Computer Vision and Image Processing – Fundamentals and Applications
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Lecture – 07
Image Formation - Geometric Camera Model - III

Welcome to NPTEL MOOCS course on Computer Vision and Image Processing, Fundamentals and Applications. I have been discussing about camera projection technique. The first technique is the perspective projection. In perspective projection, distant objects appear smaller and after this suppose if the relative depth of a group of points on a particular object are much smaller than the distance to the COP.

Then in this case I will be getting the weak perspective projection. So, weak perspective projection approximate perspective projection and finally I have another projection technique that is orthographic projection that is nothing, but parallel projection. So, x, y, z coordinate is mapped into x, y coordinate in the image plane that is the orthographic projection. Today, I am going to discuss about this projections and finally I will discuss the concept of camera calibration.

So, camera has intrinsic and the extrinsic parameters. So, I have to estimate these parameters for camera calibration. So, what are the intrinsic parameters and what are the extrinsic parameters that concept I am going to discuss today and based on this I can do camera calibration.

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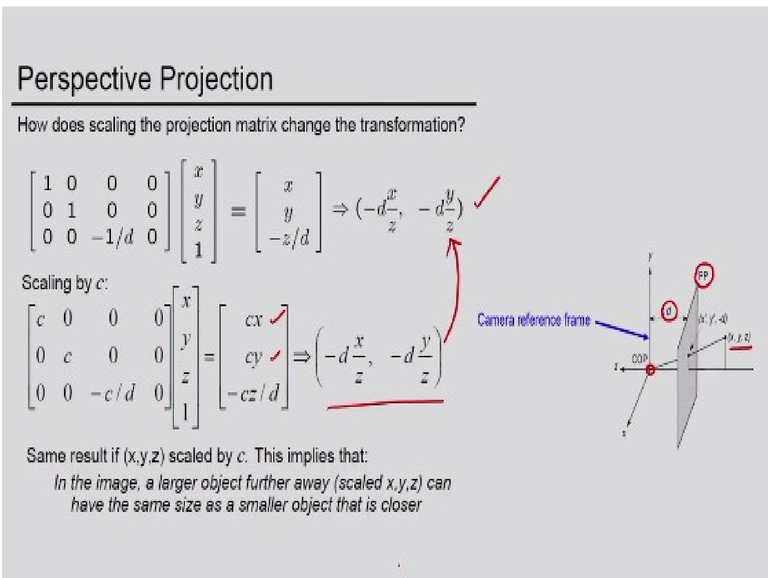

So, let us first see the concept of the projections that already I have discussed in my last class the perspective projections, weak perspective projections and the orthographic projection. So, first one is you can see the perspective projection and the main concept is the distant object appear smaller in perspective projection. So, in this case I have shown in the figure, I shown the COP that is the center of projection and PP means the plane of projection that is nothing, but the image plane.

So, I have shown the PP that is the plane of projection and also the object is x, y, z that object I have shown and this is the camera reference frame the x, y, z I have shown that is the camera reference frame and d is the distance between the COP and the PP so that is nothing, but the focal length. So d is the distance between the COP and the PP the plane of projection and in this equations the first transformation I have shown that is corresponding to the perspective projection.

And in the second case what I am doing the scaling of the x coordinate, scaling of the y coordinate and also the scaling of the z coordinate I am doing the scaling and finally after doing the scaling I am getting this result and this is the same as that of this that is the perspective projection that concept I have already discussed in my last class. So, what is the interpretation of this mathematics?

So that means a larger object further away that is further away means the scale of x coordinate, y coordinate and the z coordinate. So x coordinate is scaled, y coordinate is scaled and the z coordinate is scaled can have the same size as a smaller object that is closer. The meaning is the distant object appear smaller in the perspective projection.

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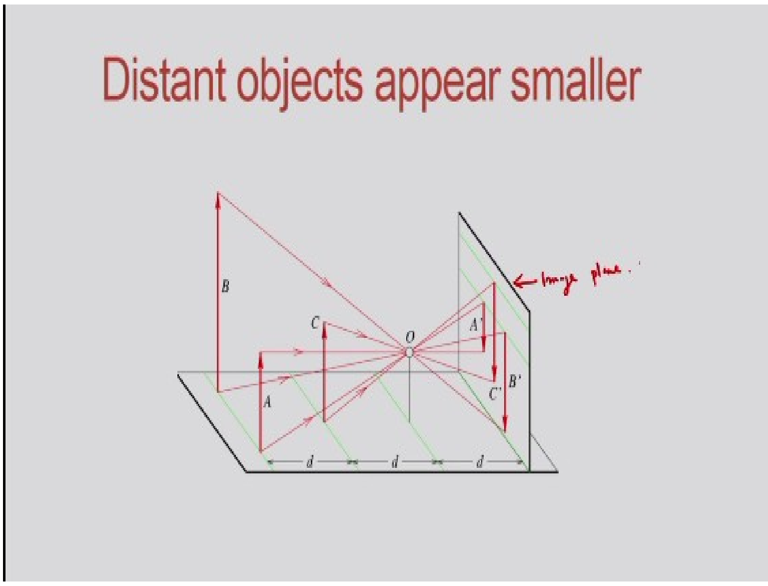

So, in this figure I have shown this concept that is the distant object appears smaller. You can see in this figure. So, I have shown the image plane so you can see this is the image plane and I have shown the projection of the objects one is A another one is B another one is C like this I have shown and you can see the projections in the image plane.

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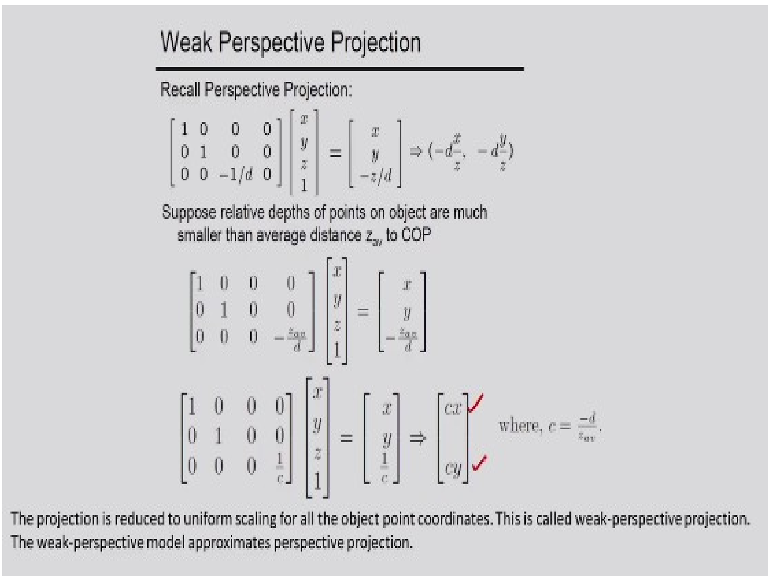

Already this concept I have discussed that is the weak perspective projection. So weak perspective projections the concept is if the relative depth of points on a particular object are much smaller than the average distance to COP the center of projections then I will be getting the weak perspective projections and actually the weak perspective projections approximates perspective projection. So, in weak perspective projection it is nothing, but the scaling of the x coordinate and scaling of the y coordinate that is the weak perspective projection.

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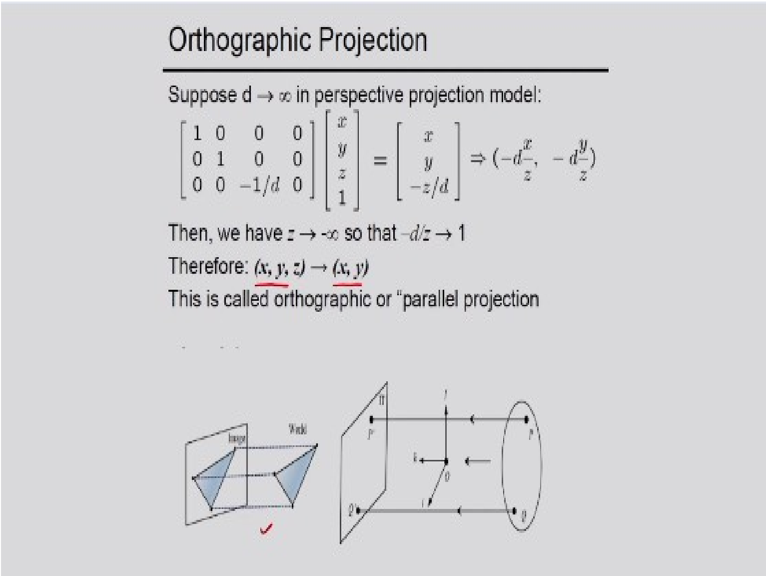

And finally another projection technique is the orthographic projection. So, it is nothing but the parallel projection orthographic projection. The x, y, z point that is mapped into x, y point in the image plane that is the parallel projection. So, you can see in this figure I have shown the world coordinates and corresponding to this I have shown the image coordinates and this is nothing, but the mapping of x, y, z coordinate into x, y coordinate in the image plane.

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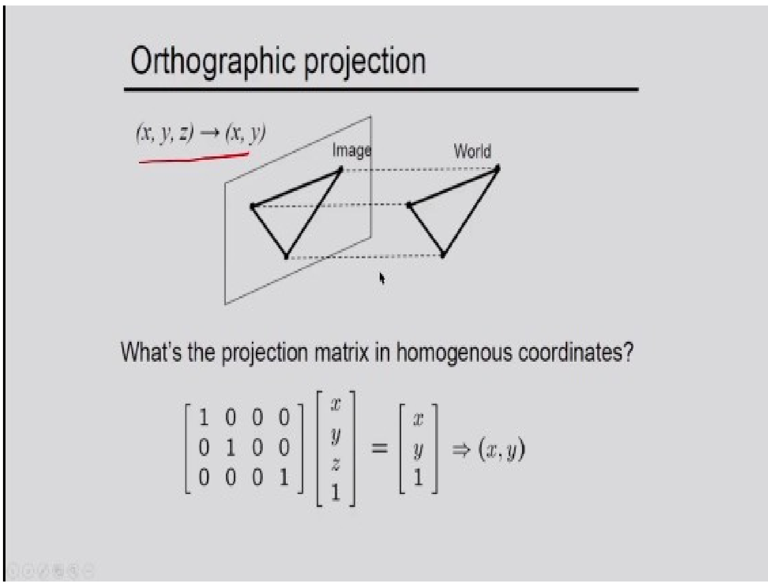

And here also I have shown the orthographic projection and in this case it is nothing but the mapping from x, y, z into x, y coordinate that is the concept of the parallel projection the orthographic projection.

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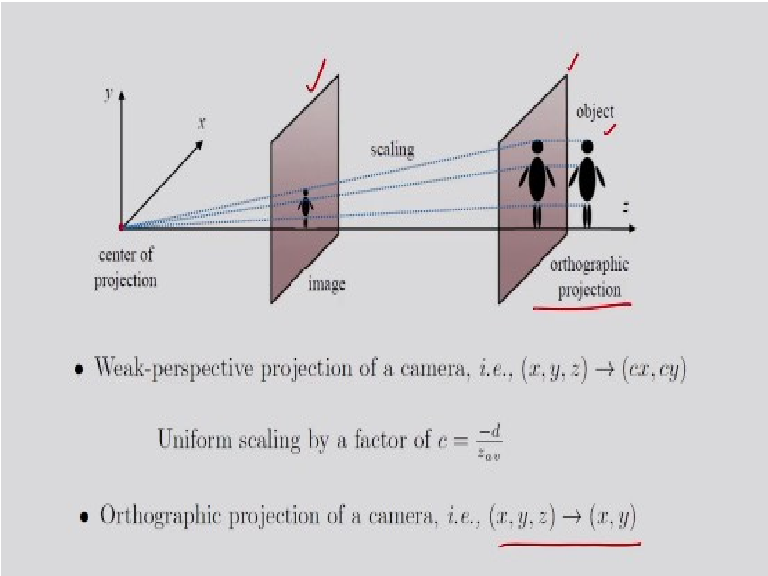

And in this figure I want to show the distinction between the orthographic projection and the weak perspective projection. So, in the first case you can see this is the object and corresponding to this image plane the image plane is far away from the center of projection and corresponding to this I have the orthographic projection so it is nothing, but the x, y, z point is mapped into x, y coordinate in the image plane. T

he first one is the orthographic projection I have shown and in case of the perspective projection that is nothing, but the scaling of the x coordinate and the scaling of the y coordinate that I have shown in the image plane and I have shown the weak perspective projection and that is nothing, but the scaling of the x coordinate and the scaling of the y coordinate.

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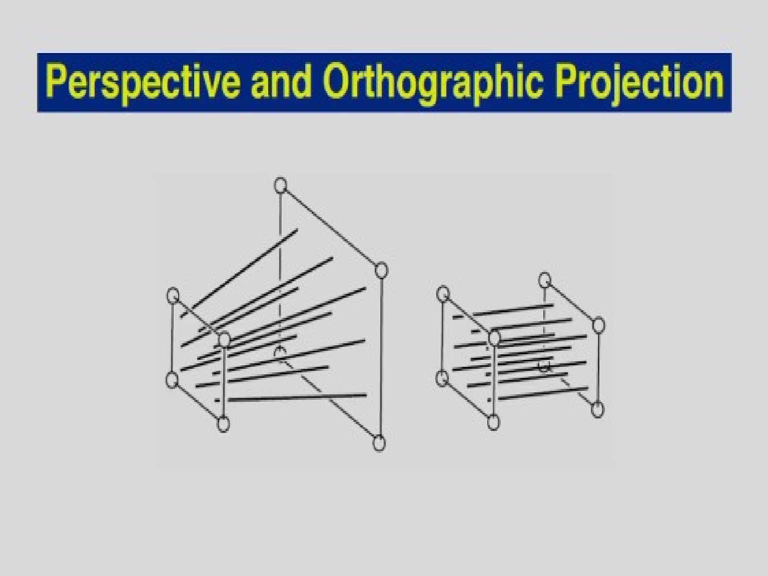

And in this figure I have shown the perspective projection another one is the orthographic projections.

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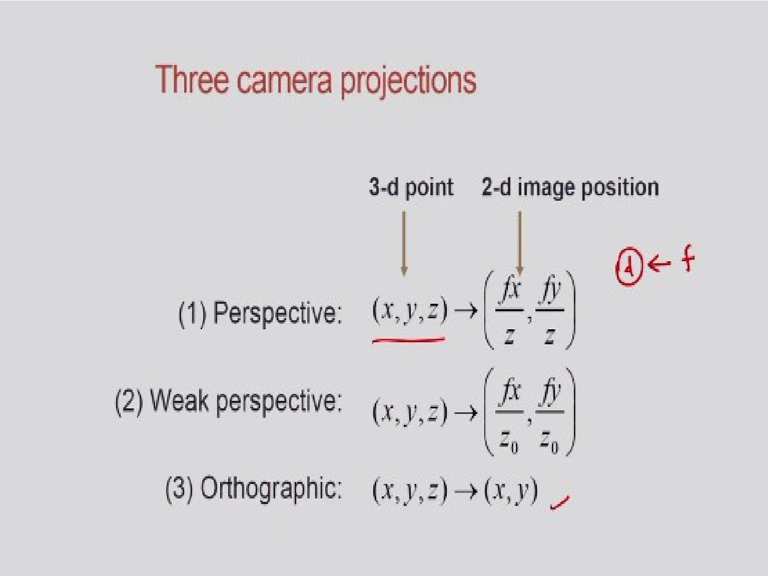

And in summary what I can show in perspective projection the 3 d point is x, y, z and corresponding to this the 2 d image position will be fx divided by z so f is the focal length and fy divided by z. In my figure, I put d in place of f, but now I am considering f in place of d. So, in place of d I am considering f that is the focal length in this equation. So corresponding to the perspective projection it is nothing, but fx divided by z, fy divided by z.

In case of the weak perspective projection it is nothing but the scaling of the x coordinate and the scaling of the y coordinate and in case of the orthographic projection the point x, y, z is mapped into x, y point in the image plane. So, in summary I can show like this one is the perspective projection, one is the weak perspective projection, one is the orthographic projection.

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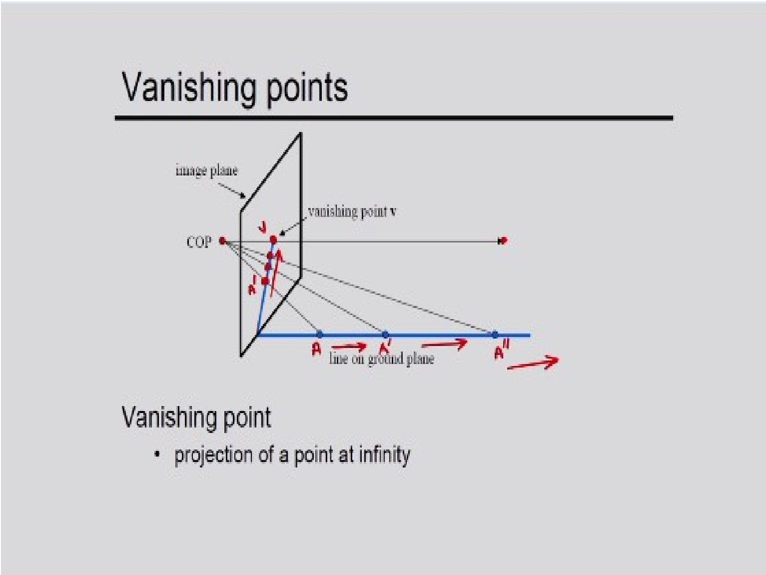

Now there is a concept of the vanishing point. So what is the definition of the vanishing point? So what is the definition of the vanishing point? You can see here I have shown the image plane in the figure and also I have shown the COP is the center of projection. Suppose, if I consider one point at the infinity suppose one point at the infinity corresponding to this if I see the projection of this point in the image plane.

Then I will be getting this point and that point is the point is v that is the vanishing point. Now, let us consider another point the point is suppose A so corresponding to the point A so what is the projection in the image plane? In the image plane the projection is A dash that is this point. Suppose, the point A is moved in this direction and suppose this point is A dash and corresponding to this A dash I have the projection in the image plane.

So corresponding point is this that is the projection of the point A dash and like this if I move the points in this direction and suppose this point is A double dash. So, corresponding to this my projection point in the image plane will be this so this is the projection point. So that means if I move the point A to infinity in this direction then what will happen this projection point moves closer to the vanishing point the v is the vanishing point.

So, ultimately corresponding to the point suppose the point A moves to infinity then corresponding to this corresponding to this my projection point will be the vanishing point that means the projection of a point at infinity is nothing, but the vanishing point. So, corresponding to two parallel lines I have the same vanishing point so that I can show in my next figure.

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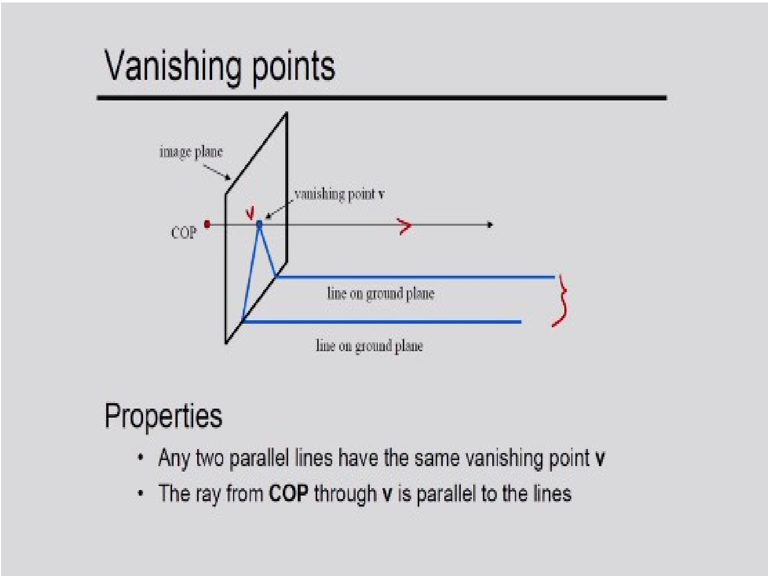

So in the next figure I have shown the vanishing point is v and I have shown the COP, COP is the center of projection and I have shown the image plane and you can see the two parallel lines have the same vanishing point v. So this is my vanishing point that vanishing point is v and I have these two parallel lines.

I have the same vanishing point v and the ray from the COP through v is parallel to this lines. So, suppose if I consider one ray that is this ray is this ray from the COP through the vanishing point is parallel to the lines this two lines. So, from this figure you can understand the concept of the vanishing point. So that means the two parallel lines have the same vanishing point.

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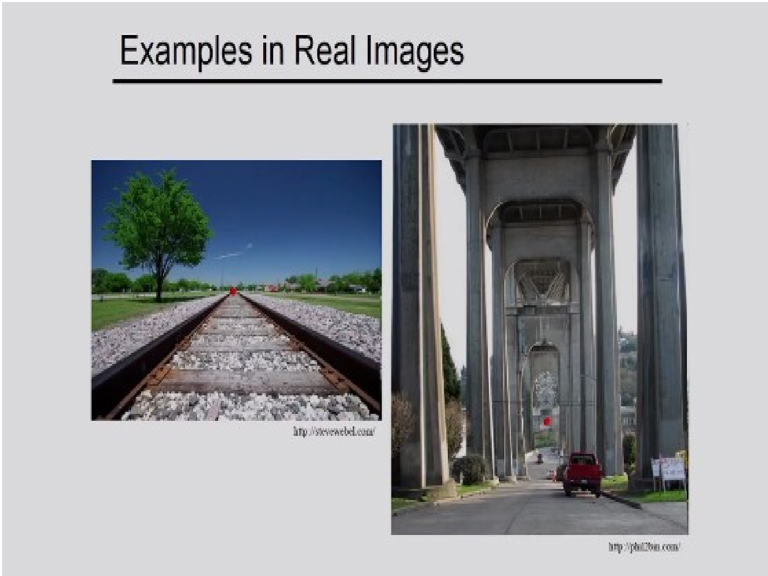

And in this figure I have shown some real images and corresponding vanishing point you can see the first one is the railway tracks I have shown and you can see this is the parallel lines. So, I may have the vanishing point at this point. Similarly, in this figure the second figure I have the parallel plane and corresponding to this I may have the vanishing point somewhere like this.

So, I have the vanishing points corresponding to the parallel lines. So, this vanishing point is quite important one application I can say in robotic path planning robot can determine this vanishing point and based on this vanishing point robot can do robotic path planning it can be done based on the vanishing point.

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And here also I have shown one vanishing point you can see the vanishing point here and corresponding to the parallel lines.

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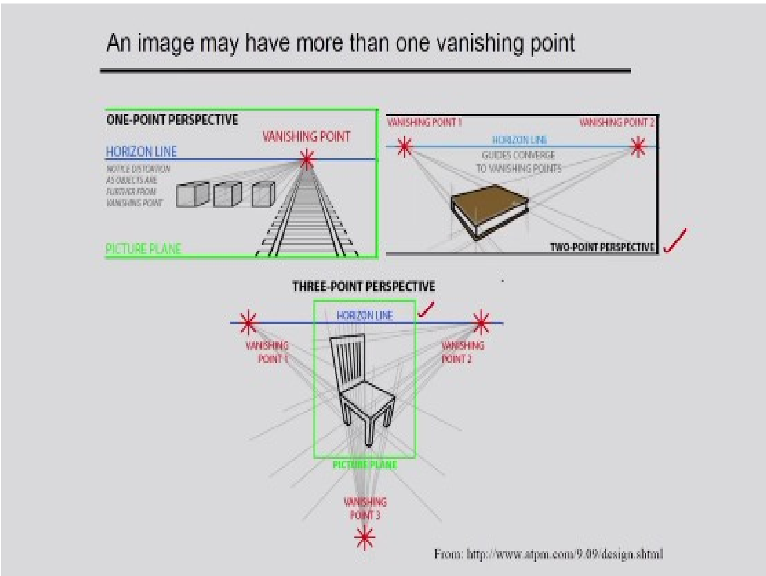

And in this case I have shown one point perspective, two point perspective and the three point perspective and in this case I have shown one vanishing point corresponding to one point perspective. The next one is two vanishing points corresponding to two point perspective and third one is three vanishing points corresponding to three point perspective and you can see the horizon line you can see. So, I will define what is the horizon line in the next slide.

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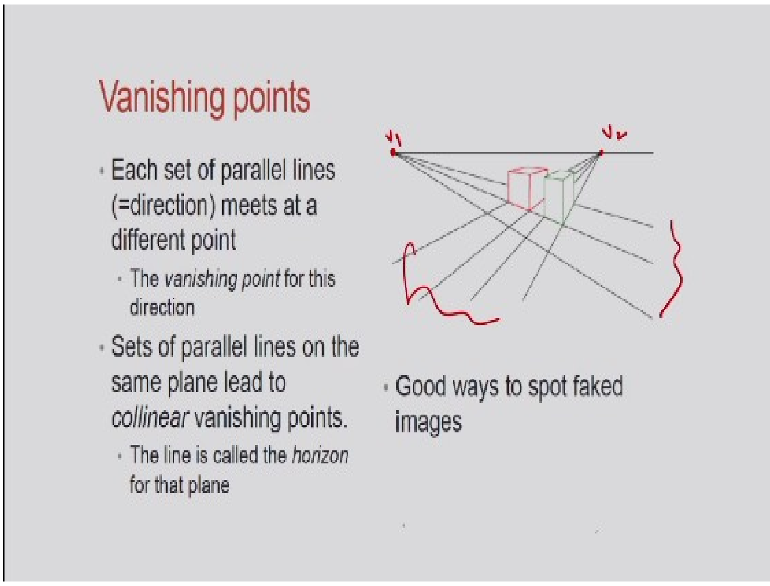

So, here you can see the each set of parallel lines meets at different point that is the vanishing point. So, if I consider suppose this is the parallel lines all these are parallel lines they will meet at a vanishing point the vanishing point is suppose v1. Similarly, if I consider another set of parallel lines suppose this set of parallel lines they will meet at another vanishing point. The vanishing point is suppose v2 and this parallel lines are on the same plane.

So, this all the parallel lines are on the same plane and in this case I will be getting collinear vanishing points that means the point v1 and v2 are collinear. So I am getting two vanishing points in this case v1 and v2 then in this case I will be getting the collinear vanishing points and the line is called the horizon for that plane. So that line joining v1 and v2 it is called the horizon line that is the definition of the horizon line.

In this case, you can see I have two vanishing points and one important application of the vanishing point that determination of the fake images. So, suppose if I consider copy and paste forgery. So, suppose one portion of the image is cut and pasted it on another image then based on the vanishing points I can determine whether that image is the forge of the original image.

The forging I am considering the copy and paste forgery. So I can determine the vanishing points and based on the vanishing points I can say whether it is the original image or the forged image that I can determine.

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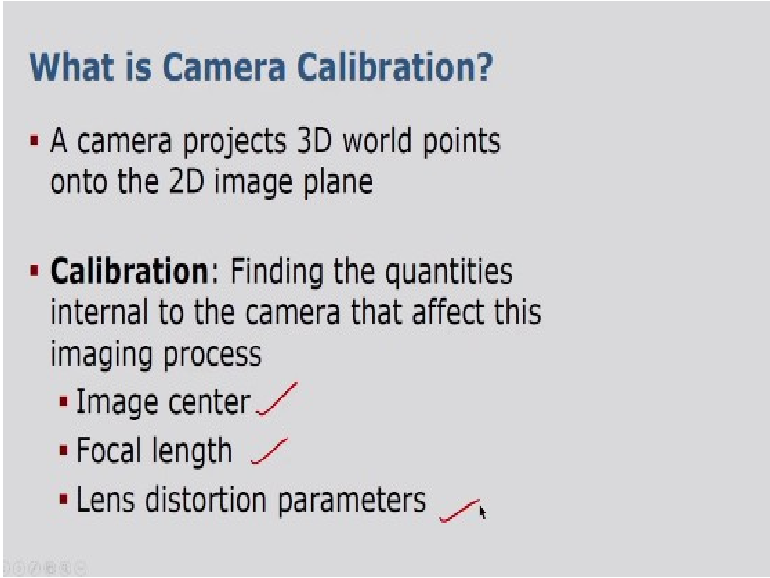

Now what is camera calibration? So in case of the camera projections it is nothing, but the 3D to 2D projections. 3D world points are projected on to the 2D image plane. So, camera calibration is the process of estimating intrinsic or extrinsic parameter. So what do you mean by intrinsic parameters? The intrinsic parameters deal with the cameras internal characteristics such as focal length of the camera, distortion parameters of the camera and the image centers and what are the extrinsic parameters?

Extrinsic parameters describes each positions that is the camera positions and the orientation in the world. So that means I can estimate the intrinsic parameters like the focal length, distortion parameters, image centers I can estimate and also I can estimate the extrinsic parameters that is nothing, but the position and the orientation of the camera with respect to the world coordinate.

So, in this case you can see finding the quantities internal to the camera that affects the image process. So like this image center I can consider the focal length I can consider, the lens distortion parameters I can consider these are the internal parameters.

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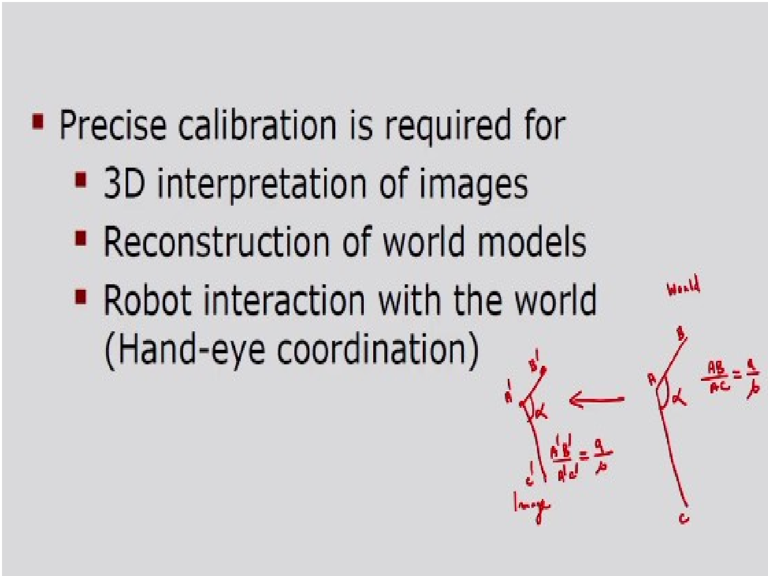

Now this precise calibration is required for 3D interpretation of images, reconstruction of world models and robot interactions with the world. So, I can give one example in the real world I have two lines suppose this segment is A, B and C this is the world coordinate in the world coordinate and this ratio AB divided by AC I have some ratio it is a divided by b and suppose this angle is alpha.

So, corresponding to this I will be getting one image in the image plane so I will be getting one image plane and in this case this I am considering the ratio a by b and also the angle alpha I am considering. So, I have the image in the image plane corresponding to this scene in the world. So this characteristics should be preserved in the image what are the characteristics I am considering the ratio, ratio is a by b and also angle is alpha.

So in the image also this angle should be alpha and the ratio should be maintained the ratio between this point and this point that is A dash B dash and it is C dash. So that ratio should be maintained A dash B dash divided by A dash C dash that should be also maintained in the image. So this characteristics should be preserved in the image. So, in case of the robot interactions robot take image so robot captures images.

And what information available in the image that should be perfectly match with the real world scene so that is important for hand eye coordination in case of the robot interactions. So, I am repeating this because the robot takes images by using the camera and what information available in the image that should be perfectly matched with the real world scene that is the case. So, for all this cases I need camera calibration.

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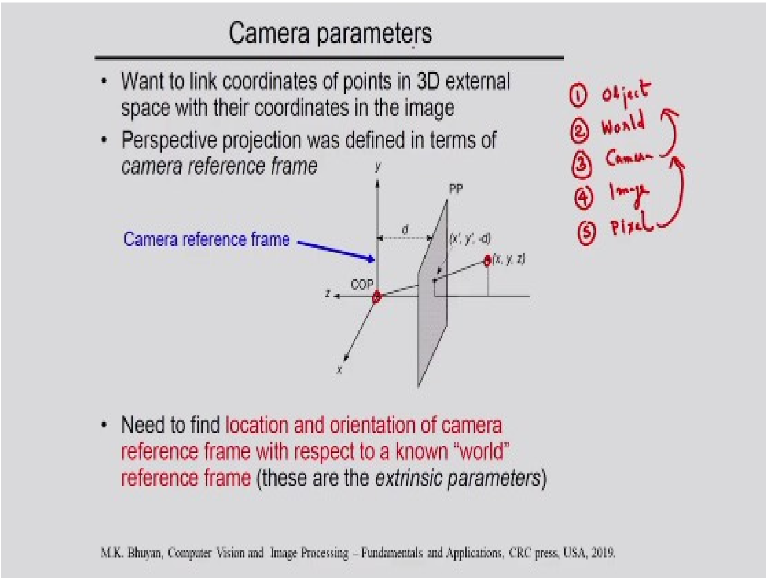

So the parameters of the camera already I have mentioned the extrinsic parameters and the intrinsic parameters of the camera and in this figure you can see that I have shown the camera reference frame, I have shown the COP, COP is the center of projections and PP is the plane of projections I have shown and our object is nothing, but the x, y, z is the object. So, what is the extrinsic parameters?

Extrinsic parameters define the location and the orientation of the camera reference frame with respect to the known world reference frame. I am repeating this so extrinsic parameters define the location and orientation of the camera reference frame with respect to a known world reference frame that is the extrinsic parameters and what about the intrinsic parameters?

The intrinsic parameters link the pixel coordinates of an image point with the corresponding coordinates in the camera reference frame and in this case I can show this case so what is the meaning of the intrinsic and the extrinsic parameters? So, I have the coordinate one is the object coordinate system the number one is the object coordinate system and again I have the another coordinate system that is the world coordinate system.

And because I am the camera so I have the camera reference frame the camera reference coordinate system that is the camera reference frame. Number four, I have the image plane that is the image reference frame and after this I have the pixels coordinates, the pixel reference frame. So I have this reference frames one is the object reference frame, one is the world reference frame another one is the camera reference frame, one is the image reference frame and another one is the pixel coordinate.

So extrinsic parameters means that is the location and the orientation of the camera reference frame with respect to the known world reference frame that is the extrinsic parameters and the intrinsic parameters link the pixel coordinate of an image point with a corresponding coordinate in the camera reference frame. So I have the pixel coordinates and I can find the correspondence between the pixel coordinate and the camera reference frame.

So this is about the extrinsic parameters and the intrinsic parameters. In my next slide you can understand this concept.

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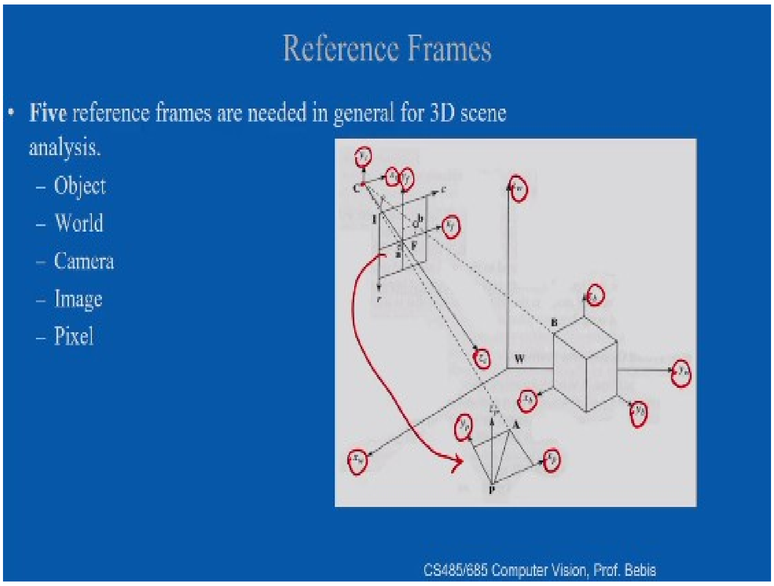

So, first one is I have the object reference frame. So here I have shown the object reference frame so the coordinates are xb, yb and zb that is the object reference frame I have shown and also I have shown the world reference frame that here I have shown the coordinate is xw, yw and zw that is the world coordinate reference frame and I have the camera reference frame so you can see the COP the center of projection here.

So, I have the camera reference frame xc, yc and the zc that is the camera reference frame and if I consider the image frame image is nothing, but the 2D. So, in the image frame the image reference frame is xf and one is yf. If I consider the object frame, world frame and the camera frame they are 3D, but if I consider the image reference frame that is nothing but the 2D so I have x coordinate and the y coordinate.

And after this I have the pixel coordinates so corresponding to this image plane I have the pixel so this corresponds to the pixel coordinates so you can see xp and the yp the pixel coordinates. So, now I will show all this one by one. One is the object reference frame, one is the world reference frame, one is the camera reference frame. One is the image reference frame, one is the pixel reference frame.

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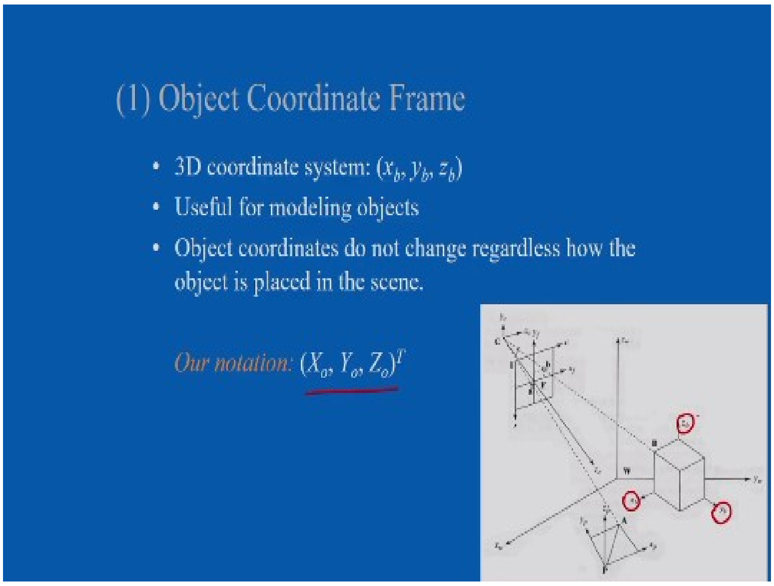

The first one is the object coordinate frame. So I am considering the notation X0 Y0 Z0 and it is useful for modeling objects. Son in this figure I have shown the object coordinate frame that already I have shown that is xb, yb and the zb.

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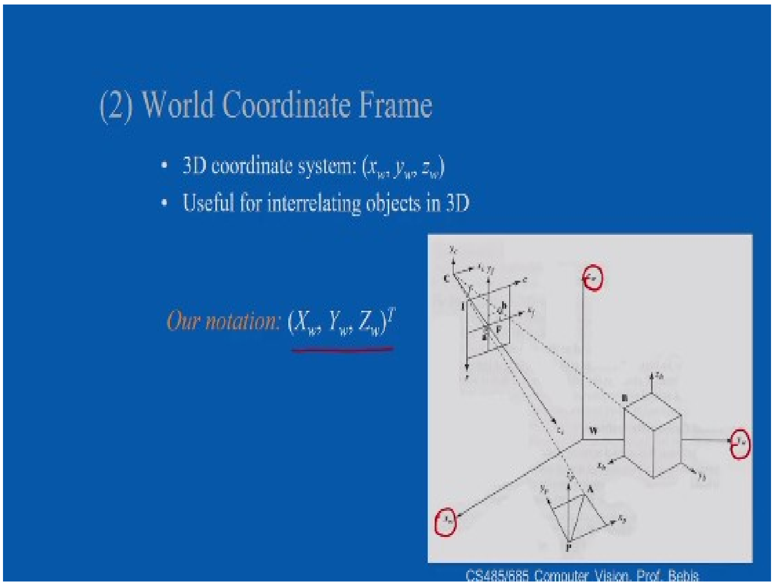

The next one is the world coordinate frame. So my notation is Xw, Yw and Zw. So, in the figure you can see one is Xw, one is Yw and one is Zw that is the world coordinate frame and it is useful for interrelating objects in 3D that is the world coordinate frame.

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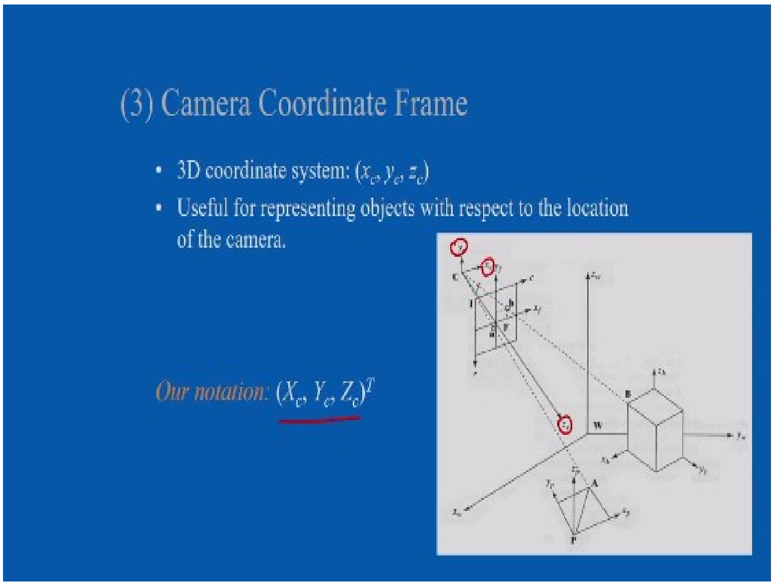

The next one is the camera coordinate frame. So the 3D coordinate system for the camera and my notations are Xc, Yc and Zc for the camera coordinate frame. So, here in the figure you can see one is xc, yc and the zc for the camera coordinate systems and it is useful for representing objects with respect to the location of the camera.

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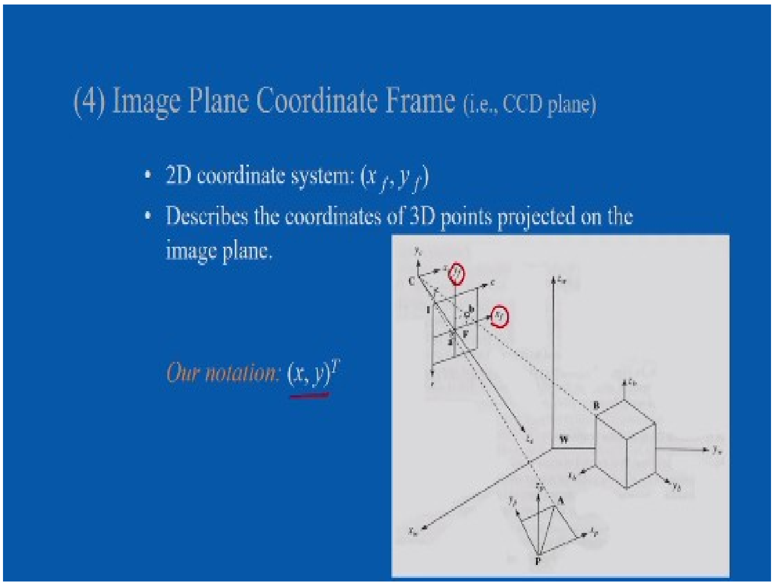

The next one is the image plane coordinate because in case of the image plane I have the CCD sensors the charge coupled devices are available and we have the 2D coordinate system and in this case you can see my x coordinate is xf and y coordinate is yf and my notation is x, y corresponding to the image plane coordinate frame. So, it describes the coordinate of the 3D points projected on the image plane.

So that is the coordinate of the image plane coordinate that is the coordinate of the image plane.

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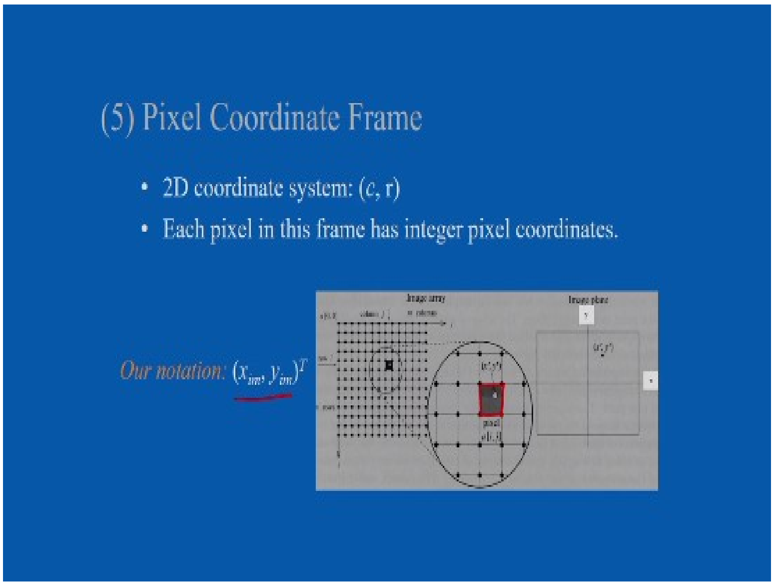

And finally I have the pixel coordinates. So you can see the pixels and you can see the size of the pixels also so if you see this is a pixel, this is the size of the pixels and corresponding to this pixel my notation is x im and y im that is the pixel coordinate. So, this is the pixel coordinate I have the x im and the y im in the pixel coordinate frame. So that means I have five frames.

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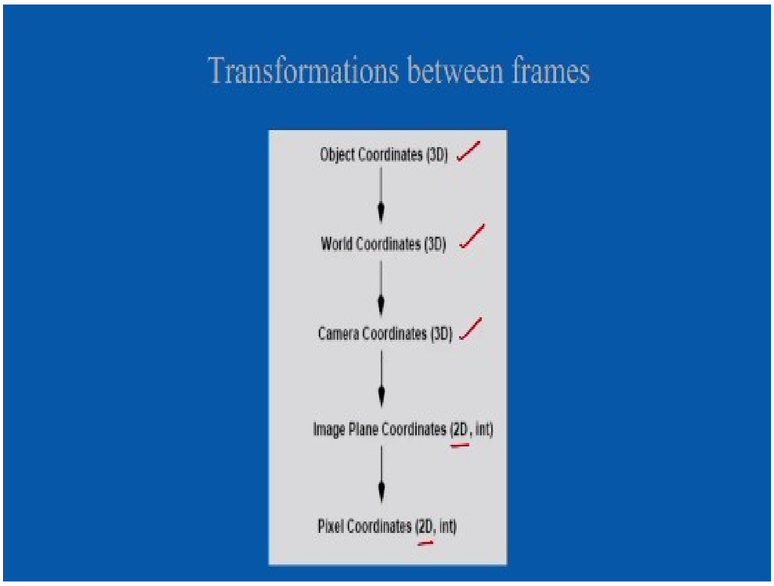

So, in this case in summary I can show you the first one is object coordinates that is the 3D coordinate systems the next one is the world coordinates that is also the 3D after this we have the camera coordinates this is also the 3D coordinate system and these are mainly the object coordinates, world coordinates and the camera coordinates.

These are the external parameters and after this I have the image plane coordinates that is the 2D and the pixel coordinate that is also 2D these are the internal parameters of the camera. The external parameters mainly considers the world coordinates and the camera coordinates.

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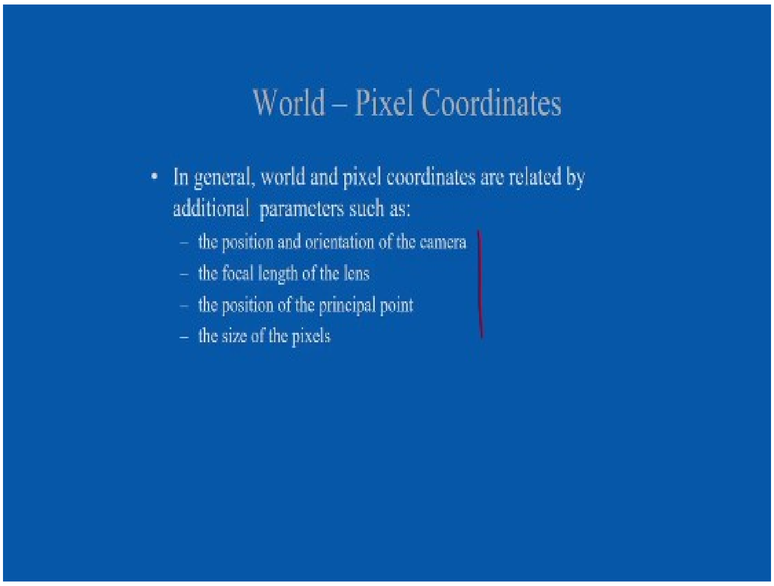

So, the world to pixel coordinates if you see the world and the pixel coordinates are related by some additional parameters. The additional parameters are the position and the orientation of the camera, the focal length of the lens of the camera, the position of the principal points in the image plane and also the size of the pixels.

So, these are the parameters so that is the relationship between world and the pixel coordinates. So, for this I need the position and the orientation of the camera, the focal length of the camera lens. The position of the principal points in the image plane and the size of the pixels.

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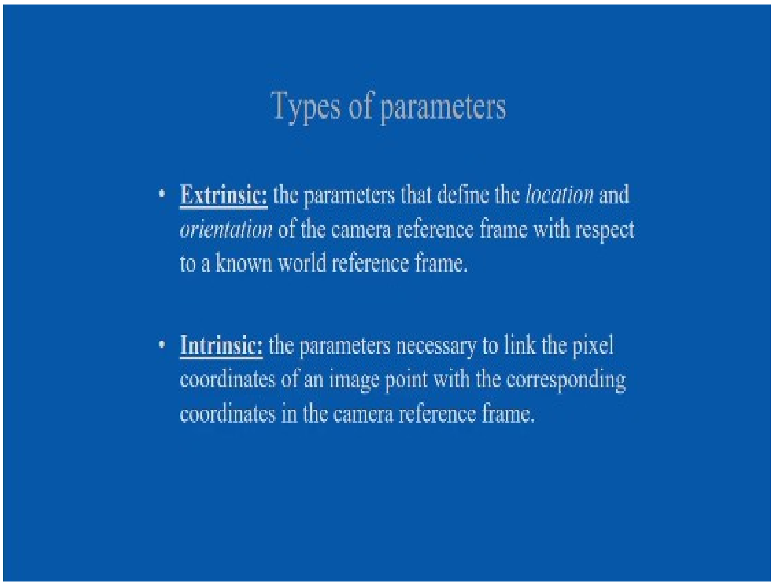

And already I have defined what are the extrinsic parameters and the intrinsic parameters? Extrinsic parameters that define the location and the orientation of the camera reference frame with respect to a known world reference frame that is the definition of the extrinsic parameters and for intrinsic parameters, what is the definition of the intrinsic parameters?

The intrinsic parameters necessary to link the pixel coordinates of an image point with a corresponding coordinates in the camera reference frame. So that is the meaning of the intrinsic parameters.

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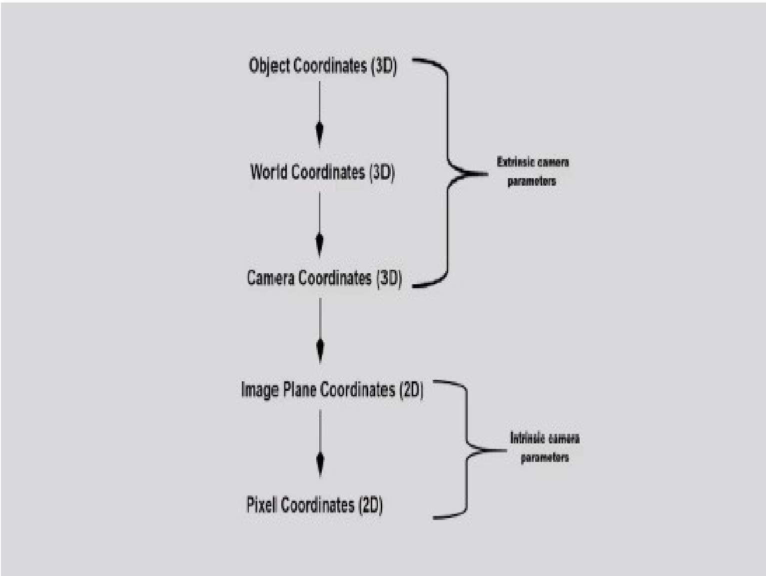

Now, in the next slide you can see I have shown the extrinsic parameters and the intrinsic parameters. So, I have shown the object coordinates, world coordinates, camera coordinates and the image plane coordinates and the pixel coordinates. So, in this figure also you can see the extrinsic parameters and the intrinsic parameters.

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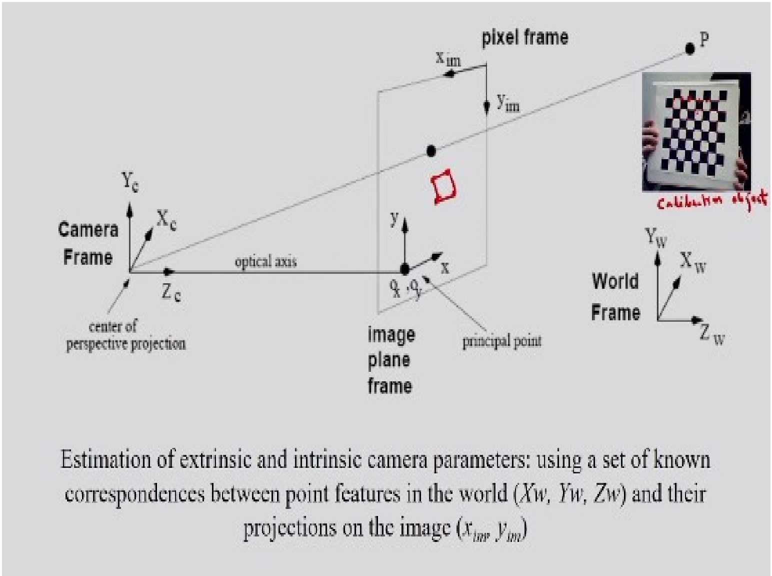

Now in this case in this figure I have shown how to estimate the extrinsic and the intrinsic parameters? So, for this a set of known correspondence between a point features in the world and their projections on the image I have to determine.

So, in this case you can see I am considering one calibration object here this is the calibration object I am considering and in this case I have to estimate the intrinsic and the extrinsic parameters by considering a set of known correspondence between point features that I have shown in the calibration object in the world and their projections are on the image.

So, corresponding to this known feature point if you see this point I will be getting one image in the image plane so I will be getting one image plane and corresponding to this features I can find the correspondence between the world coordinate and the camera coordinates that I can find. So, what is the calibration object?

The parameters of the corners are estimated using an object with known geometry so what is the calibration object? The parameters of the camera are estimated using an object with known geometry. So, in this case I have considered the calibration object and I know the geometry of this object. So from this I can estimate the parameters.

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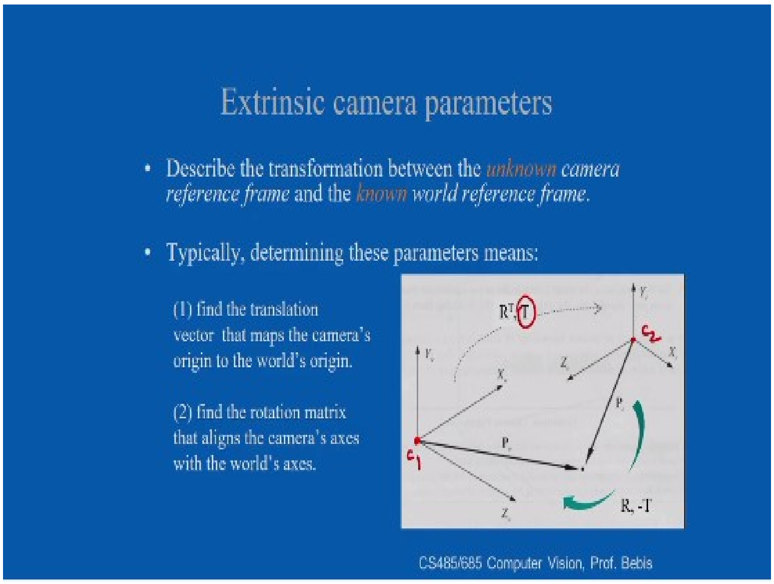

Now in this case first I am considering the concept of the extrinsic parameters. So, for this what I have to do I have to develop some transformation between the unknown camera reference frame and the known world reference frame. So, in this figure I have shown two reference frames one is the world reference frames. So, first one is the world reference frames suppose the center is C1 and another one is the camera reference frame.

And suppose the center is C2. So, if I want to align the camera reference frame with the world reference frame for this what I have to do? First, I have to translate the origin of the camera reference frame to the world reference frame. So that both the origins will be coincided exactly that means first I have to do the translation operation so that the origin of the camera reference frame will be coincided with the origin of the world reference frame.

So, that means I am repeating it again so if I want to align the camera reference frame with the world reference frame I have to translate the origin of the camera reference frame to the world reference frame so that both the origins will be coincided exactly. So, first I have to do the translation after this I have to do rotation of the camera reference frame so that it aligns with the world reference frame.

So that means first I have to do the translation so that I can do the matching that is the C1 and C2 will be matched it will be overlapping match means it is overlapping and after this I have to do the rotation so that it aligns the camera axes with the world's axes. So this operations I have to do. First, I have to do the translation and after this I have to do the rotation. So this transformation I have to do for alignment of this two coordinate reference frame. One is the world reference frame another one is the camera reference frame.

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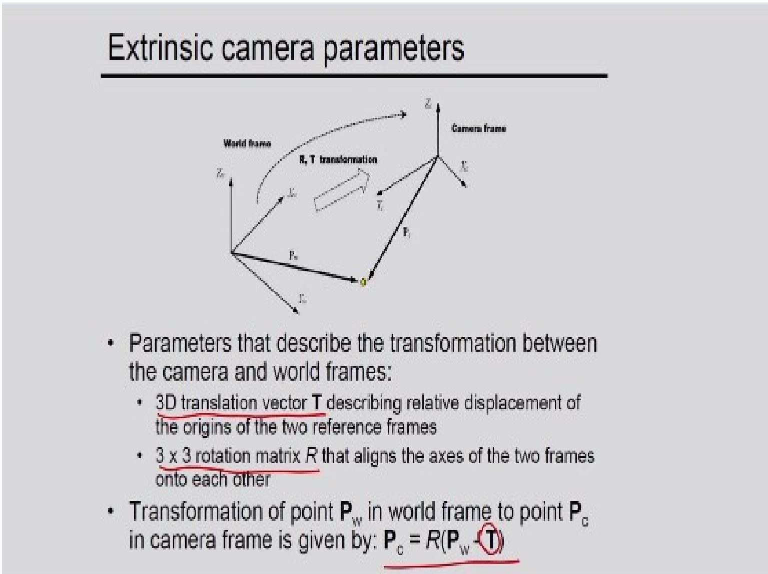

In this figure also I have shown the same concept. So, for alignment of this two coordinate systems first I am doing the translation and after this I am considering rotation. So that means the 3D translation I am considering that is describing relative displacement of the origin of the two reference frames because I have to overlap the origins C1 and C2 so for this I have to do the translation.

And after this for aligning the axes of this two coordinate systems I have to do the rotation. So for this I am considering the 3 by 3 rotation matrix I am considering. So you can see what is the transformation from the camera to world. So Pw is the world point and Pc is the camera point in the camera reference frame. So, first I have to do the translation of the world reference frame and after this I have to do the rotation.

So, first I am doing the translation of the world reference frame and after this I am doing the rotation of this frame world frame so that the world reference frame will be coinciding with the camera reference frame.

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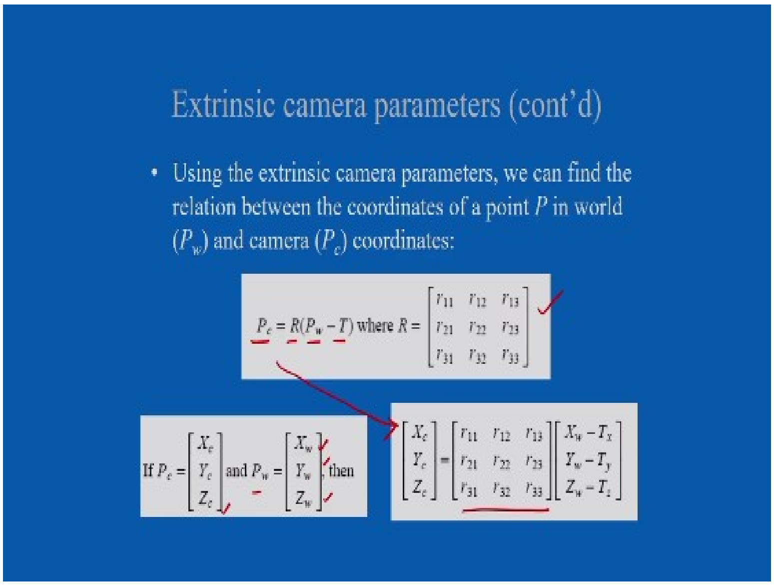

So in this case the same thing I am showing here so Pc is the camera coordinates and this Pw is the world coordinates and what transformation I have to do? I have to do the translation first and after this I have to do the rotation. So, here I have shown the 3 by 3 rotation matrix and I have shown the Pc this is the points of the camera the x coordinate, y coordinate and the z coordinate I have shown corresponding to the camera coordinates.

And corresponding to the world coordinate I have shown the X coordinate, Y coordinate and the Z coordinate. So, from this equation I can write in the matrix form like this so Xc, Yc, Zc are the camera coordinates and I am considering the 3 by 3 rotation matrix you can see this is the rotation matrix. The elements are r11, r12, r13 like this. It is a 3 by 3 matrix and Xw, Yw, Zw these are the world coordinates. And I am doing the translation along the x direction, translation along the y direction, translation along the z directions.

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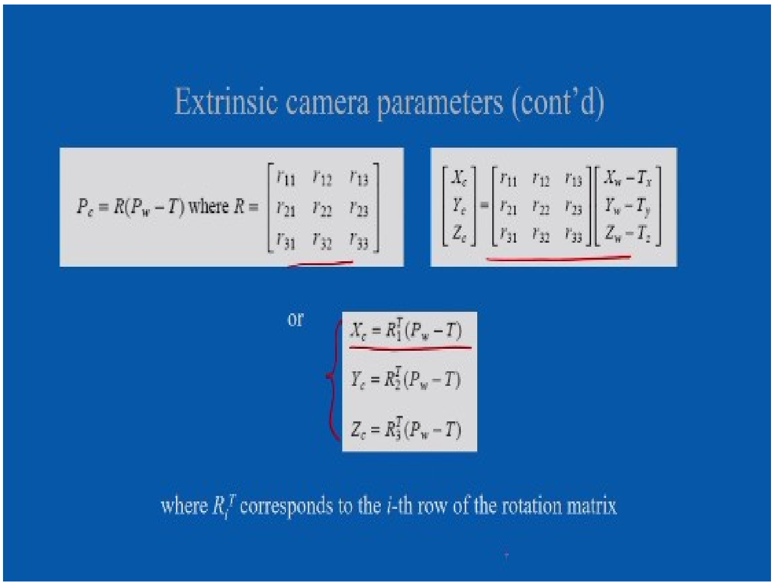

Same thing I am showing that Pc is equal to R Pw minus T and I have the rotation matrix and already I have defined this one. So what is Xc what is the X coordinate of the camera this is nothing but R1 T, T is the transpose Pw minus T that is the X coordinate. What about the Y coordinate is also R2 T Pw minus T that is Y coordinate and similarly I can get the Z coordinate so I can get the X coordinate, Y coordinate and the Z coordinate. This Ri T corresponds to the i-th row of the rotation matrix that I am considering.

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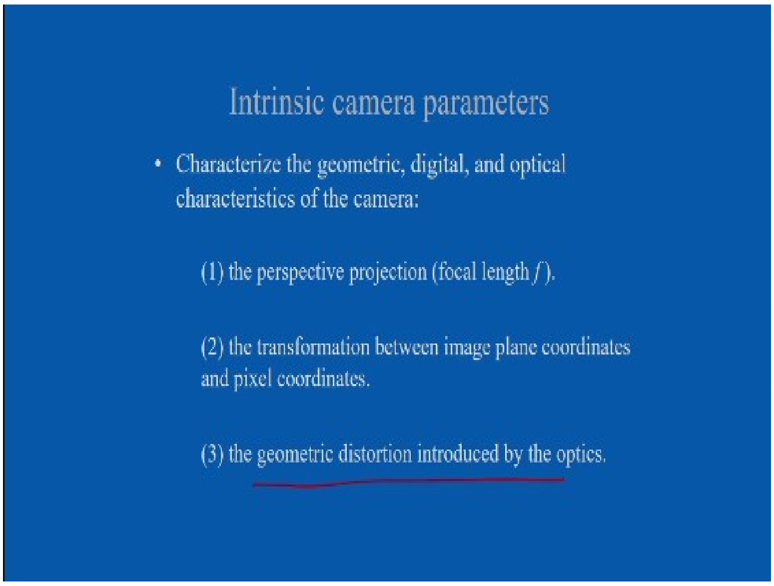

Now, in case of the intrinsic parameters I have to consider the focal length of the camera and also I have to do the transformation between the image plane coordinates and the pixel coordinates and also I have to consider geometric distortions introduced by the optics that I have to consider.

So, these are the internal parameters of the camera one is the perspective projections that I need the focal length for perspective projection equation I need the information of the focal length and also the transformation between image plane coordinates and the pixel coordinates and also I need to consider the geometric distortion introduced by the lens that I need to consider.

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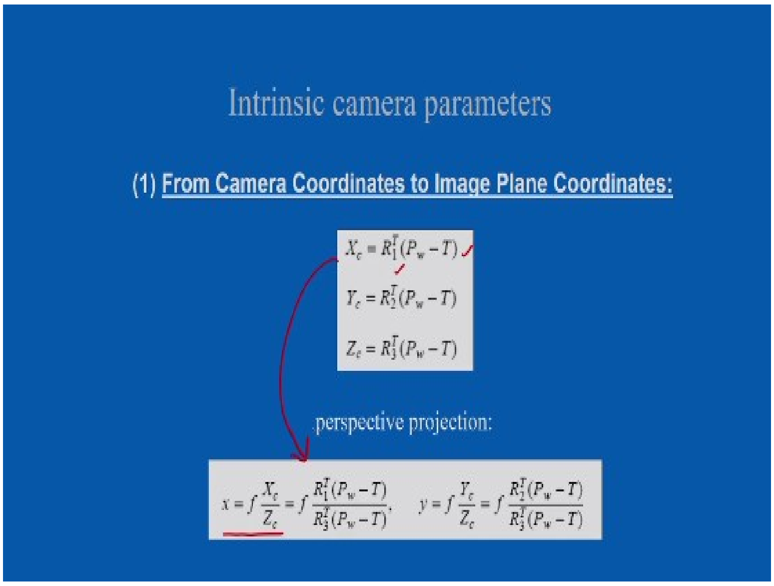

So, for camera coordinates to the image plane coordinates what transformation I have to do already I have shown here. So, again I have to do the translation and also I have to do the rotation so in this case already I have defined the camera coordinates Xc, Yc, Zc and the world coordinates are I have defined that is Pw I have considered and I am doing the translation and the rotation.

And if I consider the perspective projections so if I put the perspective projection equation this is the perspective projection equations x is equal to f Xc divided by Zc so corresponding to this if I put this equations here into this equation I will be getting this one. So, just putting the value of Xc and Zc I am putting this in the perspective projection equations.

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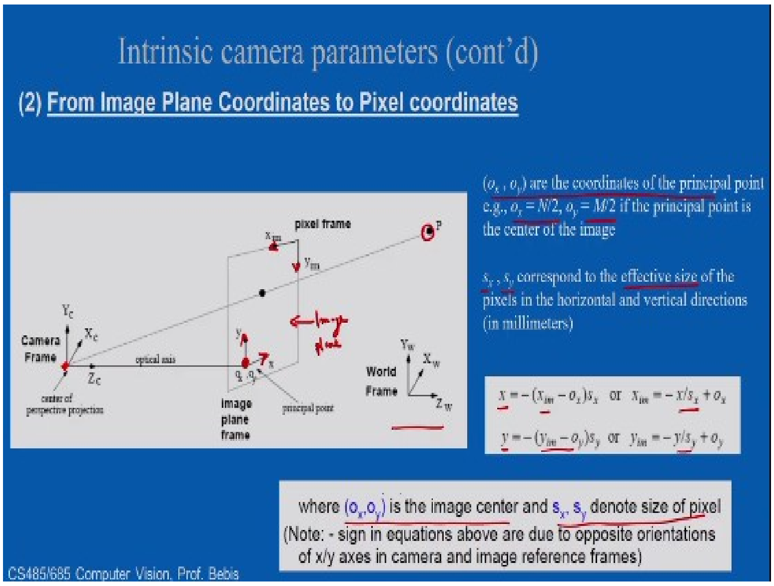

And from image plane coordinates to pixel coordinates I need to consider and in this case I am considering the ox, oy that is the coordinates of the principal point. So, principal point in the image plane that I can consider as the center of the image. So, if the size of the image is M cross N or by M cross M the ox will be N divided by 2 and oy will be M divided by 2. The size of the image is N cross M that is the principal point in the center of the image.

And also I need to consider the effective size of the pixels so that I need to consider. So, in this case I am considering sx and sy I am considering that corresponds to the effective size of the pixels in the horizontal in the vertical directions that is in millimeters. So, that means in this figure you can see I am considering the scene point the point is P in the world and I have shown the image plane.

So, this is the image plane and also I have shown the pixels if you have seen the pixels so I have shown the pixels and principal point I am considering ox, oy and I have shown the camera projection centers. So this is the camera reference frame I have considered. So, considering to this you can see I have the coordinates x and y coordinates I will be getting and in this case it is nothing.

But x im minus ox because I have to do the translation corresponding to the principal point in the image point and similarly for the y also I am doing and also I have to do the scaling you can see here I am doing the scaling of the x coordinate and the y coordinates because I am considering the effective size of the pixels I am considering. So, in this case ox, oy is the image center I am considering and sx and sy corresponds to the size of the pixels.

And in this case I am considering the negative sign in the equations are due to the opposite orientation of the xy axes in the camera and the image reference frame. So, in the figure you can see the opposite orientations of this two coordinate system here you see in the image the direction of x coordinate is this, y coordinate is this, but in the pixel coordinate what I am showing x direction is this and y direction is this that is opposite.

So, for this I am considering the negative sign. So that is the intrinsic parameters I am considering and from image plane coordinate to the pixel coordinate.

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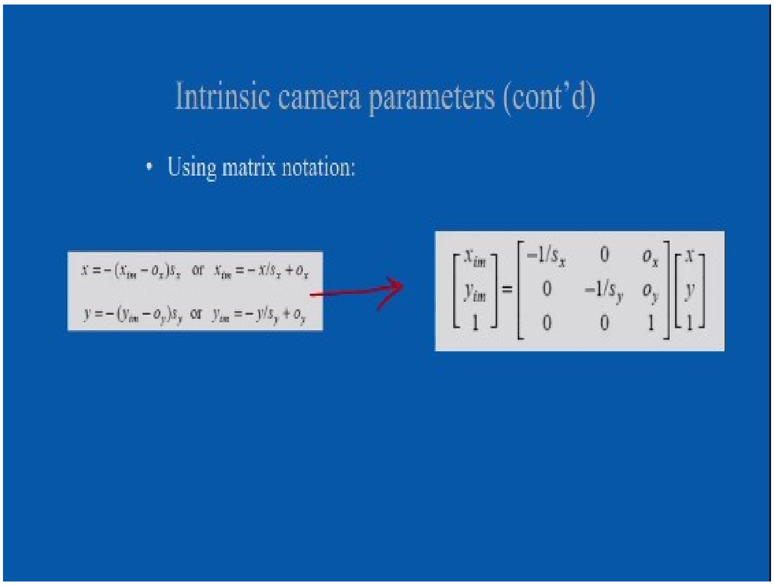

So by using this matrix notation because already we have defined this equations and in matrix form I can write like this that is x im y im 1 I can write like this and in this case sx and sy I am considering because of the size of the pixel that means I am doing the scaling.

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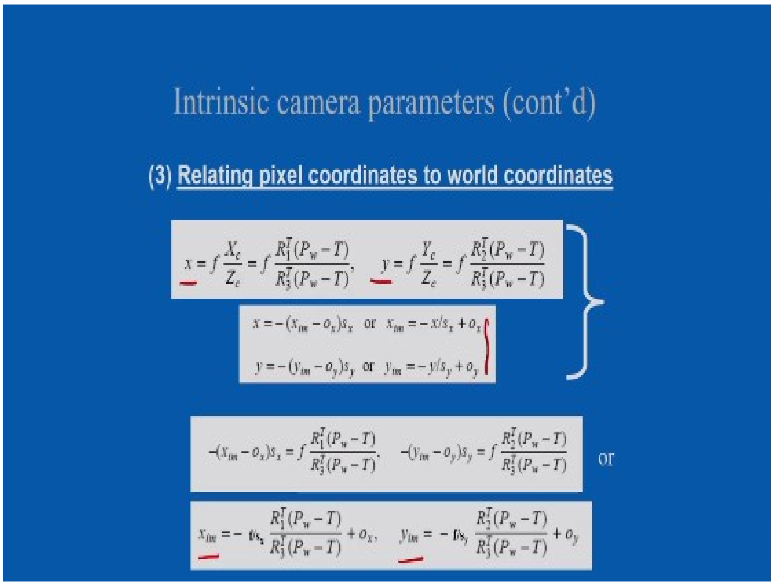

And finally the relating pixel coordinates with the world coordinate and already I have defined this equations that is the perspective projection equations and that is the perspective projection equations and we know this already I have defined about this the image and the pixel coordinates. So from this you can get x im and the y im you can see this equations so I will be getting x im and the y im from this equations.

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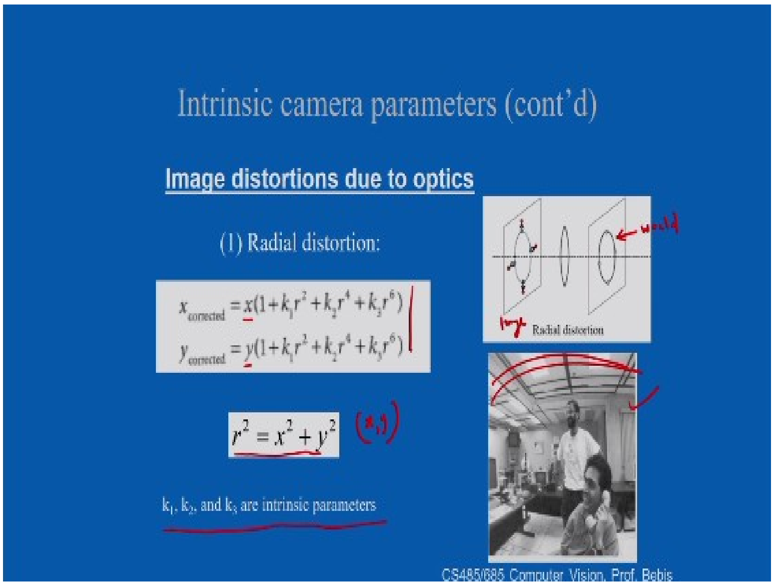

And also I have to consider that image distortions due to the optics that is the lens so I have to consider. So, in this case I have shown the radial distortion and this is in the world frame of reference and this is in the image. In world frame of reference this is a circle, but in the image plane I am not getting the circle because of the radial distortion. So in this case x, y that is I am considering x, y that coordinate of the distorted pixels in the equation.

You can see x and y I am considering and I am showing that this is the equation of the circle x square plus y square is equal to r square it is the equation of the circle I am considering. So that means in the world it is a circle, but because of the radial distortions I am not getting the circle in the image plane and what is x, y? X, y is the coordinates of the distorted points. The distortion is a radial displacement of the image points.

So this is the distortion that distortion is called the radial distortion and the distortion is the radial displacement of the image point. The displacement is null there is no displacement at the image center and it increases with the distance of the point from the image center and the optics introduces image distortions that become evident at the periphery of the image. So, this distortion I can model by using this equations.

So, I am doing the corrections so x corrected and the y corrected I am getting by modeling of this the radial distortions and I am considering the parameters k1, k2, k3 these are the intrinsic parameters and if you see this image the optics introduces image distortions that become evident at the periphery of the image. You can see in the periphery this distortion is evident that is the radial distortion is evident.

And so you can see the k1, k2, k3 these are the intrinsic parameters of the camera. So that I need to consider during the camera calibration.

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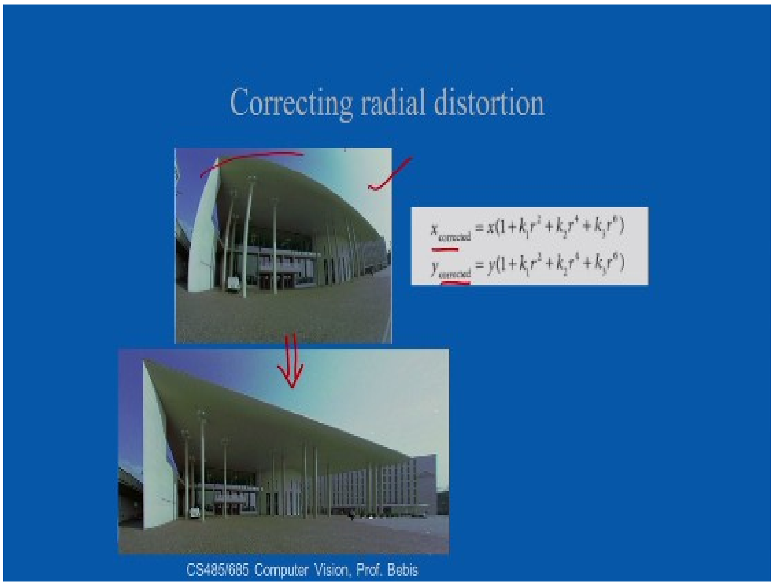

And you can see here I have shown the distorted image because of the radial distortion and after this it is corrected by using this equation that is I am doing the modeling. So, by using this it is corrected the x coordinate is corrected and y coordinate is corrected and you can see the corrected image here.

The second image is the corrected image the first image is the distorted image because of the radial distortion. And you can see the distortion is evident at the periphery of the image. So, at the periphery of the image the distortion is visible.

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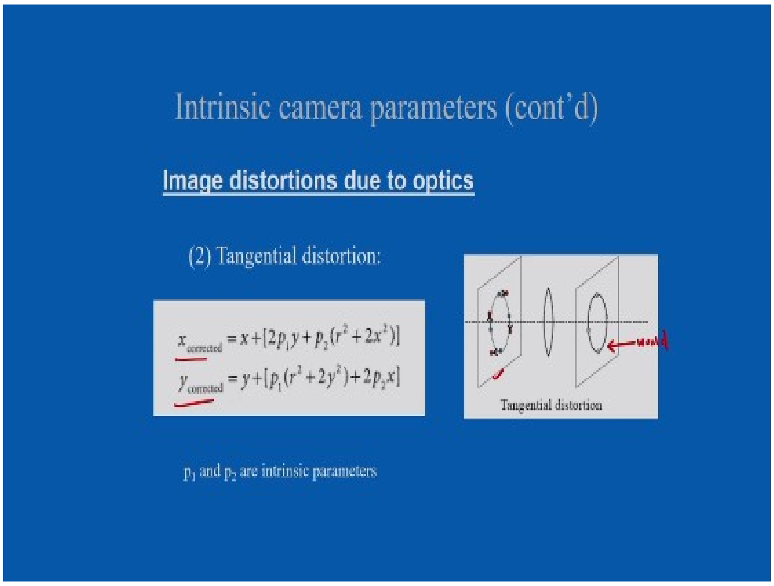

And there is another distortion due to the camera lens the distortion in the tangential direction that is the tangential distortions. So, again in the figure I have shown this is in the world frame of the reference and image I have shown this is the image plane of reference I have shown and by using this equations I can do the corrections of the x coordinates and the y coordinates.

I am not going to discuss about the tangential distortions, but these are the distortions one is the tangential distortion another one is the radial distortions that is because of the optics.

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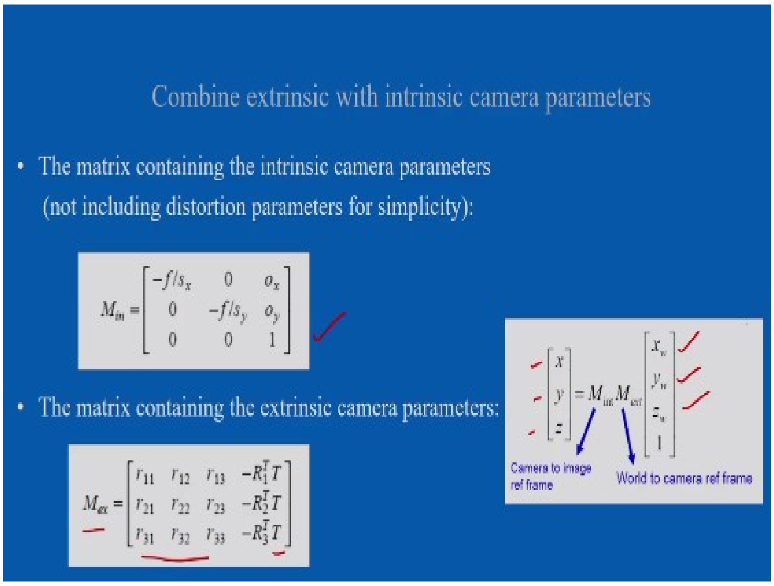

And after this I am combining extrinsic with the intrinsic parameters. So I have shown the matrix the matrix is M in that is the internal parameters of the camera I am considering the matrix M in and also I am considering this matrix M ex that is nothing, but I am considering the external parameters. So for external parameters I am considering the rotation and the translation and after combining these two I have the image coordinate x, y, z is the image coordinate and I have the world coordinate the world coordinate is xw, yz, zw.

And I am considering the matrix M int that is the internal parameters I am considering that is the transformation for camera to image reference frame and I am considering M external that is the external parameters of the camera extrinsic parameters of the camera that is world to camera reference frame.

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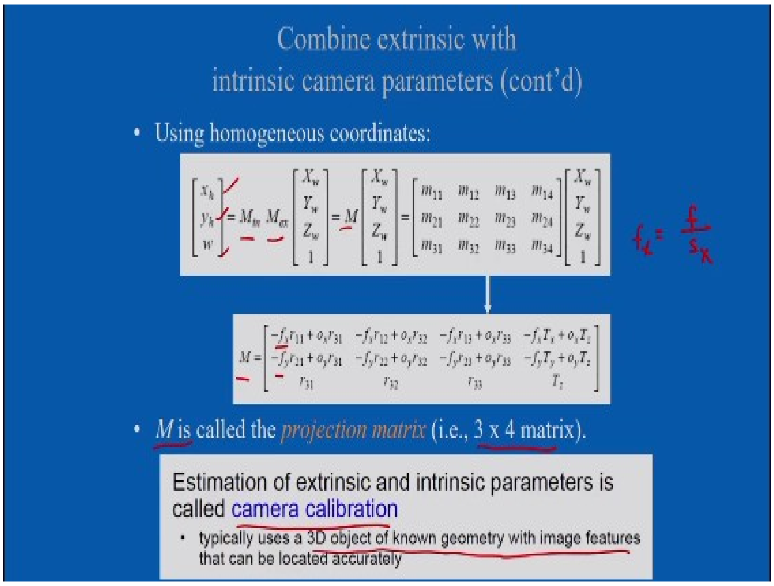

And if I use the homogeneous coordinate system I can write like this xh, yh, w and I am considering the matrix M in and M ex and in this case you can see the elements of the matrix M I am combining this two matrix one is the M in and another one is M ex I am combining these two matrices and I am getting the matrix M and what are the elements of the M? m11, m12, m13, m14 m21, m22, m23, m24.

These are the elements of the matrix and finally from the previous equations you can get the matrix M and in this case you can see I am defining fx and the fy I am considering fx and fy. What is fx? Fx is nothing, but f divided by sx. So fx is the focal length expressed in the effective horizontal pixel size that is the focal length in the horizontal pixels and similarly I can define fy.

So I am repeating this what is fx? Fx is the focal length expressed in the effective horizontal pixel size. So, pixel size I am considering sx and the sy so that is why f divided by sx that is the focal length in horizontal pixels that I am considering. Now I am considering the projection matrix the projection matrix is M is projection matrix and is the 3 by 4 matrix. In projection matrix I am considering this two matrices one is the internal parameters.

I am considering another one is the external parameter matrix I am considering that is M in and M ex I am considering and I am combining these two I am getting the projection matrix the projection matrix is M. So, I have to estimate the extrinsic and the intrinsic parameters of the camera and that is called the camera calibration.

So all these parameters I have to determine the parameters are here you can see m11, m12, m13 these are the parameters I have to define. And already I have defined that typically I can consider the 3D objects of the known geometry with image features that I can consider for calibration.

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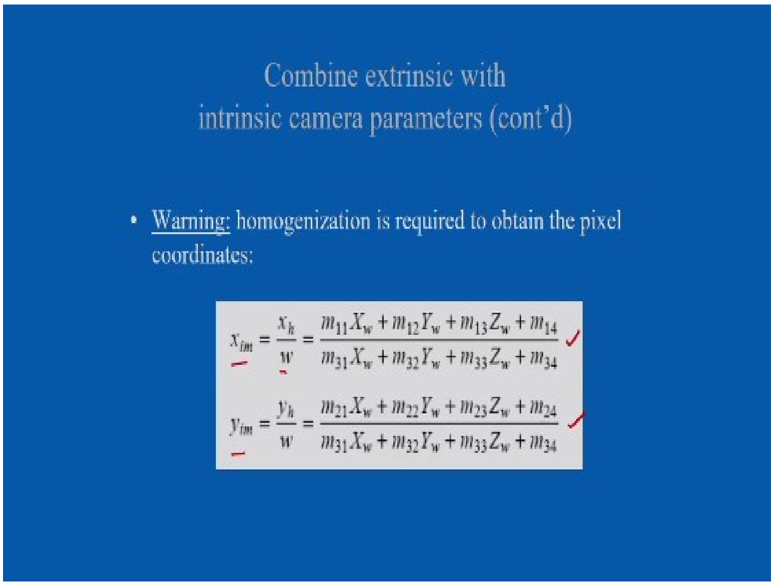

So, for camera calibrations I will explain some methods, but before going to this you can see first I have to consider the homogenous coordinate systems and you can see I have the pixel coordinates x im and the y im and that is divided by w so I will be getting x im and the y im by using this equations. So, I am combining extensive parameters with the intrinsic parameters of the camera and finally I am getting x im and the y im I am getting and I am using the homogenous coordinate system.

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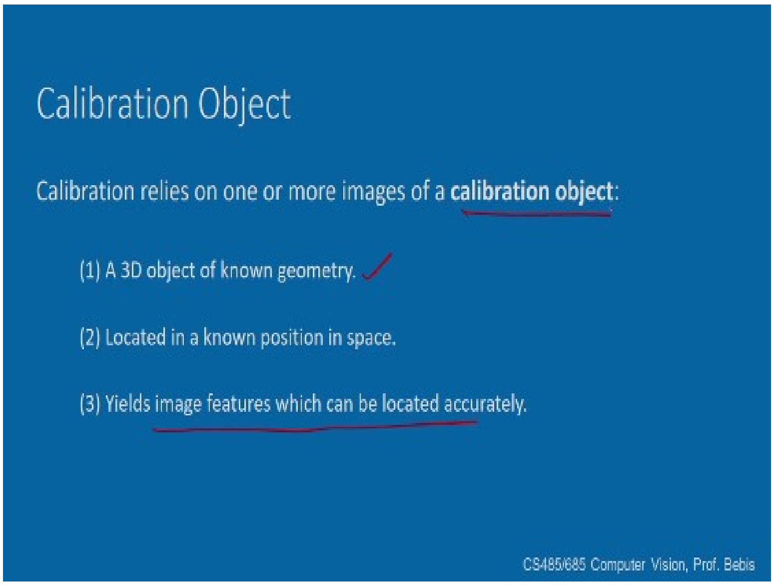

And for calibration already I have defined I have defined I need calibration object that means a 3D object of know geometry I need and it should be located in a known position in space and from this I can extract image features which can be located accurately. So, from this object I can extract image features and based on this calibration object I can do the calibration that is the meaning of the calibration object.

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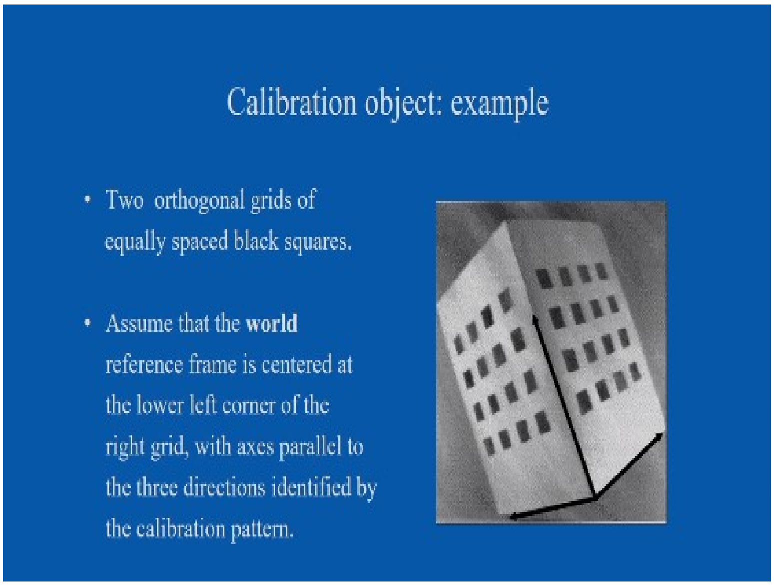

So, in my next slide I have shown one calibration object so this is the calibration object and two orthogonal grids of equally spaced black squares I can consider as a calibration object and assume that the world reference frame is centered at the lower left corner of the right grid with axes parallel to the three directions identified the calibration pattern.

So, I can show the world reference frame. You can see the world reference frame is this is centered at the lower left corner of the right grid and with axes parallel to the three directions identified by the calibration pattern.

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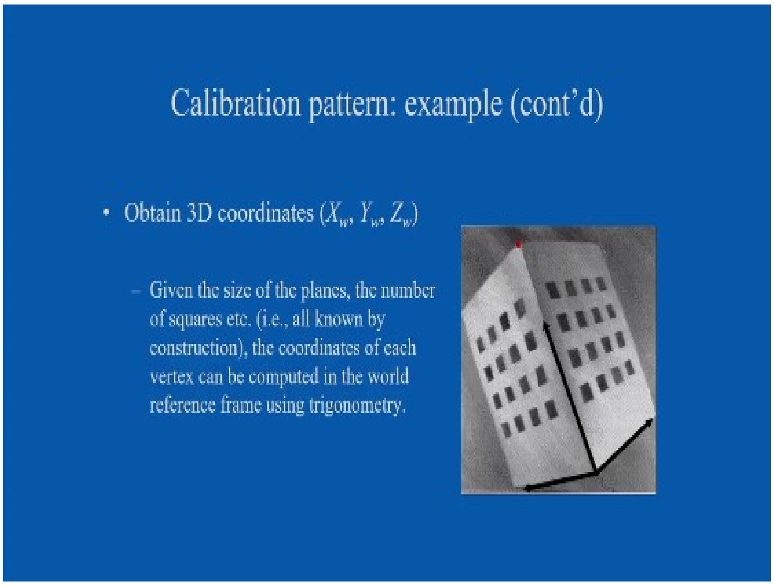

And from this I can obtain the coordinates the 3D coordinates I can obtain Xw, Yw, Zw I can obtain. So, given the size of the planes the number of squares so all known by constructions the coordinate of each vertex can be computed in the world reference frame by simple mathematics. So, I can determine all the coordinates I can determine by simple mathematics.

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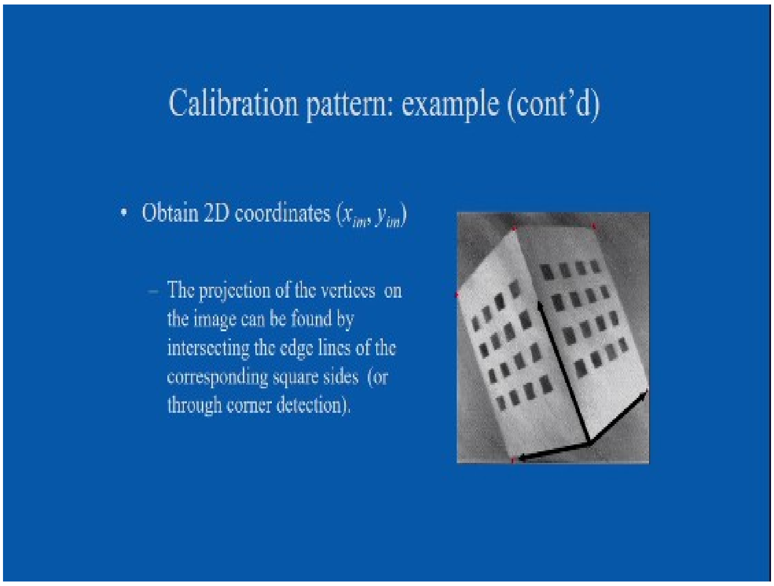

And after this obtain 2D coordinates that is nothing, but x im and the y im I can obtain. The projection of the vertices on the image can be found by intersecting the edge lines of the corresponding square side or maybe by using the corner detection principles. So after detecting the edges and the corners I can obtain the 2D image coordinates. So that means I am considering the calibration object for camera calibration.

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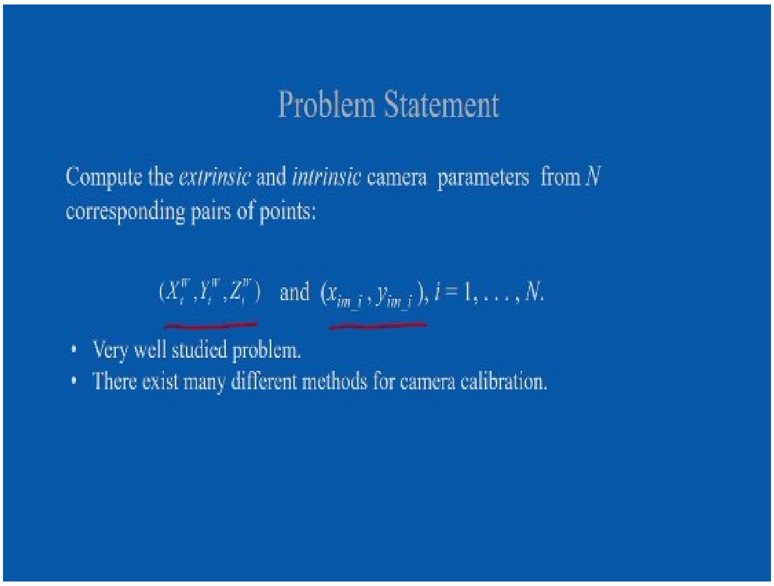

So, the problem of the camera calibration is this. Compute the extrinsic and the intrinsic camera parameters from N corresponding pairs of points. So, what are the corresponding points? I have the world coordinates points and also I have the image coordinate points the corresponding points. So, from this I have to estimate the extrinsic and the intrinsic camera parameters.

So this corresponding pair of points I can obtain from the calibrating object so I can determine this the corresponding pair of points I can determine from the calibrating object. So there are many methods for camera calibrations, but in this class I will only discuss briefly two process. One is the direct camera calibration technique and another one is the indirect camera calibration technique.

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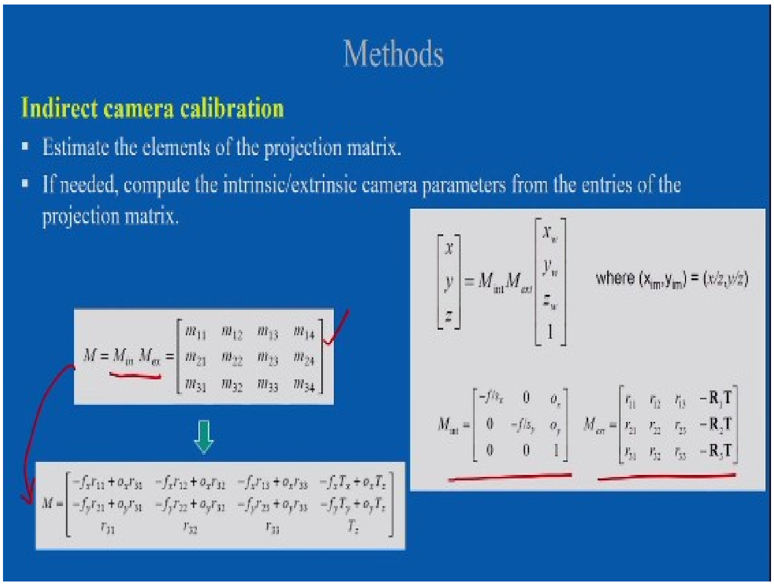

So, in the indirect camera calibration estimate the elements of the projection matrix if needed compute the intrinsic and the extrinsic camera parameters from the entries of the projection matrix. So you have seen the projection matrix this is the 3 by 4 matrix and I am considering both the internal parameters and the external parameters I have considered. So, you can see what is M int?

This is M int I am considering the internal parameters in this case I am considering the focal length the size of the pixels I am considering, but I am not considering the camera distortions parameters. So, if I consider the camera distortion parameters the computation will be difficult. So that is why I am not considering the camera distortion parameters, but here only I am considering the focal length and the size of the pixels.

And this matrix if you see M external I am considering the rotation and the translation that already I have explained and corresponding to this you can see corresponding to the projection matrix I have this matrix M matrix. So, I have to estimate the elements of the projection matrix in case of the indirect camera calibration. The indirect camera calibration is somewhat easier as compared to direct camera calibration.

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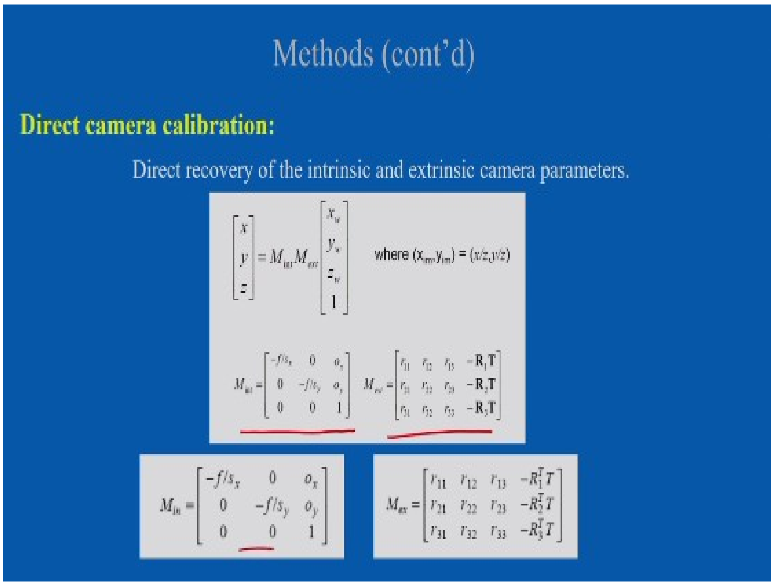

In case of direct camera calibrations so it is nothing, but direct recovery of the intrinsic and the extrinsic camera parameters. So, this is the intrinsic parameters and these are the extrinsic parameters and I can directly recover the intrinsic and the extrinsic parameters of the camera. So, briefly I will explain this both the principle one is the indirect camera calibration one is the direct camera calibration.

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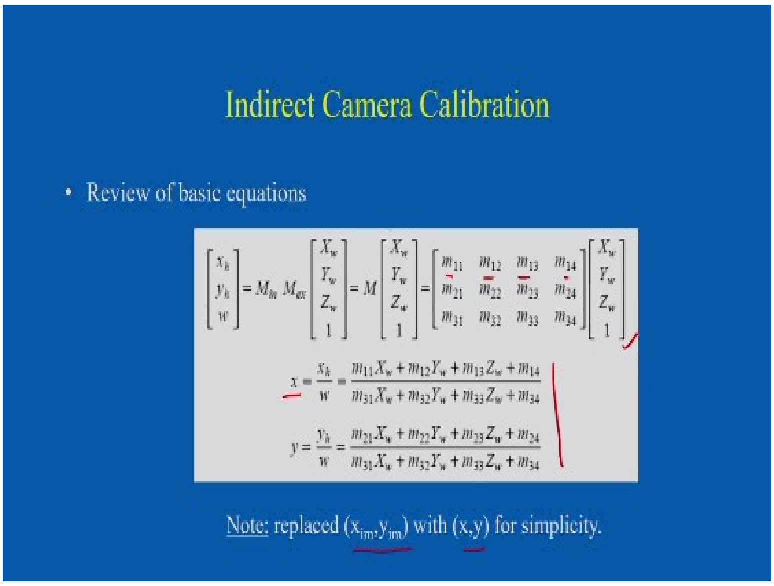

The first one is the indirect camera calibration I am showing so here you can see this equation already I have defined in my previous slide. So I have the projection matrix and the projection matrix I have this one m11, m12, m13 these are the elements of the projection matrix and you can see I have the image coordinates x and y I am considering. So, just I am replacing x im y im with x, y for simplicity. So already I have defined this equations in my previous slide.

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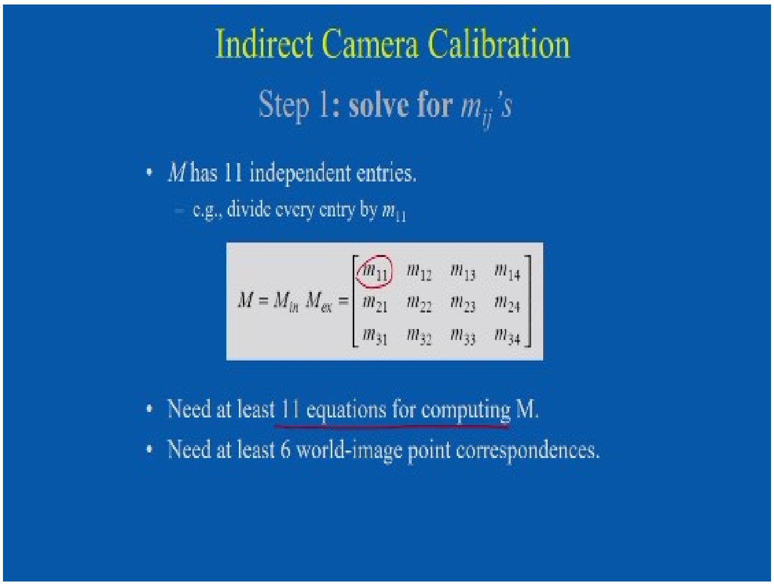

So, in this projection matrix you can see if I divide every entry by m11. So each and every element is divided by m11 that means I have only 11 independent entries. So, for this I need at least 11 equations for computing the projection matrix. So I need only 11 equations for computing the projection matrix and we need at least 6 world image point correspondence. So you can understand this so this is the projection matrix.

And this projection matrix has 11 independent entries because the other elements are divided by m11 and for this I need 11 equations for computing M.

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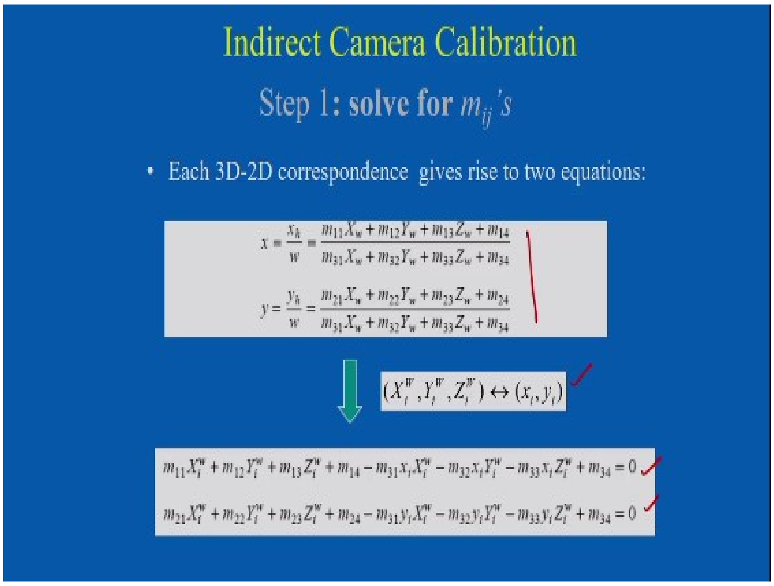

And step 1 so already you have this equations and I have this correspondence one is the world coordinates and the other one is the image coordinate I have this information and from this equation you will be getting this one. If you see from this equation you will be getting this you can see by simple mathematics you will be getting this one.

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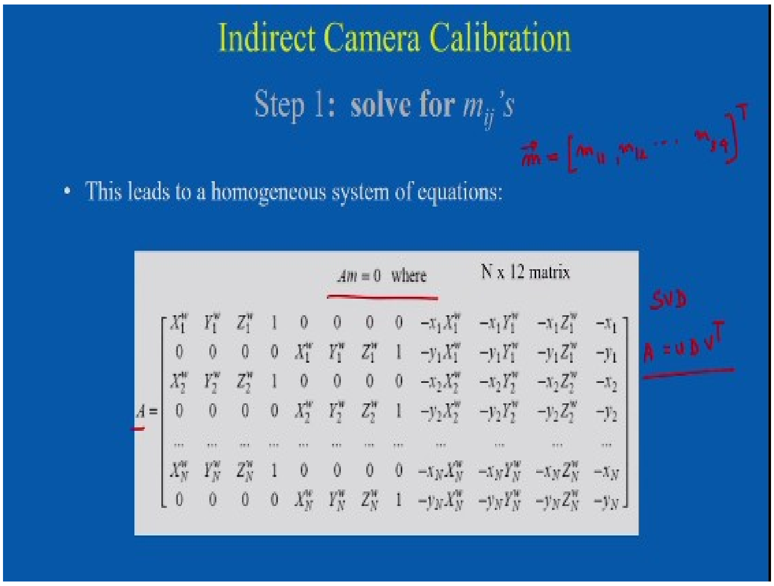

And from this equations I will be getting one homogenous system of equations so I will be getting one homogenous system of equations. So, N number of rows and 12 number of columns I am having the matrix and I am considering the solution Am is equal to 0 that I am considering this. So, what is m here? M is nothing but the vector I am considering the vector m is a vector suppose.

And what are the elements of the vector m11, m12 like this m34 it is the transpose and in this case since the A has the rank 11 so this matrix has the rank 11 the vector M can be recovered by using some techniques like singular value decomposition. So, by using SVD the singular value decomposition I can estimate that vector M can be recovered from the singular value decomposition technique.

So, the singular value decomposition technique is you know A is equal to U D V T. So in this case the column of v corresponds to the 0 singular value of A, so that means the vector M can be recovered from singular value decomposition as the column of V corresponding to the 0 singular value of A and in this case I am considering A is equal to U D V transpose I am considering.

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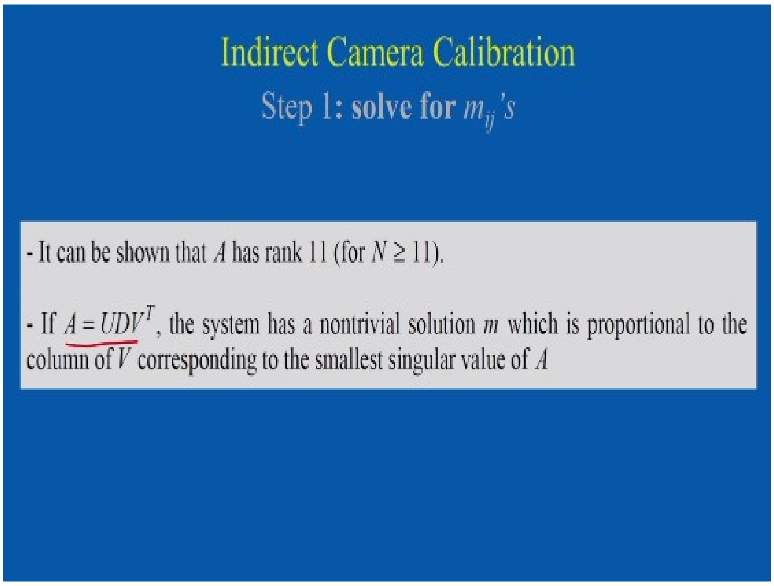

So that means already I have explained so the matrix A has the rank of 11 so N is greater than equal to 11 and I can apply this singular value decomposition technique and by this I can recover the vector M.

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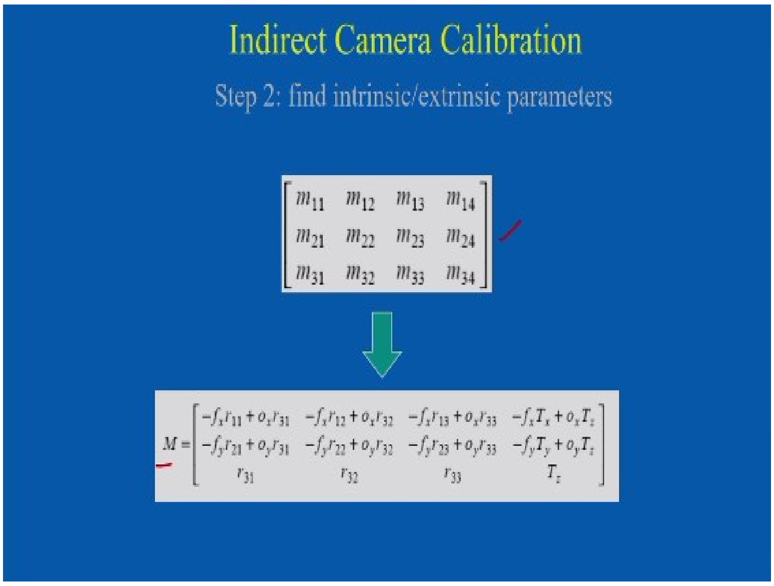

And after this I can find the intrinsic and the extrinsic parameters, the step number 2. So, I can find the values of m11, m22 like this I can find so this is the projection matrix.

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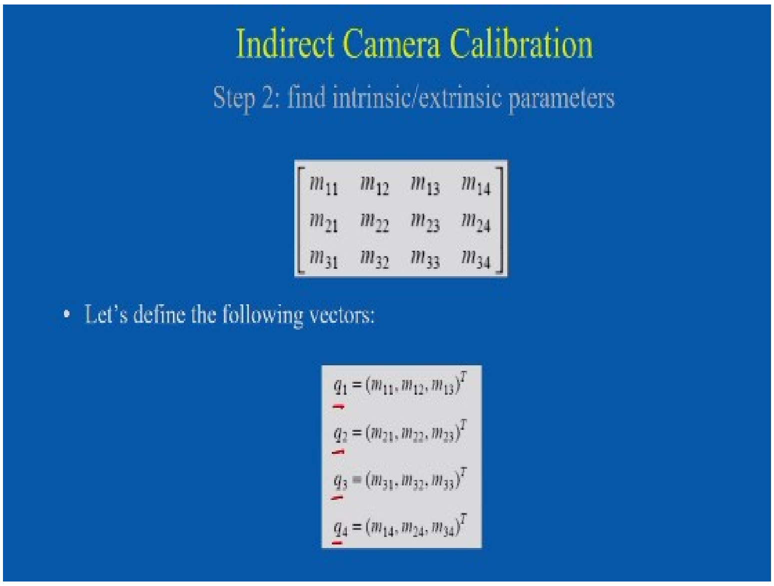

So, this corresponding to this m11, m12, m13, m14 I am defining following vectors, so vector is suppose q1, q2, q3, q4. So, what is q1? m11, m12, m13 I am considering. For q2, I am considering m21, m22, m23 I am considering and similarly q3 and q4 I am considering.

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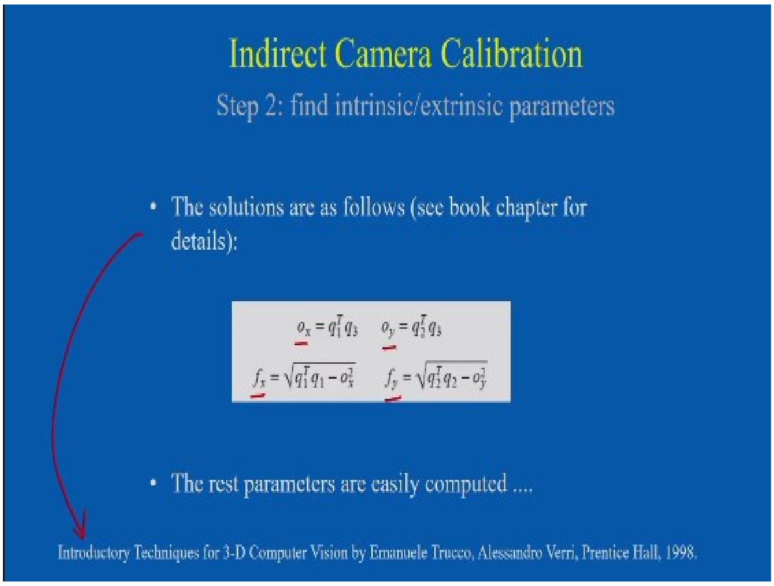

And after this the solutions will be getting because I have to compute this parameters ox I can determine oy I can determine and the fx and the fy also I can determine by using this equations. For solutions you can see this book this book chapter you can see so this book you can download also from the net and you can see the solution of this you will be getting the fx and the fy that is the focal length you can determine that is the intrinsic parameters of the camera. This is about the intrinsic camera calibration. So, this is about the indirect camera calibration. The next I am going to discuss the direct camera calibration.

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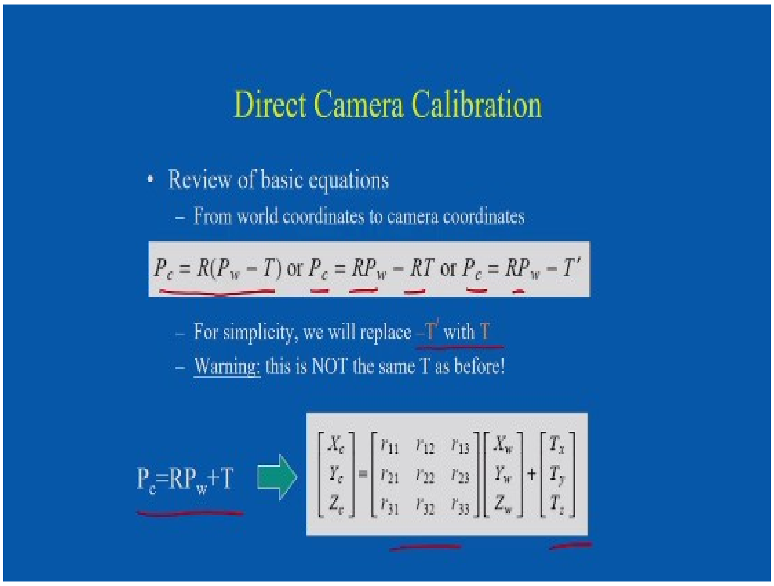

So, already I have defined this equation the Pc is equal to R Pw minus T and Pc is equal to RP w minus RT and Pc is equal to RP w minus T transpose. Now for simplicity we replace minus T transpose with T. So for simplicity I am replacing minus T transpose with T and corresponding to this Pc I am getting the Pc is the camera coordinates, R is the rotation matrix, Pw is the world coordinates plus T is the translation vector.

So, I am considering the translations and the rotational matrix I am considering. So, this transformation I am getting that is the transformation for the camera coordinates and the world coordinates.

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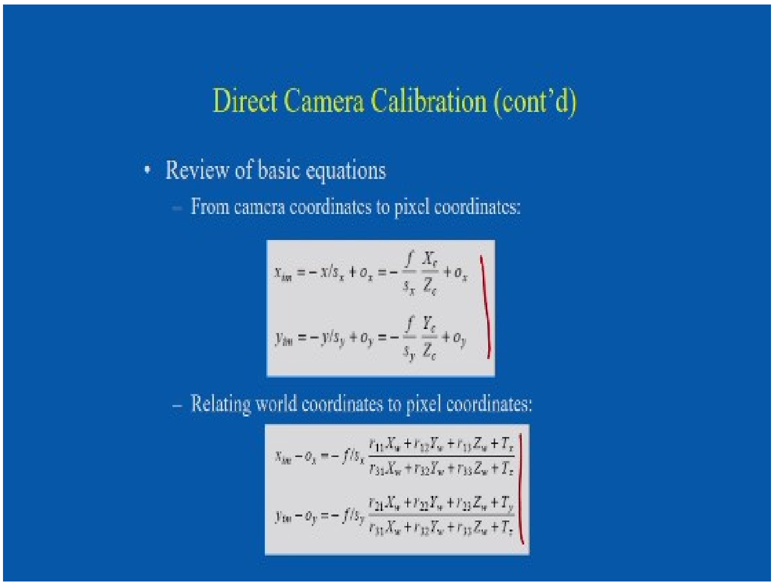

And from camera coordinates the pixel coordinates already I have obtained this equations in my previous slide and from this I can get the relation between the world coordinates and the pixel coordinates. So this I am getting. So, I am getting the relationship between the world coordinates and the pixel coordinates you can see this equations.

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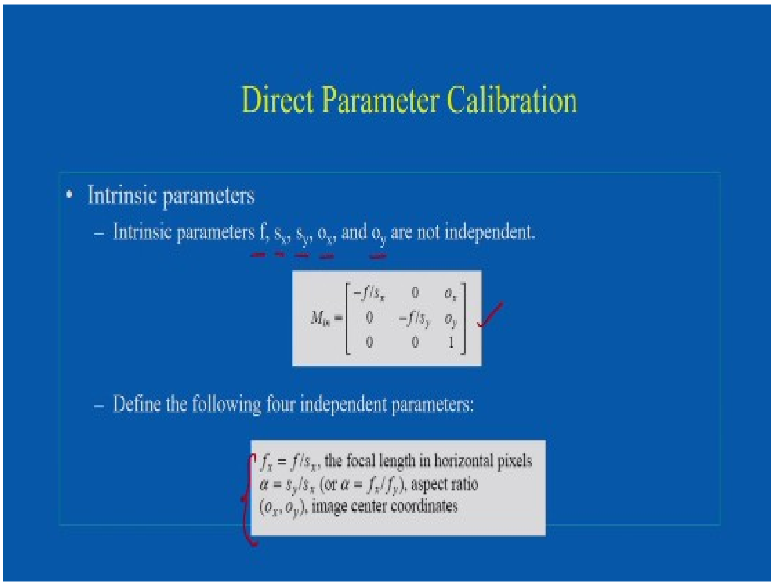

And after this the intrinsic parameters, what are the intrinsic parameters? The focal length sx and the sy that is the size of the pixels and in this case ox and the oy that is the center point I am considering and you can see this is my matrix M in that is I am considering the internal parameters of the camera that is the intrinsic parameters of the camera and I can define four independent parameters.

I can define like this fx equal to f divided by sx that is the focal length in horizontal pixels alpha that is the aspect ratio I can define that is nothing but sy divided by sx and also the image center coordinates already I have defined that is ox, oy I can define.

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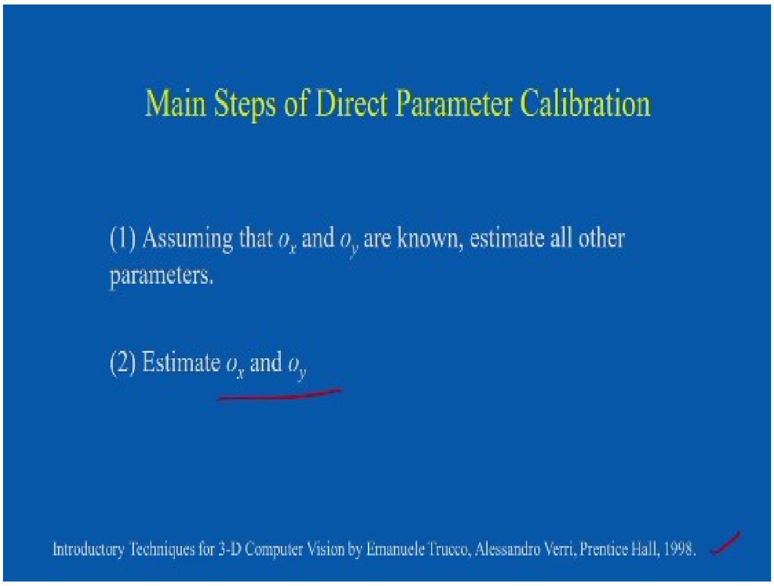

So the main steps so assuming ox and oy are known and we have to estimate all the parameters and finally I can estimate ox and oy. So, detail derivations you can see from this book so already I told you that this book you can download from the net and you can see so how to do the camera calibration by using the indirect camera calibration method and another one is the direct camera calibration method.

But in my class I have explained this two techniques one is the concept of the camera calibration and why the camera calibration is important and after this I discussed the indirect camera calibration and direct camera calibration techniques.

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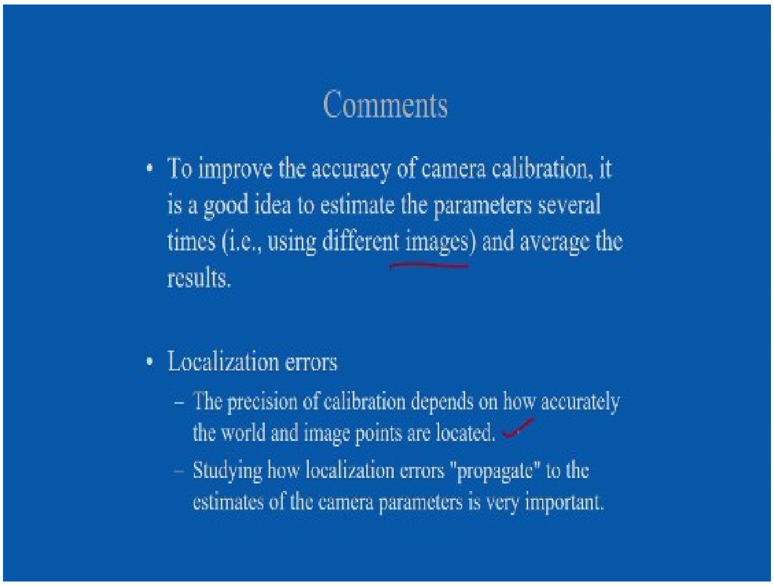

So, finally the comments to improve the accuracy of camera calibration it is a good idea to estimate the parameters several times using different images and average the results that I can consider. Different images mean I can consider the calibrating objects. The precise calibration depends on how accurately the world and the image points are located so that is the localization errors.

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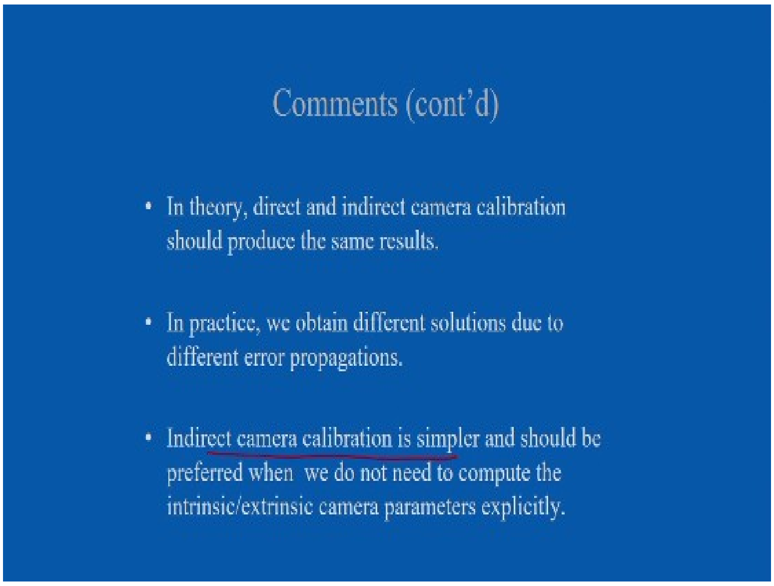

And you can see in theory the direct and the indirect camera calibration should produce the same results, but in practice we obtain different solution due to different error propogations and one point is important the indirect camera calibration is simpler and should be preferred when we do not need to compute the intrinsic and the extrinsic camera parameters explicitly. So, indirect camera calibration is simpler as compared to the direct camera calibration that I want to explain.

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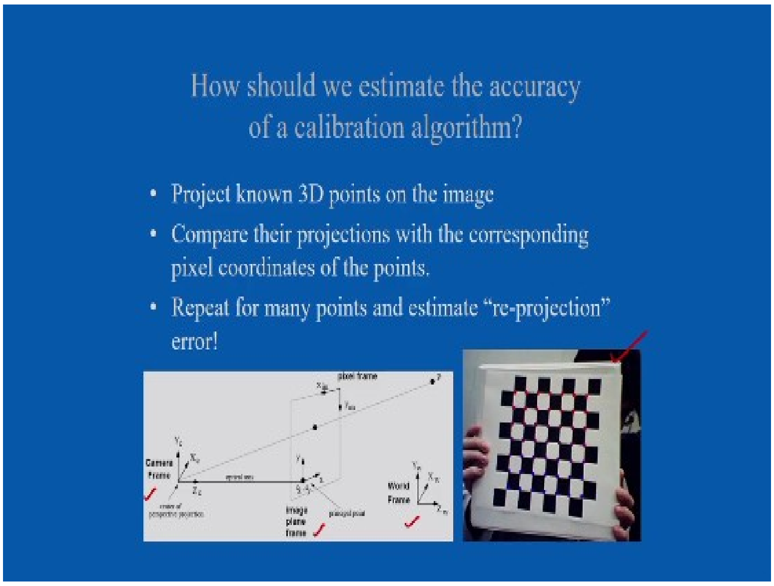

So, already I have explained this concept how to do the camera calibrations. In this figure you can see I am considering the calibrating objects so this is my calibrating object I have the known 3D points of the object and after this I have to compare their projections with the corresponding pixel coordinates of the points so I have to compare. Repeat for many points and estimate re—projection error that I can do.

So, repeat for many points and estimate the re-projection error I can estimate. So, in this case I am doing the calibration by considering one calibrating object. So, in this figure also I have shown the world frame, the image plane and the camera frame I have shown and by using the calibrating objects I can do the camera calibration.

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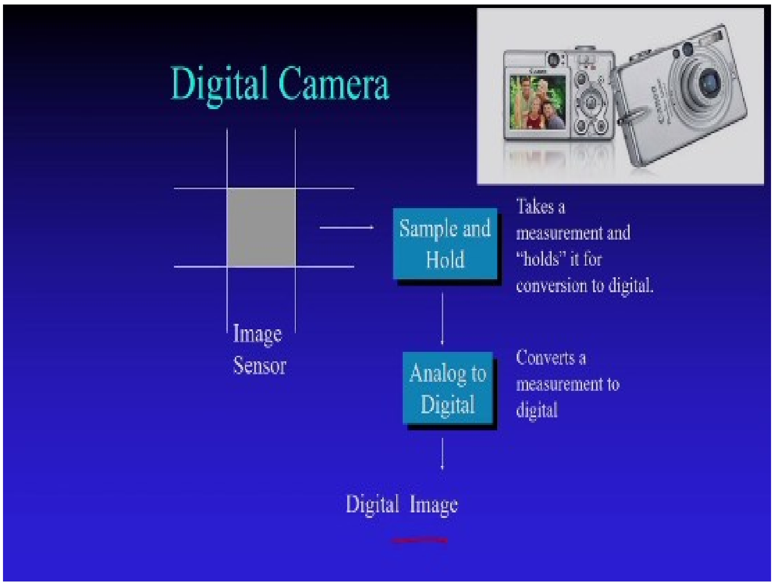

And this is about the camera calibrations. Now I will discuss one or two issues of the digital camera. So, in this case I have shown the digital camera sensor that is the image senor that converts light photon into electrical signal. So, I will be getting the analog signal. The analog signal I can convert into digital signal that is the digital image by the process of sampling and the quantization and I will be getting the digital image.

So, I am showing the image sensor corresponding to a digital camera. So that may be the charge couple device may be present or maybe the CMOS complementary metal oxide semiconductor or may be the charge couple device I can consider in the image consider as image sensor.

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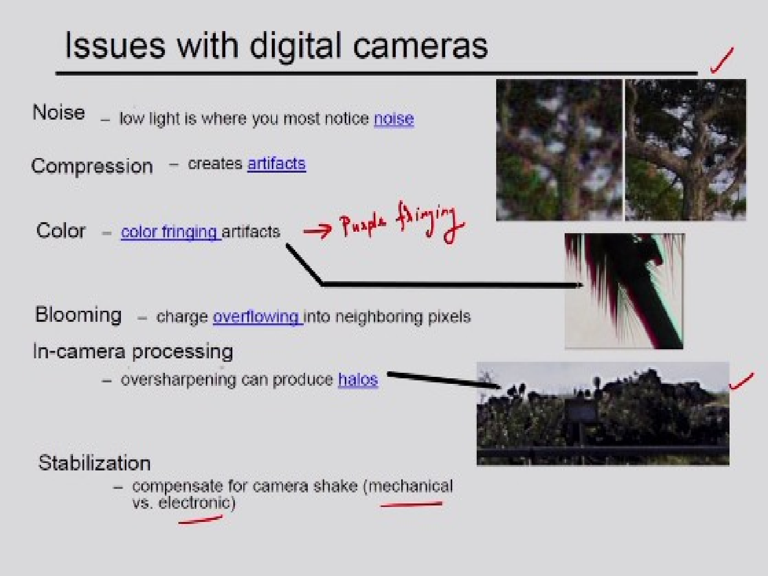

And some of the issues with digital cameras. First one is the noise because of the low light we will be observing noise. In the image you can see here in the figure you can see the image is blur and we will be getting the noise because of the low light conditions and if I do the compression of the image than I will be getting the artifacts. So, suppose if I consider a Zetec compression or Zetec 2000.

Then also I will be getting the artifacts because of the compression and one is the color fringing artifacts is called the chromatic aberration. So, what is the chromatic aberration that is the range of different wavelength focus in different planes. This is also called the purple fringing or a chromatic aberration that I will explain later on. Purple fringing so that means the rays of different wave length focused in different planes.

And this is called the purple fringing or the chromatic aberration and a blooming is mainly the charge overflowing into neighborhood pixels this is because of the charge overflow and also if I do the over sharpening then over sharpening can produce halos as shown in the figure and for stabilization I can consider a mechanical stabilization, the mechanical compensation or maybe the electronic compensation.

I can consider that is corresponding to the camera shave. So, these are the issues with the digital cameras.

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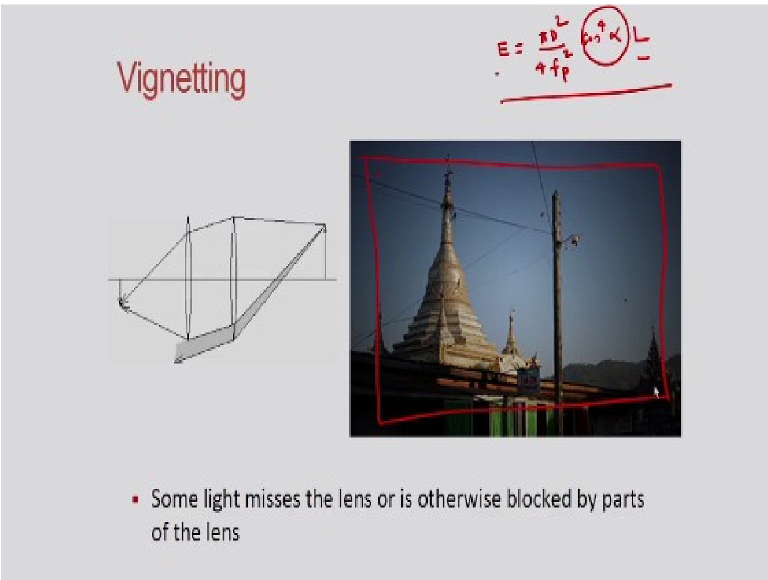

And one effect I discussed in my radiometry class that is the concept of the vignetting. In that class I define the radiometry for the thin lens the E is equal to pi d square divided by 4 fp square cos 4 alpha L. So, we derive this equation in radiometry. So, in this case E is the E irradiance and L is the radiance. So, that is you can see that is irradiance is proportional to the scene radiance.

That means the gray value of an image depends on L is the radiance and in this case the term cos 4 alpha this is the term cos 4 alpha indicates a systematic lens optical defect and that is called the vignetting effect. So, this is the vignetting effects actually depends on the term the cos 4 alpha. So, what is the interpretation of this cos 4 alpha? So, optical rays with larger span of angle alpha the span of angle is alpha are attenuated more.

Hence, the pixel closer to the image borders will be darker. So, you can see the image pixels in the boundary in the border it is darker because of the vignetting effect. So, that is the optical rays with larger span of angle the angle is suppose alpha the span of angle is alpha are attenuated more and for this the pixels closer to the image borders will be darker and this vignetting effect can be compensated by a radiometrically calibrated lens. So this vignetting effect already I have discussed in my radiometry class.

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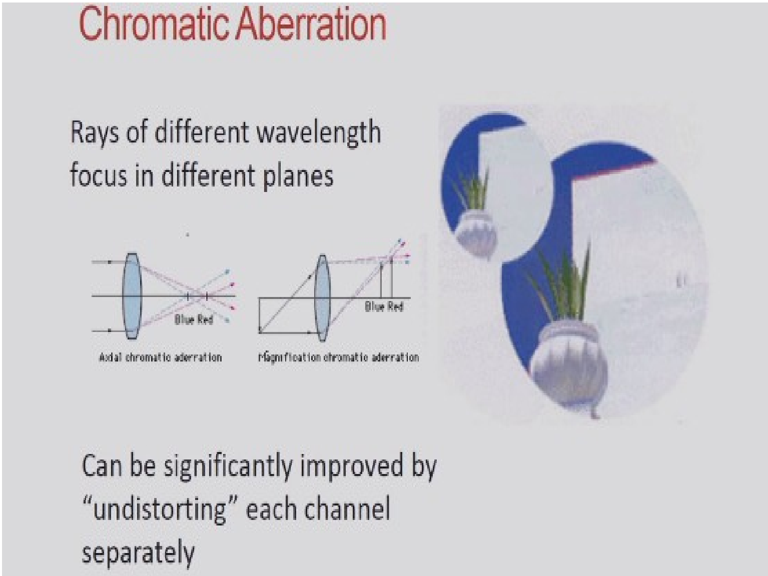

And this chromatic aberrations the definition is the rays of different wavelength focus in different planes. Here you can see I am showing the different wavelength the red, the blue they focus in different planes and in this case you can see because of this I am getting the radius boundary here maybe something like this I am getting because of the chromatic aberration that is the purple fringing. So, this is mainly the rays of different wavelength focus in different planes.

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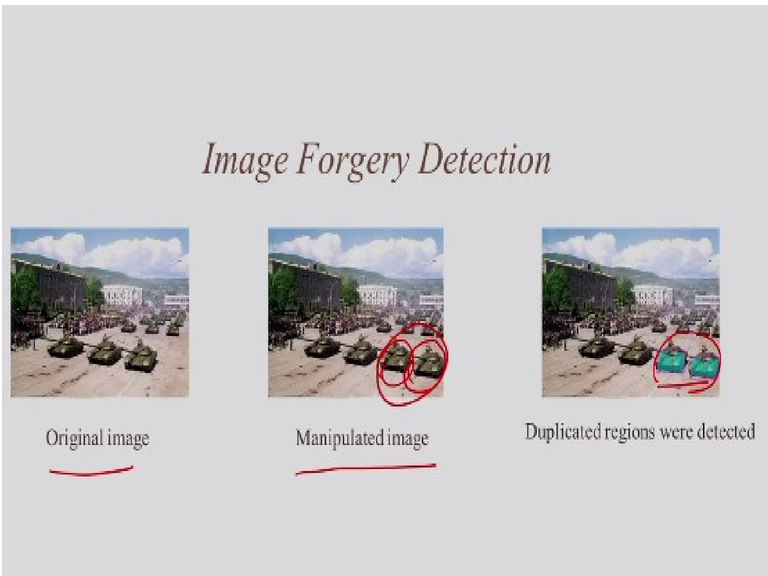

And one application of this I can give one application the image forgery detections by considering the chromatic aberration that is the purpose fringing. So, you can see one is the original image and after this I am considering the manipulated image you can see I am doing the manipulation of this and the duplicated regions were detected you can see the duplicated regions are detected because it is a copy and paste forgery that I am taking the one part of the image from other images.

And for that camera the purple fringing effect will be different from that of the original camera and based on this property I can detect the image forgery. So, that means this portion is taken from another camera and that camera has different the purple fringing characteristics, the chromatic aberration characteristics and based on this characteristics I can identify whether this image is the original image or the forged image.

So, this is one application of the image forgery detection based on chromatic aberration. So in this class, I discussed the concept of camera calibrations. So, how to determine intrinsic parameters and the extrinsic parameters of the camera. I have discussed two methods one is the direct camera calibration method another one is the indirect camera calibration methods. So, briefly I discussed how to do the camera calibrations.

And how to estimate the camera parameters and how to find a projection matrix. So, this is about this camera calibrations and let me stop here today. Thank you.