we have considered the stereo image pairs that can be rectification by image rectification procedure.

After this, we have to find the disparity map for this we have to find the disparity between the pixels. If I determine the disparity values for all the pixels of the image then in this case I can determine the disparity map. After finding the disparity map, we can determine and from the depth map depth map can be applied for any applications.

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So summary you can see so we have considered a stereo vision and after this we considered two problems. One is the matching problem and next one is the reconstruction problems and for matching we have considered two approaches one is the area based approach another one is the feature based approach. So, this is about the stereo vision concept.

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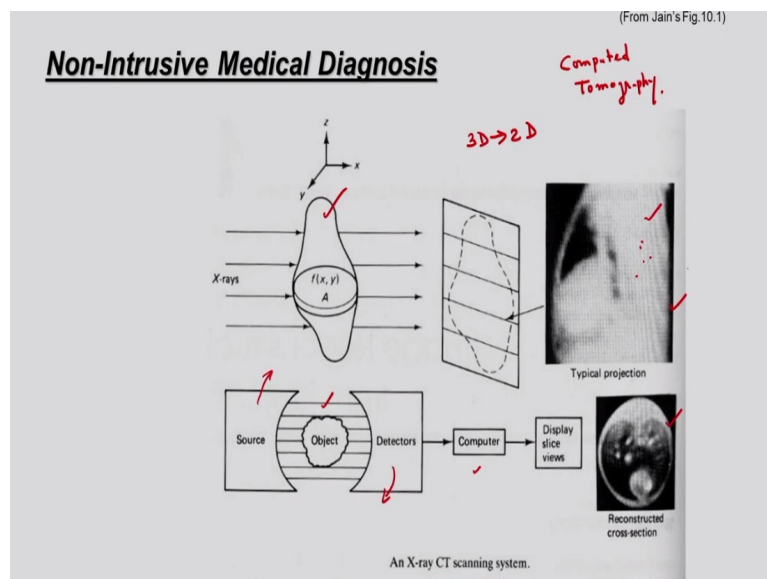
So, next topic is in this class I am going to discuss that image reconstruction from series of projections and that is very important topic, the image reconstruction from a series of projections that is nothing but 3D representation of a particular object from number of

projections. So, we will be taking number of projections and from this projection I can get the 3D representation of the object for better visualization.

This concept is applied in the case of CT Scan, CT Scan means the computed tomography. In x-ray imaging if you see it is nothing, but the 3D to 2D projections. So that means we are losing information in x-ray imaging, but in the CT Scan we can get the 3D representation of the internal part of the organ or maybe some objects based on the projections. So, for this I will be discussing one concept that called the Radon transform.

So by using the Radon transform you can find the projections at different angles and after getting the projections we have to do the back projections that is the inverse Radon transform we have to do and so that we can get the 3D representation of the object. So, first one is the projection by using the Radon transform and after this we have to consider the inverse Radon transform for reconstruction. So, this principle is called the image reconstruction from a series of projections and mainly one application I have highlighted that is the CT scan.

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So, here you see the non intrusive medical diagnosis the first example I have shown the x-rays. So x-ray is nothing, but it is 3D to 2D projections. So, if you see this is the x-ray image so in the x-ray image the intensity of the x-ray image is mainly depends on the x-ray energy that is mainly depends on the absorption of the x-ray by the object.

So, this is my object so this intensity of the pixels of this x-ray image depends on the amount of x-ray, amount of light absorbed by the object that depends on the absorption. So that

means it is nothing but the 3D to 2D projections. Suppose, in the second case what I am considering the object is this and I have the source the x-ray source and we have the detectors the photo detectors.

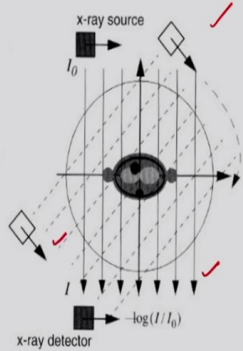
Now, in this case the source is moving the source is rotating and also the detector is rotating like this. So, in this case I will be getting number of projections I can get 10 projections, I can get 15 projections so I will be getting number of projections. From this projections, I want to get the 3D representation of the object.

So here you can see this is a computer for processing and after this, this is nothing but the reconstructed cross section that is the 3D representation of the object from the projections and in this case if you compare this image and this image the x-ray image and this image you can see visually the second image has more information as compared to the first image. So the second principle is the CT scanning the CT scan means the computed tomography. So, this is the principle of the x-ray and the CT scan.

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Non-Intrusive Medical Diagnosis

- Observe a set of projections (integrations) along different angles of a cross-section
- Want to recover inner structure from the projections
 - “Computerized Tomography” (CT)



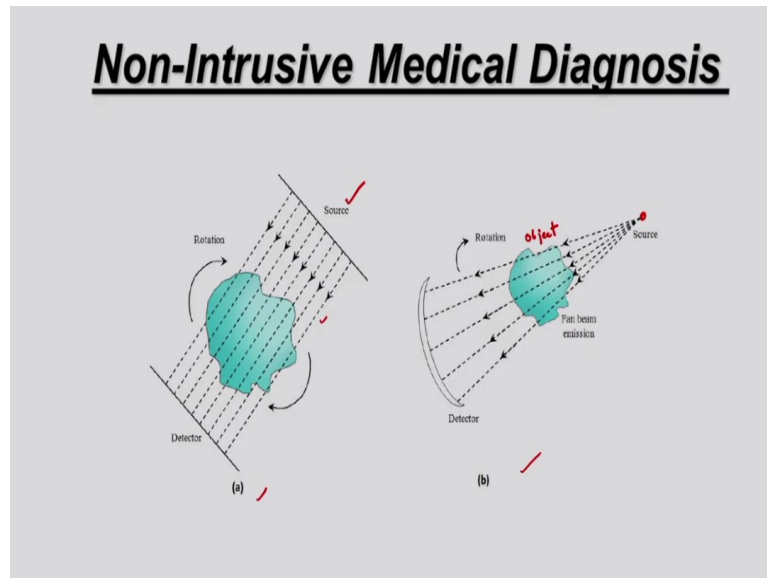
(From Bovik's Handbook Fig.10.2.1)

Here also I am showing the same principle. The source is rotating and also the detector is rotating then in this case I will be getting number of projections. From these projections, my objective is to do the reconstruction that is after reconstruction it is nothing, but the 3D representation of the object.

So in this arrangement in this configuration I am getting number of projections because the source is rotating and the detector is rotating. And in this case I will be getting number of

projections like this, this is one projection and this is another projection so like this I will be getting number of projections.

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The same principle I am showing here the source is rotating and you can see the detector is also rotating. This is my source and the detector is rotating. So, in this case you can see I am getting one projection and suppose it is rotated by a particular angle and corresponding to this I am getting another projection like this I am getting the number of projections. This is mainly the rotation of the source and the detectors.

This arrangement if I compare this arrangement another arrangement shown in the b the second arrangement. In this case the source is not moving instead I am considering the fan beam emission the beam is rotating. This is the source the x-ray source now the beam is rotating and this is my object and this is the detector. So, in this case also I am getting number of projections.

The first configuration and the second configuration the concept is very similar, but the principle is slightly different. In the second case the source is not moving, but instead we are considering the fan beam emission. So, the beam is rotating and in this case I am getting number of projections. So, from the projections my objective would be 3D representation of the object that is 3D reconstruction.

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Radon Transform

The projection of an object $f(x, y)$ along a particular line is given by:

$g(s, \theta) = \int_L f(x, y) du$

 $\int_{-L}^L \delta(u) ds = 1$

 $\delta(u) = 1$ for $u=0$

 $= 0$ otherwise.

$\textcircled{1} \Rightarrow x \cos \theta + y \sin \theta = 0$

 $\therefore g(0, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(x \cos \theta + y \sin \theta) dx dy$

 $\textcircled{2} \Rightarrow (x - s \cos \theta) \cos \theta + (y - s \sin \theta) \sin \theta = 0$

 $\Rightarrow x \cos \theta + y \sin \theta - s = 0$

- A linear transform $f(x, y) \rightarrow g(s, \theta)$
 - Line integral or "ray-sum"
 - Along a line inclined at angle θ from y-axis and s away from origin
- Fix θ to get a 1-D signal $g_\theta(s)$

Now let us consider the Radon transform. What is Radon transform? In this case I am showing the object first after this the coordinate system corresponding to the object is x and y coordinate. One coordinate is x another coordinate is y so this is corresponding to the object. This is the direction of the x-ray I am considering the x-ray in this direction. So, in this case I am considering another coordinate system that is s and u .

In polar coordinate I am considering s and θ . So one is s another one is θ . So s θ represents the coordinates of the x-ray relative to the object. So, in this diagram I have shown two coordinate system one is x and y coordinate system another one is the s and θ coordinate system. What is s and θ means I am considering s and θ coordinate system and another one is x, y .

In polar coordinate is s θ in rectangular coordinate in Cartesian coordinate it will be s, u . So, I have two coordinate system one is x, y coordinate system another one is the s θ coordinate system. So s θ represents the coordinate of the x-ray relative to the object. Now in this case you can see corresponding to this x-ray I am getting the projection, the projection is represented by $g(s, \theta)$.

So what is the image reconstruction problem? The image reconstruction problem is to determine $f(x, y)$ from $g(s, \theta)$. What is $g(s, \theta)$? $g(s, \theta)$ is the projection I am considering corresponding to the object the object is $f(x, y)$ and in this case I am getting the projection the

projection is $g(s, \theta)$. So, it is a linear transformation the projection from $f(x, y)$ to $g(s, \theta)$ and in this case considering the particular angle the angle is θ .

At a particular angle θ I am determining the $g(s, \theta)$, $g(s, \theta)$ is the projection that means I am getting one projection. If I change θ then in this case I will be getting another projection so like this I can get number of projections. So fix θ to get 1D signal that is $g(s, \theta)$ that means I am getting one projection that is $g(s, \theta)$ corresponding to a particular angle.

Now what is $g(s, \theta)$? $g(s, \theta)$ is nothing, but it is the line integral $f(x, y)$ along the direction the direction is in the direction of u along this direction that is the definition of the $g(s, \theta)$. The projection of an object $f(x, y)$ along a particular line, line is I am considering this line suppose line number is 1 and second line is number 2 I have two lines 1 and 2. So, projection of an object $f(x, y)$ along a particular line is given by $g(s, \theta)$ so it is $\int f(x, y) du$ that is the line integral.

And already I told you what is the image reconstruction problem? The image reconstruction problem is determination of $f(x, y)$ from $g(s, \theta)$ that is the image reconstruction problem. Now the concept of the Radon transform is like this the value of the 2D function at an arbitrary point is uniquely obtained by the integrals along the lines of all directions passing through the points.

So that means in this case I am considering the line integral. Now what is the equation of the line the line number is 1 suppose the equation of the line number 1 is $x \cos \theta + y \sin \theta = s$. In this case what is the line 1? Line passing through the origin and whose normal vector is in the θ direction. The normal vector is this is the normal vector whose normal vector is in the θ direction.

What is the line number 2? The line number 2 is the line whose normal vector is in the θ direction and whose distance from the origin is s . So, corresponding to line number 1 the equation is $x \cos \theta + y \sin \theta = s$ that is the equation of the line and corresponding to this the s value will be 0 you can see the x value is 0 and I have the θ value 0 θ because θ I am considering particular θ the projection is $g(0, \theta)$.

The $g(0, \theta)$ is double integration from minus infinity to plus infinity $f(x, y)$ is the object delta function $x \cos \theta + y \sin \theta = s$ $dx dy$. Why I am considering the delta function? Delta

function is the dirac delta function, dirac delta properties delta 0 suppose d s is equal to 1 this is the property of the dirac delta function that means delta n is equal to 1 for n is equal to 0 is equal to 0 otherwise. This is the dirac delta function.

So, in this case I am considering this dirac delta function because I want to determine the integration along this particular line the line is this line that means along this line the value of the dirac delta function will be 1 because $x \cos \theta + y \sin \theta$ is equal to 0. So that means the delta 0 will be 1. I want to determine the integration along this particular line. So, that is why I am considering the dirac delta function.

What is the equation of the second line? The equation of the second line the number two line you can see $x \cos \theta + y \sin \theta$ so finally I will be getting the equation of the second line. The equation of the second line is $x \cos \theta + y \sin \theta - s = 0$. So this is the equation of the second line. The second line is the line whose normal vector is in the θ direction.

And whose distance from the origin is s so that means this line is in the θ direction and what is the distance from the origin? The distance from the origin is s so equation of the line number two is $x \cos \theta + y \sin \theta - s = 0$ that is the line number 2.

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Radon Transform

Object $f(x,y)$ is shown in the $x-y$ plane. A line is drawn at an angle θ from the x -axis. The distance from the origin to this line along the normal direction is s . The projection of the object onto this line is $g(s,\theta)$.

The equation of the line is $x \cos \theta + y \sin \theta - s = 0$.

The Radon transform is defined as:

$$g(s, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(x \cos \theta + y \sin \theta - s) dx dy$$

Coordinate transformation:

$$\begin{bmatrix} s \\ u \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Rotation matrix:

$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

Coordinate transformations:

$$\begin{aligned} s &= x \cos \theta + y \sin \theta & \text{and} & \quad x = s \cos \theta - u \sin \theta \\ u &= -x \sin \theta + y \cos \theta & \text{and} & \quad y = s \sin \theta + u \cos \theta \end{aligned}$$

Derivation of the line equation:

$$\begin{aligned} \therefore x \cos \theta + y \sin \theta - s &= (s \cos \theta - u \sin \theta) \cos \theta + (s \sin \theta + u \cos \theta) \sin \theta - s \\ \Rightarrow x \cos \theta + y \sin \theta - s &= s(\cos^2 \theta + \sin^2 \theta) - u \sin \theta \cos \theta + u \sin \theta \cos \theta - s = 0 \end{aligned}$$

Corresponding to the line number 2 then in this case I have the $g(s, \theta)$ so this is my $g(s, \theta)$ corresponding to line number 2 so I have the $g(s, \theta)$. $G(s, \theta)$ is $f(x, y)$ and I am considering again the dirac delta function $dx dy$. So, this is called the Radon transform. This expression is

called the Radon transform. So corresponding to the line number 2 here this is the line number two this is the Radon transform.

And already I have define the Radon transform for 1 that is $g(s, \theta)$ corresponding to the line number 1 and if I consider the line number 2 the $g(s, \theta)$ is this expression that is the Radon transform. This $g(s, \theta)$ I can display as an image and that is something like this I can put like an image so it is s and θ suppose θ maybe 0 degree, 90 degree, 180 degree. So, I can plot this one that is θ versus s I can plot like an image.

And this is called the Sinogram $g(s, \theta)$ is displayed as an image. So, this is called the Sinogram. Now in this case I am considering one case that means suppose already I have explained the rotation case. Rotation of a particular point or vector suppose so one vector is suppose x, y and suppose this is r at an angle θ_0 and this vector is rotated x' y' the new position is x' y' this vector is rotated and this angle is θ .

So, corresponding to this rotation, rotation of this vector what is my transformation matrix? The transformation matrix you can determine $\begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ this is my transformation matrix for the rotation. So you know about this rotation matrix suppose the axes rotation I am considering one is the x, y coordinate system that is the x, y coordinate systems and I have another coordinate system s and u suppose.

So, how I am getting the s and u coordinates system by rotation of the x, y coordinate system by an angle θ . So, I have shown the θ here this is θ . So by rotation I am getting the s and u coordinate system. So, I have two coordinate system one is x and y coordinate system another one is the s and u coordinate system. Corresponding to this, my transformation matrix the rotation matrix will be simply $\begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ you can verify this one.

In the first case I am considering rotation of vector in the second case I am considering the axes rotation. So, here I have shown I have the x, y coordinate system. How to get the s and u coordinate system? You can see here in this diagram also x and y coordinate and I have the another coordinate system and that is s and u . So how to get s and u coordinate system? If I rotate x, y coordinate system by an angle θ then in this case I will be getting s and u coordinate system that is the axes rotation.

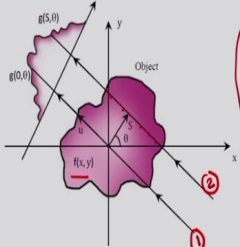
So corresponding to this, this is the transformation equation the transformation matrix is $\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$. So from this equation you can see what will be the value of s ? S is $x \cos \theta + y \sin \theta$ and x is equal to $s \cos \theta - u \sin \theta$, u is equal to $-x \sin \theta + y \cos \theta$ and y equal to $s \sin \theta + u \cos \theta$.

Now based on this I want to verify whether $x \cos \theta + y \sin \theta - s$ should be equal to 0 because you know that corresponding to the line number 2 it is $x \cos \theta + y \sin \theta$ is equal to s this is the equation of the second line the line number 2 so that I want to verify. So you know $x \cos \theta + y \sin \theta - s$ that should be equal to 0. From this you can see $x \cos \theta + y \sin \theta - s$ should be equal to 0.

So, if I put this value here I am just putting the value of the x , I am putting the value of y and just after doing some simplification I am getting 0 that means my equation is correct equation of $x \cos \theta + y \sin \theta - s$ is equal to 0 that is the equation of the line number 2.

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Transition from (x, y) coordinate to (s, u) yields no expansion or shrinkage, and so, $dx dy = ds du$.



Radon Transform

$$g(s, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(x \cos \theta + y \sin \theta - s) dx dy$$

$$x \cos \theta + y \sin \theta - s = 0$$

$$\begin{cases} x = s \cos \theta - u \sin \theta \\ y = s \sin \theta + u \cos \theta \end{cases}$$

$$g(s, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(s \cos \theta - u \sin \theta, s \sin \theta + u \cos \theta) \delta(0) ds du$$

Ray Sum $\rightarrow \therefore g(s, \theta) = \int_{-\infty}^{\infty} f(s \cos \theta - u \sin \theta, s \sin \theta + u \cos \theta) du$

Now the transition from x, y coordinate to s and u coordinate system there is no expansion or there is no shrinkage that means the area will be same $dx dy$ is equal to $ds du$. There is no expansion and there is no shrinkage because of this transition from xy coordinate to s and u coordinate. So that means in this case we have this equation you can know this is the equation Radon transform equation $g(s, \theta) = \int_{-\infty}^{\infty} f(x, y) \delta(x \cos \theta + y \sin \theta - s) dx dy$.

I am considering the delta function $\delta(x \cos \theta + y \sin \theta - s)$ $dx dy$ this is called the Radon transform, this is the Radon transform equation and corresponding to the

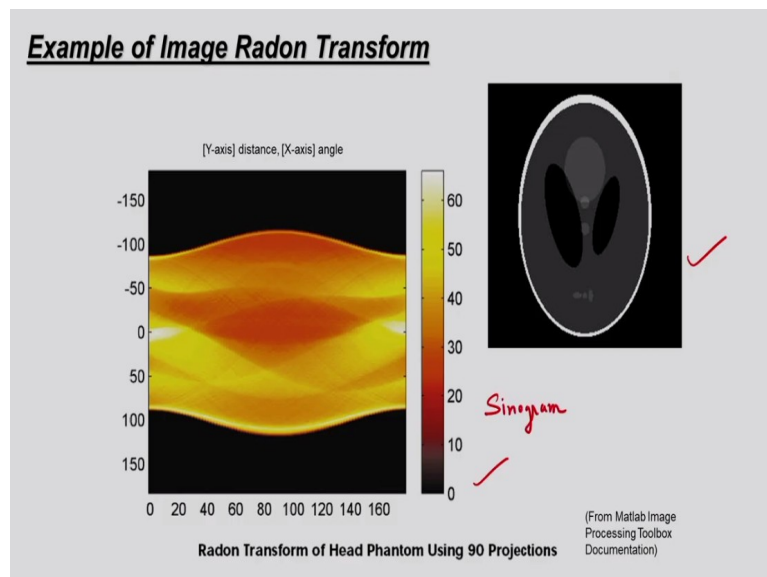
line number 2 my equation $x \cos \theta + y \sin \theta - s = 0$ and we have this the value of the x and y coordinate x is equal to $s \cos \theta - u \sin \theta$ y is equal to $s \sin \theta + u \cos \theta$.

So, putting all this value in this equation the Radon transform equation $g(s, \theta)$ I am just putting the value of x and y here and this is x this is y and what is this? $x \cos \theta + y \sin \theta - s$ that is equal to 0 so I am considering $\delta(s - x \cos \theta - y \sin \theta)$. What is the δ value? δ value is integration δ value is 1 and also you can see I am just changing the $dx dy$ into $ds du$.

So because the transition from xy coordinate to su coordinates yields no expansion or no shrinkage so that is why $dx dy$ is equal to $ds du$. So just I am putting $ds du$ here then in this case I have the final expression. The expression is $g(s, \theta) = \int_{-\infty}^{\infty} f(x \cos \theta - y \sin \theta, s \sin \theta + u \cos \theta) du$.

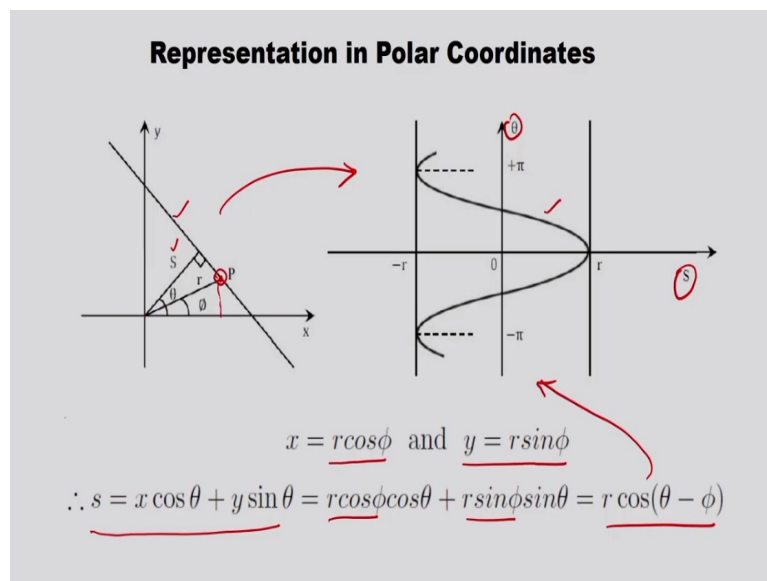
This equation is called the Ray Sum equation. This is a very important equation that means I am getting one projection along this particular line the line is number 2. So that means just I am doing the summation along this particular line. So this is the Ray sum equations.

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Now this diagram I have shown that is one example of the image Radon transform this is called the Sinogram. So corresponding to this image, I have the Radon transform the Radon transform you can display like an image and this is called the Sinogram.

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Now representation in polar coordinate system. So how to represent in the polar coordinate system? So we have the x and y coordinate systems and suppose I am considering this line and in this case you know s coordinate and the theta coordinate you know that is the direction of the normal so theta is this and suppose if I consider this is the point P and suppose if I consider this vector the vector is r.

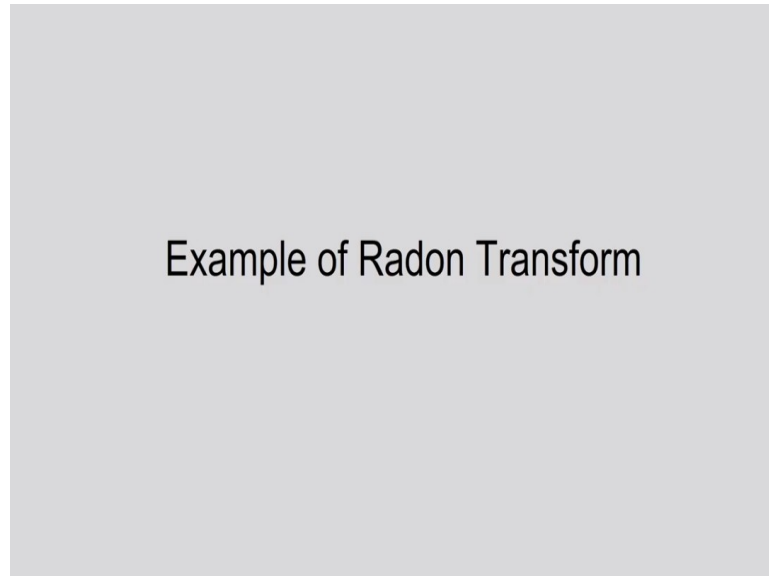
So what will be the x if I do the projection of here x will be r cos phi I can determine the x component and also I can determine the y component r sin phi and what is s my equation for s is equal to x cos theta plus y sin theta and after this I can put the value of x just putting the value of x r cos theta and just putting the value of y r sin phi and finally I will be getting r cos theta minus phi. So corresponding to this that means corresponding to this I have this one.

So, this equation that is r cos theta minus phi I am showing here r cos theta is this equation I am showing this diagram I have shown that is the cosine function I am showing cos theta minus phi. So, I can write this the point P the point is mapped into a sinusoid in the s theta plane. So, in this case I have shown this is the s theta plane this is s and this is theta. So that means the point P is mapped into a sinusoid in the s theta plane that is the case I am showing the mapping.

So, the point P this point P is mapped into a sinusoid the sinusoid is represented by cos theta minus phi in the s theta plane and for a fix point r phi we have a locus of all the points in the s theta. So, this is the representation of this equation s is equal to x cos theta plus y sin theta in

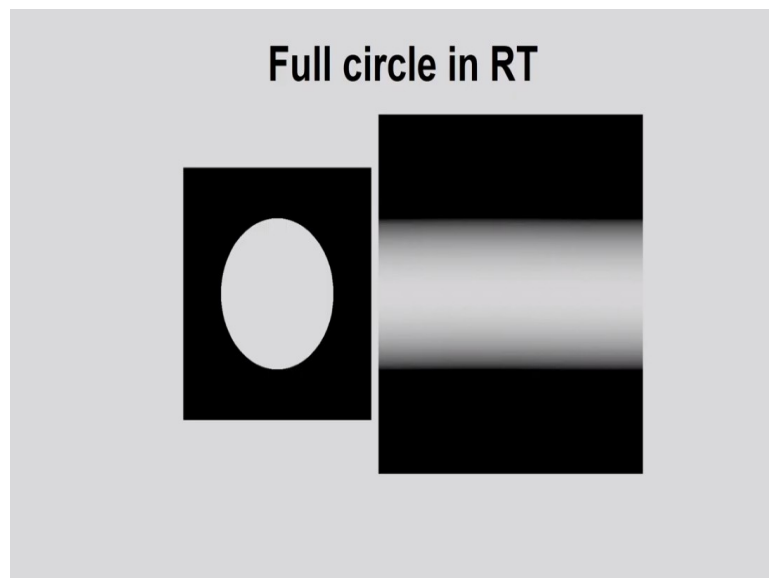
a polar coordinate. So, in a polar coordinate I can represent this equation s is equal to $x \cos \theta$ plus $y \sin \theta$.

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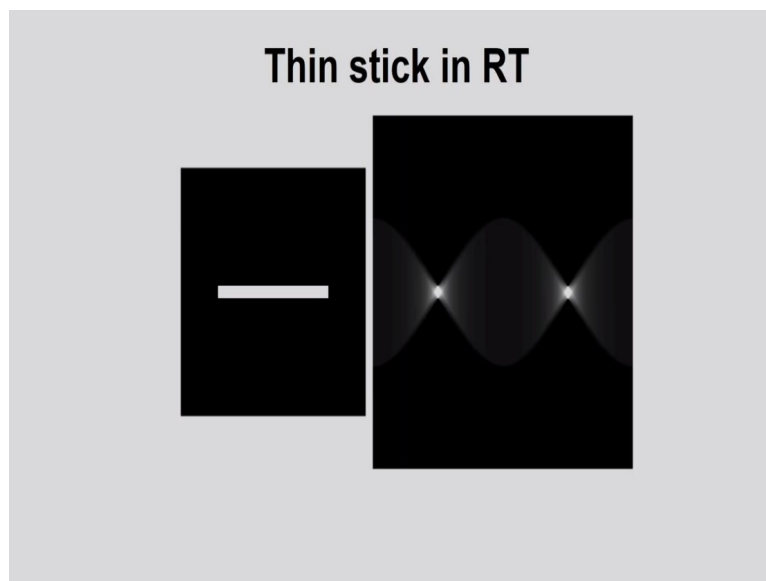
So here I want to show some examples of the Radon transform.

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The first one is the Radon transform corresponding to this image that is the full circle in Radon transform and corresponding to this I have shown the Sinogram.

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In the second example also I am considering the thin stick bar and corresponding to this I can find the Radon transform in this equation. So, you can write compute program for the Radon transform. So, up till now I discussed the concept of the Radon transform. So how to determine the Radon transform that concept I have discussed that means I am just determining the projection along a particular line.

I have shown the equation of the line two line, I have shown one is $x \cos \theta + y \sin \theta$ is equal to 0 and what is the equation of the second line? The equation of the second line is $x \cos \theta + y \sin \theta$ is equal to s . So, just I am defining the projection along a particular line. So first I have define the Radon transform and after this I have considered some equations to find the Ray Sum equation.

So, Ray Sum equation also I have explained. So, that means this concept I am explaining that is how to get the projection of an object. The object is $f(x, y)$ and corresponding to this $f(x, y)$ the projection is represented by $g(s, \theta)$. So, in my next class I will discuss about the image reconstruction that means how to reconstruct the image $f(x, y)$ from $g(s, \theta)$, $g(s, \theta)$ is the projection.

So, I can apply some techniques like the back projection technique or another technique is the Fourier transform technique. There are some other techniques also mainly I will discuss these two techniques, one is the back projection technique and another is the Fourier transform technique. So, how to reconstruct $f(x, y)$ from $g(s, \theta)$; that is $f(x, y)$ the object, so I can

determine the projection along these lines, along the line 1, along the line 2 for a particular angle.

So, like this if I change the angles then in this case I will be getting a number of projections. The theta angle I can change and corresponding to each and every theta I will be getting one projection. Now, the next class I will be discussing how to reconstruct $f(x, y)$ from $g(s, \theta)$. So, let me stop here today. Thank you.